4 - families of discrete distributions

bernoulli distribution

a random variable with two possible values $1 \to$ success $\Rightarrow P(1) = p$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $0 \to$ failures $\Rightarrow P(0) = 1-p = q$

☆ any experiment with a binary outcome is called bernoulli trial.

$E(x) = 1 \cdot p + 0 \cdot (1-p) = p \qquad E(x^2) = 1^2 \cdot p + 0^2(1-p) = p$

$V(x) = E(x^2) - E(x)^2 = p - p^2 = p(1-p) = p \cdot q$

☆ the distribution with $p = 0.5$ carries the highest level of uncertainty (Var is max)

$\quad\hookrightarrow$ when $p=0$ or $p=1$ the variance is zero

binomial distribution

number of successes in a sequence of independent bernoulli trials

n: number of trials

p: probability of success $\qquad P(x=x) = f_x(x) = \binom{n}{x} p^x q^{n-x}$

$\qquad\qquad\qquad\qquad\qquad\qquad E(x) = np$

$\qquad\qquad\qquad\qquad\qquad\qquad Var(x) = npq$

example: tossing a coin $\quad H \to 0.3 \brace T \to 0.7$ among 50 times

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ probability of 10 are H

$P(x=10) = \binom{50}{10} \cdot (0.3)^{10} \cdot (0.7)^{50} {}^{n-x}$

$\qquad\qquad\qquad\quad x \qquad p \qquad\quad q$

<u>using distribution tables</u>

from table we can compute as $P(x) = F(x) - F(x-1)$, because table shows the cumulative distribution

$\hookrightarrow$ same question: $P(10) = F(10) - F(9) \quad n=50 \quad p=0.3$
   using table

example: 20% are selected from all. among 10 people, the probability of at least 4 of them are selected

$\hookrightarrow P(x \geq 4) = ?$

geometric distribution

the number of bernoulli trials needed to get the first success

☆ the number of trials needed is not limited by a specific number
　↳ tossing a die → you can never get 3 $[1, 2, 3, \cdots \infty)$
☆ p = the probability of success　　x = # of trials

$$P(x) = (1-p)^{x-1} \cdot p = p q^{(x-1)}$$

☆ unbounded random variable

$$E(x) = \frac{1}{p}$$

☆ $\sum_x P(x) = \sum_{x=1}^{\infty} (1-p)^{x-1} p = \frac{(1-p)^0}{1-(1-p)^1} \cdot p =$

$$Var(x) = \frac{1-p}{p^2}$$

(from geometric)
series sum

negative binomial distribution

the number of bernoulli trials needed to get k successes

$P(x) = P\{$ the xth trial results in the k-th success $\}$

$$P(x) = \binom{x-1}{k-1} (1-p)^{x-k} p^k$$

☆ when $k=1$, it is geometric distribution

$$E(x) = \frac{k}{p}$$

$$Var(x) = \frac{k(1-p)}{p^2}$$

☆ it is like sum of independent geometric variables (generalized)
$$X = X_1 + X_2 + \cdots + X_k$$

poisson distribution

the number of rare events occuring within a fixed period of time
↳ it has a single parameter

$$P(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

λ = frequency,
average number of events

$$E(x) = \lambda$$

$$Var(x) = \lambda$$

poisson approximation of binomial distribution

when n is large and p is small: $n \geq 30$ $p \leq 0.05$

$np = \lambda$

Table A2. Binomial distribution

$$F(x) = P\{X \le x\} = \sum_{k=0}^{x} \binom{n}{k} p^k (1-p)^{n-k}$$

n	x	.050	.100	.150	.200	.250	.300	.350	.400	.450	.500	.550	.600	.650	.700	.750	.800	.850	.900	.950
1	0	.950	.900	.850	.800	.750	.700	.650	.600	.550	.500	.450	.400	.350	.300	.250	.200	.150	.100	.050
2	0	.903	.810	.723	.640	.563	.490	.423	.360	.303	.250	.203	.160	.123	.090	.063	.040	.023	.010	.003
	1	.998	.990	.978	.960	.938	.910	.878	.840	.798	.750	.698	.640	.578	.510	.438	.360	.278	.190	.098
3	0	.857	.729	.614	.512	.422	.343	.275	.216	.166	.125	.091	.064	.043	.027	.016	.008	.003	.001	.000
	1	.993	.972	.939	.896	.844	.784	.718	.648	.575	.500	.425	.352	.282	.216	.156	.104	.061	.028	.007
	2	1.0	.999	.997	.992	.984	.973	.957	.936	.909	.875	.834	.784	.725	.657	.578	.488	.386	.271	.143
4	0	.815	.656	.522	.410	.316	.240	.179	.130	.092	.063	.041	.026	.015	.008	.004	.002	.001	.000	.000
	1	.986	.948	.890	.819	.738	.652	.563	.475	.391	.313	.241	.179	.126	.084	.051	.027	.012	.004	.000
	2	1.0	.996	.988	.973	.949	.916	.874	.821	.759	.688	.609	.525	.437	.348	.262	.181	.110	.052	.014
	3	1.0	1.0	.999	.998	.996	.992	.985	.974	.959	.938	.908	.870	.821	.760	.684	.590	.478	.344	.185
5	0	.774	.590	.444	.328	.237	.168	.116	.078	.050	.031	.018	.010	.005	.002	.001	.000	.000	.000	.000
	1	.977	.919	.835	.737	.633	.528	.428	.337	.256	.188	.131	.087	.054	.031	.016	.007	.002	.000	.000
	2	.999	.991	.973	.942	.896	.837	.765	.683	.593	.500	.407	.317	.235	.163	.104	.058	.027	.009	.001
	3	1.0	1.0	.998	.993	.984	.969	.946	.913	.869	.813	.744	.663	.572	.472	.367	.263	.165	.081	.023
	4	1.0	1.0	1.0	1.0	.999	.998	.995	.990	.982	.969	.950	.922	.884	.832	.763	.672	.556	.410	.226
6	0	.735	.531	.377	.262	.178	.118	.075	.047	.028	.016	.008	.004	.002	.001	.000	.000	.000	.000	.000
	1	.967	.886	.776	.655	.534	.420	.319	.233	.164	.109	.069	.041	.022	.011	.005	.002	.000	.000	.000
	2	.998	.984	.953	.901	.831	.744	.647	.544	.442	.344	.255	.179	.117	.070	.038	.017	.006	.001	.000
	3	1.0	.999	.994	.983	.962	.930	.883	.821	.745	.656	.558	.456	.353	.256	.169	.099	.047	.016	.002
	4	1.0	1.0	1.0	.998	.995	.989	.978	.959	.931	.891	.836	.767	.681	.580	.466	.345	.224	.114	.033
	5	1.0	1.0	1.0	1.0	1.0	.999	.998	.996	.992	.984	.972	.953	.925	.882	.822	.738	.623	.469	.265
7	0	.698	.478	.321	.210	.133	.082	.049	.028	.015	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000
	1	.956	.850	.717	.577	.445	.329	.234	.159	.102	.063	.036	.019	.009	.004	.001	.000	.000	.000	.000
	2	.996	.974	.926	.852	.756	.647	.532	.420	.316	.227	.153	.096	.056	.029	.013	.005	.001	.000	.000
	3	1.0	.997	.988	.967	.929	.874	.800	.710	.608	.500	.392	.290	.200	.126	.071	.033	.012	.003	.000
	4	1.0	1.0	.999	.995	.987	.971	.944	.904	.847	.773	.684	.580	.468	.353	.244	.148	.074	.026	.004
	5	1.0	1.0	1.0	1.0	.999	.996	.991	.981	.964	.938	.898	.841	.766	.671	.555	.423	.283	.150	.044
	6	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.996	.992	.985	.972	.951	.918	.867	.790	.679	.522	.302
8	0	.663	.430	.272	.168	.100	.058	.032	.017	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000	.000
	1	.943	.813	.657	.503	.367	.255	.169	.106	.063	.035	.018	.009	.004	.001	.000	.000	.000	.000	.000
	2	.994	.962	.895	.797	.679	.552	.428	.315	.220	.145	.088	.050	.025	.011	.004	.001	.000	.000	.000
	3	1.0	.995	.979	.944	.886	.806	.706	.594	.477	.363	.260	.174	.106	.058	.027	.010	.003	.000	.000
	4	1.0	1.0	.997	.990	.973	.942	.894	.826	.740	.637	.523	.406	.294	.194	.114	.056	.021	.005	.000
	5	1.0	1.0	1.0	.999	.996	.989	.975	.950	.912	.855	.780	.685	.572	.448	.321	.203	.105	.038	.006
	6	1.0	1.0	1.0	1.0	1.0	.999	.996	.991	.982	.965	.937	.894	.831	.745	.633	.497	.343	.187	.057
	7	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.996	.992	.983	.968	.942	.900	.832	.728	.570	.337
9	0	.630	.387	.232	.134	.075	.040	.021	.010	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000
	1	.929	.775	.599	.436	.300	.196	.121	.071	.039	.020	.009	.004	.001	.000	.000	.000	.000	.000	.000
	2	.992	.947	.859	.738	.601	.463	.337	.232	.150	.090	.050	.025	.011	.004	.001	.000	.000	.000	.000
	3	.999	.992	.966	.914	.834	.730	.609	.483	.361	.254	.166	.099	.054	.025	.010	.003	.001	.000	.000
	4	1.0	.999	.994	.980	.951	.901	.828	.733	.621	.500	.379	.267	.172	.099	.049	.020	.006	.001	.000
	5	1.0	1.0	.999	.997	.990	.975	.946	.901	.834	.746	.639	.517	.391	.270	.166	.086	.034	.008	.001
	6	1.0	1.0	1.0	1.0	.999	.996	.989	.975	.950	.910	.850	.768	.663	.537	.399	.262	.141	.053	.008
	7	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.991	.980	.961	.929	.879	.804	.700	.564	.401	.225	.071
	8	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.995	.990	.979	.960	.925	.866	.768	.613	.370
10	0	.599	.349	.197	.107	.056	.028	.013	.006	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.914	.736	.544	.376	.244	.149	.086	.046	.023	.011	.005	.002	.001	.000	.000	.000	.000	.000	.000
	2	.988	.930	.820	.678	.526	.383	.262	.167	.100	.055	.027	.012	.005	.002	.000	.000	.000	.000	.000
	3	.999	.987	.950	.879	.776	.650	.514	.382	.266	.172	.102	.055	.026	.011	.004	.001	.000	.000	.000
	4	1.0	.998	.990	.967	.922	.850	.751	.633	.504	.377	.262	.166	.095	.047	.020	.006	.001	.000	.000
	5	1.0	1.0	.999	.994	.980	.953	.905	.834	.738	.623	.496	.367	.249	.150	.078	.033	.010	.002	.000
	6	1.0	1.0	1.0	.999	.996	.989	.974	.945	.898	.828	.734	.618	.486	.350	.224	.121	.050	.013	.001
	7	1.0	1.0	1.0	1.0	1.0	.998	.995	.988	.973	.945	.900	.833	.738	.617	.474	.322	.180	.070	.012
	8	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.995	.989	.977	.954	.914	.851	.756	.624	.456	.264	.086
	9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.994	.987	.972	.944	.893	.803	.651	.401

n	x	.050	.100	.150	.200	.250	.300	.350	.400	.450	.500	.550	.600	.650	.700	.750	.800	.850	.900	.950
																			p	
11	0	.569	.314	.167	.086	.042	.020	.009	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.898	.697	.492	.322	.197	.113	.061	.030	.014	.006	.002	.001	.000	.000	.000	.000	.000	.000	.000
	2	.985	.910	.779	.617	.455	.313	.200	.119	.065	.033	.015	.006	.002	.001	.000	.000	.000	.000	.000
	3	.998	.981	.931	.839	.713	.570	.426	.296	.191	.113	.061	.029	.012	.004	.001	.000	.000	.000	.000
	4	1.0	.997	.984	.950	.885	.790	.668	.533	.397	.274	.174	.099	.050	.022	.008	.002	.000	.000	.000
	5	1.0	1.0	.997	.988	.966	.922	.851	.753	.633	.500	.367	.247	.149	.078	.034	.012	.003	.000	.000
	6	1.0	1.0	1.0	.998	.992	.978	.950	.901	.826	.726	.603	.467	.332	.210	.115	.050	.016	.003	.000
	7	1.0	1.0	1.0	1.0	.999	.996	.988	.971	.939	.887	.809	.704	.574	.430	.287	.161	.069	.019	.002
	8	1.0	1.0	1.0	1.0	1.0	.999	.998	.994	.985	.967	.935	.881	.800	.687	.545	.383	.221	.090	.015
	9	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.994	.986	.970	.939	.887	.803	.678	.508	.303	.102
	10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.991	.980	.958	.914	.833	.686	.431
12	0	.540	.282	.142	.069	.032	.014	.006	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.882	.659	.443	.275	.158	.085	.042	.020	.008	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000
	2	.980	.889	.736	.558	.391	.253	.151	.083	.042	.019	.008	.003	.001	.000	.000	.000	.000	.000	.000
	3	.998	.974	.908	.795	.649	.493	.347	.225	.134	.073	.036	.015	.006	.002	.000	.000	.000	.000	.000
	4	1.0	.996	.976	.927	.842	.724	.583	.438	.304	.194	.112	.057	.026	.009	.003	.001	.000	.000	.000
	5	1.0	.999	.995	.981	.946	.882	.787	.665	.527	.387	.261	.158	.085	.039	.014	.004	.001	.000	.000
	6	1.0	1.0	.999	.996	.986	.961	.915	.842	.739	.613	.473	.335	.213	.118	.054	.019	.005	.001	.000
	7	1.0	1.0	1.0	.999	.997	.991	.974	.943	.888	.806	.696	.562	.417	.276	.158	.073	.024	.004	.000
	8	1.0	1.0	1.0	1.0	1.0	.998	.994	.985	.964	.927	.866	.775	.653	.507	.351	.205	.092	.026	.002
	9	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.992	.981	.958	.917	.849	.747	.609	.442	.264	.111	.020
	10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.992	.980	.958	.915	.842	.725	.557	.341	.118
	11	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.994	.986	.968	.931	.858	.718	.460
13	0	.513	.254	.121	.055	.024	.010	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.865	.621	.398	.234	.127	.064	.030	.013	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000
	2	.975	.866	.692	.502	.333	.202	.113	.058	.027	.011	.004	.001	.000	.000	.000	.000	.000	.000	.000
	3	.997	.966	.882	.747	.584	.421	.278	.169	.093	.046	.020	.008	.003	.001	.000	.000	.000	.000	.000
	4	1.0	.994	.966	.901	.794	.654	.501	.353	.228	.133	.070	.032	.013	.004	.001	.000	.000	.000	.000
	5	1.0	.999	.992	.970	.920	.835	.716	.574	.427	.291	.179	.098	.046	.018	.006	.001	.000	.000	.000
	6	1.0	1.0	.999	.993	.976	.938	.871	.771	.644	.500	.356	.229	.129	.062	.024	.007	.001	.000	.000
	7	1.0	1.0	1.0	.999	.994	.982	.954	.902	.821	.709	.573	.426	.284	.165	.080	.030	.008	.001	.000
	8	1.0	1.0	1.0	1.0	.999	.996	.987	.968	.930	.867	.772	.647	.499	.346	.206	.099	.034	.006	.000
	9	1.0	1.0	1.0	1.0	1.0	.999	.997	.992	.980	.954	.907	.831	.722	.579	.416	.253	.118	.034	.003
	10	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.989	.973	.942	.887	.798	.667	.498	.308	.134	.025
	11	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.995	.987	.970	.936	.873	.766	.602	.379	.135
	12	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.990	.976	.945	.879	.746	.487
14	0	.488	.229	.103	.044	.018	.007	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.847	.585	.357	.198	.101	.047	.021	.008	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.970	.842	.648	.448	.281	.161	.084	.040	.017	.006	.002	.001	.000	.000	.000	.000	.000	.000	.000
	3	.996	.956	.853	.698	.521	.355	.220	.124	.063	.029	.011	.004	.001	.000	.000	.000	.000	.000	.000
	4	1.0	.991	.953	.870	.742	.584	.423	.279	.167	.090	.043	.018	.006	.002	.000	.000	.000	.000	.000
	5	1.0	.999	.988	.956	.888	.781	.641	.486	.337	.212	.119	.058	.024	.008	.002	.000	.000	.000	.000
	6	1.0	1.0	.998	.988	.962	.907	.816	.692	.546	.395	.259	.150	.075	.031	.010	.002	.000	.000	.000
	7	1.0	1.0	1.0	.998	.990	.969	.925	.850	.741	.605	.454	.308	.184	.093	.038	.012	.002	.000	.000
	8	1.0	1.0	1.0	1.0	.998	.992	.976	.942	.881	.788	.663	.514	.359	.219	.112	.044	.012	.001	.000
	9	1.0	1.0	1.0	1.0	1.0	.998	.994	.982	.957	.910	.833	.721	.577	.416	.258	.130	.047	.009	.000
	10	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.989	.971	.937	.876	.780	.645	.479	.302	.147	.044	.004
	11	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.994	.983	.960	.916	.839	.719	.552	.352	.158	.030
	12	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.992	.979	.953	.899	.802	.643	.415	.153
	13	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.993	.982	.956	.897	.771	.512
15	0	.463	.206	.087	.035	.013	.005	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.829	.549	.319	.167	.080	.035	.014	.005	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.964	.816	.604	.398	.236	.127	.062	.027	.011	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000
	3	.995	.944	.823	.648	.461	.297	.173	.091	.042	.018	.006	.002	.000	.000	.000	.000	.000	.000	.000
	4	.999	.987	.938	.836	.686	.515	.352	.217	.120	.059	.025	.009	.003	.001	.000	.000	.000	.000	.000
	5	1.0	.998	.983	.939	.852	.722	.564	.403	.261	.151	.077	.034	.012	.004	.001	.000	.000	.000	.000
	6	1.0	1.0	.996	.982	.943	.869	.755	.610	.452	.304	.182	.095	.042	.015	.004	.001	.000	.000	.000
	7	1.0	1.0	.999	.996	.983	.950	.887	.787	.654	.500	.346	.213	.113	.050	.017	.004	.001	.000	.000
	8	1.0	1.0	1.0	.999	.996	.985	.958	.905	.818	.696	.548	.390	.245	.131	.057	.018	.004	.000	.000
	9	1.0	1.0	1.0	1.0	.999	.996	.988	.966	.923	.849	.739	.597	.436	.278	.148	.061	.017	.002	.000
	10	1.0	1.0	1.0	1.0	1.0	.999	.997	.991	.975	.941	.880	.783	.648	.485	.314	.164	.062	.013	.001
	11	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.982	.958	.909	.827	.703	.539	.352	.177	.056	.005
	12	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.989	.973	.938	.873	.764	.602	.396	.184	.036
	13	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.995	.986	.965	.920	.833	.681	.451	.171
	14	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.995	.987	.965	.913	.794	.537

n	x	.050	.100	.150	.200	.250	.300	.350	.400	.450	.500	.550	.600	.650	.700	.750	.800	.850	.900	.950
16	1	.811	.515	.284	.141	.063	.026	.010	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.957	.789	.561	.352	.197	.099	.045	.018	.007	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000
	3	.993	.932	.790	.598	.405	.246	.134	.065	.028	.011	.003	.001	.000	.000	.000	.000	.000	.000	.000
	4	.999	.983	.921	.798	.630	.450	.289	.167	.085	.038	.015	.005	.001	.000	.000	.000	.000	.000	.000
	5	1.0	.997	.976	.918	.810	.660	.490	.329	.198	.105	.049	.019	.006	.002	.000	.000	.000	.000	.000
	6	1.0	.999	.994	.973	.920	.825	.688	.527	.366	.227	.124	.058	.023	.007	.002	.000	.000	.000	.000
	7	1.0	1.0	.999	.993	.973	.926	.841	.716	.563	.402	.256	.142	.067	.026	.007	.001	.000	.000	.000
	8	1.0	1.0	1.0	.999	.993	.974	.933	.858	.744	.598	.437	.284	.159	.074	.027	.007	.001	.000	.000
	9	1.0	1.0	1.0	1.0	.998	.993	.977	.942	.876	.773	.634	.473	.312	.175	.080	.027	.006	.001	.000
	10	1.0	1.0	1.0	1.0	1.0	.998	.994	.981	.951	.895	.802	.671	.510	.340	.190	.082	.024	.003	.000
	11	1.0	1.0	1.0	1.0	1.0	1.0	.999	.995	.985	.962	.915	.833	.711	.550	.370	.202	.079	.017	.001
	12	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.989	.972	.935	.866	.754	.595	.402	.210	.068	.007
	13	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.998	.993	.982	.955	.901	.803	.648	.439	.211	.043
	14	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.990	.974	.937	.859	.716	.485	.189
	15	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.990	.972	.926	.815	.560
18	1	.774	.450	.224	.099	.039	.014	.005	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.942	.734	.480	.271	.135	.060	.024	.008	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	3	.989	.902	.720	.501	.306	.165	.078	.033	.012	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000
y	4	.998	.972	.879	.716	.519	.333	.189	.094	.041	.015	.005	.001	.000	.000	.000	.000	.000	.000	.000
	5	1.0	.994	.958	.867	.717	.534	.355	.209	.108	.048	.018	.006	.001	.000	.000	.000	.000	.000	.000
	6	1.0	.999	.988	.949	.861	.722	.549	.374	.226	.119	.054	.020	.006	.001	.000	.000	.000	.000	.000
	7	1.0	1.0	.997	.984	.943	.859	.728	.563	.391	.240	.135	.058	.021	.006	.001	.000	.000	.000	.000
	8	1.0	1.0	.999	.996	.981	.940	.861	.737	.578	.407	.253	.135	.060	.021	.005	.001	.000	.000	.000
	9	1.0	1.0	1.0	.999	.995	.979	.940	.865	.747	.593	.422	.263	.139	.060	.019	.004	.001	.000	.000
	10	1.0	1.0	1.0	1.0	.999	.994	.979	.942	.872	.760	.609	.437	.272	.141	.057	.016	.003	.000	.000
	11	1.0	1.0	1.0	1.0	1.0	.999	.994	.980	.946	.881	.774	.626	.451	.278	.139	.051	.012	.001	.000
	12	1.0	1.0	1.0	1.0	1.0	1.0	.999	.994	.982	.952	.892	.791	.645	.466	.283	.133	.042	.006	.000
	13	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.995	.985	.959	.906	.811	.667	.481	.284	.121	.028	.002
	14	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.988	.967	.922	.835	.694	.499	.280	.098	.011
	15	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.997	.992	.976	.940	.865	.729	.520	.266	.058
	16	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.995	.986	.961	.901	.776	.550	.226
20	1	.736	.392	.176	.069	.024	.008	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.925	.677	.405	.206	.091	.035	.012	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
y	3	.984	.867	.648	.411	.225	.107	.044	.016	.005	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
	4	.997	.957	.830	.630	.415	.238	.118	.051	.019	.006	.002	.000	.000	.000	.000	.000	.000	.000	.000
	5	1.0	.989	.933	.804	.617	.416	.245	.126	.055	.021	.006	.002	.000	.000	.000	.000	.000	.000	.000
	6	1.0	.998	.978	.913	.786	.608	.417	.250	.130	.058	.021	.006	.002	.000	.000	.000	.000	.000	.000
	7	1.0	1.0	.994	.968	.898	.772	.601	.416	.252	.132	.058	.021	.006	.001	.000	.000	.000	.000	.000
	8	1.0	1.0	.999	.990	.959	.887	.762	.596	.414	.252	.131	.057	.020	.005	.001	.000	.000	.000	.000
	9	1.0	1.0	1.0	.997	.986	.952	.878	.755	.591	.412	.249	.128	.053	.017	.004	.001	.000	.000	.000
	10	1.0	1.0	1.0	.999	.996	.983	.947	.872	.751	.588	.409	.245	.122	.048	.014	.003	.000	.000	.000
	11	1.0	1.0	1.0	1.0	.999	.995	.980	.943	.869	.748	.586	.404	.238	.113	.041	.010	.001	.000	.000
	12	1.0	1.0	1.0	1.0	1.0	.999	.994	.979	.942	.868	.748	.584	.399	.228	.102	.032	.006	.000	.000
	13	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.979	.942	.870	.750	.583	.392	.214	.087	.022	.002	.000
	14	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.979	.945	.874	.755	.584	.383	.196	.067	.011	.000
	15	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.994	.981	.949	.882	.762	.585	.370	.170	.043	.003
	16	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.995	.984	.956	.893	.775	.589	.352	.133	.016
25	2	.873	.537	.254	.098	.032	.009	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	3	.966	.764	.471	.234	.096	.033	.010	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	4	.993	.902	.682	.421	.214	.090	.032	.009	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	5	.999	.967	.838	.617	.378	.193	.083	.029	.009	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000
	6	1.0	.991	.930	.780	.561	.341	.173	.074	.026	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000
	7	1.0	.998	.975	.891	.727	.512	.306	.154	.064	.022	.006	.001	.000	.000	.000	.000	.000	.000	.000
	8	1.0	1.0	.992	.953	.851	.677	.467	.274	.134	.054	.017	.004	.001	.000	.000	.000	.000	.000	.000
	9	1.0	1.0	.998	.983	.929	.811	.630	.425	.242	.115	.044	.013	.003	.000	.000	.000	.000	.000	.000
	10	1.0	1.0	1.0	.994	.970	.902	.771	.586	.384	.212	.096	.034	.009	.002	.000	.000	.000	.000	.000
	11	1.0	1.0	1.0	.998	.989	.956	.875	.732	.543	.345	.183	.078	.025	.006	.001	.000	.000	.000	.000
	12	1.0	1.0	1.0	1.0	.997	.983	.940	.846	.694	.500	.306	.154	.060	.017	.003	.000	.000	.000	.000
	13	1.0	1.0	1.0	1.0	.999	.994	.975	.922	.817	.655	.457	.268	.125	.044	.011	.002	.000	.000	.000
	14	1.0	1.0	1.0	1.0	1.0	.998	.991	.966	.904	.788	.616	.414	.229	.098	.030	.006	.000	.000	.000
	15	1.0	1.0	1.0	1.0	1.0	1.0	.997	.987	.956	.885	.758	.575	.370	.189	.071	.017	.002	.000	.000
	16	1.0	1.0	1.0	1.0	1.0	1.0	.999	.996	.983	.946	.866	.726	.533	.323	.149	.047	.008	.000	.000
	17	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.999	.994	.978	.936	.846	.694	.488	.273	.109	.025	.002	.000
	18	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.993	.974	.926	.827	.659	.439	.220	.070	.009	.000
	19	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.991	.971	.917	.807	.622	.383	.162	.033	.001
	20	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	.998	.991	.968	.910	.786	.579	.318	.098	.007

Table A3. Poisson distribution

$$F(x) = P\{X \le x\} = \sum_{k=0}^{x} \frac{e^{-\lambda}\lambda^k}{k!}$$

x	λ														
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
0	.905	.819	.741	.670	.607	.549	.497	.449	.407	.368	.333	.301	.273	.247	.223
1	.995	.982	.963	.938	.910	.878	.844	.809	.772	.736	.699	.663	.627	.592	.558
2	1.00	.999	.996	.992	.986	.977	.966	.953	.937	.920	.900	.879	.857	.833	.809
3	1.00	1.00	1.00	.999	.998	.997	.994	.991	.987	.981	.974	.966	.957	.946	.934
4	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.998	.996	.995	.992	.989	.986	.981
5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.998	.998	.997	.996
6	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999
7	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

x	λ														
	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.202	.183	.165	.150	.135	.122	.111	.100	.091	.082	.074	.067	.061	.055	.050
1	.525	.493	.463	.434	.406	.380	.355	.331	.308	.287	.267	.249	.231	.215	.199
2	.783	.757	.731	.704	.677	.650	.623	.596	.570	.544	.518	.494	.469	.446	.423
3	.921	.907	.891	.875	.857	.839	.819	.799	.779	.758	.736	.714	.692	.670	.647
4	.976	.970	.964	.956	.947	.938	.928	.916	.904	.891	.877	.863	.848	.832	.815
5	.994	.992	.990	.987	.983	.980	.975	.970	.964	.958	.951	.943	.935	.926	.916
6	.999	.998	.997	.997	.995	.994	.993	.991	.988	.986	.983	.979	.976	.971	.966
7	1.00	1.00	.999	.999	.999	.999	.998	.997	.997	.996	.995	.993	.992	.990	.988
8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.999	.999	.998	.998	.997	.996
9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.999	.999
10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table A3, continued. Poisson distribution

x	11	12	13	14	15	16	17	18	19	20	22	24	26	28	30
0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
2	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3	.005	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4	.015	.008	.004	.002	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
5	.038	.020	.011	.006	.003	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000
6	.079	.046	.026	.014	.008	.004	.002	.001	.001	.000	.000	.000	.000	.000	.000
7	.143	.090	.054	.032	.018	.010	.005	.003	.002	.001	.000	.000	.000	.000	.000
8	.232	.155	.100	.062	.037	.022	.013	.007	.004	.002	.001	.000	.000	.000	.000
9	.341	.242	.166	.109	.070	.043	.026	.015	.009	.005	.002	.000	.000	.000	.000
10	.460	.347	.252	.176	.118	.077	.049	.030	.018	.011	.004	.001	.000	.000	.000
11	.579	.462	.353	.260	.185	.127	.085	.055	.035	.021	.008	.003	.001	.000	.000
12	.689	.576	.463	.358	.268	.193	.135	.092	.061	.039	.015	.005	.002	.001	.000
13	.781	.682	.573	.464	.363	.275	.201	.143	.098	.066	.028	.011	.004	.001	.000
14	.854	.772	.675	.570	.466	.368	.281	.208	.150	.105	.048	.020	.008	.003	.001
15	.907	.844	.764	.669	.568	.467	.371	.287	.215	.157	.077	.034	.014	.005	.002
16	.944	.899	.835	.756	.664	.566	.468	.375	.292	.221	.117	.056	.025	.010	.004
17	.968	.937	.890	.827	.749	.659	.564	.469	.378	.297	.169	.087	.041	.018	.007
18	.982	.963	.930	.883	.819	.742	.655	.562	.469	.381	.232	.128	.065	.030	.013
19	.991	.979	.957	.923	.875	.812	.736	.651	.561	.470	.306	.180	.097	.048	.022
20	.995	.988	.975	.952	.917	.868	.805	.731	.647	.559	.387	.243	.139	.073	.035
21	.998	.994	.986	.971	.947	.911	.861	.799	.725	.644	.472	.314	.190	.106	.054
22	.999	.997	.992	.983	.967	.942	.905	.855	.793	.721	.556	.392	.252	.148	.081
23	1.00	.999	.996	.991	.981	.963	.937	.899	.849	.787	.637	.473	.321	.200	.115
24	1.00	.999	.998	.995	.989	.978	.959	.932	.893	.843	.712	.554	.396	.260	.157
25	1.00	1.00	.999	.997	.994	.987	.975	.955	.927	.888	.777	.632	.474	.327	.208
26	1.00	1.00	1.00	.999	.997	.993	.985	.972	.951	.922	.832	.704	.552	.400	.267
27	1.00	1.00	1.00	.999	.998	.996	.991	.983	.969	.948	.877	.768	.627	.475	.333
28	1.00	1.00	1.00	1.00	.999	.998	.995	.990	.980	.966	.913	.823	.697	.550	.403
29	1.00	1.00	1.00	1.00	1.00	.999	.997	.994	.988	.978	.940	.868	.759	.623	.476
30	1.00	1.00	1.00	1.00	1.00	.999	.999	.997	.993	.987	.959	.904	.813	.690	.548

x	3.5	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0	10.5
0	.030	.018	.011	.007	.004	.002	.002	.001	.001	.000	.000	.000	.000	.000	.000
1	.136	.092	.061	.040	.027	.017	.011	.007	.005	.003	.002	.001	.001	.000	.000
2	.321	.238	.174	.125	.088	.062	.043	.030	.020	.014	.009	.006	.004	.003	.002
3	.537	.433	.342	.265	.202	.151	.112	.082	.059	.042	.030	.021	.015	.010	.007
4	.725	.629	.532	.440	.358	.285	.224	.173	.132	.100	.074	.055	.040	.029	.021
5	.858	.785	.703	.616	.529	.446	.369	.301	.241	.191	.150	.116	.089	.067	.050
6	.935	.889	.831	.762	.686	.606	.527	.450	.378	.313	.256	.207	.165	.130	.102
7	.973	.949	.913	.867	.809	.744	.673	.599	.525	.453	.386	.324	.269	.220	.179
8	.990	.979	.960	.932	.894	.847	.792	.729	.662	.593	.523	.456	.392	.333	.279
9	.997	.992	.983	.968	.946	.916	.877	.830	.776	.717	.653	.587	.522	.458	.397
10	.999	.997	.993	.986	.975	.957	.933	.901	.862	.816	.763	.706	.645	.583	.521
11	1.00	.999	.998	.995	.989	.980	.966	.947	.921	.888	.849	.803	.752	.697	.639
12	1.00	1.00	.999	.998	.996	.991	.984	.973	.957	.936	.909	.876	.836	.792	.742
13	1.00	1.00	1.00	.999	.998	.996	.993	.987	.978	.966	.949	.926	.898	.864	.825
14	1.00	1.00	1.00	1.00	.999	.999	.997	.994	.990	.983	.973	.959	.940	.917	.888
15	1.00	1.00	1.00	1.00	1.00	.999	.999	.998	.995	.992	.986	.978	.967	.951	.932
16	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998	.996	.993	.989	.982	.973	.960
17	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998	.997	.995	.991	.986	.978
18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.998	.996	.993	.988
19	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.998	.997	.994
20	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998	.997

31	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998	.996	.992	.973	.932	.859	.752	.619
32	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998	.995	.983	.953	.896	.805	.685
33	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.997	.989	.969	.925	.850	.744
34	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.999	.994	.979	.947	.888	.797
35	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.996	.987	.964	.918	.843
36	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.998	.992	.976	.941	.880
37	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.995	.984	.959	.911
38	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.997	.990	.972	.935
39	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.998	.994	.981	.954
40	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.996	.988	.968
41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998	.992	.978
42	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.995	.985
43	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.997	.990
44	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.998	.994
45	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.996
46	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999	.998
47	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999
48	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999
49	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.999
50	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00