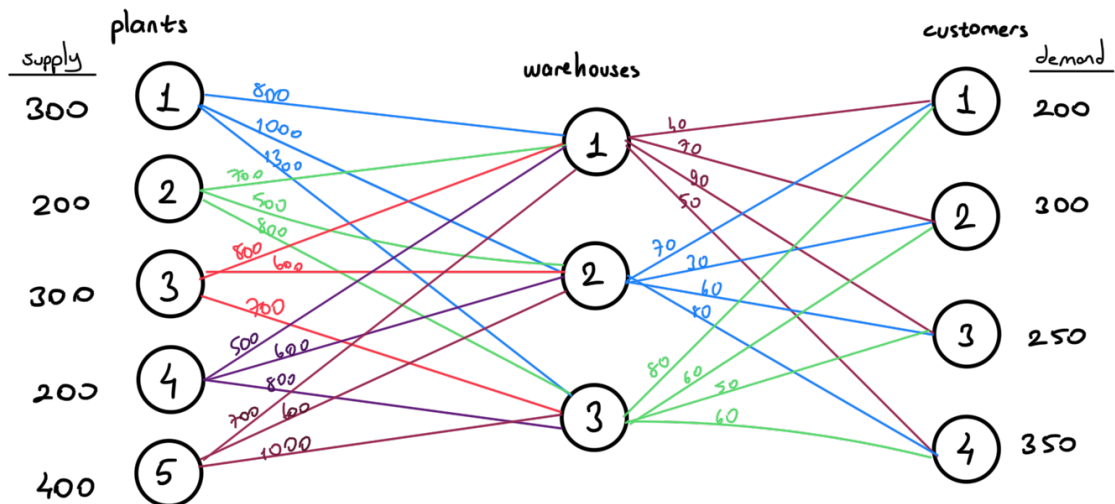


a)



decision variables:

- $x_{ij}$ : amount of sauce in tons, shipped to warehouse  $j$  from plant  $i$
- $y_{jk}$ : amount of sauce in tons, delivered to customer  $k$  from warehouse  $j$

objective cost function to minimize:

$$800x_{11} + 1000x_{12} + 1300x_{13} + 700x_{21} + 500x_{22} + 800x_{23} + 800x_{31} + 600x_{32} + 700x_{33} + 500x_{41} + 600x_{42} + 800x_{43} + 700x_{51} + 600x_{52} + 1000x_{53} + 20y_{11} + 70y_{12} + 90y_{13} + 50y_{14} + 70y_{21} + 70y_{22} + 60y_{23} + 80y_{24} + 80y_{31} + 60y_{32} + 50y_{33} + 60y_{34}$$

constraints:

↳ supply constraints:

$$\begin{aligned} x_{11} + x_{12} + x_{13} &\leq 300 \\ x_{21} + x_{22} + x_{23} &\leq 200 \\ x_{31} + x_{32} + x_{33} &\leq 300 \\ x_{41} + x_{42} + x_{43} &\leq 200 \\ x_{51} + x_{52} + x_{53} &\leq 400 \end{aligned}$$

↳ demand constraints:

$$\begin{aligned} y_{11} + y_{21} + y_{31} &\geq 200 \\ y_{12} + y_{22} + y_{32} &\geq 300 \\ y_{13} + y_{23} + y_{33} &\geq 250 \\ y_{14} + y_{24} + y_{34} &\geq 350 \end{aligned}$$

↳ warehouses supply/demand constraints

$$\begin{aligned} x_{11} + x_{21} + x_{31} + x_{41} + x_{51} &= y_{11} + y_{12} + y_{13} + y_{14} \\ x_{12} + x_{22} + x_{32} + x_{42} + x_{52} &= y_{21} + y_{22} + y_{23} + y_{24} \\ x_{13} + x_{23} + x_{33} + x_{43} + x_{53} &= y_{31} + y_{32} + y_{33} + y_{34} \end{aligned}$$

↳ sign constraints:

$$x_{ij}, y_{jk} \geq 0$$

plant	p1	p2	p3	p4	p5	
tons	300	200	300	200	400	
	0	200	300	200	400	

	w1	w2	w3
p1	800	1000	1300
p2	700	500	800
p3	800	600	700
p4	500	600	800
p5	700	600	1000

	c1	c2	c3	c4
w1	40	70	90	50
w2	70	30	60	80
w3	80	60	50	60

customer	c1	c2	c3	c4
demand	200	300	250	350
	200	300	250	350

$x_{ij}$

	w1	w2	w3
p1	0	0	0
p2	0	200	0
p3	0	300	0
p4	200	0	0
p5	0	400	0

$y_{jk}$

	c1	c2	c3	c4
w1	0	0	0	200
w2	200	300	250	150
w3	0	0	0	0

objective func value

min
680000

equal constraints	
200	200
900	900
0	0

$w_1$   
 $w_2$   
 $w_3$

b)

additional decision variables:

$p_1, p_2, p_3, p_4, p_5 \rightarrow 0$  if plant is not used  
 $\rightarrow 1$  if plant is used  
 $w_1, w_2, w_3 \rightarrow 0$  if warehouse is not used  
 $\rightarrow 1$  if warehouse is used

} binary

additional constraints:

$$x_{11} + x_{12} + x_{13} \leq 300 p_1$$

$$x_{21} + x_{22} + x_{23} \leq 200 p_2$$

$$x_{31} + x_{32} + x_{33} \leq 300 p_3$$

$$x_{41} + x_{42} + x_{43} \leq 200 p_4$$

$$x_{51} + x_{52} + x_{53} \leq 400 p_5$$

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} \leq 1400 w_1$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} \leq 1400 w_2$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} \leq 1400 w_3$$

new cost value:

cost in previous question  $+ 40000 p_1 + 50000 p_2 + 45000 p_3 + 50000 p_4 + 45000 p_5 +$   
 $30000 w_1 + 40000 w_2 + 30000 w_3$

plant	p1	p2	p3	p4	p5	
tons	300	200	300	200	400	400
	0	200	300	200	400	400
$p_{1,2,3,4,5} \leftarrow$	0	1	1	1	1	1
	40000	50000	45000	50000	45000	

	w1	w2	w3	
p1	800	1000	1300	0
p2	700	500	800	200
p3	800	600	700	300
p4	500	600	800	200
p5	700	600	1000	400

	c1	c2	c3	c4
w1	40	70	90	50
w2	70	30	60	80
w3	80	60	50	60

customer	c1	c2	c3	c4
demand	200	300	250	350
	200	300	250	350

	1400	300000	40000	30000
	0	1	0	0
p1	0	0	0	0
p2	0	200	0	0
p3	0	300	0	0
p4	0	200	0	0
p5	0	400	0	0
	0	1400	0	0

	c1	c2	c3	c4
w1	0	0	0	0
w2	200	300	250	350
w3	0	0	0	0

objective func value

min
936000

equal constraints
0
1100
0

c)

$$\begin{aligned}
 y_{11} &= z_{11}b_1 + z_{12}b_2 + z_{13}b_3 \rightarrow 100z_{12} + 1400z_{13} \\
 f_1(y_{11}) &= z_{11}f_1(b_1) + z_{12}f_1(b_2) + z_{13}f_1(b_3) \rightarrow 4000z_{12} + 43000z_{13} \\
 f_1(y_{11}) &= \begin{cases} \text{if } y_{11} \leq 100 & 40y_{11} \rightarrow m_{11}=1 \\ \text{else} & 30(y_{11}-100) + 40000 \rightarrow m_{12}=1 \end{cases} \\
 z_{11} + z_{12} + z_{13} &= 1 \quad z_{11}, z_{12}, z_{13} \geq 0 & z_{11} \leq m_{11} \\
 m_{11} + m_{12} &= 1 \quad m_{11}, m_{12} \rightarrow \text{binary} & z_{12} \leq m_{11} + m_{12} \\
 & & z_{13} \leq m_{12}
 \end{aligned}$$

$$\begin{aligned}
 y_{12} &= z_{21}b_1 + z_{22}b_2 + z_{23}b_3 \rightarrow 100z_{22} + 1400z_{23} \\
 f_2(y_{12}) &= z_{21}f_2(b_1) + z_{22}f_2(b_2) + z_{23}f_2(b_3) \rightarrow 7000z_{22} + 46000z_{23} \\
 f_2(y_{12}) &= \begin{cases} \text{if } y_{12} \leq 100 & 70y_{12} \rightarrow m_{21}=1 \\ \text{else} & 30(y_{12}-100) + 7000 \rightarrow m_{22}=1 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 z_{21} + z_{22} + z_{23} &= 1 \quad z_{21}, z_{22}, z_{23} \geq 0 & z_{21} \leq m_{21} \\
 m_{21} + m_{22} &= 1 \quad m_{21}, m_{22} \rightarrow \text{binary} & z_{22} \leq m_{21} + m_{22} \\
 & & z_{23} \leq m_{22}
 \end{aligned}$$

$$\begin{aligned}
 y_{13} &= z_{31}b_1 + z_{32}b_2 + z_{33}b_3 \rightarrow 100z_{32} + 1400z_{33} \\
 f_3(y_{13}) &= z_{31}f_3(b_1) + z_{32}f_3(b_2) + z_{33}f_3(b_3) \rightarrow 9000z_{32} + 48000z_{33} \\
 f_3(y_{13}) &= \begin{cases} \text{if } y_{13} \leq 100 & 90y_{13} \\ \text{else} & 30(y_{13}-100) + 9000 \end{cases} \\
 z_{31} + z_{32} + z_{33} &= 1 \quad z_{31}, z_{32}, z_{33} \geq 0 & z_{31} \leq m_{31} \\
 m_{31} + m_{32} &= 1 \quad m_{31}, m_{32} \rightarrow \text{binary} & z_{32} \leq m_{31} + m_{32} \\
 & & z_{33} \leq m_{32}
 \end{aligned}$$

$$\begin{aligned}
 y_{14} &= z_{41}b_1 + z_{42}b_2 + z_{43}b_3 \rightarrow 100z_{42} + 1400z_{43} \\
 f_4(y_{14}) &= z_{41}f_4(b_1) + z_{42}f_4(b_2) + z_{43}f_4(b_3) \rightarrow 5000z_{42} + 44000z_{43} \\
 f_4(y_{14}) &= \begin{cases} \text{if } y_{14} \leq 100 & 50y_{14} \\ \text{else} & 30(y_{14}-100) + 5000 \end{cases} \\
 z_{41} + z_{42} + z_{43} &= 1 \quad z_{41}, z_{42}, z_{43} \geq 0 & z_{41} \leq m_{41} \\
 m_{41} + m_{42} &= 1 \quad m_{41}, m_{42} \rightarrow \text{binary} & z_{42} \leq m_{41} + m_{42} \\
 & & z_{43} \leq m_{42}
 \end{aligned}$$

additional constraint for using one of the warehouse  $\rightarrow w_1 + w_2 + w_3 = 1$

new objective cost function to minimize:

$$\begin{aligned} & 800x_{11} + 1000x_{12} + 1300x_{13} + 700x_{21} + 500x_{22} + 800x_{23} + 800x_{31} + 600x_{32} + 700x_{33} + \\ & 500x_{41} + 600x_{42} + 800x_{43} + 700x_{51} + 600x_{52} + 1000x_{53} + \\ & f_1(y_{11}) + f_2(y_{12}) + f_3(y_{13}) + f_4(y_{14}) + 70y_{21} + 30y_{22} + 60y_{23} + 80y_{24} + 80y_{31} + 60y_{32} + \\ & 50y_{33} + 60y_{34} + \\ & 40000w_1 + 50000w_2 + 45000w_3 + 50000p_4 + 45000p_5 + 30000w_1 + 40000w_2 + \\ & 30000w_3 \end{aligned}$$

plant	p1	p2	p3	p4	p5	
tons	300	200	300	200	400	400
	0	200	300	200	400	
	0	1	1	1	1	
	40000	50000	45000	50000	45000	

	w1	w2	w3	
p1	800	1000	1300	0
p2	700	500	800	200
p3	800	600	700	300
p4	500	600	800	200
p5	700	600	1000	400
	c1	c2	c3	c4
w1	40	70	90	50
w2	70	30	60	80
w3	80	60	50	60

customer	c1	c2	c3	c4
demand	200	300	250	350
	200	300	250	350

	1400	300000	40000	30000
	0	1	0	
	w1	w2	w3	
p1	0	0	0	
p2	0	200	0	
p3	0	300	0	
p4	0	200	0	
p5	0	400	0	
	0	1400	0	

	c1	c2	c3	c4
w1	0	0	0	0
w2	200	300	250	350
w3	0	0	0	0

final objective func value

min
936000

equal constraints
0
1100
0

	z1	z2	z3		m1	m2	$m_1 + m_2$
	0	100	1400				
y11	1	0	0	1	1	1	0
y12	1	0	0	1	1	1	0
y13	1	0	0	1	1	1	0
y14	1	0	0	1	1	1	0
f1(y11)	0	4000	43000	0			
f2(y12)	0	7000	46000	0			
f3(y13)	0	9000	48000	0			
f4(y14)	0	5000	44000	0			