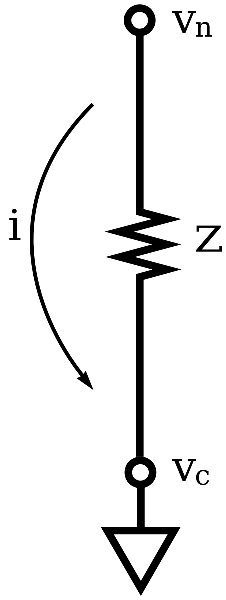
Deriving the S_{max} Algorithm for Non-unity Power Factors



Overview

The maximum feed-in for a photovoltaic system (PV) to be installed at a customer location is desired. Each location is calculated separately, assuming no other PV systems are connected. The simple calculation for feed-in for unity power factor is not applicable when the power factor is a constant but non-unity value.

Given

At the location under test, the supplied PV power is:

$$S = S_{max}\cos\phi + S_{max}\sin\phi\, j = v_c \times i^* \tag{1}$$

where S_{max} is the apparent power, ϕ is the power factor angle $pf = \cos(\phi)$ and i^* is the complex conjugate of the current.

The limiting voltage is normally a maximum of 3% over the nominal, a factor we can call *k*. *The SWING* voltage is assumed to be all real, i.e. $v_n = v_{re} + 0j$. Then:

$$|v_c| \leqslant (1 + threshold)\, v_n = k v_n$$

This complex voltage can be expressed as a function of the unknown voltage phasor (a function of angle θ) at the node under test:

$$v_c = k v_n(\cos(\theta) + \sin(\theta)\, j)$$

Derivation

The current is calculated as the voltage difference divided by the complex impedance:

$$i = \frac{v_\Delta}{Z} = \frac{v_c - v_n}{Z} = \frac{k v_n(\cos\theta + \sin\theta\, j) - v_n}{Z} = v_n((k\cos\theta - 1) + k\sin\theta\, j) \times (g + bj)$$

by using admittance:

$$\frac{1}{Z} = \frac{1}{r + xj} = \frac{r}{r^2 + x^2} - \frac{x}{r^2 + x^2}\, j = \frac{r}{|Z|^2} - \frac{x}{|Z|^2}\, j = g + bj = Y$$

Expanding:

$$i = v_n[(k\cos\theta g - k\sin\theta b - g) + (k\cos\theta b - k\sin\theta g)\, j]$$

Then

$$\begin{aligned} S = v_c \times i^* &= k v_n(\cos(\theta) + \sin(\theta)\, j) \times v_n[(k\cos\theta g - k\sin\theta b - g) - (k\cos\theta b - k\sin\theta g)\, j] \\ &= k v_n^2[(kg - \cos\theta g - \sin\theta b) + (-kb + \cos\theta b - \sin\theta g)\, j] \end{aligned} \tag{2}$$

Equating real and imaginary parts of (1) and (2):

$$S = S_{\Re} + S_{\Im}\, j = S_{max}\cos\phi + S_{max}\sin\phi\, j = kv_n^2[(kg - \cos\theta g - \sin\theta b) + (-kb + \cos\theta b - \sin\theta g)\, j]$$
$$S_{max}\cos\phi = kv_n^2(kg - \cos\theta g - \sin\theta b)$$
$$S_{max}\sin\phi = kv_n^2(-kb + \cos\theta b - \sin\theta g)$$
$$S_{max} = kv_n^2\frac{(kg - \cos\theta g - \sin\theta b)}{\cos\phi}$$
$$S_{max} = kv_n^2\frac{(-kb + \cos\theta b - \sin\theta g)}{\sin\phi}$$
$$equating:$$
$$kv_n^2\frac{(kg - \cos\theta g - \sin\theta b)}{\cos\phi} = kv_n^2\frac{(-kb + \cos\theta b - \sin\theta g)}{\sin\phi}$$
$$\frac{(kg - \cos\theta g - \sin\theta b)}{\cos\phi} = \frac{(-kb + \cos\theta b - \sin\theta g)}{\sin\phi}$$
$$\tan\phi = \frac{(-kb + \cos\theta b - \sin\theta g)}{(kg - \cos\theta g - \sin\theta b)} \tag{3}$$
$$\phi = \arctan[\frac{(-kb + \cos\theta b - \sin\theta g)}{(kg - \cos\theta g - \sin\theta b)}]$$

$as\, terms\, of\, S$

$$\widetilde{S_{\Re}} = \frac{S_{\Re}}{kv_n^{\;2}} = (-kb + \cos\theta b - \sin\theta g)$$
$$\widetilde{S_{\Im}} = \frac{S_{\Im}}{kv_n^{\;2}} = (kg - \cos\theta g - \sin\theta b)$$
$$\phi = \arctan(\frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}})$$

Solving

Solving the final non-linear equation in (3) for the unknown θ, has no obvious closed form solution. Instead we perform a non-linear optimization to find the voltage phasor angle θ, that minimizes, the difference between the left and right hand sides:

$$f(\theta) = (\arctan(\frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}}) - \phi)^2 = (\arctan[u(\theta)] - \phi)^2 = (\arctan[\frac{(-kb + \cos\theta b - \sin\theta g)}{(kg - \cos\theta g - \sin\theta b)}] - \phi)^2 \tag{4}$$

We choose to square the difference to avoid problems with differentiability of an absolute value function.

Partial Derivatives

To perform the optimization using Newton's method, we require the first and second partial derivatives of the function. So, using:

$$\frac{\partial\, atan(a)}{\partial x} = \frac{1}{1+a^2}\cdot\frac{\partial a}{\partial x}$$
$$\frac{\partial\frac{a}{b}}{\partial x} = \frac{\frac{\partial a}{\partial x}b - a\frac{\partial b}{\partial x}}{b^2} = \frac{1}{b}\frac{\partial a}{\partial x} - \frac{a}{b^2}\frac{\partial b}{\partial x}$$

the derivative is:

$$\frac{\partial f(\theta)}{\partial\theta} = 2(\arctan[u(\theta)] - \phi)\cdot\frac{\partial\arctan[u(\theta)]}{\partial\theta} = 2(\arctan[u(\theta)] - \phi)\cdot\frac{1}{1+u(\theta)^2}\cdot\frac{\partial u(\theta)}{\partial\theta}$$

where

$$\frac{\partial u(\theta)}{\partial \theta} = \frac{\partial \frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}}}{\partial \theta} = \frac{1}{\widetilde{S_{\Re}}} \frac{\partial \widetilde{S_{\Im}}}{\partial \theta} - \frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}^2} \frac{\partial \widetilde{S_{\Re}}}{\partial \theta} =$$

$$\frac{1}{\widetilde{S_{\Re}}}(-\sin\theta b - \cos\theta g) - \frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}^2}(\sin\theta g - \cos\theta b)$$

Finally

$$\frac{\partial f(\theta)}{\partial \theta} = 2\left(atan\left[\frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}}\right] - \phi\right) \cdot \frac{1}{1 + \left(\frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}}\right)^2} \cdot (-\sin\theta b - \cos\theta g) - \frac{\widetilde{S_{\Im}}}{\widetilde{S_{\Re}}^2}(\sin\theta g - \cos\theta b)$$