16 - homogenous equations with constant coefficients

$y^{(n)} + a_1 y^{(n-1)} + \dots + a_n y = 0$ ($a_1, a_2 \dots a_n$ are constants)

$P(\lambda) = 0$ is characteristic equation • $y^{(n)} = \lambda^n$ • $y' = \lambda^1$ • $y = 1$

$\boxed{\text{distinct eigenvalues}}$ → real and complex

$y = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} + \dots + c_n e^{\lambda_n t}$

ex: $y'' - y = 0 \longrightarrow \lambda^2 - 1 = 0 \longrightarrow y = c_1 e^{t} + c_2 e^{-t}$ (real)

$\lambda = 1, -1$

$\lambda = a \mp ib$ $y \begin{cases} e^{at} \cdot \cos(bt) \\ e^{at} \sin(bt) \end{cases}$ $\longrightarrow e^{(a+bi)t} = e^{at} \cdot e^{bit} = e^{at}\left[\cos(bt) + i\sin(bt)\right]$

ex: $y^{(4)} + y = 0 \longrightarrow \lambda^4 + 1 = 0 \longrightarrow \left.\begin{array}{l} \lambda_{1,2} = \frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}t \\ \lambda_{3,4} = -\frac{\sqrt{2}}{2} \pm \frac{\sqrt{2}}{2}t \end{array}\right\}$ $y = e^{\frac{\sqrt{2}}{2}t}\left[c_1 \cos\left(\frac{\sqrt{2}}{2}t\right) + c_2 \sin \frac{\sqrt{2}}{2}t \dots\right.$

$\boxed{\text{repeated roots}}$

$y = (c_1 + c_2 t + c_3 t^2 + \dots + c_n t^{n-1}) e^{\lambda t}$

ex: $y'' - 2y' + y = 0$, $y(0) = 1$, $y'(0) = -1 \rightarrow \lambda^2 - 2\lambda + 1 = 0$ $c_1 e^{t} + c_2 \cdot t e^{t} = 0$

$(\lambda - 1)^2 = 0$ $c_1 = 1$ $c_2 \rightarrow -2$

$y = e^{t} - 2te^{t}$