Lecture 1

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Motivation: Solving systems of linear equation

$5u + 6v + 7w = 5$ ← linear eqn.

$\underbrace{5uv}_{} + 6uw + 7vw = 5$, $5\frac{u}{v} + 6\frac{u}{w} + 7\frac{w}{u} = 5$ } not linear

Solve

Exp:
$$\begin{aligned} 2u + v + w &= 5 \quad (1) \\ 4u - 6v \quad &= -2 \quad (2) \\ -2u + 7v + 2w &= 9 \quad (3) \end{aligned}$$

3 linear eqns. in 3 unknowns: u, v, w

RMK: Geometrically each eqn. represents a plane in $\mathbb{R}^3$

Try eliminating one of the unknowns

(eq1) + (eq. 3)
$$\begin{aligned} 2u + v + w &= 5 \\ 4u - 6v \quad &= -2 \\ 0.u + 8v + 3w &= 14 \end{aligned}$$

(−2)(eq 1) + (eq 2)
$$\begin{aligned} 2u + v + w &= 5 \\ 0.u - 8v - 2w &= -12 \\ 0.u + 8v + 3w &= 14 \end{aligned}$$

(eq 2) + (eq 3)
$$\begin{aligned} 2u + v + w &= 5 \quad \Rightarrow u = 1 \\ -8v - 2w &= -12 \quad \Rightarrow v = 1 \\ w &= 2 \end{aligned}$$

$(u, v, w) = (1, 1, 2)$

This method is called "Gaussian Elimination".

Instead of writing eqns each time we will represent the system(s) by some arrays of numbers.

$$\left.\begin{aligned} 2u + v + w &= 5 \\ 4u - 6v \quad &= -2 \\ -2u + 7v + 2w &= 9 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 4 & -6 & 0 & -2 \\ -2 & 7 & 2 & 9 \end{array}\right] \to \cdots \to \left[\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & -8 & -2 & -12 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Sec 1. Matrices & Matrix Algebra

Defn: A matrix is a rectangular array of numbers. More precisely an $m\times n$ - matrix A over $\mathbb{R}$ (or $\mathbb{C}$) is a function from the set $\{(i,j) \mid i=1,2,\dots,m \quad j=1,2,\dots,n\}$ into $\mathbb{R}$ ($\mathbb{C}$)

$$A : \{(i,j) \mid i=1,2,\dots,m \quad j=1,2,\dots,n\} \longrightarrow \mathbb{R}\ (\mathbb{C})$$

$$A(i,j) = a_{ij} \quad \text{which is called } (i,j)\text{-th entry}$$

e.g. 3×4 - matrix over $\mathbb{R}$

$$A : \{(i,j) \mid i=1,2,3 \quad j=1,2,3,4\} \longrightarrow \mathbb{R}$$

$$\begin{array}{cccccc} & & & & & \\ A = & \left[\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{array}\right]_{3\times 4} & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} & \left.\vphantom{\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}}\right\} \text{rows} \\ & \underbrace{\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array}}_{\text{columns.}} & & \end{array}$$

The entries of i-th row $[a_{i1} \quad a_{i2}, \dots\dots a_{in}]$ in $A_{m\times n}$

The entries of j-th column $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$

Defn: An $n\times n$ - matrix is called a "square matrix"

Defn: In a square matrix the entries (i,i) are called "diagonal entries".

Defn: A square matrix which has diagonal entries are 1 and all other entries are zero is called "identity matrix"

$$A_{n\times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \qquad I_{n\times n} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 \cdots & 0 \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}_{n\times n}$$

- Operations on Matrices:

1 - Matrix Addition:

Defn: Given two $m \times n$ matrices A & B, define

$$(A+B)(i,j) := a_{ij} + b_{ij} \qquad i=1,\dots,m \quad j=1,\dots,n$$

Thm 1.1: A, B, C are $m \times n$ - matrices $\left(O_{m\times n} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{bmatrix}_{m\times n} \right.$

1 - $A + B = B + A$

2 - $A + (B+C) = (A+B) + C$

3 - $A + O_{m\times n} = O_{m\times n} + A$

4 - For any A we have $-A$ i.e. $A + (-A) = O_{m\times n}$ where $(-A)(i,j) = (-a_{ij})$