IE 407 - FUNDAMENTALS OF OR | FALL 2022

HOMEWORK 3

Due: 17.00 on January 3rd, 2023

YumChips has two top selling flavors 'Sweet Corn' and 'Barbeque'. Due to the market search of the marketing team, the firm realized that there is a demand for a new 'Mix of Bests' chips which will be a mixed product of a combination of 'Sweet Corn' and 'Barbeque' flavored chips. One package (100 g) of 'Sweet Corn' flavored chips contains 1.5 g protein and 400 calories. One package (100 g) of 'Barbeque' flavored chips contains 3.5 g protein and 600 calories. YumChips' marketing department has decided that each package (100 g) of 'Mix of Bests' must have at most 550 calories, at least 2.5 g protein, and the grammage difference of two types must be less than or equal to 35 g. It costs YumChips 5¢ to produce 10 g of 'Sweet Corn' flavored chips and 7¢ to produce 10 g of 'Barbeque' flavored chips.

1. Formulate a linear programming model to determine how YumChips can meet the marketing department's requirements at minimum cost and solve the model by using the Simplex method.
2. Solve the model formulated in part (1) by using a software to verify your solution in part (1).
3. What if the required protein amount per package of 'Mix of Bests' is at least 2.75 g? Solve by both Simplex method and a software program.
4. Find the shadow price of the protein constraint by using the final tableau of the simplex method in part (1) first, and then by using the sensitivity analysis report of the software program in part (2).
5. Is there a shadow price (other than 0) for another constraint? If yes, find the shadow price of the constraint by using the final tableau of the simplex method in part (1) first, and then by using the sensitivity analysis report of the software program in part (2). If no, explain why.

1. Model: Dv's:

   x1= Grams of "Sweet Corn" flavored chips in the "Mix of Bests" package

   x2= Grams of "Barbeque" flavored chips in the "Mix of Bests" package

   v= Grammage difference

   LP: min z= 0.5x1 + 0.7x2

   1. x1 + x2 =100 total grammage constraint
   2. 4x1 + 6x2 <= 550 calories constraint
   3. 1.5/100x1 + 3.5/100x2 >= 2.5 protein constraint
   4. v >= x1 -x2
   5. v >= x2- x1
   6. v <= 35
   7. x1, x2, v >=0

Convert to Standard Form and add necessary artificial variables:

z - 0.5x1 - 0.7x2 = 0

1. x1 + x2 + a1 =100 total grammage constraint
2. 4x1 + 6x2 + s2 = 550 calories constraint
3. 1.5x1 + 3.5x2 -e3 + a3 = 250 protein constraint
4. v - x1 + x2 - e4 =0
5. v + x1 - x2 - e5 =0
6. v + s6 = 35
7. x1, x2, v, a1, a3, s2, e3, e4,e5,s6 >=0

For the Big M method:

new z = 0.5 x1 + 0.7 x2 + Ma1 + Ma3

1. Simplex Method:

Initialization:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS
1	-0.5	-0.7		-M			-M				0
	1	1		1							100
	4	6			1						550
	1.5	3.5				-1	1				250
	-1	1	1					-1			0
	1	-1	1						-1		0
			1							1	35

Iteration 1:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	2.5M-0.5	4.5M-0.7		0		-M	0				350M	
	1	1		1							100	100
	4	6			1						550	91.67
	1.5	3.5				-1	1				250	71.43
	-1	1	1					-1			0	0
	1	-1	1						-1		0	0
			1							1	35	-

Iteration 2:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	7M-1.2	0	0.7-4.5M	0		-M	0	4.5M-0.7			350M	
	2	0	-1	1				1			100	50
	10	0	-6		1			6			550	55
	5	0	-3.5			-1	1	3.5			250	50
	-1	1	1					-1			0	-
	0	0	2					-1	-1		0	-
			1							1	35	-

Iteration 3:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	0	0	0.1-M	0.6-3.5M		-M	0	M-0.1			60	
	1	0	-1/2	1/2				1/2			50	100
	0	0	-1	-5	1			1			50	50
	0	0	-1	-5/2		-1	1	1			0	0
	0	1	1/2	1/2				-1/2			50	

	0	0	2					-1	-1		0	
			1							1	35	

Iteration 4:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	0	0	0	0.35-M		-0.1	0.1-M	0			60	
	1	0	0	7/4		1/2	-1/2	0			50	
	0	0	0	-5/2	1	1	-1	0			50	
	0	0	-1	-5/2		-1	1	1			0	
	0	1	0	-3/4		-1/2	1/2	0			50	
	0	0	1	-5/2		-1	1	0	-1		0	
			1							1	35	

2.

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$5	Obj	58	60

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$1	v	20	0	Contin
$C$2	x1	60	50	Contin
$C$3	x2	40	50	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$11	Obj	0	$B$11<=$D$11	Not Binding	35
$B$6	Obj	100	$B$6=$D$6	Binding	0
$B$7	Obj	500	$B$7<=$D$7	Not Binding	50
$B$8	Obj	250	$B$8>=$D$8	Binding	0
$B$9	Obj	0	$B$9>=$D$9	Binding	0
$B$10	Obj	0	$B$10>=$D$10	Binding	0

3.i) Simplex Method:

Initialization:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS
1	-0.5	-0.7		-M			-M				0

	1	1		1							100
	4	6			1						550
	1.5	3.5				-1	1				275
	-1	1	1					-1			0
	1	-1	1						-1		0
			1							1	35

Iteration 1:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	2.5M-0.5	4.5M-0.7		0		-M	0				375M	
	1	1		1							100	100
	4	6			1						550	91.67
	1.5	3.5				-1	1				275	78.57
	-1	1	1					-1			0	0
	1	-1	1						-1		0	0
			1							1	35	-

Iteration 2:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	7M-1.2	0	0.7-4.5M	0		-M	0	4.5M-0.7			375M	
	2	0	-1	1				1			100	50
	10	0	-6		1			6			550	55
	5	0	-3.5			-1	1	3.5			275	60
	-1	1	1					-1			0	-
	0	0	2					-1	-1		0	-
			1							1	35	-

Iteration 3:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	0	0	0.1-M	0.6-3.5M		-M	0	M-0.1			60+25M	
	1	0	-1/2	1/2				1/2			50	100
	0	0	-1	-5	1			1			50	50
	0	0	-1	-5/2		-1	1	1			25	25
	0	1	1/2	1/2				-1/2			50	
	0	0	2					-1	-1		0	
			1							1	35	

Iteration 4:

z	x1	x2	v	a1	s2	e3	a3	e4	e5	s6	RHS	Ratio
1	0	0	0	0.35-M		-0.1	0.1-M	0			62.5	
	1	0	0	7/4		1/2	-1/2	0			37.5	
	0	0	0	-5/2	1	1	-1	0			25	
	0	0	-1	-5/2		-1	1	1			25	
	0	1	0	-3/4		-1/2	1/2	0			62.5	
	0	0	1	-5/2		-1	1	0	-1		25	25
			1							1	35	35

3.ii)

Objective Cell (Min)

Cell	Name	Original Value	Final Value
$C$5	Obj	60	62.5

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$1	v	0	25	Contin
$C$2	x1	50	37.5	Contin
$C$3	x2	50	62.5	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$11	Obj	25	$B$11<=$D$11	Not Binding	10
$B$6	Obj	100	$B$6=$D$6	Binding	0
$B$7	Obj	525	$B$7<=$D$7	Not Binding	25
$B$8	Obj	275	$B$8>=$D$8	Binding	0
$B$9	Obj	25	$B$9>=$D$9	Not Binding	50
$B$10	Obj	25	$B$10>=$D$10	Binding	0

4.i) Row 0 value of e3 which corresponds to the protein constraint.
4.ii)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$1	v	0	0	0	0.1	0
$C$2	x1	50	0	0.5	0.2	1E+30
$C$3	x2	50	0	0.7	1E+30	0.2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$11	Obj	0	0	35	1E+30	35
$B$6	Obj	100	0.35	100	14	0
$B$7	Obj	500	0	550	1E+30	50
$B$8	Obj	250	0.1	0	0	35
$B$9	Obj	0	0	0	35	0
$B$10	Obj	0	0	0	0	1E+30

5.i) Row 0 value of a1 which corresponds to the total grammage constraint. (M=0)

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$1	v	0	0	0	0.1	0
$C$2	x1	50	0	0.5	0.2	1E+30
$C$3	x2	50	0	0.7	1E+30	0.2

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$11	Obj	0	0	35	1E+30	35
$B$6	Obj	100	0.35	100	14	0
$B$7	Obj	500	0	550	1E+30	50
$B$8	Obj	250	0.1	0	0	35
$B$9	Obj	0	0	0	35	0
$B$10	Obj	0	0	0	0	1E+30

5.ii)