

### 3- monty hall problem

Your Choice	Prize Door	Don't change	change	opened door
1	1	+	-	2/3
1	2	-	+	3
1	3	-	+	2
2	1	-	+	3
2	2	+	-	1/3
2	3	-	+	1
3	1	-	+	2
3	2	-	+	1
<u>3</u> C	<u>3</u> X	<u>+</u> 1/3	<u>-</u> 2/3	<u>1/2</u> M

$$P(X=x | C=c, M=m)$$

$m=2$   $c=1$  choice is 1

$$P(M=2 | X=1) = 0.5$$

probabilities of monty open door 2, if prize is in 1, 2, 3

$$P(X=3 | M=2) = ? \quad P(X=1 | M=2) = ?$$

probability of prize is in 3 when monty opens 2, and you chosen 1:

$$P(X=3 | M=2) = \frac{P(X=3, M=2)}{P(M=2)} = \frac{P(M=2 | X=3) \cdot P(X=3)}{\sum_{x=1}^3 P(M=2 | X=x) \cdot P(X=x)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

also consider choice is 1

$$P(M=2 | X=1) \cdot P(1) + P(M=2 | X=2) \cdot P(2) + P(M=2 | X=3) \cdot P(3)$$

$$\frac{1}{2} \cdot \frac{1}{3} + 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

winning probability if change

probability of prize is in 1 when monty opens 2, and you chosen 1:

$$P(X=1 | M=2) = \frac{P(X=1, M=2)}{P(M=2)} = \frac{P(M=2 | X=1) \cdot P(X=1)}{\frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$$

winning probability if not change

```
num_games <- 1000000
prizes <- sample(c(1, 2, 3), num_games, replace = TRUE)
picks <- sample(c(1, 2, 3), num_games, replace = TRUE)
games <- data.frame(pick = picks, prize = prizes)
```

```
monty <- function(x) {
  doors <- c(1, 2, 3)
  choices <- setdiff(doors, x) //finds different
  elemets between two sets
  if (length(choices) == 1) {
    choices # There really is no choice
  } else {
    sample(choices, 1) # Pick one of two choices
  }
}
```

```
games["monty"] <- apply(games, 1, monty)
```

```
switched <- function(x) {
  setdiff(c(1, 2, 3), c(x["pick"], x["monty"]))
}
```

```
new_pick <- apply(games, 1, switched)
games["ifswitch"] <- new_pick

sum(games$pick == games$prize) / num_games
sum(games$ifswitch == games$prize) / num_games
```