MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

CENG 384

Spring 2022

Homework 1

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw1.pdf' to the odtuclass page of the course. You need to use the given template 'hw1.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 03 April, 2022 (Sunday).
- **Late Submission:** Not allowed.

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1. (16 pts) Solve the following, showing your solution in detail.

   (a) (4 pts) Given a complex number in Cartesian coordinate system, $z = x + jy$ and $2z - 9 = 4j - \bar{z}$,

      i. find $|z|^2$ and

      ii. find and plot z on the complex plane.

   (b) (4 pts) Given $z^3 = -27j$, find z in polar form ($z = re^{j\theta}$).

   (c) (4 pts) Find the magnitude and angle of $z = \frac{(1+j)(\sqrt{3}-j)}{(\sqrt{3}+j)}$.

   (d) (4 pts) Write z in polar form where $z = -(1+j)^8 e^{j\pi/2}$.

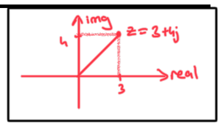
a) i. $2(x+jy) - 9 = 4j - (x-jy)$

$2x + 2jy - 9 = 4j - x + jy$

$j(y-4) + (3x-9) = 0$

$y=4 \quad x=3 \qquad |z|^2 = 16+9 = \boxed{25}$

a) ii.



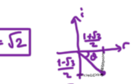
b) $(z^3)^{\frac{1}{3}} = (-27j)^{\frac{1}{3}} \qquad (-j)^{\frac{1}{3}} = j \qquad \theta = \frac{\pi}{2} \qquad \boxed{z = 3e^{\frac{j\pi}{2}}}$

$z = 3j \qquad a=3 \; b=0 \qquad r=3 \qquad 3$

c) $\frac{(1+j)(\sqrt{3}-j)(\sqrt{3}-j)}{(\sqrt{3}+j)(\sqrt{3}-j)} = \frac{(1+j)(1-\sqrt{3}j)\cdot\cancel{2}}{\cancel{4}\,2} = \frac{1+\sqrt{3}}{2} + \left(\frac{1-\sqrt{3}}{2}\right)j = z$

$|z|^2 = \left(\frac{1+\sqrt{3}}{2}\right)^2 + \left(\frac{1-\sqrt{3}}{2}\right)^2 = 2 \qquad \boxed{|z| = \sqrt{2}}$

$z = re^{\theta j} = \sqrt{2}e^{\frac{\pi}{12}j}$



$\tan\theta = \frac{1-\sqrt{3}}{1+\sqrt{3}} = \sqrt{3}-2$

$\boxed{\theta = -15° = \frac{\pi}{12}}$

d) $e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = 0 + j = j$

$((1+j)^2)^4 = 2^4 \cdot j^4 = 2^4 \qquad z = -2^4 \cdot (j) = -16j \qquad r=16 \qquad \theta = -\frac{\pi}{2} \qquad \boxed{z = 16e^{-\frac{\pi}{2}j}}$

2. (12 pts) Calculate power P and energy E of the given signals and determine whether they are Power signals, Energy signals or neither of them. $E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 \qquad P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |x[n]|^2$

   (a) (6 pts) $x[n] = nu[n]$

   (b) (6 pts) $x(t) = e^{-2t}u(t)$

$E = \sum_{n=-\infty}^{+\infty} |n.u[n]|^2 = \sum_{n=-\infty}^{0} |n.u[n]|^2 + \sum_{n=0}^{\infty} |n.u[n]|^2 = 0 + \sum_{n=0}^{\infty} |n|^2 = 0 + 1^2 + 2^2 \ldots \to \boxed{\infty}$

$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |n.u[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{0} |n.u[n]|^2 + \sum_{n=0}^{N} |n.u[n]|^2\right) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |n|^2 = \lim_{N\to\infty} \frac{1}{\cancel{2N+1}} \cdot \frac{N(N+1)\cancel{(2N+1)}}{6} = \lim_{N\to\infty} \frac{N.(N+1)}{6} \to \boxed{\infty}$

$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt \qquad P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} |x(t)|^2 dt$

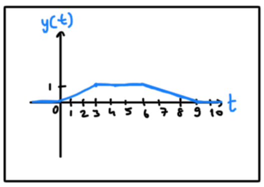
$E = \int_{-\infty}^{\infty} |e^{-2t}.u(t)|^2 dt = \int_{-\infty}^{0} |e^{-2t}.u(t)|^2 dt + \int_{0}^{\infty} |e^{-2t}.u(t)|^2 dt = 0 + \int_{0}^{\infty} e^{-4t} = \frac{e^{-4t}}{-4}\Big|_{0}^{\infty} = 0 - (-\frac{1}{4}) = \boxed{\frac{1}{4}}$

$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |e^{-2t}.u(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\left(\int_{-T}^{0} |e^{-2t}.u(t)|^2 dt + \int_{0}^{T} |e^{-2t}.u(t)|^2 dt\right) = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} e^{-4t} dt = \lim_{T\to\infty} \frac{1}{2T}\left(\frac{e^{-4t}}{-4}\Big|_{0}^{T}\right) = \lim_{T\to\infty} \frac{1}{2T} \cdot \frac{-1}{4} \cdot \left(\frac{1}{e^{4T}} - 1\right) = \lim_{T\to\infty} \left(\frac{-1}{8T.e^{T}} + \frac{1}{8T}\right) \to \boxed{0}$

3. (10 pts) Given the $x(t)$ signal in Figure 1, draw the signal $y(t) = \frac{1}{2}x(-\frac{1}{3}t + 2)$.

$$x(t) \begin{cases} -\infty < t < -1 & 0 \\ -1 < t < 0 & 2+2t \\ 0 < t < 1 & 2 \\ 1 < t < 2 & 4-2t \\ 2 < t < \infty & 0 \end{cases}$$

$$\frac{x(\frac{-t}{3}+2)}{2} \begin{cases} 9 < t < \infty & 0 & = 0 \\ 6 < t < 9 & 6-\frac{2t}{3} & = 3-\frac{t}{3} \\ 3 < t < 6 & 2 & = 1 \\ 0 < t < 3 & \frac{2t}{3} & = \frac{t}{3} \\ -\infty < t < 0 & 0 & = 0 \end{cases}$$



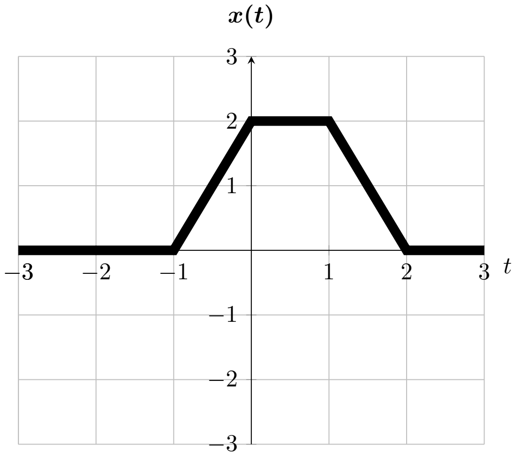


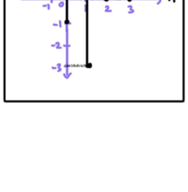
Figure 1: t vs. $x(t)$.

4. (15 pts) Given the $x[n]$ signal in Figure 2,

   (a) (10 pts) Draw $x[-2n] + x[n-2]$.

   (b) (5 pts) Express $x[-2n] + x[n-2]$ in terms of the unit impulse function.

n	
-1	$\to x[2] + x[-3] = 1+1 = 2$
0	$\to x[0] + x[-2] = 0-1 = -1$
1	$\to x[-2] + x[-1] = -1-2 = -3$
2	$\to x[-4] + x[0] = 0+0 = 0$
3	$\to x[-6] + x[1] = 0+0 = 0$



$\boxed{x[n] = 2\delta[n+1] - \delta[n] - 3\delta[n-1]}$

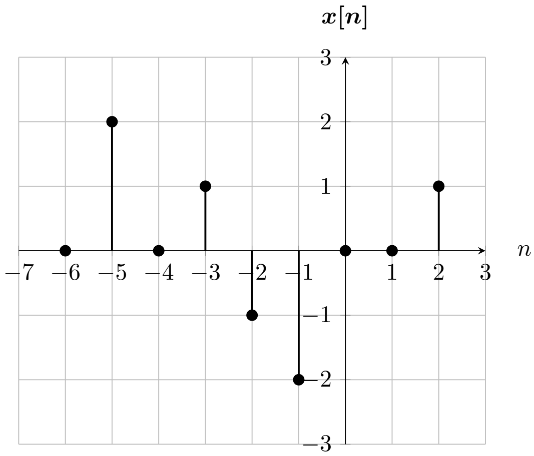


Figure 2: n vs. $x[n]$.

$$x(t) = \begin{cases} t < -1 & 0 \\ -1 < t < 0 & 2+2t \\ 0 < t < 1 & 2 \\ 1 < t < 2 & 4-2t \\ 2 < t & 0 \end{cases} \qquad x(-t) = \begin{cases} 1 < t & 0 \\ 0 < t < 1 & 2-2t \\ -1 < t < 0 & 2 \\ -2 < t < -1 & 4+2t \\ t < -2 & 0 \end{cases}$$

5. (8 pts) Determine whether the following signals are periodic and if periodic find the fundamental period.

   (a) (4 pts) $x(t) = \frac{e^{j3t}}{-j}$

   $\frac{e^{j3t}}{-j} = \frac{e^{j3(t+T_0)}}{-j}$ $\quad e^{j3t}(1-e^{j3T_0}) = 0$ $\quad 1-e^{3jT_0} = 0$ $\quad e^{3jT_0} = 1$ $\quad 2\pi k = 3T_0$ $\quad k=3$ $\boxed{T_0 = 2\pi}$

   (b) (4 pts) $x[n] = \underbrace{\frac{1}{2}\sin[\frac{7\pi}{8}n]} + \underbrace{4\cos[\frac{3\pi}{4}n - \frac{\pi}{2}]}$

   $\frac{1}{2}\sin[\frac{7\pi}{8}n] = \frac{1}{2}\sin[\frac{7\pi}{8}(n+N_0)]$ $\quad \frac{7\pi N_0}{8} = 2\pi k_1$ $\quad N_0 = \frac{16k_1}{7}$

   $4\cos[\frac{3\pi}{4}n - \frac{\pi}{2}] = 4\cos[\frac{3\pi}{4}(n+N_0) - \frac{\pi}{2}]$ $\quad \frac{3\pi}{4}N_0 = 2\pi k_2$ $\quad N_0 = \frac{8k_2}{3}$

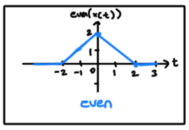
   $N_0 = \frac{16k_1}{7} = \frac{8k_2}{3}$ $\quad k_1 = 7$ $\quad k_2 = 6$ $\quad \boxed{N_0 = 16}$

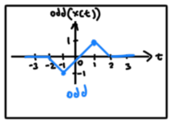
6. (15 pts) Consider the signal in Figure 1.

   (a) (5 pts) Show that the signal is neither even nor odd. Since $x(-t)$ neither equals to $x(t)$ or $-x(t)$, the function is neither even nor odd

   (b) (10 pts) Find the even and odd decompositions of the signal and draw these parts.

	$x(t)$	$x(-t)$	$\boxed{\text{even}}$ $\frac{x(t)+x(-t)}{2}$	$\boxed{\text{odd}}$ $\frac{x(t)-x(-t)}{2}$
$t < -2$	0	0	0	0
$-2 < t < -1$	0	$4+2t$	$2+t$	$-2-t$
$-1 < t < 0$	$2+2t$	2	$2+t$	t
$0 < t < 1$	2	$2-2t$	$2-t$	t
$1 < t < 2$	$4-2t$	0	$2-t$	$2-t$
$2 < t$	0	0	0	0

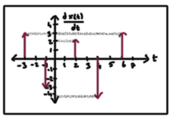




7. (12 pts) Given the $x(t)$ signal in Figure 3,

   (a) (5 pts) Express $x(t)$ in terms of the unit step function. $x(t) = 3u(t+3) - 3u(t+1) + 2u(t-2) - 4u(t-4) + 3u(t-6)$

   (b) (7 pts) Find and draw $\frac{dx(t)}{dt}$. $\frac{dx(t)}{dt} = 3\delta(t+3) - 3\delta(t+1) + 2\delta(t-2) - 4\delta(t-4) + 3\delta(t-6)$



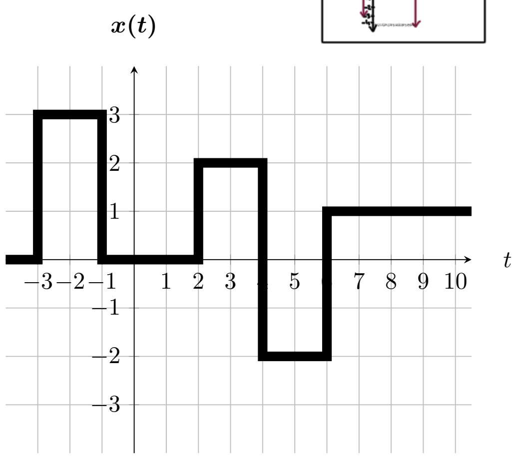


Figure 3: t vs. $x(t)$.

8. (12 pts) Analyze whether the following systems have these properties: *memory, stability, causality, linearity, invertibility, time-invariance*. Provide your answer in detail.

   (a) (6 pts) $y[n] = x[2n-2]$

   (b) (6 pts) $y(t) = tx(\frac{t}{2} - 1)$

memory: for $n=0$ $y[0] = x[-2] \rightarrow$ past; $n=3$ $y[3] = x[4] \rightarrow$ future } the present value of the output depends on past & future, so it has memory

stability: for a definite bounded input, we can get a bounded output (if we put $x[2n-2] = 3$, $y[n] = 3$ which is bounded in nature) Therefore, the system is stable

causality: for $n=3$ $y[3] = x[4] \rightarrow$ future } since the present value of the output depends on future too, the system is not causal

linearity: $x_1[n] \rightarrow x_1[2n-2] = y_1[n]$
$x_2[n] \rightarrow x_2[2n-2] = y_2[n]$
$a_1x_1[n] + a_2x_2[n] \rightarrow a_1x_1[2n-2] + a_2x_2[2n-2] \rightarrow$ superposition of inputs
$a_1y_1[n] + a_2y_2[n] = a_1x_1[2n-2] + a_2x_2[2n-2] \rightarrow$ superposition of outputs } since they are equal, the system is linear

invertibility: since there are no two inputs which produce same output for the given system, the sistem is invertible.

time invariance: if the signal is first passed through the system and then through the delay, the output will be $x[2n-2-n_0]$
if it is passed through the time delay first and then though the system, the output will be $x[2(n-n_0)-2] = x[2n-2-2n_0]$ } since the outputs are not same, the system is time variant

memory: for $n=-4$ $y(-4) = -4x(-3) \rightarrow$ future; $y(4) = 4x(3) \rightarrow$ past } the present value of the output depends on past & future, so it has memory

stability: for a finite input, we cannot expect a finite output. For example, if we will put $x(\frac{t}{2}-1) = 2 \Rightarrow y(t) = 2t$. This is not a finite value, because we do not know the value of t, so it can be ranged from anywhere. Therefore, the system is unstable

causality: for $n=-4$ $y(-4) = -4x(-3)$ future } since the present value of the output depends on future too, the system is not causal

linearity: $x_1(t) \rightarrow t x_1(\frac{t}{2}-1) = y_1(t)$
$x_2(t) \rightarrow t x_2(\frac{t}{2}-1) = y_2(t)$
$a_1x_1(t) + a_2x_2(t) \rightarrow t a_1x_1(\frac{t}{2}-1) + t a_2x_2(\frac{t}{2}-1) \rightarrow$ superposition of inputs
$a_1y_1(t) + a_2y_2(t) \rightarrow a_1 t x_1(\frac{t}{2}-1) + a_2 t x_2(\frac{t}{2}-1) \rightarrow$ superposition of outputs } since they are equal, the system is linear

invertibility: when $t=0$ $w(t) = \frac{y(t)}{t}$ is undefined. so when $t \neq 0$ since there are no two inputs which produce same output for the given system, the sistem is invertible.

time invariance: if the signal is first passed through the system and then through the delay, the output will be $(t-t_0) \times (\frac{t}{2}-1-t_0)$
if it is passed through the time delay first and then though the system, the output will be $(t-t_0) \times (\frac{t-t_0}{2}-1) = (t-t_0) \times (\frac{t}{2}-1-\frac{t_0}{2})$ } since the outputs are not same, the system is time variant