

# CENG 223

## Discrete Computational Structures

Fall '2020-2021

### Homework 3

---

Due date: 22 December 2020, 23:55

#### Question 1

Use Fermat's Little Theorem theorem the find  $(2^{22} + 4^{44} + 6^{66} + 8^{80} + 10^{110}) \bmod 11 \equiv ?$

Note: Fermat's Little Theorem is provided in our book (Kenneth H. Rosen, Discrete Mathematics and Its Applications), and it is a prerequisite for this question. This means that your solution have to use Fermat's Little Theorem.

#### Question 2

Find  $\gcd(5n + 3, 7n + 4)$  and while doing that, show the steps of Euclid's algorithm clearly.

#### Question 3

Let  $x$  be a prime number,  $\underbrace{(m-n)}_a \underbrace{(m+n)}_b = \underbrace{kx}_p$   
Suppose  $x$  does not divide  $(m-n)$   
we wish to show  $x|(m+n)$   
If  $m^2 = n^2 + kx$  where  $m$ ,  $n$ , and  $k$  are integer numbers.

Show that  $x|(m+n)$  or  $x|(m-n)$ .  $(m-n)z + (x)t = 1$   
 $m+n = kx.z +$

#### Question 4

Show that for all  $n$  such that  $n \geq 1$  the following is true:

$$1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n - 1)}{2} \quad (1)$$

Note: You have to use mathematical induction to prove that.

# 1 Regulations

1. You have to write your answers to the provided sections of the template answer file given. Other than that, you cannot change the provided template answer file. If a latex structure you want to use cannot be compiled with the included packages in the template file, that means you should not use it.
2. Do not write any other stuff, e.g. question definitions, to answers' sections. Only write your answers. Otherwise, you will get 0 from that question.
3. **Late Submission: Not allowed**
4. **Cheating: We have zero tolerance policy for cheating.** People involved in cheating will be punished according to the university regulations.
5. **Newsgroup:** You must follow the odtuclass discussions (<https://odtuclass.metu.edu.tr>) for discussions and possible updates on a daily basis.
6. **Evaluation:** Your latex file will be converted to pdf and evaluated by course assistants. The .tex file will be checked for plagiarism automatically using "black-box" technique and manually by assistants, so make sure to obey the specifications.

# 2 Submission

Submission will be done via odtuclass. Download the given template file, "hw3.tex", when you finish your exam upload the .tex file with the same name to odtuclass.

**Note: You cannot submit any other files.** Don't forget to make sure your .tex file is successfully compiled in Inek machines using the command below.

```
$ pdflatex hw3.tex
```