22 - regular singular points

$P(x)y'' + Q(x)y' + R(x)y = 0$

$y'' + \frac{Q(x)}{P(x)}y' + \frac{R(x)}{P(x)}y = 0 \qquad P(x_0) = 0 \qquad x = x_0$ is a singular point

$\boxed{\text{classification of singular points}}$

when $x = x_0$ is a singular point:

① $\lim_{x \to x_0} (x - x_0)\frac{Q(x)}{P(x)} = \alpha$

② $\lim_{x \to x_0} (x - x_0)^2 \frac{R(x)}{P(x)} = \beta$

if they are real $x_0 \to$ regular singular point

otherwise irregular singular point

we can write $\Rightarrow$ $(x - x_0)^2 y'' + (x - x_0)A(x)y' + B(x)y = 0$
when $x = x_0$

$\lim_{x \to x_0} A(x_0) = \alpha$

$\lim_{x \to x_0} B(x_0) = \beta$

example

$x^3(x-1)y'' + (x+2)y' + 4y = 0 \quad \to \quad y'' + \frac{(x+2)}{x^3(x-1)}y' + \frac{4}{x^3(x-1)}y = 0$

$x = 1 \qquad \lim_{x \to 1} (x-1)\frac{(x+2)}{x^3(x-1)} = 3 = \alpha \checkmark \qquad \lim_{x \to 1} \frac{(x-1)^2 \cdot 4}{x^3(x-1)} = 0 = \beta \checkmark \to$ $x = 1$ regular singular point

$x = 0 \qquad \lim_{x \to 0} x\frac{(x+2)}{x^3(x-1)} \Rightarrow \infty \ \times \qquad \lim_{x \to 0} \frac{x^2 \cdot 4}{x^3(x-1)} \Rightarrow \infty \ \times \to$ $x = 0$ irregular singular point