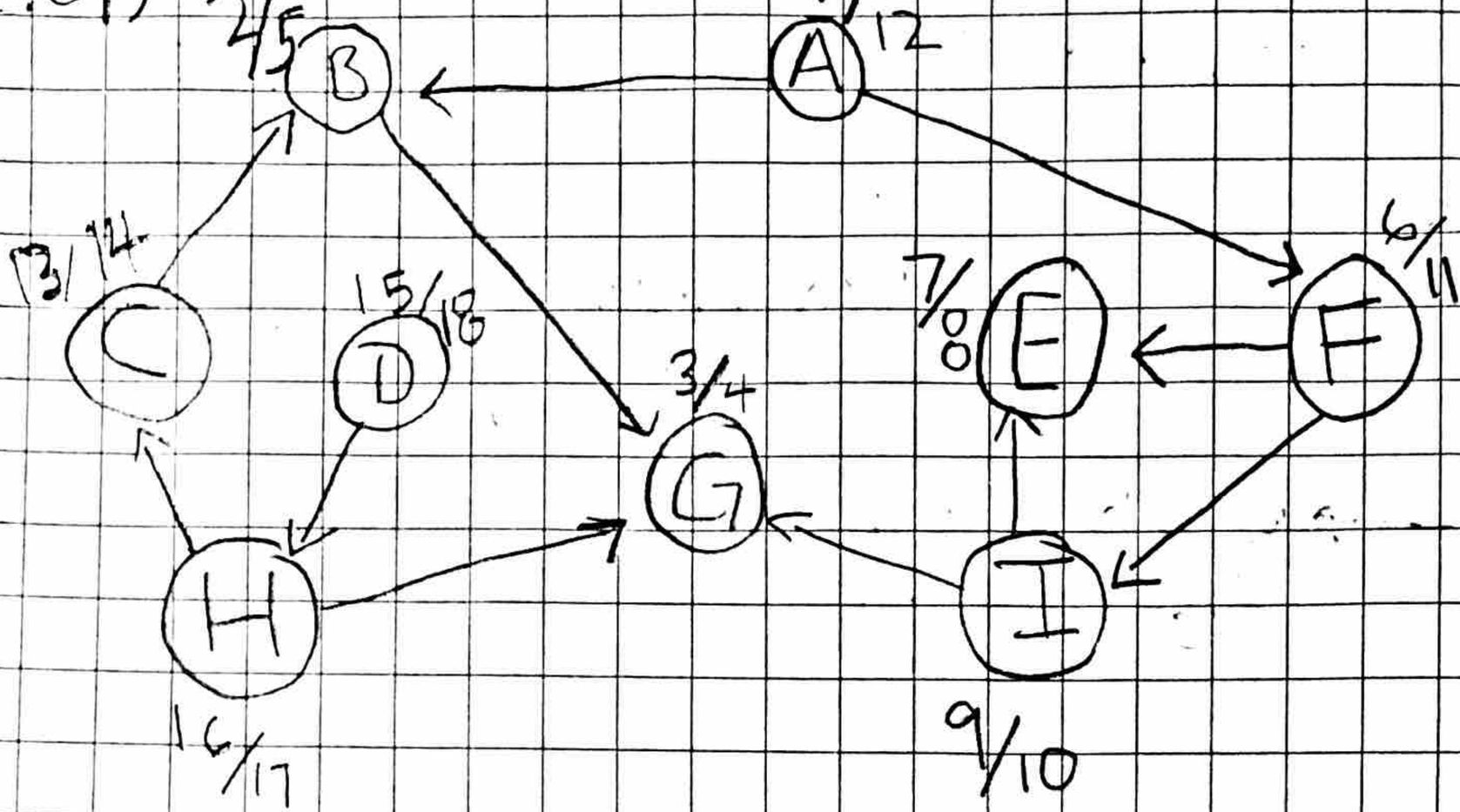
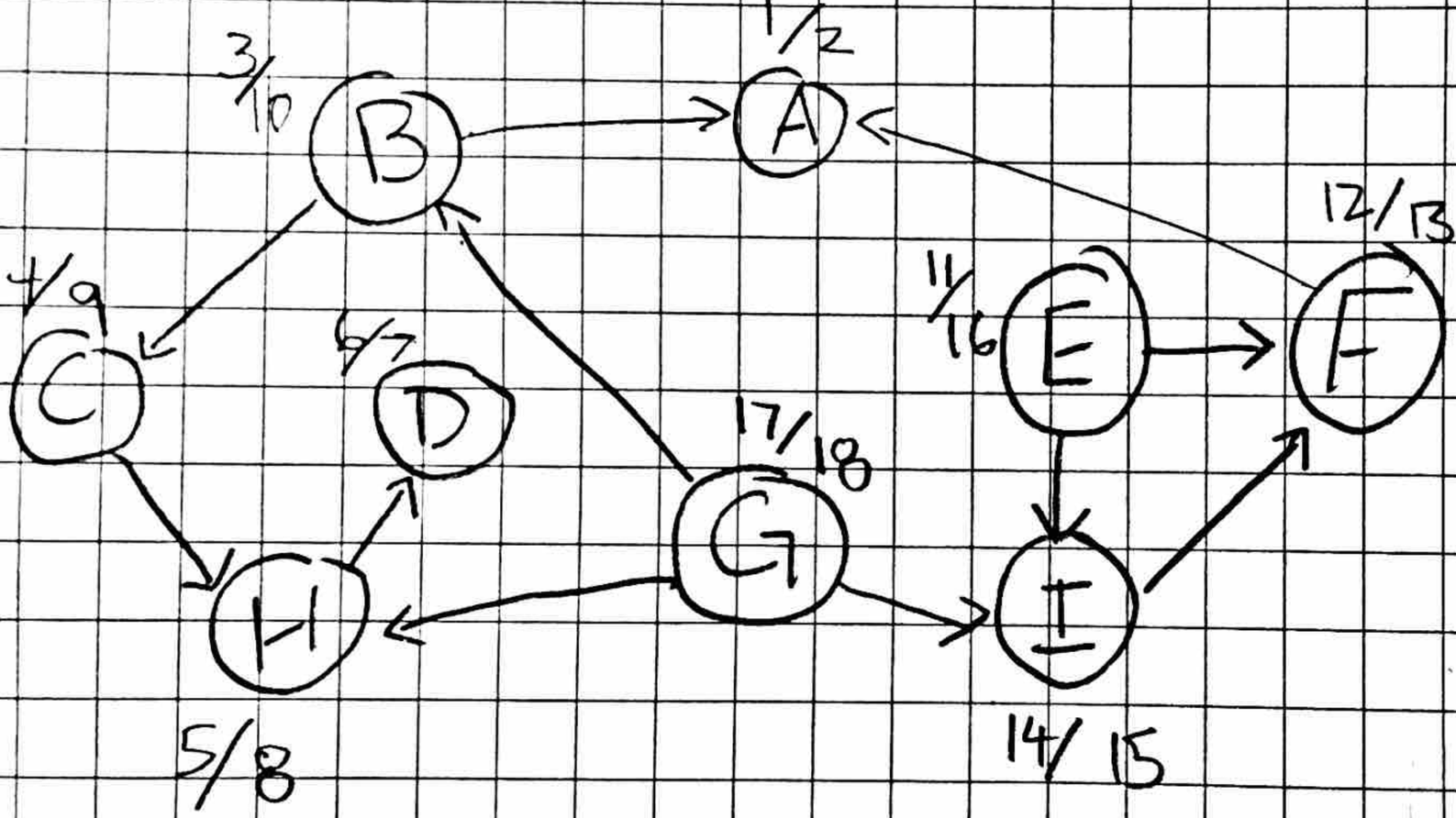
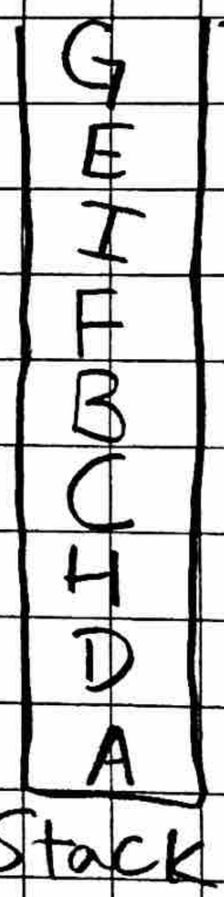
2.01)

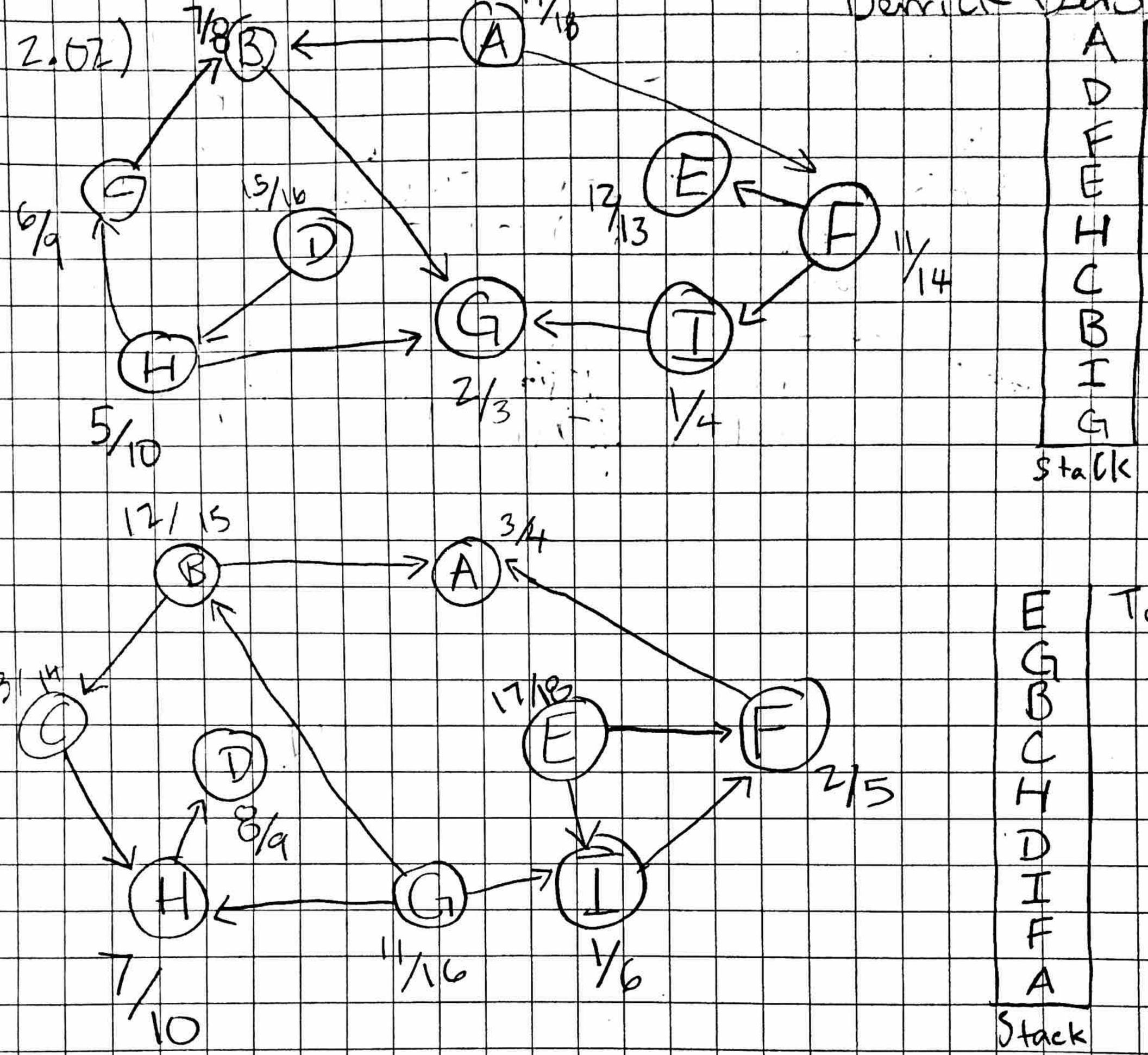
Derick DuBose







Derrick DeBose



A	top
D	
F	
E	
H	
C	
B	
I	
G	

$tack

E	Top
G	
B	
C	
H	
D	
I	
F	
A	

Stack

The topological sorder is not the same in both cases because the graph is not acyclic or a DAG

Derrick DeBose

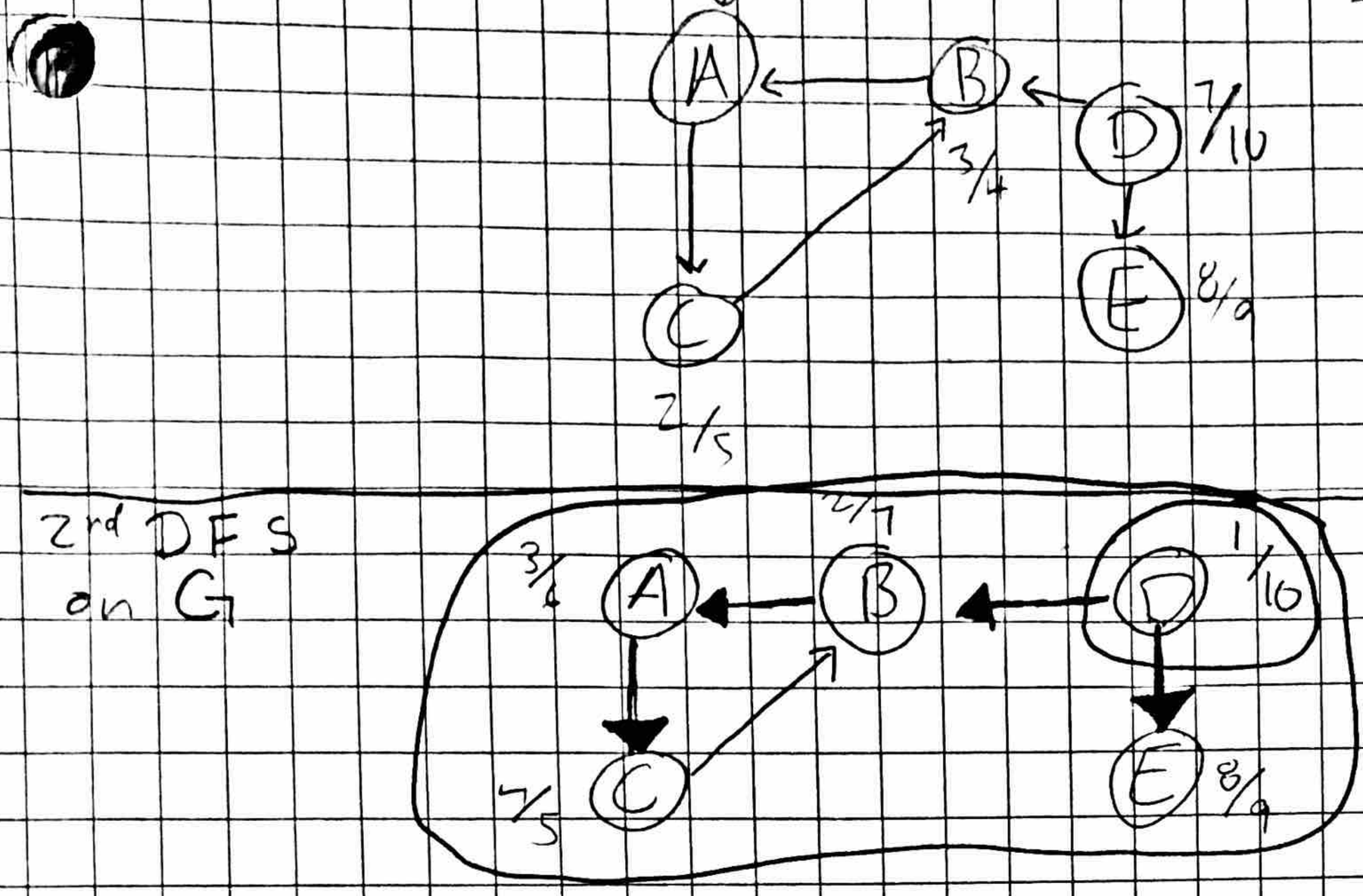
2.03) Analysis: $\Theta(n+m)$ for DFS
(linear algorithm)

We will visit each vertex once in DFS alone and call DFSTrace on each vertex.
Analysis: $O(n)$

DFSTrace is called once per vertex and we want (search every edge once. Analysis: $\boxed{O(m)}$

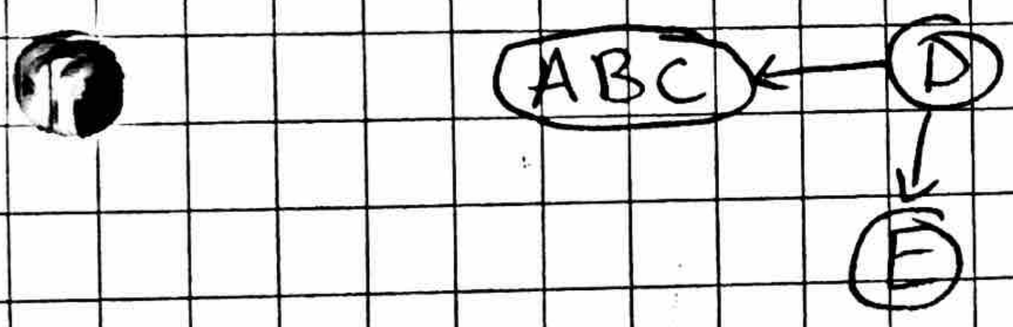
Derrick DeBose

2.04)



Top | D | ✓
E | x
A | x
C | x
B | x
Stack

The condensation graph should be with 3 leaders: A, D, E.



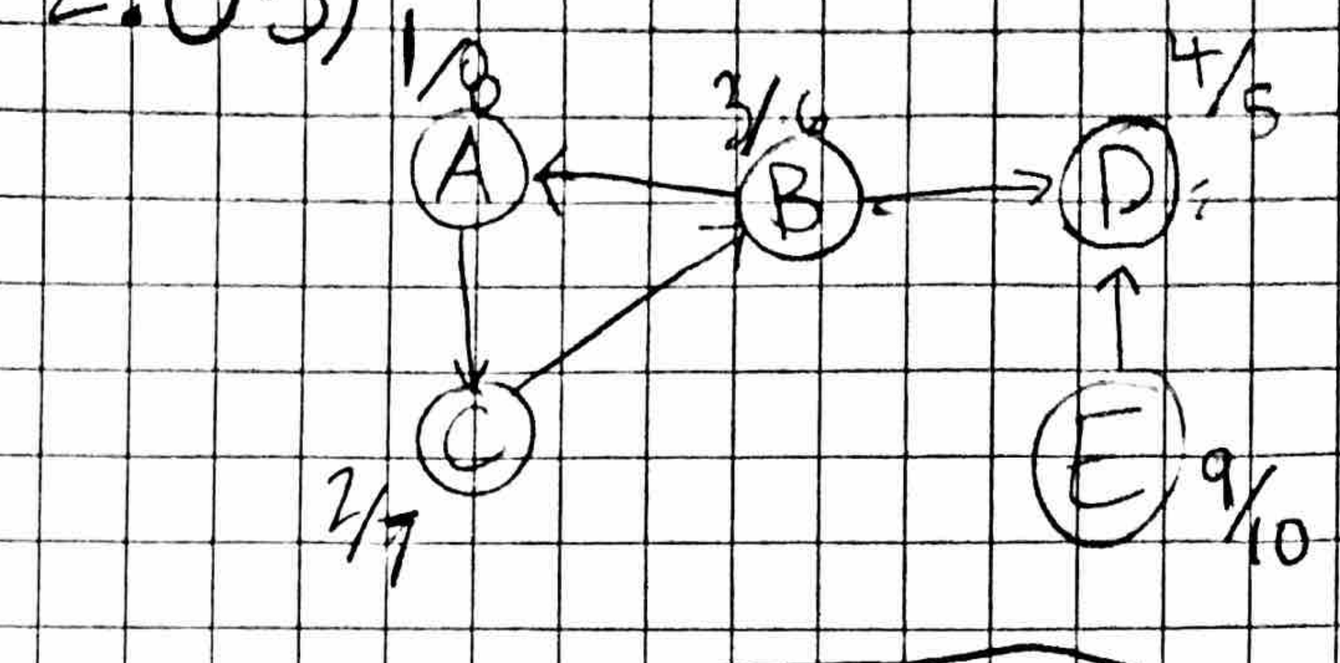
With doing 2nd DFS on G_T instead of G^T, the condensation graph is

(DBACE) with finding D as the only leader.

we know this is the incorrect SCC because ABC is the only cycle, but the 2nd DFS on G_T sees the entire graph as a cycle which is incorrect.

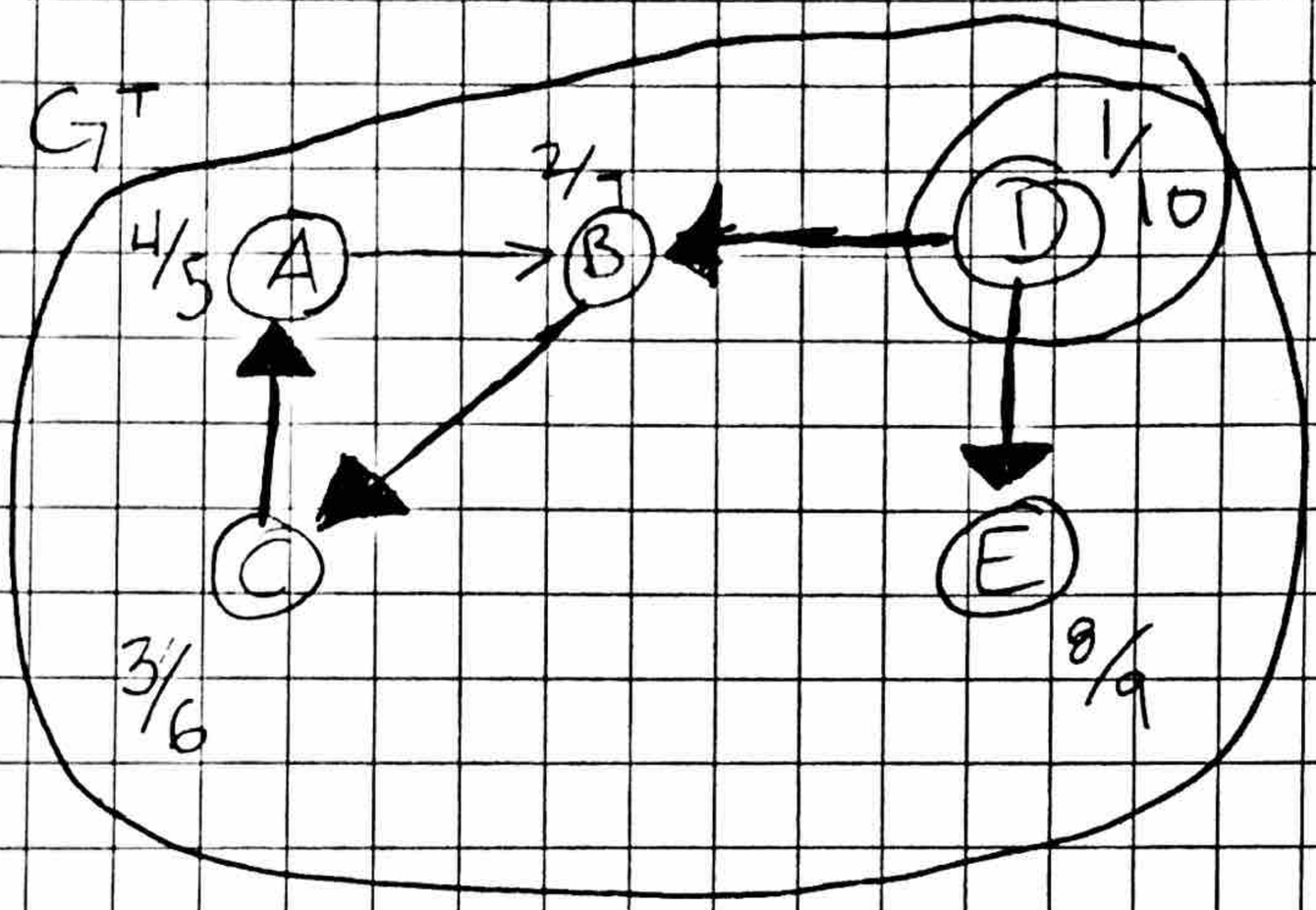
Derrick DeBose

2.05)



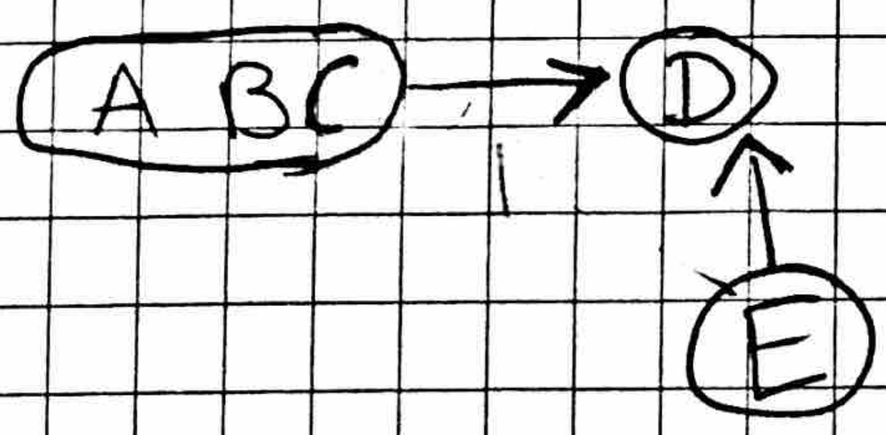
E	x
A	x
C	x
B	x
D	✓

✓ leader
x not leader



Searching vertices from bottom to top

The proper condensation graph is

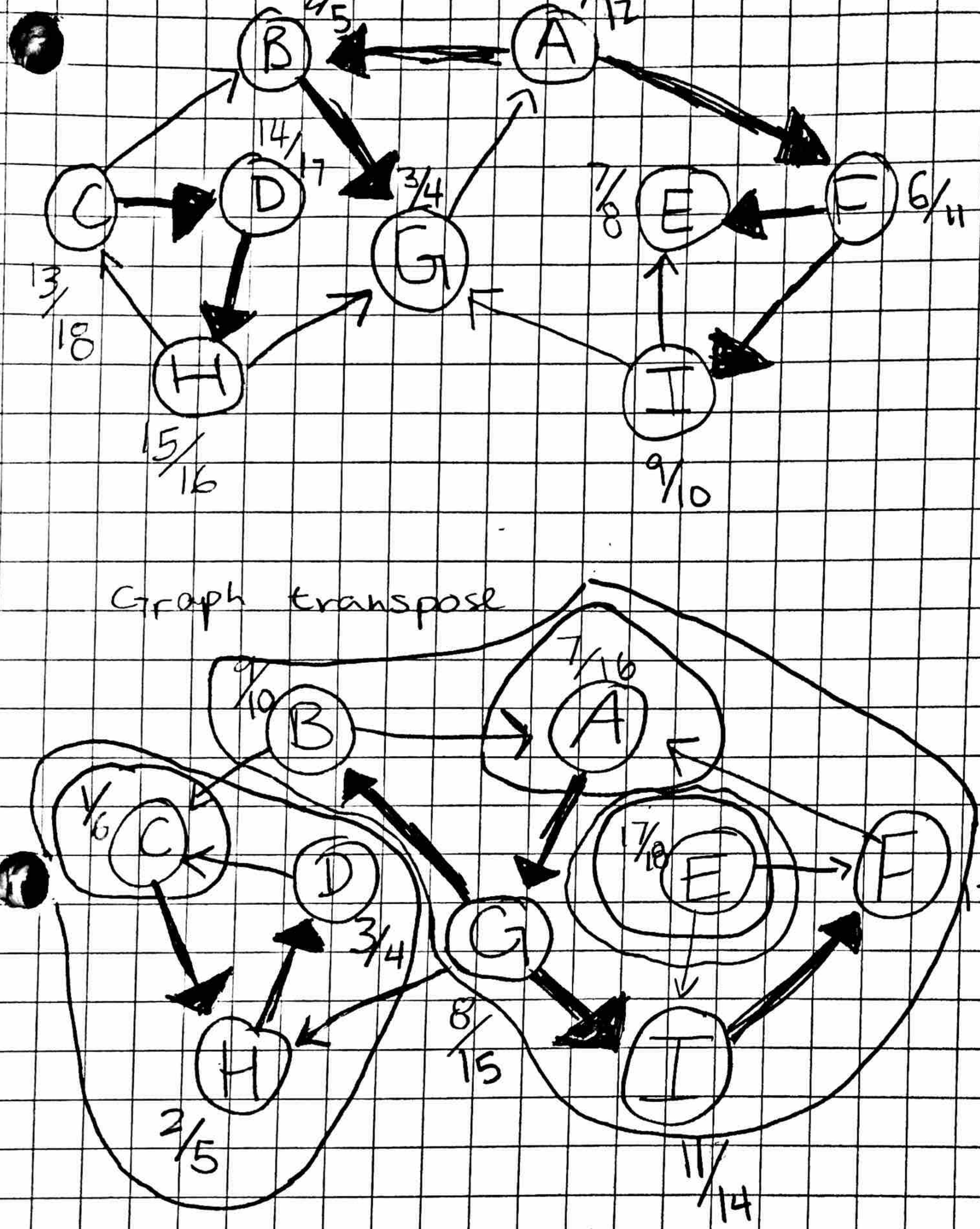


When traversing the stack from bottom to top the condensation graph is

(DACBE) finding D as the leader

This is an incorrect graph because ABC is the SCC with E & D as independent SCCs. The entire graph is the not a cycle which is incorrectly identifying the SCCs.

2.06)



Derrick DeBose

Top		
	C	✓
	D	X
	H	X
	A	✓
	F	X
	I	X
	E	✓
	B	X
	G	X
	Stack	

✓ leader
X not a leader

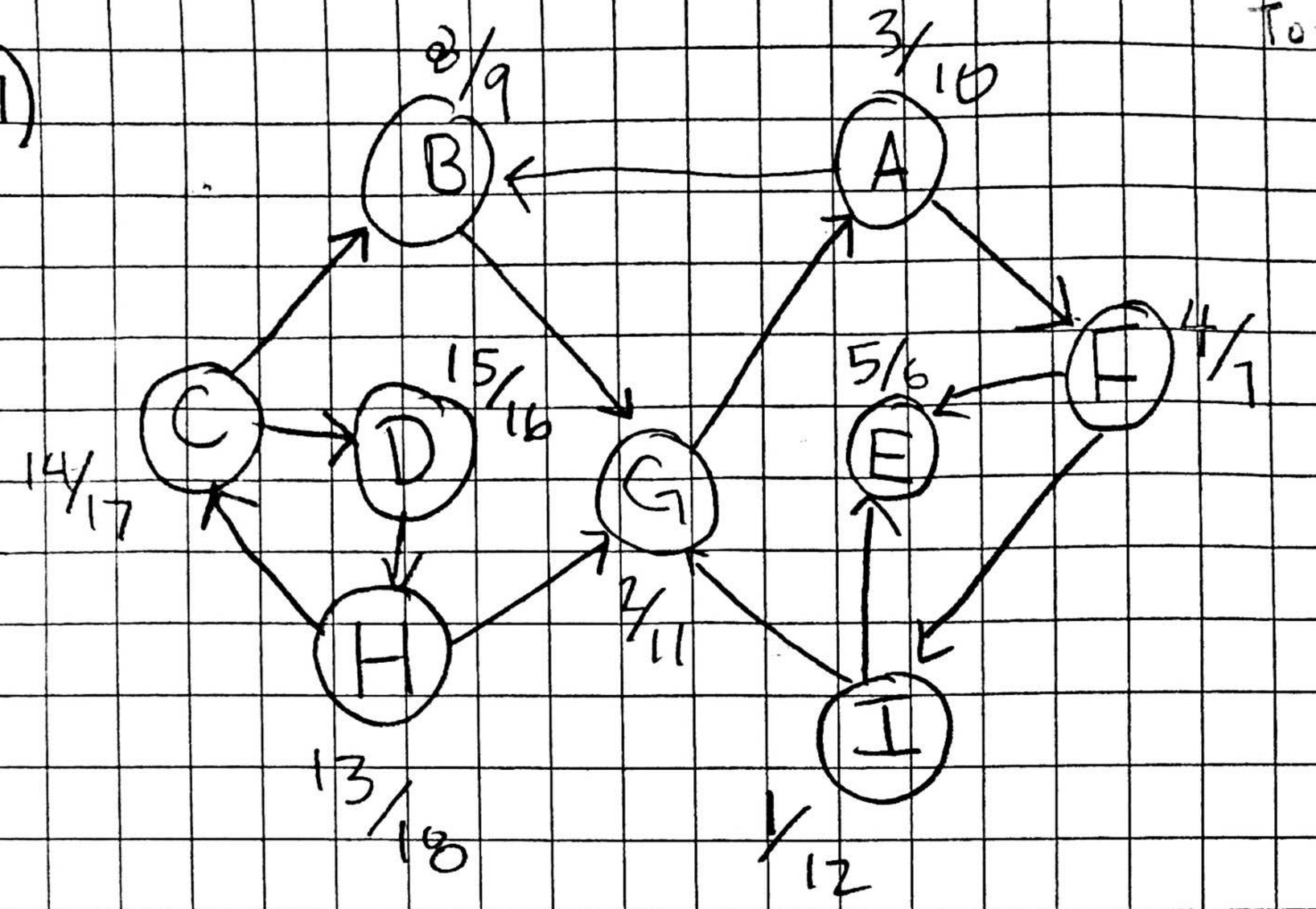
Graph transpose

E is its own leader. Separate from the A leader SCC

SCC leaders A, C, E.

Derrick DeBose

2.07)

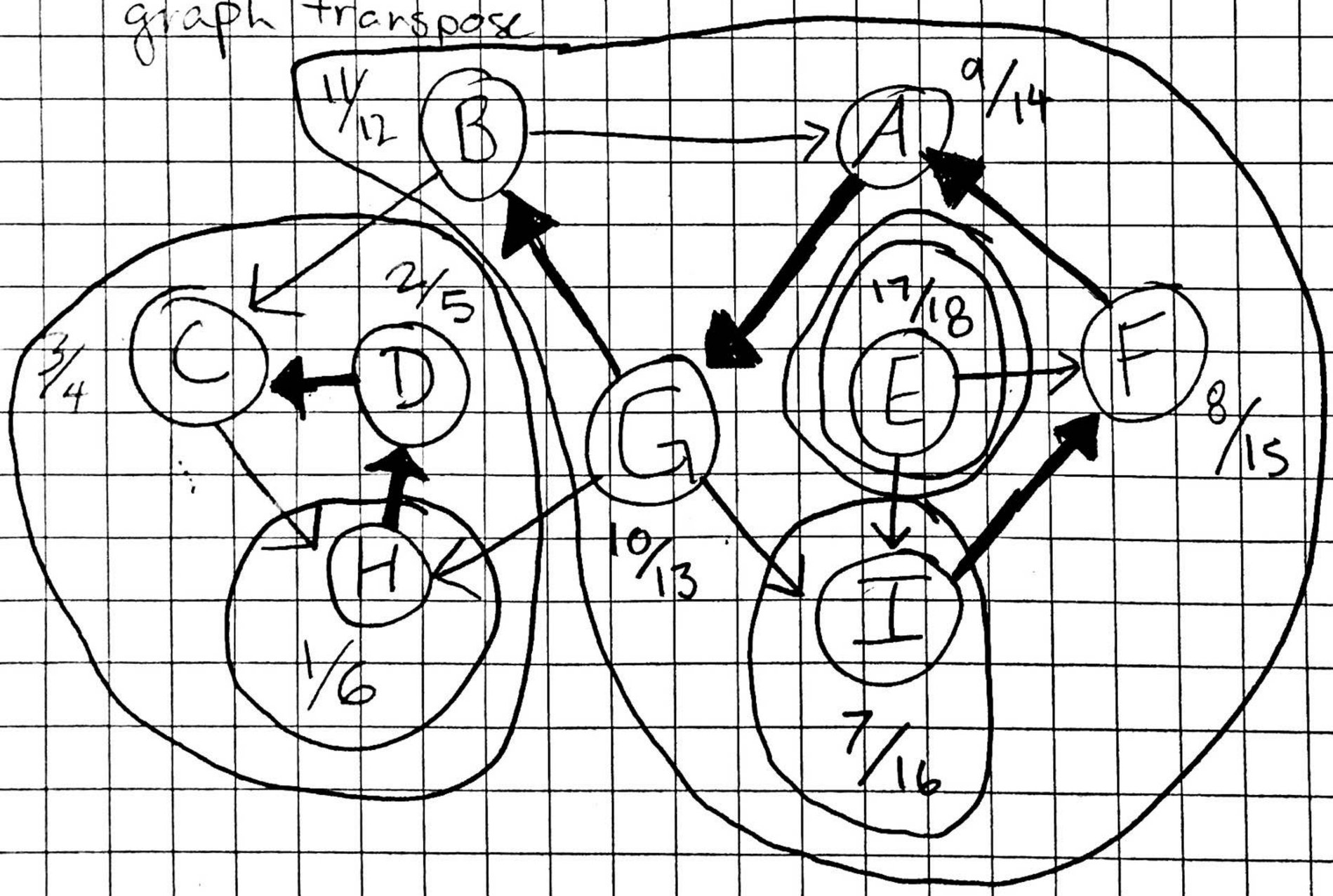


Top	
H	✓
C	X
D	X
I	✓
G	X
A	X
B	X
F	X
E	✓

Stack

✓ leader
X not a leader

graph transpose



E is its own leader
I is its own SCC
as the leader of vertices A, B, G, F

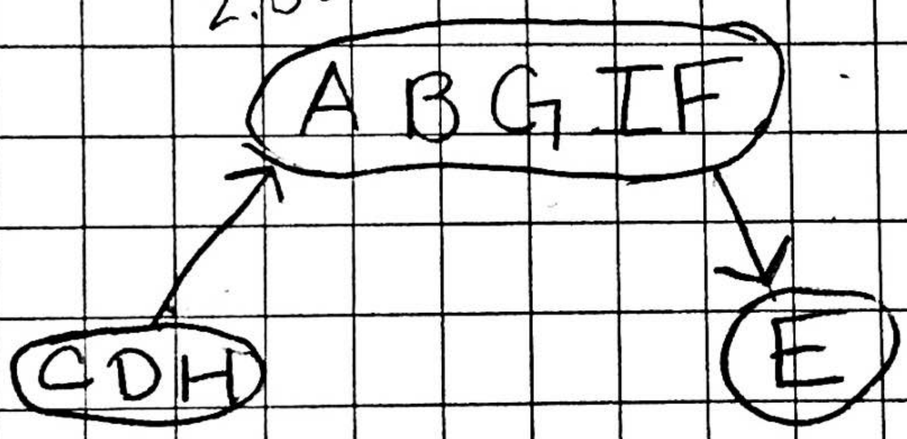
SCC leaders - H, I, E

Derrick DeBose

2.08) In ABC order, the SCC leaders are A, C, E. In ZYX order the SCC leaders are H, I, E. The leaders are different because the graphs were traversed differently so the stack was assembled different. But the leaders contain the same SCCs. The leaders are found by doing DFS on graph G and vertex v is pushed on the stack when v is finished being discovered. Then on graph G^T you do a second DFS but the graph is traversed using the stack. DFS is called when a white node is found at the top of the stack. Any white node found when popping off the stack is an SCC leader.

2.09)

2.06



2.07

The graphs are the same for ABC & ZYX order. There is only one condensation graph because traversing the graph differently doesn't change the SCCs. The leaders are the only thing that is different.

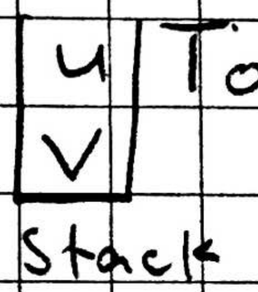
Derrick DeBose

2.10) Prove: SCC has the same leader in the first DFS (on G) and the second DFS (on G^T)

Suppose there is a white path from $u \rightarrow v$ such that v is a descendent of u. This means

discoveryTime[u] < discoveryTime[v] < finishingTime[v] < finishingTime[u]

So when 1st DFS is called on G then u must be the leader of v. v finishes before u, so v is put on the stack before u.



The 2nd DFS is called on G^T and the top of the stack is popped. If the popped vertex is a white/undiscovered vertex then DFS is called on that vertex. This process is done until the stack is empty.

Since u will be popped off the stack before v, u will <u>still</u> be the leader in the 2nd DFS.