Student Information

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Answer 1

a)

Expected Value Of A Single Blue Die Roll:
There are three different possible outcomes and their possibilities are:
$P(2) = \frac{4}{6}$, $P(3) = \frac{1}{6}$, $P(4) = \frac{1}{6}$. The expected value is $E(x) = \sum_x x.P(x) = 2.\frac{4}{6} + 3.\frac{1}{6} + 4.\frac{1}{6} = 2.5$

Expected Value Of A Single Yellow Die Roll:
There are three different possible outcomes and their possibilities are:
$P(1) = \frac{2}{6}$, $P(2) = \frac{2}{6}$, $P(3) = \frac{2}{6}$. The expected value is $E(x) = \sum_x x.P(x) = 1.\frac{2}{6} + 2.\frac{2}{6} + 3.\frac{2}{6} = 2$

Expected Value Of A Single Red Die Roll:
There are four different possible outcomes and their possibilities are:
$P(1) = \frac{2}{8}$, $P(2) = \frac{2}{8}$, $P(3) = \frac{3}{8}$, $P(5) = \frac{1}{8}$. The expected value is $E(x) = \sum_x x.P(x) = 1.\frac{2}{8} + 2.\frac{2}{8} + 3.\frac{3}{8} + 5.\frac{1}{8} = 2.5$

b)

Since $E(x+y) = E(x) + E(y)$,
Expected value of rolling 2 red and 1 yellow = Red's expected value + Red's expected value + Yellow's expected value
$2.5 + 2.5 + 2 = 7$
Expected value of rolling 2 yellow and 1 blue = Yellow's expected value + Yellow's expected value + Blue's expected value
$2 + 2 + 2.5 = 6.5$
Therefore, I would choose first option: 2 red and 1 yellow, because its expected value is greater.

c)

Then expected value of rolling 2 yellow and 1 blue would be $2 + 2 + 4 = 8$
Therefore, second option's expected value = 8 would be greater than the first one = 7, so I would choose second option 2 yellow and 1 blue to maximize the total value.

d)

Let P(R) = rolling a red die probability = $\frac{1}{3}$(since each color has equal probability in random choosing)
P(Y) = rolling a yellow die probability= $\frac{1}{3}$
P(B) = rolling a blue die probability= $\frac{1}{3}$
P(T) = probability of a rolled die's outcome is 3

We are asked $P(R|T)$
$P(R|T) = \frac{P(T|R).P(R)}{P(T)} = \frac{P(T|R).P(R)}{P(T|R).P(R)+P(T|Y).P(Y)+P(T|B).P(B)} = \frac{\frac{3}{8}\cdot\frac{1}{3}}{\frac{3}{8}\cdot\frac{1}{3}+\frac{2}{6}\cdot\frac{1}{3}+\frac{1}{6}\cdot\frac{1}{3}} = \frac{3}{7} = 0.4286$

e)

The numbers that yellow can produce are 1,2,3
The numbers that red can produce are 1,2,3,5
We can obtain 6 when yellow is 1,and red is 5, or when yellow is 3 and red is 3.
Therefore, $P(6) = P_y(1) * P_r(5) + P_y(3) * P_r(3) = \frac{2}{6} * \frac{1}{8} + \frac{2}{6} * \frac{3}{8} = \frac{8}{48} = \frac{1}{6} = 1.667$

Answer 2

a)

no electric outages in Ankara means a = 0
two electric outages in Istanbul means i = 2
Therefore, from the table it is 0.17

b)

two electric outages in Ankara means a = 2
one electric outages in Istanbul means i = 0
There is no such a outage possible from the table therefore the probability is 0.

c)

when
a= 0 and i= 2
a= 1 and i= 1
There are two electric outages in total.
Therefore, in total their probability is $0.17 + 0.11 = 0.28$

d)

Single electric outage in Ankara means when a = 1. If we add all the possibilities where a=1: $0.12 + 0.11 + 0.22 + 0.15 = 0.6$ is its probability.

e)

Let $T = A + I$ be the total number of electric outages. To find the distribution of T, we first identify its possible values, then find the probability of each value. We see that T can be as small as 0 and as large as 4. Then,
$P_T(0) = P(a + i = 0) = P(a = 0 \cap i = 0) = P(0, 0) = 0.08$,
$P_T(1) = P(a = 0 \cap i = 1) + P(a = 1 \cap i = 0) = P(0, 1) + P(1, 0) = 0.13 + 0.12 = 0.25$,
$P_T(2) = P(a = 0 \cap i = 2) + P(a = 1 \cap i = 1) = P(0, 2) + P(1, 1) = 0.17 + 0.11 = 0.28$,
$P_T(3) = P(a = 0 \cap i = 3) + P(a = 1 \cap i = 2) = P(0, 3) + P(1, 2) = 0.02 + 0.22 = 0.24$,
$P_T(4) = P(a = 1 \cap i = 3) = P(1, 3) = 0.15$

f)

To decide on the independence of the electric outages in Ankara and İstanbul, we have to check if their joint pmf factors into a product of marginal pmfs. We see that $P(A, I)(0, 1) = 0.13$ whereas
$P(a = 0) = 0.08 + 0.13 + 0.17 + 0.02 = 0.4$
$P(i = 1) = 0.13 + 0.11 = 0.24$
$P_A(0) * P_I(1) = (0.4)(0.24) = 0.96$, is a pair of A and I that violates the formula for independent random variables. $P(0, 1) \neq P_A(0) * P_I(1)$ Therefore, the numbers of errors in two modules are dependent.