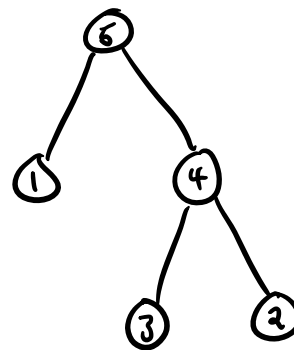
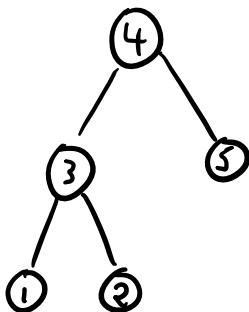
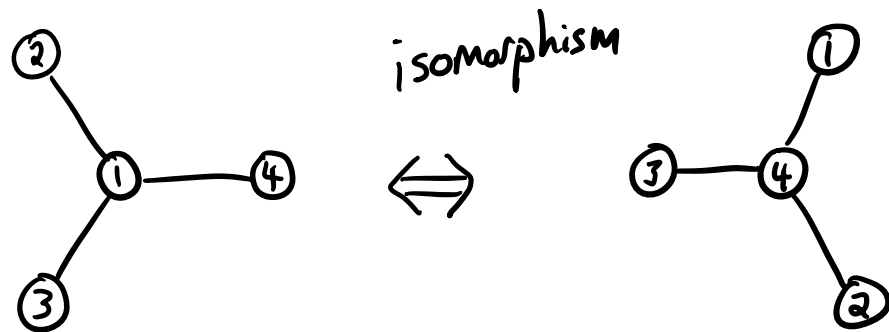


Tree Isomorphism



$\bigcirc \equiv []$

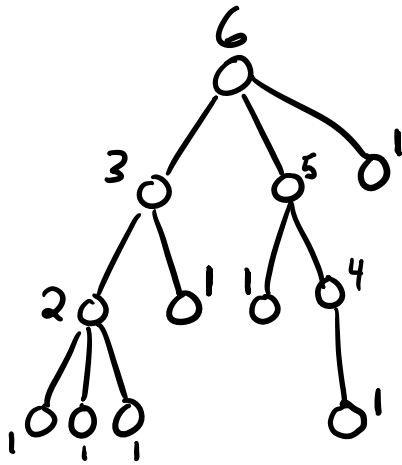
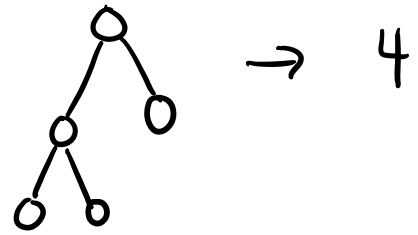
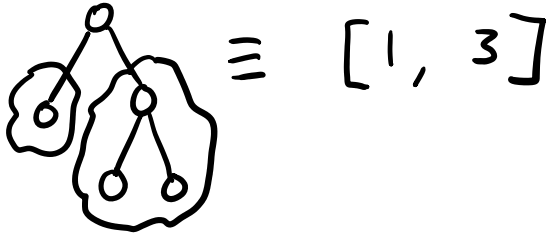
$\bigcirc - \bigcirc \equiv [1]$

$\bigcirc \rightarrow 1$

$\bigcirc - \bigcirc \rightarrow 2$

$\bigcirc - \bigcirc - \bigcirc \rightarrow 3$

0

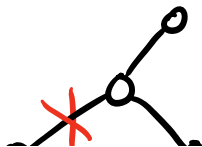


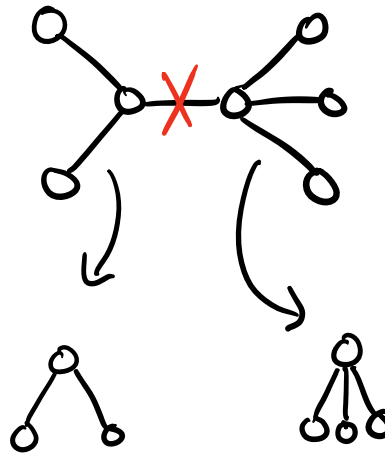
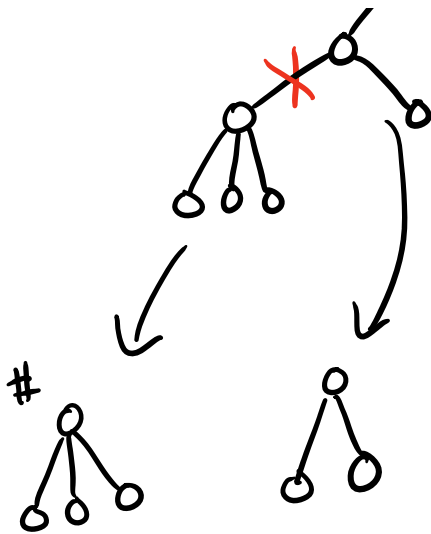
Map

7

$[3]$	\rightarrow	1
$[1, 1, 1]$	\rightarrow	2
$[1, 2]$	\rightarrow	3
$[1]$	\rightarrow	4
$[1, 4]$	\rightarrow	5
$[1, 3, 5]$	\rightarrow	6

`HashMap<ArrayList, Integer>`

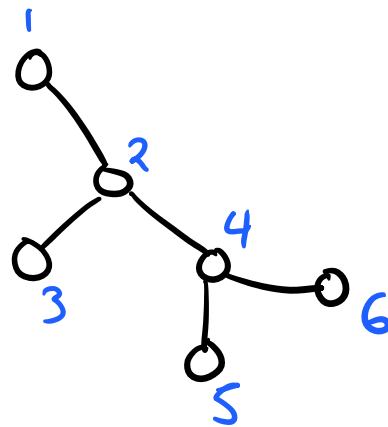
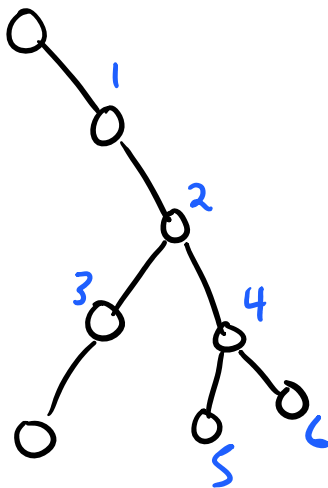




Subtree Isomorphism

2 trees

$n \leq 100$



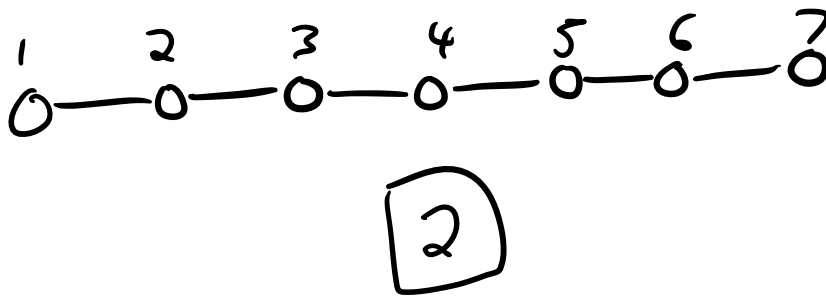
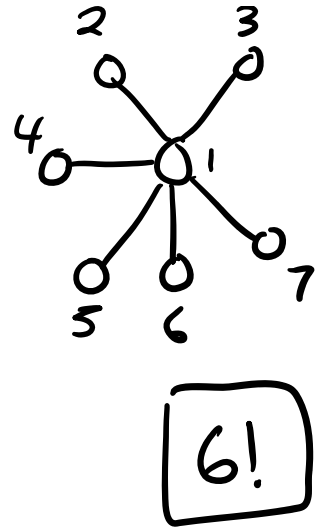
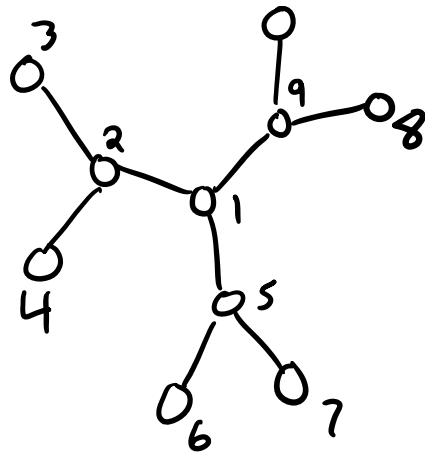
Automorphism

3
Q

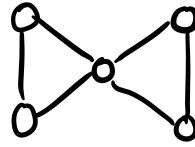
10
Q

2
Q

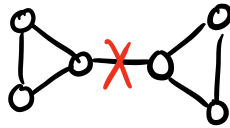
3
Q



2-edge-connected

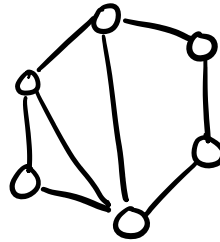


bridge

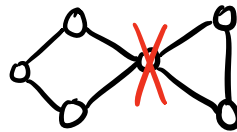


cut edge

2-vertex-connected

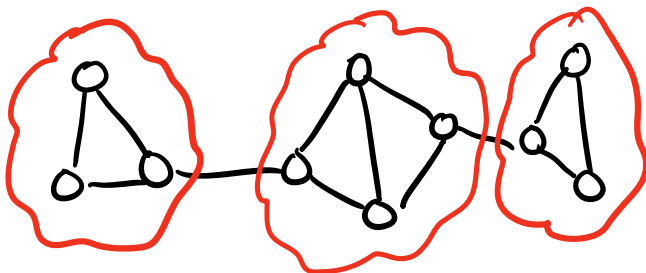


articulation point



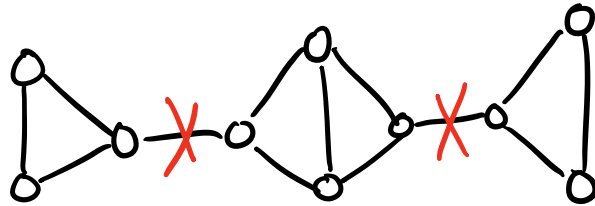
cut vertex

2-edge-connected components
* vertex set * (large as possible)



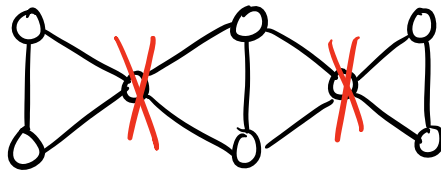
bridges

$$O(E(V+E))$$

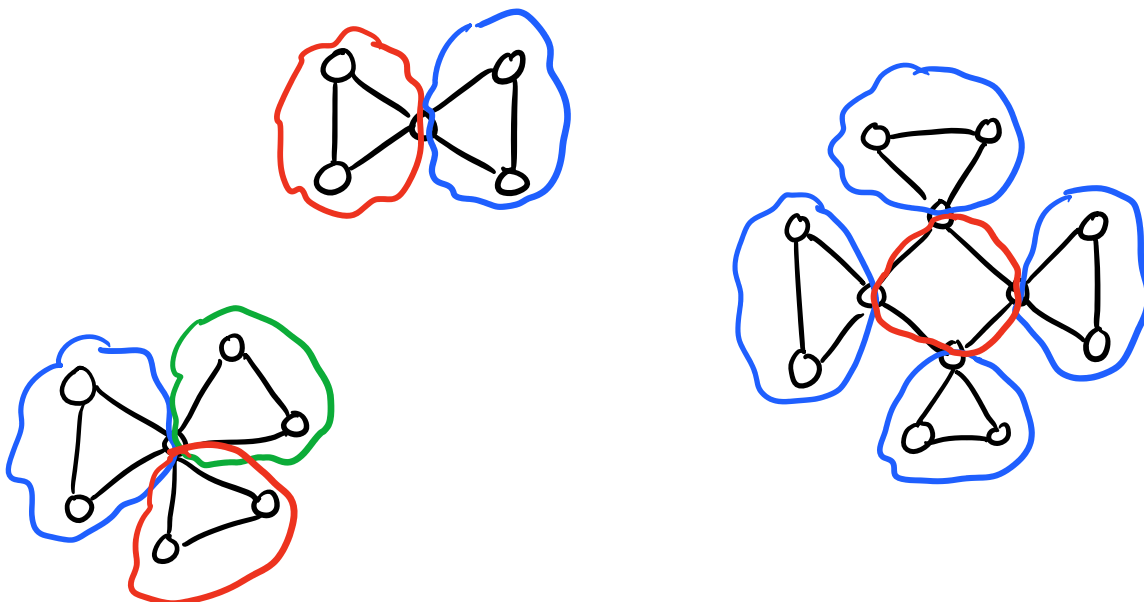


articulation points

$$O(V(V+E))$$



2-vertex-connected components
* edge set * (large as possible)



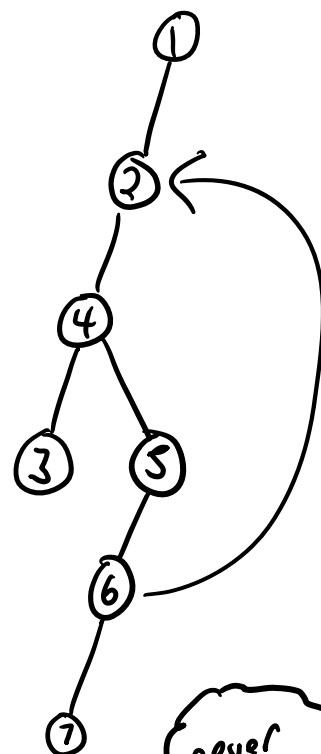
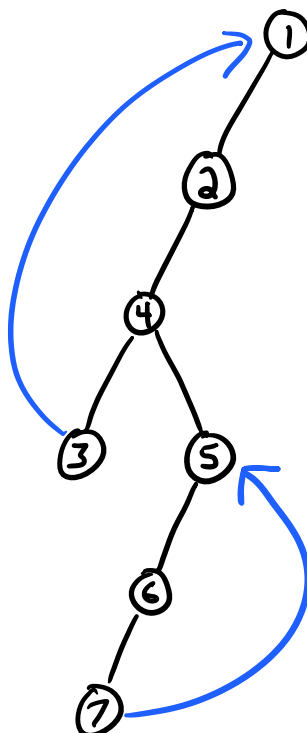
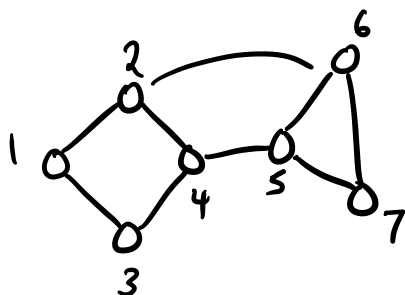
$$n \leq 10^{5.5}$$

$$m \leq 10^{5.5}$$

$$O(V+E)$$

← goal

DFS + lowlinking



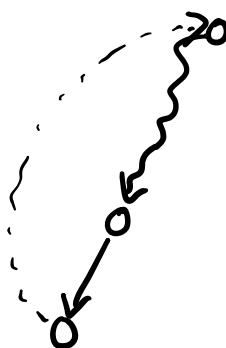
never for undirected

DFS (edges)

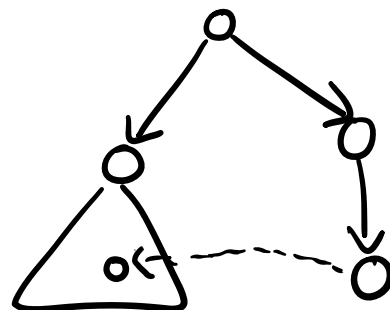
forward edge:



back edge:



cross edge:

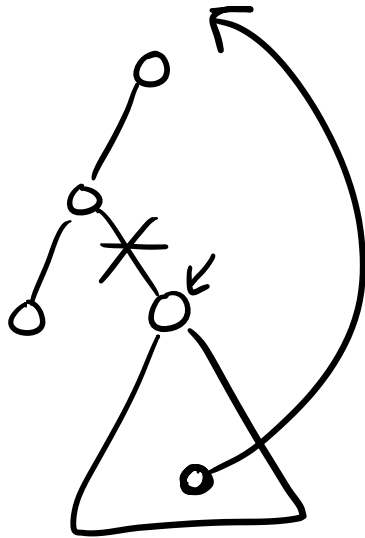


人

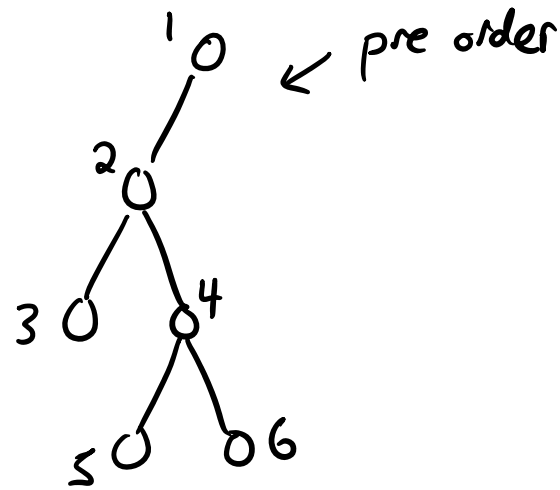
Linca



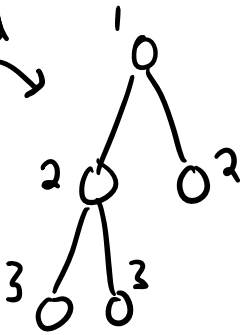
← fancy
cross
edge



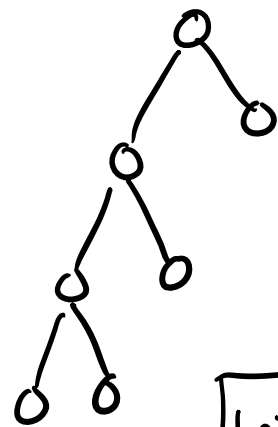
Escape the subtree



height
based

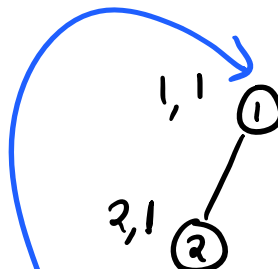
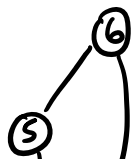
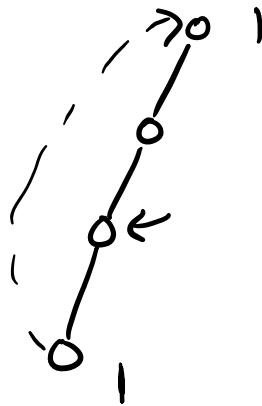
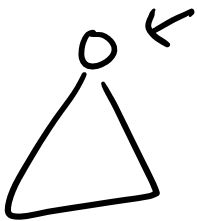


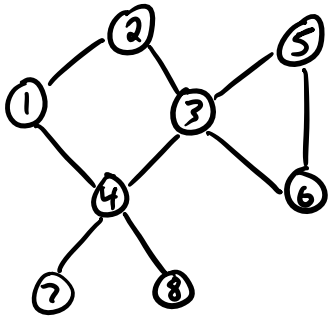
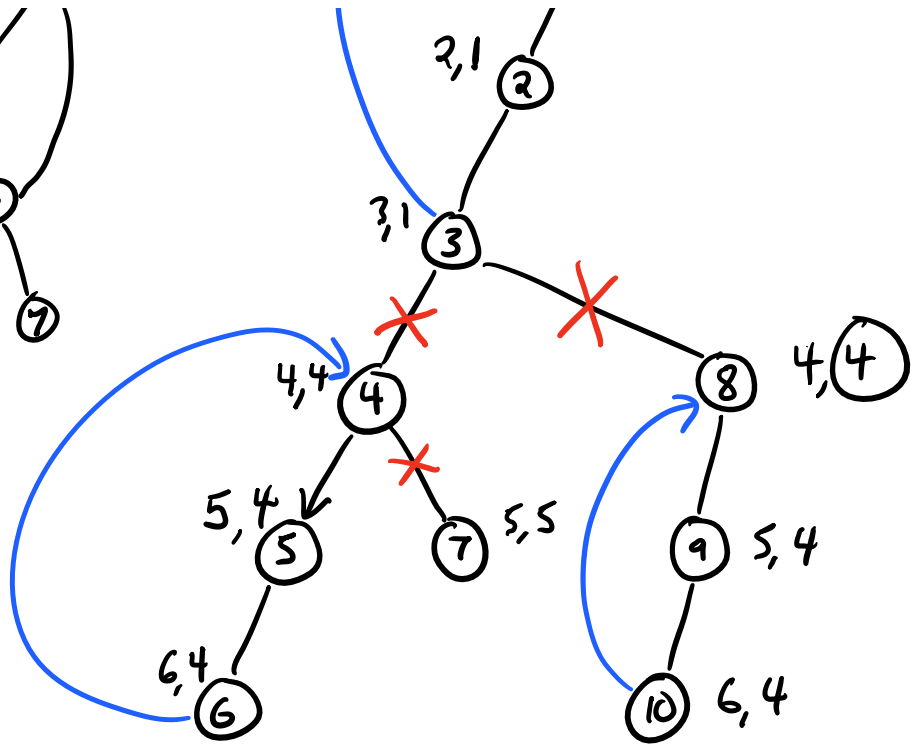
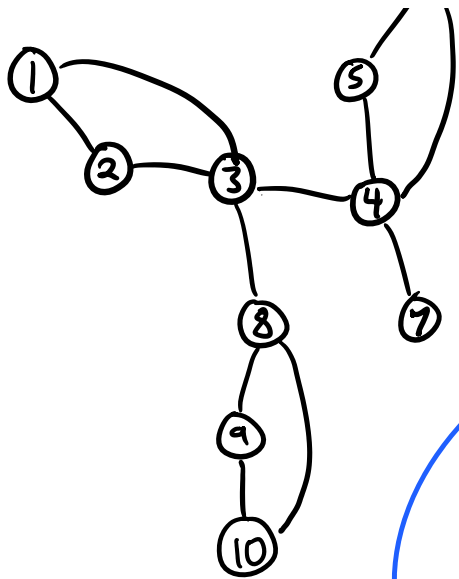
subtree
aggregates



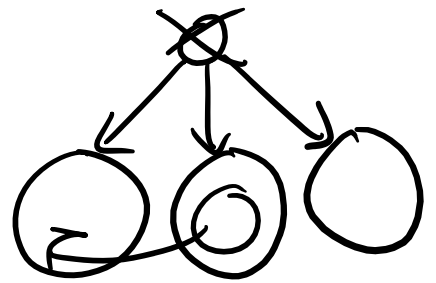
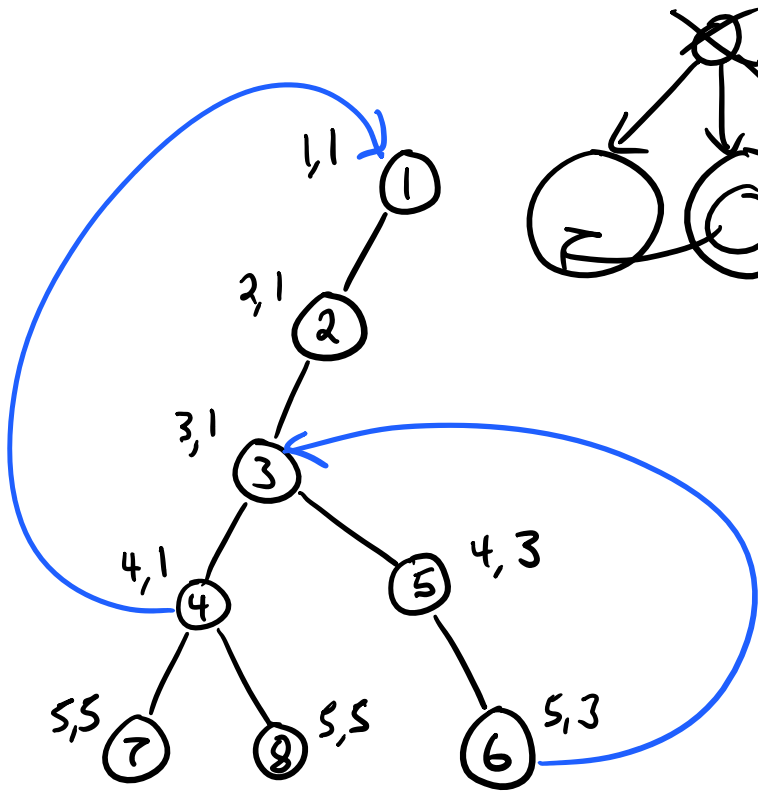
height, low

lowest
lowlink





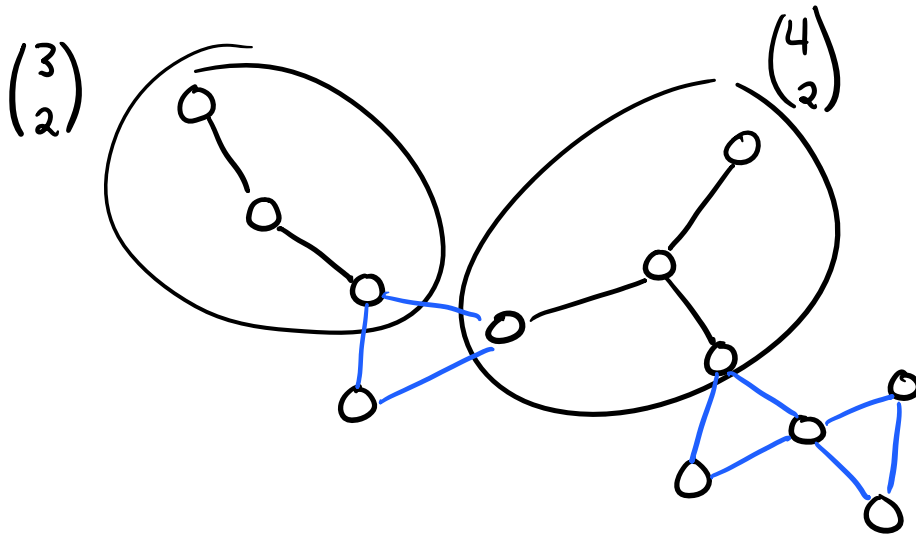
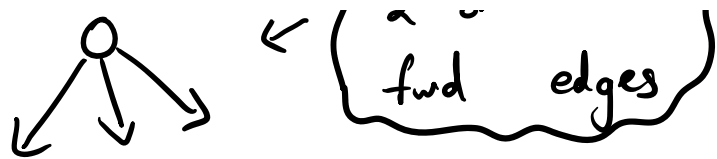
{4, 3}



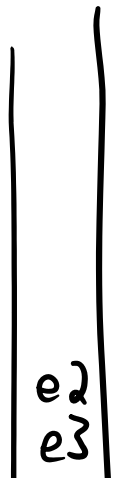
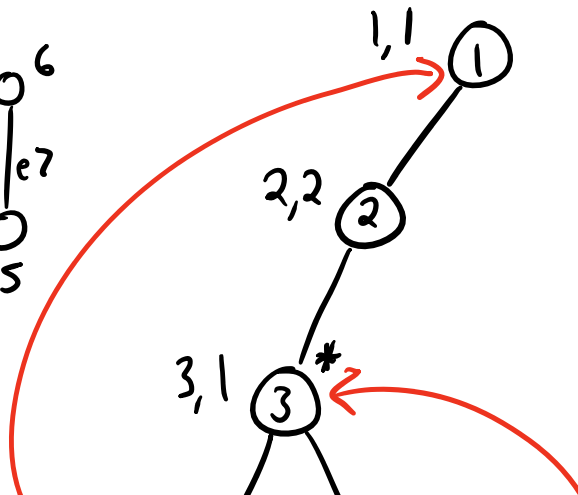
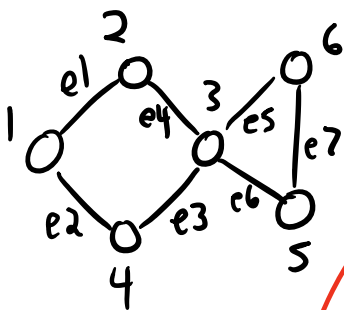
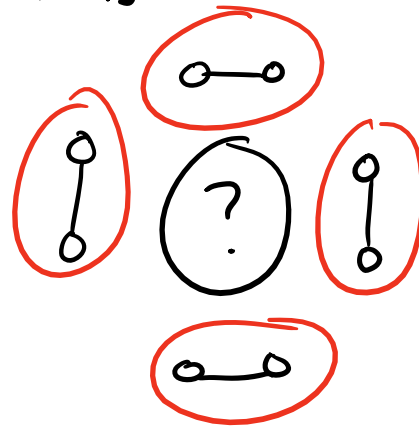
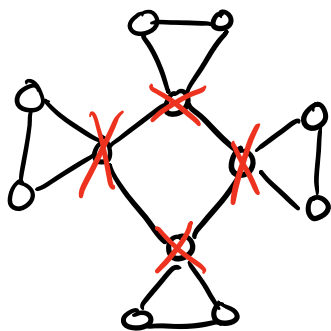
* root node is special *



← { 2 or more
fwd edges }

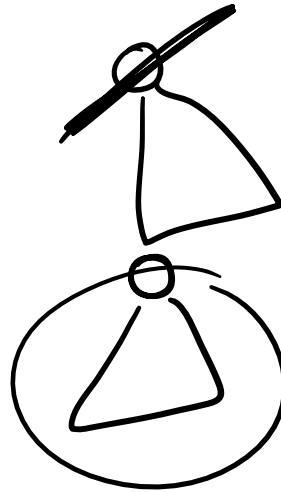
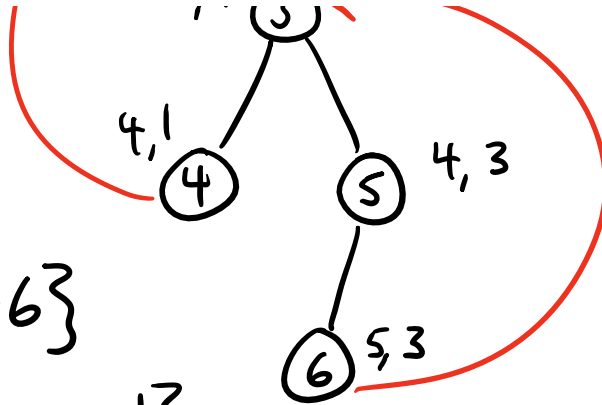


2-vertex-connected components

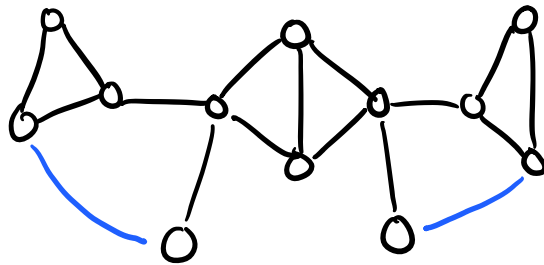


e_2
 e_3
 e_4
 e_1

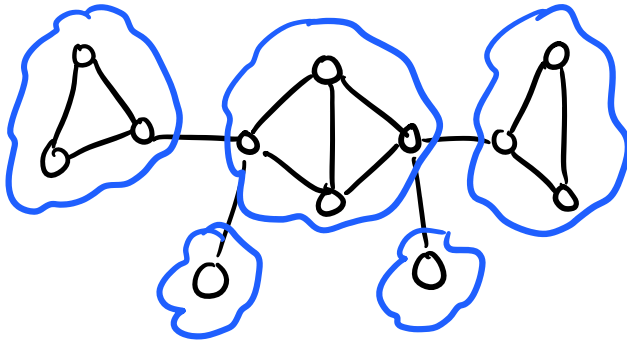
$\{e_5, e_7, e_6\}$
 $\{e_2, e_3, e_4, e_1\}$



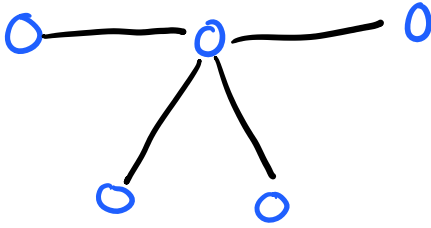
Min edges to make 2-edge-connected



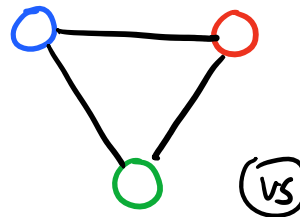
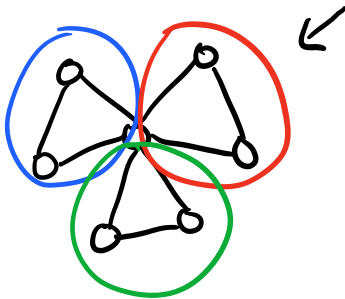
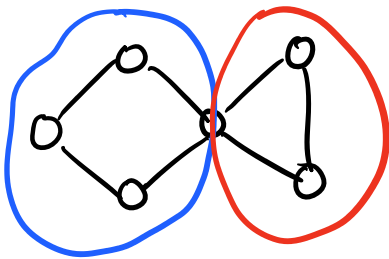
Metagraph



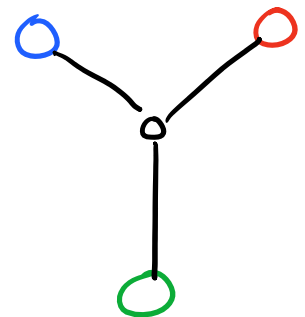
Graph → Tree
 2ECC → node
 bridge → edge



2VCC → tree?



(vs)



Graph

2VCC + Art pts \rightarrow nodes

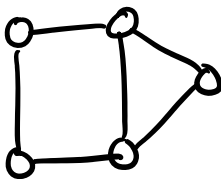
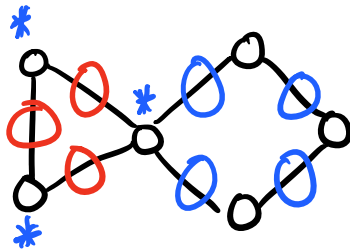
2VCC \leftrightarrow Art pts \rightarrow edges

Tree

Lucky Cities

2VCC \Leftrightarrow cycles

2ECC \Leftrightarrow circuits



abcbcd
abcd
└─┬─┘

$$1 \leq |A| \leq 1000$$

$$1 \leq |B| \leq 1000$$

abcbcd
bc bcd
c bcd
b c d
c d

all substrings
into hash set

d
cd

Suffix Tree

Suffix Array

abcbcd
bc bcd
c bcd
b c d
c d
d

perm →

abcbcd
bc bcd
b c d
c bcd
c d
d

lcp

0
2
0
1
0
0

$$\propto \cdot n$$

$$n \log n \leftarrow$$

$$n \log^2 n$$

abcd \$ abcbcd