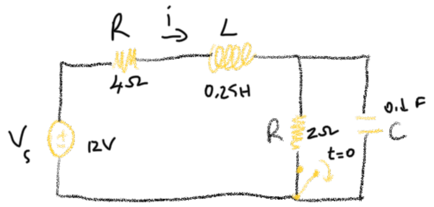
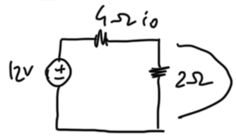
second order circuits

RLC circuits



$t<0$



$\frac{12}{6} = 2i$

$V_0 = 4V$

birden değişmezler

$L \to i(0^-) = i(0^+) = i(0)$

$C \to V(0^-) = V(0^+) = V(0)$

$\boxed{L}$

$I_L(0) = 2$

$I_\infty(0) = 0$

$\frac{dI(0)}{dt} = 0\frac{A}{s}$

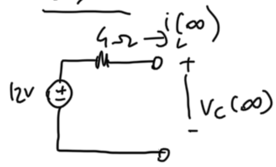
$\boxed{C}$

$V_c(0) = 4V$

$V_c(\infty) = 12V$

$\frac{dV(0)}{dt} = 20\frac{V}{s}$

$t>0$



when $t=0$ (L ve C dengeye ulaşmadan kısa an)

$\boxed{i_c = C\frac{dV_c}{dt}}$

$\boxed{V_L = L\frac{di_L}{dt}}$

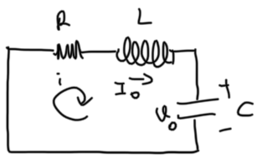
$2 = 0.1\frac{dV_c}{dt}$

$\frac{V_c}{dt} = 20\frac{V}{s}(t=0)$

$\frac{di_L}{dt} = 0\frac{A}{s}(t=0)$

natural response (without v)

L-C in series



$V_0 = V(0) = \frac{1}{C}\int_{-\infty}^{0} i(t)\,dt$

$i(0) = I_0$

$Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt = 0$

$\hookrightarrow \frac{di(0)}{dt} = -\frac{1}{L}(RI_0 + V_0)$

$\left(Ri + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt\right)' = 0 \quad dt$

$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i(t)}{C} = 0$

$\Downarrow$

$i'' + \frac{R i'}{L} + \frac{i}{LC} = 0 \quad \rightarrow$ assumption: $i(t) = Ae^{st} \rightarrow Ae^{st}\underbrace{\left(s^2 + \frac{R}{L}s + \frac{1}{LC} = 0\right)}_{\text{char eq.}}$

$s^2 + 2as + w_0^2 = 0$ ←
$\underbrace{a = \frac{R}{2L}}_{\text{damping factor (sönümleme)}}$ $\underbrace{w_0 = \frac{1}{\sqrt{LC}}}_{\text{resonance frequency}}$ roots s_1, s_2

natural frequency

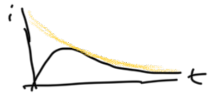
$$\boxed{i = A_1 e^{s_1 t} + A_2 e^{s_2 t}}$$

to find A_1 and A_2 put $i(0)$ and $\frac{di(0)}{dt}$

roots are negative

$a > w_0 \Rightarrow$ overdamped $\Delta > 0$ $\left(C > \frac{4L}{R^2}\right)$

$t \to \infty$
$i(t) = 0$

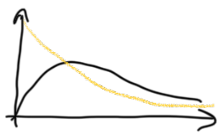


(en hızlı tepki

$a = w_0 \Rightarrow$ critically damped $\Delta = 0$ $\left(C = \frac{4L}{R^2}\right)$

$$i = A_1 e^{s_1 t} + A_2 e^{s_1 t} \cdot t$$

$t \to \infty$
$i(t) = 0$

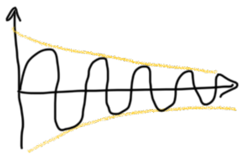


(sonra bu

$a < w_0 \Rightarrow$ underdamped $\Delta < 0$

$$s = a \pm bi$$

$$i = e^{at}(A_1 \cos bt + A_2 \sin bt)$$



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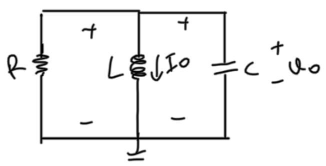
L-C in parallel



$$\left.\begin{aligned} i(0) &= I_0 = \frac{1}{L}\int_{-\infty}^{0} v(t)\,dt \\ v(0) &= V_0 \end{aligned}\right\} \quad \frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt + C\cdot\frac{dv}{dt} =$$

$$\hookrightarrow \frac{dv(0)}{dt} = -\frac{(V_0 + RI_0)}{RC}$$

$$\left(\frac{v}{R} + \frac{1}{L}\int_{-\infty}^{t} v(t)\,dt + C\cdot\frac{dv}{dt}\right)(0)'$$

$$\frac{dv}{dt}\cdot\frac{1}{R} + \frac{1}{L}\cdot v + C\frac{d^2v}{d^2t} = 0$$

$$v'' + \frac{1}{RC}v' + \frac{v}{LC} = 0 \longrightarrow \text{char eq } s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

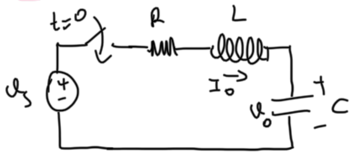
$\alpha: \frac{1}{2RC}$

$w_0 = \frac{1}{\sqrt{LC}}$

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

step response

L-C in series



$$V_s = Ri(t) + L\cdot\frac{dv(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

$$\left(i(t) = C\,\frac{dv(t)}{dt}\right)$$

$$\underbrace{v'' + \frac{R}{L}v' + \frac{v}{LC}}_{\text{char eq is same}} = \frac{V_s}{LC}$$

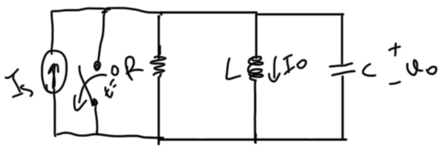
$$v(t) = \underbrace{V_{ss}(t)}_{\substack{\text{steady state}\\ t=\infty}} + \underbrace{v_t(t)}_{\text{doğal tepki}}$$

$\boxed{t \to \infty}$ son gerilim

$V_c(\infty) = V_s$

$$v(t) = V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

L-C in parallel



$$\frac{v}{R} + i + C\frac{dv}{dt} = I_s$$

$$\left(v(t) = L\,\frac{di(t)}{dt}\right)$$

$$\rightarrow\ i'' + \frac{1}{RC}i' + \frac{1}{LC} = \frac{I_s}{LC}$$

$$i(t) = \underbrace{i_t(t)}_{\text{doğal}} + \underbrace{i_{ss}(t)}_{\text{kalıcı durum}}\ I(\infty)$$

$$I(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + I_s$$

char eq

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$\alpha: \frac{1}{2RC} \qquad w_0 = \frac{1}{\sqrt{LC}}$$