Spectral Ratio of a Structure & Estimation of Shear-wave Veloctiy of a Structure

Deniz Ertuncay

The National Institute of Oceanography and Applied Geophysics (OGS)
Center for Seismological Research (CRS)

dertuncay@ogs.it

03/11/2024

Spectral Ratio

- A structure can be modelled as a single-degree-of-freedom (SDOF) or multi-degree-of-freedom (MDOF) system. For damped free vibrations, the equation of motion is written as;

$$m\ddot{u} + c\dot{u} + ku = 0 \tag{1}$$

for mass, m, damping, c, stiffness, k, and ground motion u. In the case of a system with damping (ζ),

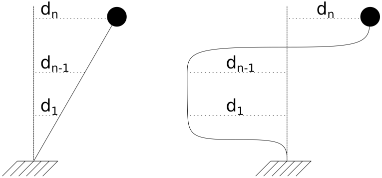
$$\ddot{u} + 2\zeta\omega_0\dot{u} + \omega_0^2 u = 0 \tag{2}$$

- In the case of the MDOF system, Eq. 6 can be written in a matrix form

$$\boldsymbol{m}\ddot{\boldsymbol{u}} + \boldsymbol{c}\dot{\boldsymbol{u}} + \boldsymbol{k}\boldsymbol{u} = 0 \tag{3}$$

- After some clever math and a bit of black magic, the response of a building under load can be estimated.

Spectral Ratio



Mode 1
$f_1 = 1.00$ Hz
$d_n > d_{n-1} > d_1$

Mode 2
$f_2 = 2.76$ Hz
$d_n < d_{n-1} = d_1$

Figure: Arbitrary modes of MDOF system for first 2 modes.

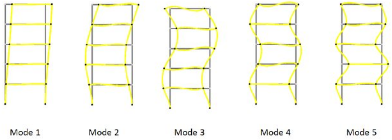


Figure: Demonstration of various modes of a structure

Spectral Ratio

- If the seismic load can be defined as a Fourier series with the complex coefficients q_n^*, then the response of the SDOF system loaded by the nth harmonic would be written as low

$$m\ddot{u}_n(t) + (c\dot{u}_n(t) + ku_n(t)) = q_n^* e^{i\omega_n t} \tag{4}$$

The response of the system can be related to the loading by

$$u_n(t) = H(\omega_n) q_n^* e^{i\omega_n t} \tag{5}$$

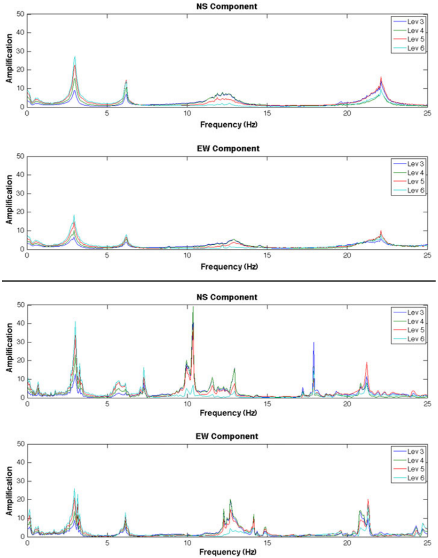
- Here $H(\omega_n)$ is the transfer function. It can be viewed as a filter that acts upon an input (seismic) signal to produce an output signal.

Spectral Ratio

- For a known $H(\omega_n)$ and obtained Fourier series of a seismic load output motions of the known system at the location.
- It is a rather simple process of multiplication of Fourier series with the transfer function in each frequency.
- This approach can be used for ground response, soil-structure analysis, and building response studies.

Excitation of Modes

Figure: Falkenhof Tower, Potsdam, Germany



Excitation of Modes

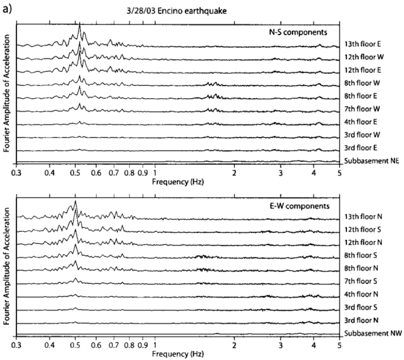


Figure: Encino, California $M_L = 2.9$ ($R_{epi} = 9km$)

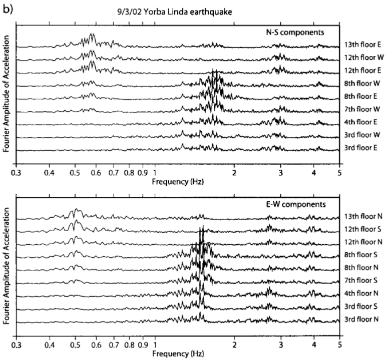


Figure: Yorba Linda, California $M_L = 4.7$ ($R_{epi} = 64km$)

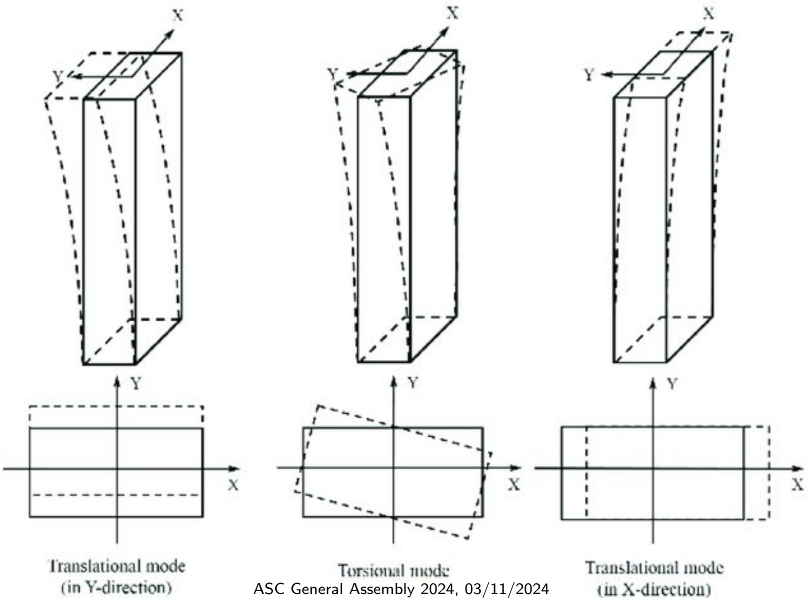
Excitation of Modes

Figure: UCLA Doris and Louis Factor building, CA, USA

Earthquake	First horizontal mode (Hz)	First torsional mode (Hz)	Second horizontal mode (Hz)	Third horizontal mode (Hz)
Encino N-S	0.52	~0.7	1.50–1.80	2.70–3.05
Encino E-W	0.50	~0.7	1.40–1.65	2.45–2.80
Yorba Linda N-S	0.57	~0.7	1.60–1.75	2.75–3.10
Yorba Linda E-W	0.51	~0.7	1.45–1.55	2.65–2.85

Ambient vibration recording	First horizontal mode (Hz)	First torsional mode (Hz)	Second horizontal mode (Hz)	Third horizontal mode (Hz)
16 November 2002 N-S	0.58	-	1.70–1.95	2.95–3.25
16 November 2002 E-W	0.54	-	1.50–1.70	2.70–3.00
29 March 2003 N-S	0.58	0.80	1.70–1.95	2.95–3.15
29 March 2003 E-W	0.54	0.80	1.50–1.70	2.75–2.90
28 May 2003 N-S	0.58	0.81	1.70–1.90	3.00–3.20
28 May 2003 E-W	0.55	0.81	1.60–1.75	2.75–2.95
4 June 2003 N-S	0.60	0.80	1.70–1.95	3.00–3.20
4 June 2003 E-W	0.56	0.80	1.55–1.75	2.80–3.00
22 July 2003 N-S	0.59	0.80	1.75–1.90	3.05–3.20
22 July 2003 E-W	0.55	0.80	1.60–1.80	2.80–3.00

Building modes



Frequency Domain Decomposition Method (FDD)

- Assume the standard input $X(t)$ - output $Y(t)$ relationship, based on the Fourier Transform: Fourier Transform:;

$$Y(\omega) = H(\omega)X(\omega) \tag{6}$$

Assuming that this is a stationary process i.e. properties do not depend on the time at which the series is observed.

$$G_{yy}(\omega) = H^{*}(\omega)G_{xx}(\omega)H^{T}(\omega) \tag{7}$$

- where G_{xx} is the $(r \times r)$ power spectral density (PSD) matrix of the input, r is the number of inputs, G_{yy} is the $(m \times m)$ PSD matrix of the responses, m is the number of responses, H_w I the $(m \times r)$ frequency response function matrix. Superscript, T is the transpose, and $*$ is the complex conjugate.
- The method can be used when the PSD input constant, i.e. input is like a white noise such as ambient noise).

Frequency Domain Decomposition Method (FDD)

Steps to calculate FDD:

1. Estimate PSD matrix, $G_{yy}(\omega)$
2. Decompose $G_{yy}(\omega)$ by taking the Singular Value Decomposition (SVD) of the matrix.
3. For each discrete frequency ω_i:

$$G_{yy}(\omega_i) = U_i \Sigma_i U_i^H$$

4. Near a peak corresponding to the k-th mode in the spectrum, that mode will dominate. In this case, the first singular vector u_{i1} estimates the mode shape and corresponding singular values the auto-PSD function of the corresponding SDOF system.

- From the piece of the SDOF density function obtained around the peak of the PSD, the natural frequency and the damping can be obtained. Today we only focus on obtaining the natural frequency.

Basics and properties of SVD

Suppose **A** is a $m \times n$ matrix with complex or real values

$$\boldsymbol{A} = \boldsymbol{U \Sigma V^H}$$

where, $\boldsymbol{U}$ is a $m \times n$ unitary matrix, $\boldsymbol{UU^H} = \boldsymbol{I_m}$, $\boldsymbol{\Sigma}$ is a $m \times n$ diagonal matrix with $\Sigma_{ii} \geq 0$, and $\boldsymbol{V^H}$ is a $n \times n$ unitary matrix. Σ_{ii} are called the singular values of **A**.

$$\boldsymbol{A^H A} = (\boldsymbol{V \Sigma^H U^H})(\boldsymbol{U \Sigma V^H}) = \boldsymbol{V}(\boldsymbol{\Sigma^H \Sigma})\boldsymbol{V^H} \tag{8}$$

$$\boldsymbol{AA^H} = \boldsymbol{U \Sigma V^H V \Sigma^H U^H} = \boldsymbol{U}(\boldsymbol{\Sigma \Sigma^H})\boldsymbol{U^H} \tag{9}$$

The columns of **V** are eigenvectors of $\boldsymbol{A^H A}$. So, for each discrete frequency ω_i:

$$\boldsymbol{G_{yy}}(\boldsymbol{\omega_i}) = \boldsymbol{U_i \Sigma_i U_i^H}$$

where $\boldsymbol{U_i} = [\boldsymbol{U_{i1}}, \ldots, \boldsymbol{U_{im}}]$

Deconvolution

- To fully understand the response of the building, one needs to unravel the properties of the building itself from the coupling of the building to the ground.
- Deconvolution gives a response that is independent of the excitation and that it does not depend on the coupling of the building with the ground.
- Deconvolved waves can be interpreted as propagating waves. By using the waves, one can determine the shear velocity of the structure.

Deconvolution

- The motion of a building depends on the excitation, the coupling of the building to the ground, and the mechanical properties of the building.
- Building's response can be separated from the excitation and the ground coupling by deconvolving the motion recorded at different levels in the building and apply this to recordings of the motion.
- Time delay of the pulse between sensors gives the pulse-wave velocity, considering an equivalent homogenous medium.

Deconvolution

The deconvolution of two signals $u_1(x)$ and $u_2(x)$ is in the frequency domain given by

$$D(\omega) = \frac{u_1(\omega)}{u_2(\omega)} \tag{10}$$

which would be very unstable where u_2 has near zero values. A stabilizer (ϵ) is added to Eq. 10 to stabilise the deconvolution.

$$D(\omega) = \frac{u_1(\omega)u_2^*(\omega)}{\left|u_2(\omega)\right|^2 + \epsilon} \tag{11}$$

$*$ is complex conjugate and when $\epsilon = 0$ Eq. 11 is identical to Eq. 10.

Deconvolution

The deconvolution by the top sensor provides a clear up-going and down-going pulse wave to assess time delays of the pulse travel time (Δt) by picking the time of the maxima of the deconvolved traces in time. Pulse-wave velocity (β) is calculated as follows:

$$\beta = \frac{H}{\Delta t} \tag{12}$$

with H the inter-station distance

Deconvolution

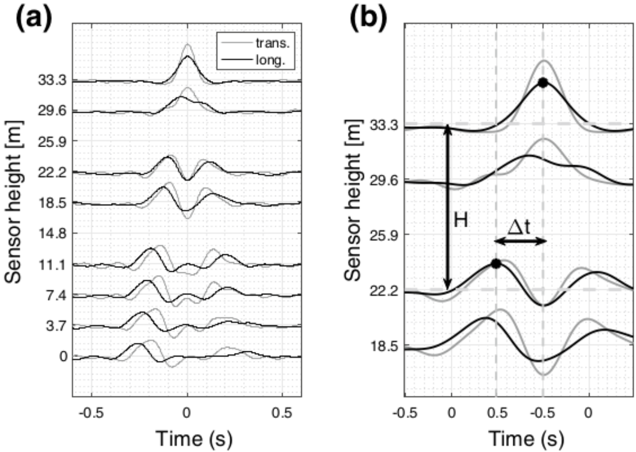
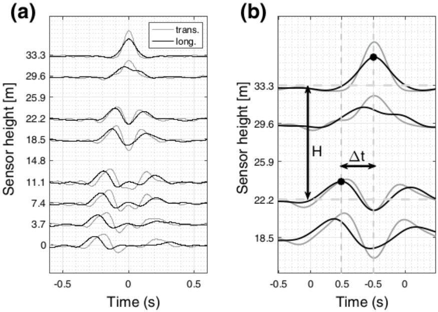


Figure: An example from acceleration time-history considering Cook Strait earthquake (M 6.5) recorded at Te Puni Apartments in New Zealand.

Deconvolution



- Deconvolved waves are acausal and consist of the superposition of one upgoing wave and one downgoing wave.
- Acausal upgoing wave and a causal downgoing wave; both waves are visible
- The deconvolved waveforms are computed from the full waveforms. It is, however, not necessary to use the full waveforms. Today, we will also use the whole waveform.

References:

- Kramer, Steven L. Geotechnical earthquake engineering. Vol. 653. Prentice Hall, 1996.
- Ditommaso, R., Parolai, S., Mucciarelli, M. et al. Monitoring the response and the back-radiated energy of a building subjected to ambient vibration and impulsive action: the Falkenhof Tower (Potsdam, Germany). Bull Earthquake Eng 8, 705–722 (2010)
- Roel Snieder, Erdal Safak; Extracting the Building Response Using Seismic Interferometry: Theory and Application to the Millikan Library in Pasadena, California. Bulletin of the Seismological Society of America 2006; 96 (2): 586–598.
- Skłodowska, A.M., Holden, C., Guéguen, P. et al. Structural change detection applying long-term seismic interferometry by deconvolution method to a modern civil engineering structure (New Zealand). Bull Earthquake Eng 19, 3551–3569 (2021).