9 - parameter estimation

method of moments

kth population moment = MK = E(Xk)

I'm sample moment = ME = 1 5 xk (estimates from the sample X1, X2-,Xn)

central moments

 k^{th} population certral moment = $M_k = E(X-\mu_1)^k$ $(k \ge 2)$ $(k=1 \Rightarrow 0)$ k^{th} sample central moment = $M_k = \frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^k$

Ly first sample moment = the sample mean -X Is second population central noment = variance V(x) (instead)



· Simply mathing the first & paratemets of population and sample.

ex: in poisson \rightarrow one parameter $M_1 = E(x) = \lambda$ Lythus, one equation: $\mu_1 = \lambda = m_1 = x$

exin gamma > or and x,

2 equations
$$M_1 = E(x) = \frac{\alpha}{\lambda} = m_1$$
 $\alpha = \frac{m_1}{m_2} = 3.4227$
 $M_2 = Var(x) = \frac{\alpha}{\lambda^2} = \frac{m_2}{\lambda^2}$ $\alpha = \frac{m_1}{m_2} = 0.0710$

$$m_1 = \overline{X} = 48.233$$
 $m_2' = 5^2 = 679.7122$
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$$M_2 = Vor(x) = \alpha = m_2$$

$$\hat{\gamma} = m_1/m_2 = 0.0710$$

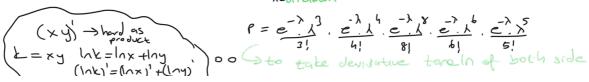
ex. cdf $F(x) = 1 - \left(\frac{x}{x}\right)^{-\theta}$ two paracters $\frac{1}{2} \int_{0}^{\pi} f(x) = F'(x) = \frac{1}{2} \int_{0}^{\pi} f(x) dx$ $M_1 = E(x) = \int_0^x x f(x) dx = \frac{\partial G}{\partial x^2} = m_1$ $M_2 = E(x^2) = \int_0^x x^2 f(x) dx = \frac{\partial G}{\partial x^2} = m_2$ $\int_0^x m_0 m = \sqrt{\frac{m_2}{m_2 - m_1^2}} + 1$ $\int_0^x m_0 m = \sqrt{\frac{m_2}{m_2 - m_1^2}} + 1$

method of maximum likehood

maximum likelihood estimator = the parameter value that maximize the likelihood of the observed sample

discrete distribution = maximize the joint pmf of data P(X1, ..., Xn) continuous distribution = maximize the point density of (x1, ..., Xn)

Sample: 3 4 8 6 5 $\rightarrow P(x=3) \cdot P(x=4) \cdot P(x=6) \cdot P(x=5)$ (indepted) $A = un k_1 u u u u$



$$P\{x=(x_1, x_2, ... x_n)\} = P(x) = \prod_{i=1}^{n} P(x_i) \quad \frac{\text{to } maximize}{\text{likehood}} \text{ derivative } \longrightarrow \underbrace{\frac{\partial P(x)}{\partial D}} = 0$$

$$\lim_{i=1}^{n} P(x_i) = \sum_{i=1}^{n} \ln P(x_i) \quad \frac{\text{telke}}{\text{derivative } make} \quad \text{find unbown paperses}$$

estimation of standard errors

$$\sigma(\hat{\lambda}) = \int V_{Ar}(\hat{\lambda}) = \int E(\hat{\lambda}^2) - E^2(\hat{\lambda})$$

$$\downarrow \qquad \qquad \downarrow$$

$$k=2 \qquad \qquad \downarrow = 1$$

var(x) = E(x) - E(y)