Student Information

Name : Solution

ID :

Answer 1

a)

Expected value of a die is calculated by dividing the sum of all face values to the number of faces.
For blue die; $E(B) = \dfrac{15}{6} = 2.5$

For yellow die; $E(Y) = \dfrac{12}{6} = 2$

For red die; $E(R) = \dfrac{20}{8} = 2.5$

b)

Using the expected values obtained in part a, $2E(R) + E(Y) = 7$ and $2E(Y) + E(B) = 6.5$. The first option is better, since it has a greater expected value.

c)

In this case expected value of blue die has become 4. So $2E(Y) + E(B)$ becomes 8, making second one a better option than rolling two red dice and a yellow die.

d)

Let $P(A)$ be the probability of rolling a red die and $P(B)$ be the probability of rolling 3. The given question asks the conditional probability $P(A|B)$. Using Bayes rule;

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Since all colors have an equal probability, $P(A) = \frac{1}{3}$. Among eight faces of a red die only three have the value 3, so $P(B|A) = \frac{3}{8}$. For $P(B)$, we must find the probability of obtaining a 3 for all colors. That is;

$$P(B) = (\tfrac{3}{8})(\tfrac{1}{3}) + (\tfrac{1}{6})(\tfrac{1}{3}) + (\tfrac{1}{3})(\tfrac{1}{3}) = \tfrac{7}{24}$$

This makes;

$$P(A|B) = \frac{(\frac{3}{8})(\frac{1}{3})}{(\frac{7}{24})} = \tfrac{3}{7} = 0.428$$

If the result is 3, with approximately 43% probability rolled die is red.

e)

Let $P(a, b)$ denote the probability of obtaining a on yellow die and b on red die. To get a total of 6, only possible options are $P(3, 3)$ and $P(1, 5)$. So;

$P(3,3) = (\frac{1}{3})(\frac{3}{8}) = \frac{1}{8}$
$P(1,5) = (\frac{1}{3})(\frac{1}{8}) = \frac{1}{24}$
$P(3,3) + P(1,5) = \frac{1}{6}$, with a roll of a single red and yellow die, there is approximately 17% probability that the total value will be 6.

Answer 2

a)

$P(A = 0, I = 2) = 0.17$

b)

$P(A = 2, I = 0) = 0$. It is not possible that two electric outages will occur in Ankara.

c)

Two possible options are $P(A = 1, I = 1)$ and $P(A = 0, I = 2)$. So the probability of two electric outages in total is $P(2) = 0.11 + 0.17 = 0.28$.

d)

For this we need to consider only $P(A = 1)$, regardless of $P(I)$. So, $0.12+0.11+0.22+0.15 = 0.6$

e)

The possible range of values are from 0 to 4.

$P(0) = P(A = 0, I = 0) = 0.08$
$P(1) = P(A = 1, I = 0) + P(A = 0, I = 1) = 0.12 + 0.13 = 0.25$
$P(2)$ was already calculated in part c, so $P(2) = 0.28$.
$P(3) = P(A = 0, I = 3) + P(A = 1, I = 2) = 0.02 + 0.22 = 0.24$
$P(4) = P(A = 1, I = 3) = 0.15$

f)

For independence, we need to consider joint probability and marginal probabilities. A single counterexample would be sufficient for independence.

$P(A = 1) = 0.6$ and $P(I = 0) = 0.2$, however $P(A = 1, I = 0) = 0.11$. Since $0.6 \times 0.2 \neq 0.11$, electric outages in Ankara and Istanbul are independent.