

### 3 - combinational circuits - analysis

$$\text{combinational circuit} = \text{input variables} + \text{logic gates} + \text{output variables}$$

☆ storage elements of previous inputs and feedback paths are in sequential circuits

analysis

## literal analysis

from the schema and the truth table  $\rightarrow$  we assign values to the inputs

↳ trace the results

↳ repeat until reach output that matches with the output in the truth table

## Symbolic analysis

from the schema  $\rightarrow$  truth table  
 $\rightarrow$  logic expression

→ we assign expression instead of values

↳ we get the output function and expression

↳ we construct the truth table as we go

☆ symbolic analysis is more work but gives us complete information.

standard (canonical) expression forms

① sum of products:

sum form

disjunctive normal form

or of and terms

$$F[A, B, C] = (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (\bar{A} \cdot B \cdot C) + (A \cdot B \cdot \bar{C}) + (A \cdot B \cdot C)$$

minterms

each minterm  $\rightarrow$  one '1' in the truth table

② product of sums:

product form

conjunctive normal form

and of or terms

$$F[A, B, C] = (A + B + \bar{C}) \cdot (A + \bar{B} + C) \cdot (\bar{A} + B + C) \cdot (\bar{A} + B + \bar{C})$$

maxterms

each maxterm  $\rightarrow$  one '0' in the truth table

☆ in both forms, each 1st level operator corresponds to one row of truth table

- ☆ each variable appears exactly once

example =  $F(A, B, C) = ?$

	F	using minterms (m)
0	1	$\bar{A} \cdot \bar{B} \cdot \bar{C} \quad m_0$
1	0	
2	0	
3	1	$\bar{A} B C \quad m_3$
4	0	
5	0	
6	1	$A B \bar{C} \quad m_6$
7	1	$+ A B C \quad m_7$

using maxterms (M)

$$A + B + \bar{C} \quad M_1$$

$$A + \bar{B} + C \quad M_2$$

$$\bar{A} + B + C \quad M_4$$

$$\bar{A} + B + \bar{C} \quad M_5$$

$$F(A, B, C) = (\bar{A} \cdot \bar{B} \cdot \bar{C}) + (\bar{A} B C) + (A B \bar{C}) + (A B C) = (A + B + \bar{C}) \cdot (A + \bar{B} + C) \cdot (\bar{A} + B + C) \cdot (\bar{A} + B + \bar{C})$$

$$m_0 + m_3 + m_6 + m_7 = M_1 \cdot M_2 \cdot M_4 \cdot M_5$$

$$\sum m(0, 3, 6, 7) = \prod M(1, 2, 4, 5)$$

### Karnaugh map (k-map) minimization

- ★ application of adjacency
- ★ guarantees a minimal expression

adjacent terms = differ in one variable

AB \ CD	00	01	11	10
00				
01				
11				
10				

ABCD

- group size is a power of 2, and they are rectangular (1, 2, 4, 8)

★ the inputs are always arranged in Gray code sequence

★ it is simply rearranged truth table

1's are grouped

0's are grouped

expression form:

$$abc + ab\bar{c} + a\bar{b}c$$

$$(m+n+k) \cdot (m+\bar{n}+k) \cdot (\bar{m}+\bar{n}+k)$$

variable value is 1 :

A

$\bar{A}$

variable value is 0 :

$\bar{B}$

B

★ if variable changes within the group do not include in both

★ they both will give the same answer.

implicants = single cells or groups that could be part of a larger group

prime implicant = a group that is as large as possible

1	1
0	0

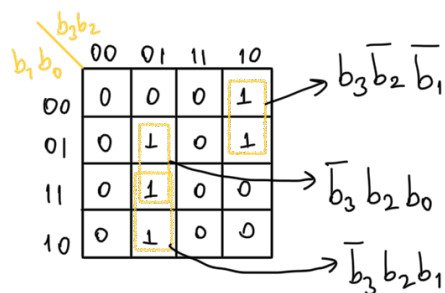
essential prime implicant = there is at least single 1 which cannot be combined in any other way

secondary / nonessential prime implicant = a prime implicant that is not essential

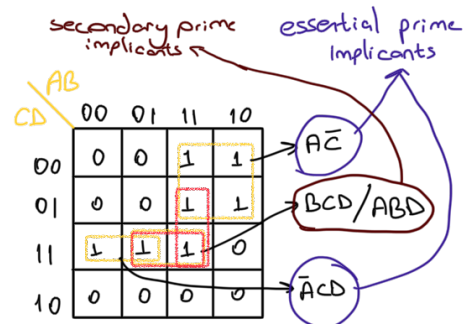
☆ the smallest set of prime implicants that covers all values forms a minimal expression for the desired function

↳ there may be more than one minimal set

examples

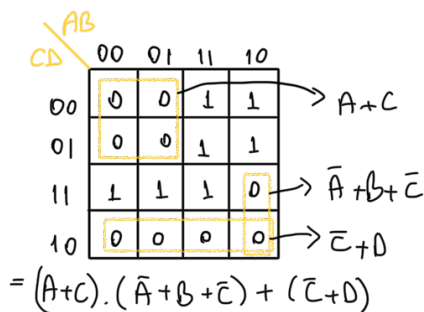


$$= b_3\bar{b}_2\bar{b}_1 + \bar{b}_3b_2b_0 + \bar{b}_3b_2b_1$$

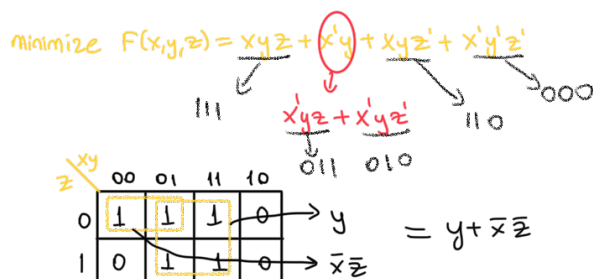
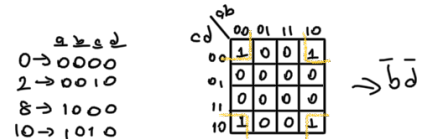


$$= \bar{A}\bar{C} + \bar{A}CD + BCD$$

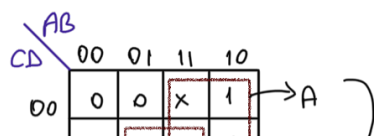
$$= \bar{A}\bar{C} + \bar{A}CD + ABD$$



minimize  $F(a,b,c,d) = \sum m(0,2,8,10)$



don't cares = - / x can be assigned 0 or 1



01	0	1	x	1	→ BD
11	0	1	x	x	→ BC
10	0	1	x	x	

} = A + BD + BC