Stratified-Medium Sound Speed Profiling for CPWC Ultrasound Imaging

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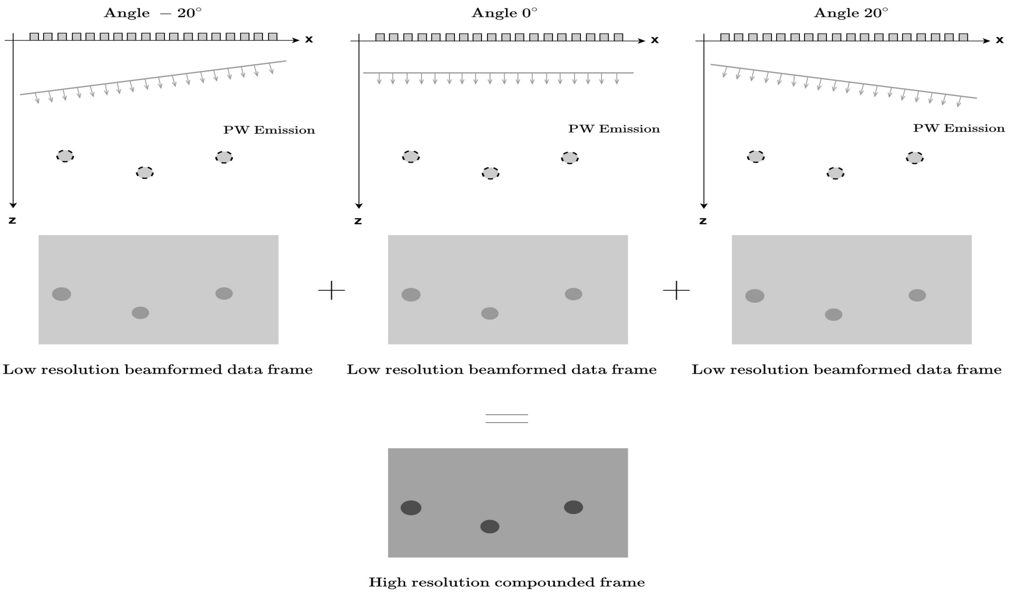
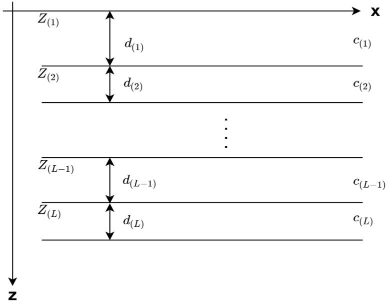
Outline

- Background and Contribution
- Proposed SOS Profiling Approach
- Computational Considerations
- Evaluation Results
- Conclusion and Future Work

Coherent Plane Wave Compounding (CPWC)



Phase-shift migration for PW imaging [1]



- Let the layers be indexed by $l = 1, 2, ..., L$
- Let $d_{(l)}$ and $c_{(l)}$ denote the thickness and sound speed of layer l, respectively
- Let $\boldsymbol{\theta} = \{\theta_w, w = 1, 2, ..., W\}$ be the plane wave angles used
- PSM gives the wavefield at the time of explosion $P_\theta(x, z, t = 0)$ using the recorded wavefield at the surface $P_\theta(x, z = 0, t)$

Motivation

- Existing PSM is sensitive to speed and thickness mismatches
- In a stratified inhomogenous medium, errors in top layers propagate down to lower layers causing increased error in lower layers
- Incorrect sound speed and layer thickness values can result in overmigration or undermigration
- Can cause misregistration of point targets, poor contrast, degraded resolution, etc.

Our objective is to estimate the **SOS profile**, i.e., speed and thickness values of each layer in the stratified medium, and use these values to carry out PSM for image reconstruction.

Our contribution

Some of the related existing approaches:

- Krucker *et al.* [2] estimated the average sound speed using ray acoustics, where overlapping nonzero-angle images obtained using different speeds were automatically registered over a zero-angle image to achieve maximum correlation
- Qu *et al.* [3] relied on speckle analysis to estimate the average sound speed using the same pre-beamformed data with different assumed speeds to identify the image with the best focus quality

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Our two-stage layer-by-layer approach:

- The **first stage** produces a sound speed estimate using cosine similarity metric, and the **second stage** produces a thickness estimate using boundary detection
- We use only raw RF data corresponding to two PW emission angles
- It enables self-calibrated migration of multi-angle raw RF data

Problem formulation

- Let $\mathbf{c}_{(l)} = \{c_{(l)}^{\min}, c_{(l)}^{\min} + \Delta c_{(l)}, c_{(l)}^{\min} + 2\Delta c_{(l)}, ..., c_{(l)}^{\max}\}$ represent a set of distinct sound speed values for a given layer l
- Let $\mathbf{d}_{(l)} = \{d_{(l)}^{\min}, d_{(l)}^{\min} + \Delta d_{(l)}, d_{(l)}^{\min} + 2\Delta d_{(l)}, ..., d_{(l)}^{\max}\}$ represent a set of distinct thickness values for a given layer l
- Let $\{\theta_a, \theta_b | \theta_a = 0°, \theta_b \in \boldsymbol{\theta}, \theta_a \neq \theta_b\}$ represent the two angles used for SOS profiling

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The layer-by-layer sound speed estimation problem can be stated as follows:

Inputs: $\Psi_{\theta_a}(k_x, Z_{(l)}, f),\ \Psi_{\theta_b}(k_x, Z_{(l)}, f),\ \mathbf{c}_{(l)},\ \ d_{(l)}^{\max}$

Output: $c_{(l)}^{*} \in \mathbf{c}_{(l)}$

Objective: Max $\rho\left(\mathbf{s}_{\theta_a}(c_{(l)}^{*}), \mathbf{s}_{\theta_b}(c_{(l)}^{*})\right) = \dfrac{\mathbf{s}_{\theta_a}(c_{(l)}^{*}) \cdot \mathbf{s}_{\theta_b}(c_{(l)}^{*})}{\left\|\mathbf{s}_{\theta_a}(c_{(l)}^{*})\right\|_2 \cdot \left\|\mathbf{s}_{\theta_b}(c_{(l)}^{*})\right\|_2}$

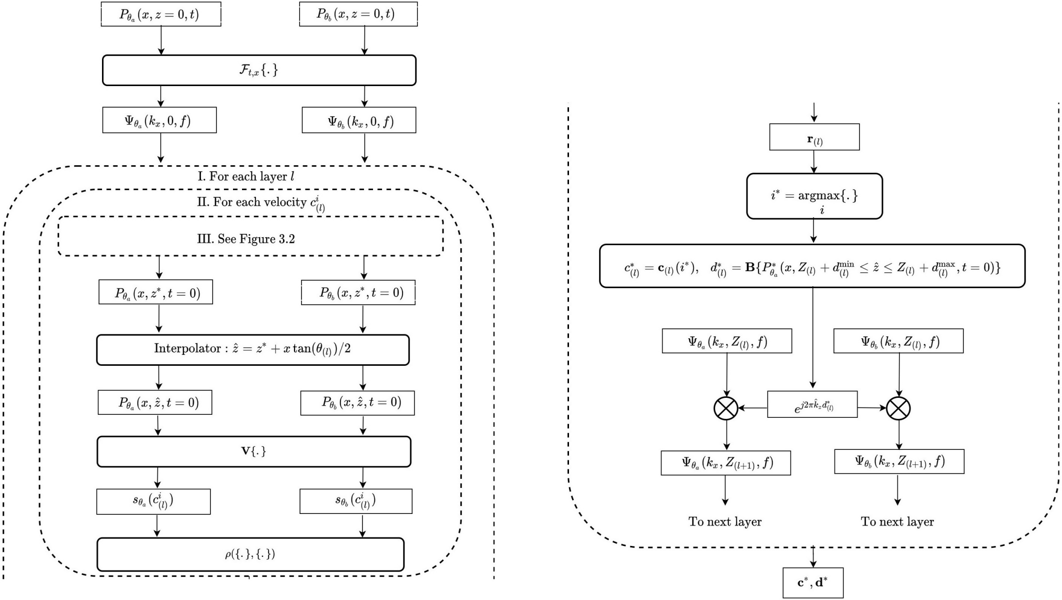
Problem formulation (cont'd)

- $\rho\left(\mathbf{s}_{\theta_a}(c^*_{(l)}), \mathbf{s}_{\theta_b}(c^*_{(l)})\right)$ represents the *cosine similarity* between two vectors $\mathbf{s}_{\theta_a}(c^*_{(l)})$ and $\mathbf{s}_{\theta_b}(c^*_{(l)})$

- $\{\mathbf{s}_{\theta_k}, k \in \{a, b\}\}$ are obtained from migrated data $P_{\theta_k}(x, \hat{z}, t = 0)$ by performing a vectorization operation $\mathbf{V}\{.\}$

For thickness estimation, we apply a certain boundary detection operation $\mathbf{B}\{.\}$ on $P^*_{\theta_a}(x, \hat{z}, t = 0)$ (obtained using sound speed estimate $c^*_{(l)}$) over the depth range from $Z_{(l)} + d^{\min}_{(l)}$ to $Z_{(l)} + d^{\max}_{(l)}$.

Proposed SOS profiling method



Proposed SOS profiling method (cont'd)

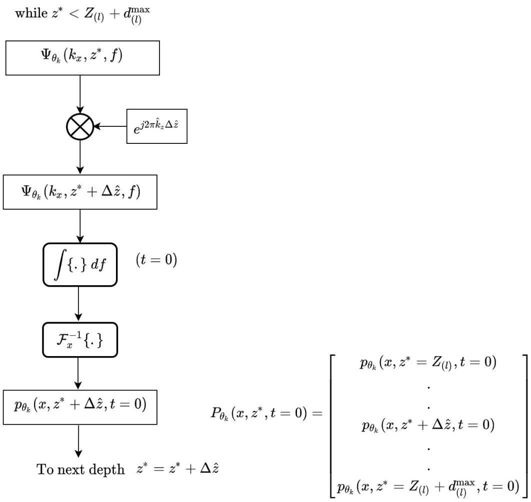


Figure: Downward extrapolation during SOS profiling.

Sound speed estimation

1. Apply PSM to $\{\Psi_{\theta_k}(k_x, Z_{(l)}, f), k \in \{a, b\}\}$, to extrapolate from $Z_{(l)}$ to $Z_{(l)} + d_{(l)}^{\max}$ to get $P_{\theta_k}(x, \hat{z}, t)$
2. Apply $\mathbf{V}\{.\}$ to convert $P_{\theta_k}(x, \hat{z}, 0)$ to vector $\mathbf{s}_{\theta_k}(c_{(l)}^i)$. This can be done in two ways:
   a) $P_{\theta_k}(x, \hat{z}, 0)$ is **summed** across the x-axis to form a z-axis vector
   b) $P_{\theta_k}(x, \hat{z}, 0)$ is **stacked** into a vector by concatenating its columns
3. Compute cosine similarity $\rho\left(\mathbf{s}_{\theta_a}(c_{(l)}^i), \mathbf{s}_{\theta_b}(c_{(l)}^i)\right)$

- The above operations are performed for each sound speed value $c_{(l)}^i$ in $\mathbf{c}_{(l)}$, yielding a vector of cosine similarities $\mathbf{r}_{(l)}$
- The index of maximum value in $\mathbf{r}_{(l)}$ gives the location of sound speed estimate $c_{(l)}^*$ in vector $\mathbf{c}_{(l)}$

Layer thickness estimation (using beamformed data)

Line detection:

1. Restrict $\hat{z}$ in $P^*_{\theta_a}(x, \hat{z}, 0)$ to the depth range $[Z_{(l)} + d^{\min}_{(l)}, Z_{(l)} + d^{\max}_{(l)}]$
2. Perform max-normalization, averaging, binary thresholding, and morphological processing
3. Apply Hough transform to detect z-axis location corresponding to the end-of-layer boundary giving estimate $d^*_{(l)}$

Peak detection:

1. Restrict $\hat{z}$ in $P^*_{\theta_a}(x, \hat{z}, 0)$ to the depth range $[Z_{(l)} + d^{\min}_{(l)}, Z_{(l)} + d^{\max}_{(l)}]$
2. Sum restricted $P^*_{\theta_a}(x, \hat{z}, 0)$ along the x-axis to get vector $s_{\theta_a}(\hat{z})$
3. Find the peak position in this vector which gives us $d^*_{(l)}$

Computational considerations

- If Ψ_{θ_k} is of size $P \times Q$, then each $\Delta\hat{z}$ within l entails $P \times Q$ complex multiplications
- For most values in $\Psi_{p,q} = \Psi_{\theta_k}(k_x, 0, f)$ we can skip phase shifting since they have relatively small magnitude
- We use cumulative sum vector $[\sigma_s]_{S\times 1}$ of the sorted vector $[\psi_r]_{S\times 1}$ ($S = P \times Q$ elements of $|\Psi_{p,q}|^2$, arranged in decreasing order of their values):

$$\sigma_s = \sum_{r=1}^{s} \psi_r$$

- Given the total energy $E = \sigma_S = \sum_{p,q} |\Psi_{p,q}|^2$, we define the threshold energy $T_e = \eta E$, where η is a fraction of energy to be retained in the spectrum

Computational considerations (cont'd)

– We find the first element in the cumulative sum vector $[\sigma_s]_{S\times 1}$, denoted by $\bar{\sigma}$, such that $\bar{\sigma} > T_e$, which gives us a binary threshold matrix **T** to accompany $\Psi_{p,q}$:

$$\mathbf{T}(p,q) = \begin{cases} 0, & \text{if } |\Psi_{p,q}|^2 < \bar{\sigma} \\ 1, & \text{otherwise} \end{cases}$$

Operation	**Computational complexity**
2D Fourier transform	$O(N_x N_t \log(N_x N_t))$
Phase shifting	$O(N_x N_t^2) \rightarrow O(N_1 N_t)$
1D inverse Fourier transform	$O(N_x N_t \log(N_x))$
Linear interpolation	$O(N_x N_t)$
End-of-layer phase shift	$O(N_x N_t L) \rightarrow O(N_1 L)$
Cosine similarity	$O(N_x N_t N_c)$
Peak-based boundary detection	$O(N_x N_t)$

Note: N_1 is the number of 1's in matrix $\mathbf{T}(p,q)$.

SOS profiling in CPWC imaging

- Nine-angle CPWC data produced by the K-WAVE simulation of ultrasound propagation in the three-layer medium, mimicking tissue-bone-tissue layer arrangment
- True values of sound speed: **1540** m/s, **3198** m/s, and **1540** m/s, respectively
- True values of thickness: **5** mm, **7** mm, and **51** mm, respectively

Case	**Vectorization method**	**Boundary detection method**
1	Stacking	Line detection in migrated data
2	Summing	Line detection in migrated data
3	Stacking	Peak detection in migrated data
4	Summing	Peak detection in migrated data
5	Stacking	Line detection in raw data
6	Summing	Line detection in raw data
7	Stacking	Peak detection in raw data
8	Summing	Peak detection in raw data

Table: Different cases for SOS profiling in CPWC imaging.

SOS profiling in CPWC imaging (cont'd)

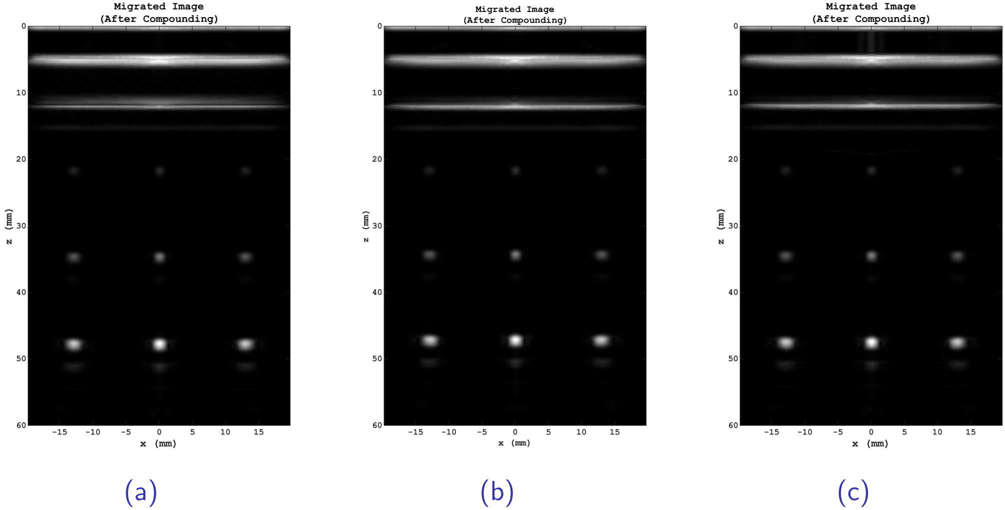


Figure: Migration results using estimated SOS profile, case 4. (a) True SOS profile, (b) Original (PWPSM), (c) Low-cost (LCPWPSM).

SOS profiling in CPWC imaging (cont'd)

Layer	Original				Low-cost			
	Speed		Thickness		Speed		Thickness	
	est., m/s	error, %	est., mm	error, %	est., m/s	error, %	est., mm	error, %
I	1550	0.65	5.00	0.00	1550	0.65	5.00	0.00
II	3320	3.81	6.78	3.14	3260	1.93	6.78	3.14
III	1510	1.94	51.66	0.43	1530	0.65	51.66	0.43

Table: Estimated speed and thickness, case 4

Structural similarity index (SSIM)								
Case	1	2	3	4	5	6	7	8
Original	0.944	0.944	0.947	0.964	0.962	0.945	0.961	0.935
Low-cost	0.941	0.940	0.945	0.958	0.956	0.940	0.956	0.931

Table: CPWC image SSIM when using estimated vs. true SOS profile.

Conclusion

Average error in estimates (%)						
Estimation type	Evaluation type	PWPSM	LCPWPSM	SAPSM	LCSAPSM	single-layer CPWC
Speed	Stacking	1.36	1.46	–	–	0.60
Speed	Summing	1.01	1.22	–	–	0.88
Thickness	Line detection	1.38	1.40	0.48	0.48	–
Thickness	Peak detection	1.59	1.59	0.51	0.51	–
Thickness	Beamformed data	1.42	1.43	0.45	0.45	–
Thickness	Raw RF data	1.52	1.53	0.54	0.54	–

- Overall speed estimation errors are within 4% for PWPSM and LCPWPSM and within 1% for single-layer CPWC imaging using PW Stolt's migration

- Overall thickness estimation errors are within 4% for PWPSM and LCPWPSM and within 0.6% for SAPSM and LCSAPSM

Future work

- Investigating methods to adaptively determine the energy threshold settings
- Investigating efficient polar approximations during downward extrapolation
- Merging sound speed and thickness estimation into a single optimization problem
- Using deep learning approaches to reconstruct migrated data

References

[1] M. Albulayli, "Migration-based image reconstruction methods for plane-wave ultrasound imaging," Ph.D. dissertation, University of Victoria, Victoria, Canada, 2018.

[2] J. Krucker, J. B. Fowlkes, and P. L. Carson, "Sound speed estimation using automatic ultrasound image registration," *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control*, vol. 51, no. 9, pp. 1095–1106, 2004.

[3] X. Qu, T. Azuma, J. T. Liang, and Y. Nakajima, "Average sound speed estimation using speckle analysis of medical ultrasound data," *International Journal of Computer Assisted Radiology and Surgery*, vol. 7, no. 6, pp. 891–899, 2012.

Thank You