10 - linear independence

$\boxed{\text{linear independence}}$

$v_1, v_2, v_3 ..$ are linearly independent iff $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + .. = 0$ has only solution: $c_1 = c_2 = c_3 = .. = 0$

ex: $\vec{v}_1 = \begin{bmatrix}1\\0\\-1\end{bmatrix}_{c_1}$ $\vec{v}_2 = \begin{bmatrix}0\\1\\1\end{bmatrix}_{c_2}$ $\vec{v}_3 = \begin{bmatrix}0\\1\\0\end{bmatrix}_{c_3}$ $\rightarrow$ $c_1 = 0 \rightarrow c_2 = 0,\ c_3 = 0$

$c_2 + c_3 = 0$

$-c_1 + c_2 = 0$

$\begin{bmatrix}1&0&0\\0&1&1\\-1&1&0\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \rightarrow$ linearly independent ✓

$$\underbrace{\begin{array}{ccc} \vec{v}_1 & \vec{v}_2 & \\ \left[\begin{matrix} v_{11} & v_{21} & \cdots \\ v_{12} & v_{22} & \cdots \\ \vdots & \vdots & \vdots \end{matrix}\right. & & \left.\right] \end{array}}_{X} \rightarrow \text{linearly independent} \iff \det(X) \neq 0$$

ex: $e^t, e^{-t}, 1 \rightarrow \quad c_1 e^t + c_2 e^{-t} + c_3 = 0$ give values to t to construct 3×3 matrix.

$$\begin{array}{lc} t=0 & c_1 + c_2 + c_3 = 0 \\ t=1 & c_1 e + \frac{c_2}{e} + c_3 = 0 \\ t=-1 & \frac{c_1}{e} + c_2 e + c_3 = 0 \end{array} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ e & \frac{1}{e} & 1 & 0 \\ \frac{1}{e} & e & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & \frac{1}{e}-e & 1-e & 0 \\ 0 & 0 & 2-e-\frac{1}{e} & 0 \end{array}\right]$$

$\rightarrow$ they are independent (only solution: $c_1 = c_2 = c_3 = 0$)

another way to test functions indepence $\rightarrow$ Wronskian determinant $\neq 0$

$$\begin{bmatrix} e^t & e^{-t} & 1 \\ (e^t)' & (e^{-t})' & 1' \\ (e^t)'' & (e^{-t})'' & 1'' \end{bmatrix} = \begin{bmatrix} e^t & e^{-t} & 1 \\ e^t & -e^{-t} & 0 \\ e^t & e^{-t} & 0 \end{bmatrix} \Rightarrow$$

det $= 2 \neq 0$

so they are independent