**1. Introduction**

The aim of this study was to investigate whether a device produced by the manufacturer would reduce the emissions when placed on a motor vehicle engine. As the manufacturer claimed, after using the device hydrocarbon and carbon monoxide (CO) emissions are supposed to be steadily decreased, while carbon dioxide (CO2) emissions should be steadily increased.

* 1. Study Design:

The analysis was focused on using the car owned by the manufacturer. We operated the vehicle under the same conditions on 13 different days and performed 4 replicates of the experiment on each of the testing days. The device was placed on the engine from the third day to the eleventh day. In the first two days and the last two days, there was no device on the engine. It took nearly six months to complete the tests.

* 1. Variables:

In this investigation, three emissions were observed for the manufacturer’s car. Among them the unit of hydrocarbon emission is ppm, which is the abbreviation of parts per million. Other factors recorded were DAY, REP and DEV. Details concerning these variables are summarized below:

Table 1 Variables in the study

|  |  |  |
| --- | --- | --- |
| Variable | Definition | Type |
| HC | Hydrocarbon (ppm) | Quantitative Measures |
| CO | Carbon Monoxide (% of the Volume) | Quantitative Measures |
| CO2 | Carbon Dioxide (% of the Volume) | Quantitative Measures |
| DAY | Test Day: 1, 2, … ,13 | Categorical Factors |
| REP | Replicate: 1, 2, 3, 4 | Categorical Factors |
| DEV | Device: 1/0 (Present/Absent) | Categorical Factors |

**2. Methodology:**

A statistical analysis of this project was performed using the statistical software R. Four statistical procedures were used in this analysis: exploratory data analysis (EDA) was used to summarized the data, two-sample t-tests were used to detect differences between groups, analysis of variance (ANOVA) was used to detect significant factor effects and Hotelling’s test was used to test all three emissions simultaneously. Details about these approaches are briefly given below.

2.1 Exploratory Data Analysis:

EDA techniques are used to summarize the data. Frequency tables, bar charts, and histograms effectively display the distribution of the variables under consideration. Bar charts were employed for the categorical variables: DAY, REP and DEV; histograms for the quantitative variables: HC, CO and CO2.

2.2 ANOVA:

To test if the device has the effect to reduce engine emissions, one-way ANOVA can be performed. The data would be separated into 6 groups: HC(DEV=0), HC(DEV=1), CO(DEV=0), CO(DEV=1), CO2(DEV=0) and CO2(DEV=1). However, there are three main assumptions that the test makes. Firstly, the dependent variable is normally distributed in each group that is being compared in the one-way ANOVA. Secondly, there is homogeneity of variance, which means that the population variance in each group are equal. Thirdly, observations are independent.

Thus, before performing ANOVA, we did Shapiro-Wilk normality test six times to check if the data of each group is normally distributed. Similarly, Bartlett’s test was used three times to check if the variance of two groups that should be compared is same. For the third assumption, it’s necessary to determine that our observations are not independent on the study design (e.g. day/rep), so we can use ANOVA to determine whether the DAY and REP are important.

If the results of these tests are good, we can use one-way ANOVA to determine the effect of the device.

2.3 Two-sample t-tests:

To investigate the manufacturer's claim that once the device is placed on an engine, the engine is irrevocably changed, we can do a two-sample t-test for equality of mean emissions using the data from Days 1 and 2, versus those of Days 12 and T13. Then we can find if there is evidence to support this claim.

In addition, we should consider the fact that new spark plugs were placed on the car after Day 7, which may have an impact on the experimental results. So another two-sample t-test is performed for equality of mean emissions using the data from Day 3, 4, 5, 6 and 7, versus those of Day 8, 9, 10 and 11.

2.4 Hotelling’s test:

An alternative approach is to consider all three emissions simultaneously, then perform a two-sample Hotelling’s test. The potential advantage of this approach is that the correlation between the responses is accounted for, which is expected according to the manufacturer's claim. Thus, we can see that if the multivariate approach makes any difference. Also, before Hotelling’s test multivariate Shapiro-Wilk tests should be performed to check the normality.

**3. Results:**

Table 2A and Table 2B presents some summary statistics for the HC, CO and CO2 variables overall, and with respect to DAY. Since the standard deviations is relatively large, so the accuracy of the measuring instrument has little impact on the statistical analysis.

The results of normality tests and variance tests are presented in Table 3A and Table 3B. It was found that these assumptions are true, except that one group of hydrocarbon seemed not to be normal. However, the one-way ANOVA is considered a robust test against the normality assumption. This means that it tolerates violations to its normality assumption rather well.

The following may have an impact on the measures. The test days were not necessarily contiguous, or the car was driven for approximately 700 to 2,000 miles between tests. In addition, testing began in November and was completed the following May. But from the results of analysis of the effects of DAY and REP using ANOVA shown in the Table 4, DAY and REP have no statistically significant effect since p-values are larger than 0.05. In other words, it can be considered as a completely randomized design. Perhaps some factors can affect our measures, but the impact is very insignificant. So we can still use the ANOVA.

Table 5 shows the results of ANOVA to check the effect of DEV respectively to each emission. Since all the p-values are greater than 0.05, we cannot reject the null hypothesis. The device has no statistically significant effect to three emissions, which is not as same as the company claimed.CO

The results of two-sample t-tests are shown in Table 6A and Table 6B. Since p-values are greater than 0.05, there is no statistically significant difference between the emissions of the first two days and the last two days. The claim made by the manufacturer that the device could reduce the emissions once placed on the engine can be rejected. Also, from Table 5B the effect of changing spark plugs is not statistically significant except CO2.

Table 7 shows the results of Hotelling’s test. The p-value is 0.34, which is much larger than 0.05. The null hypothesis is rejected as well, so the result makes no difference. When consider all three emissions simultaneously, there is still no statistically significant effect of the device to emissions.

**4. Conclusions**

Firstly, the SCP found that there were no significant results for the variables and REP, which suggests that the days and replicates did not differ significantly with respect to these variables. Then, there was a no significant DEV effect for the variables HC, CO and CO2, suggesting that hydrocarbon and carbon monoxide emissions cannot be steadily decreased and carbon dioxide emissions cannot be steadily increased after the device was placed on the engine. The certain results claimed by the manufacturer was not achieved by us. Therefore, a conclusion can be safely drawn that the claim made by the manufacturer should be rejected because there was no statistically significant difference that the device made to the emissions.

**References:**

[1]<https://statistics.laerd.com/statistical-guides/one-way-anova-statistical-guide-4.php>

[2]<https://www.statmethods.net/stats/anova.html>

**Appendix: Tables**

Table 2A Mean Emissions for Manufacturer's Car

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Device | Day | HC (ppm) | CO2 (%) | CO (%) |
| Absent | 1 | 15.30 | 15.83 | 0.25 |
| 2 | 10.75 | 14.41 | 0.11 |
| Present | 3 | 5.46 | 14.85 | 0.11 |
| 4 | 6.04 | 13.48 | 0.16 |
| 5 | 8.66 | 14.62 | 0.08 |
| 6 | 13.18 | 13.38 | 0.13 |
| 7 | 25.89 | 13.75 | 0.21 |
| 8 | 4.45 | 15.95 | 0.20 |
| 9 | 13.18 | 15.52 | 0.10 |
| 10 | 1.11 | 15.42 | 0.11 |
| 11 | 12.71 | 14.85 | 0.14 |
| Absent | 12 | 11.31 | 15.61 | 0.13 |
| 13 | 13.54 | 14.24 | 0.15 |
| Absent | Avg | 12.72 | 15.02 | 0.16 |
| Present | Avg | 10.07 | 14.65 | 0.15 |

Table 2B Summary Statistics of Emissions

Total Observation Number = 52

|  |  |  |  |
| --- | --- | --- | --- |
|  | HC (ppm) | CO2 (%) | CO (%) |
| Min | 0.170 | 11.14 | 0.0000 |
| 1st Qu. | 5.438 | 13.94 | 0.0800 |
| Median | 10.130 | 14.75 | 0.1400 |
| Mean | 10.891 | 14.76 | 0.1460 |
| 3rd Qu. | 13.932 | 15.54 | 0.2025 |
| Max | 29.440 | 17.11 | 0.3200 |
| Std. Dev | 6.9312 | 1.2472 | 0.0822 |

Table 3A Results of Shapiro-Wilk normality Tests

|  |  |  |
| --- | --- | --- |
|  | DEV = 0 | DEV = 1 |
| HC | W = 0.95 p-value = 0.47 | W = 0.90, p-value = 0.0038 |
| CO2 | W = 0.97, p-value = 0.88 | W = 0.95, p-value = 0.12 |
| CO | W = 0.93, p-value = 0.28 | W = 0.98, p-value = 0.88 |

Table 3B Results of Bartlett’s Tests

|  |  |
| --- | --- |
| HC | Bartlett's K-squared = 6.47, p-value = 0.01096 |
| CO2 | Bartlett's K-squared = 0.033, p-value = 0.856 |
| CO | Bartlett's K-squared = 1.70, p-value = 0.1918 |

Table 4 Results of ANOVA of DAY and REP

|  |  |
| --- | --- |
| CO1 ~ DAY | Df Sum Sq Mean Sq F value Pr(>F)  1 0.0028 0.0028 0.417 0.522  50 0.3418 0.0068 |
| CO1 ~ REP | Df Sum Sq Mean Sq F value Pr(>F)  1 0.0004 0.0004 0.061 0.086  50 0.3442 0.0069 |
| CO2 ~ DAY | Df Sum Sq Mean Sq F value Pr(>F)  1 0.9800 0.9807 0.626 0.433  50 78.35 1.5670 |
| CO2 ~ REP | Df Sum Sq Mean Sq F value Pr(>F)  1 1.50 1.497 0.962 0.331  50 77.83 1.557 |
| HC ~ DAY | Df Sum Sq Mean Sq F value Pr(>F)  1 1 1.02 0.021 0.886  50 2449 48.98 |
| HC ~ REP | Df Sum Sq Mean Sq F value Pr(>F)  1 43.6 43.65 0.907 0.346  50 2406.5 48.13 |

Table 5 Results of ANOVA of DEV

|  |  |
| --- | --- |
| HC ~ DEV | Df Sum Sq Mean Sq F value Pr(>F)  1 77.6 77.61 1.636 0.207  50 2372.5 47.45 |
| CO ~ DEV | Df Sum Sq Mean Sq F value Pr(>F)  1 0.0050 0.00497 0.731 0.396  50 0.3397 0.006794 |
| CO2~ DEV | Df Sum Sq Mean Sq F value Pr(>F)  1 1.55 1.546 0.993 0.324  50 77.79 1.556 |

Table 6A Two-sample t-tests for DAY 1 ~ 2 and DAY 12 ~ 13

|  |  |  |  |
| --- | --- | --- | --- |
|  | t | p-value | Mean in group 1 / 2 |
| HC | 0.28 | 0.79 | 13.02 / 12.42 |
| CO | 0.67 | 0.51 | 0.18 / 0.14 |
| CO2 | 0.31 | 0.76 | 15.12 / 14.92 |

Table 6B Two-sample t-tests for old spark plugs and new spark plugs

|  |  |  |
| --- | --- | --- |
|  | t | p-value |
| HC | 1.56 | 0.13 |
| CO | 0.0049 | 0.99 |
| CO2 | -4.03 | 0.0003 |

Table 7 Result of Hotelling’s two-sample T2-test

|  |  |
| --- | --- |
| Shapiro-Wilk normality test | W = 0.95308, p-value = 0.1307 |
| Hotelling's two sample T2-test | T2 = 1.1363, df1 = 3, df2 = 48, p-value = 0.3439 |