

CENG 280

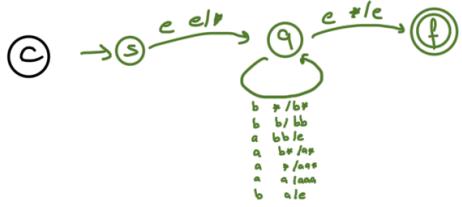
Fundamental Structures for Computation

Spring 2021-2022

Homework 2

Regulations

1. The homework is **due by May 14th 2022, 23:59. Late submission is not allowed.**
2. This homework consists of two parts: Graded Questions and Self-Study Questions. As the name implies, Self-Study Questions will not be graded (but you are recommended to solve them). Therefore, **please only submit answers to graded questions.**
3. **Submissions must be made via ODTUClass.** Do not send your homework via e-mail, or do not bring any hardcopy.
4. You can use any typesetting tool (LaTex, Word, etc.) or handwriting while writing the homework. However, **you must upload your solutions as a pdf file.** Other formats will not be considered for grading.
5. Name pdf files you will submit as **<yourstudentid>_hw2** (e.g. **2345678_hw2.pdf**). Write your name and student id number at the top of the page In case you violate the naming convention, you will receive a penalty of 5 points (over 100).
6. Send an e-mail to **garippler@metu.edu.tr** if you need to get in contact.
7. **This is an individual homework, which means you have to answer the questions on your own.** Any contrary case will be considered as cheating and university regulations about cheating will be applied.



Graded Questions

$$\textcircled{a} \bullet S \rightarrow S a S b S b S | S b S a S b S | S b S b S a S | e \\ \bullet S \rightarrow a S b S b | b S a S b | b S b S a | S S | e$$

$$\textcircled{c} L_1 = (\mathcal{L}, \Sigma, \Gamma, \Delta, s, F)$$

Question 1 $\textcircled{b} S \rightarrow a S b | a S b b | e$

- a. Write a context-free grammar for the language

$$L_1 = \{w \mid w \in \{a, b\}^* \wedge w \text{ has twice as many } b's \text{ as } a's\}$$

- b. Write a context-free grammar for the language

$$L_2 = \{a^n b^m \mid m, n \in \mathbb{N} \wedge m \leq n \leq 2m\}$$

- c. (*) Formally define and draw a PDA that accepts L_1 .

- d. (**) Write a context-free grammar for the language $L_3 = L_1 \cup L_2$.

$$\begin{aligned} S &\rightarrow A \mid B \\ A &\rightarrow aAbAb \mid bAaAb \mid bAbAa \mid AA \mid e \\ B &\rightarrow aBb \mid bBb \mid e \end{aligned}$$

* Use the construction given at the proof of the Lemma 3.4.1 in your textbook.

** Use the construction given at the proof of the Theorem 3.5.1 in your textbook.

Question 2

Given $G_1 = \{V, \Sigma, R, S\}$ where $V = \{S, A, 0, 1\}$, $\Sigma = \{0, 1\}$, and $R = \{S \rightarrow AS \mid e, A \rightarrow A1 \mid 0A1 \mid 01\}$

$$\begin{aligned} S &\rightarrow AS \rightarrow A \rightarrow 0A1 \rightarrow 0A11 \rightarrow 00111 \\ S &\rightarrow AS \rightarrow A \rightarrow A1 \rightarrow 0A11 \rightarrow 00111 \end{aligned}$$

- a. Show that G_1 is ambiguous.

- b. Give an unambiguous grammar for $L(G_1)$. (i.e. disambiguate the given grammar.) $\begin{array}{l} \text{a} = \sum S \rightarrow AS \mid e, A \rightarrow 0A1 \mid 0, 0 \rightarrow 01 \mid 01 \\ \text{b} = \sum S \rightarrow AS \rightarrow 0A1 \rightarrow 0111 \end{array}$

- c. Give the leftmost derivation of the string 00111 from the grammar you have constructed at part-b

and draw the corresponding parse tree.

$$\begin{array}{l} \text{a} = \sum S \rightarrow AS \mid e, A \rightarrow 0A1 \mid 0, 0 \rightarrow 01 \mid 01 \\ \text{b} = \sum S \rightarrow AS \rightarrow 0A1 \rightarrow 0111 \end{array}$$

Self-Study Questions

Exercise 1

For the string $abbbaabb$, show an accepting computation of the PDA you have drawn at Question 1-c.

Exercise 2

Use the strong version of the pumping theorem to prove that the language $L = \{a^{n!} \mid n \geq 0\}$ is not context-free.

Exercise 3

Show that $L = \{\omega \in \{a, b, c\}^* \mid \omega \text{ has equal numbers of } a's, b's, \text{ and } c's\}$ is not context-free.

(Hint: Use the fact that intersection of a context-free language with a regular language is context-free.)