

chinese remainder theorem

$$\begin{aligned}x &\equiv 1 \pmod{3} \\x &\equiv 4 \pmod{5} \\x &\equiv 6 \pmod{7}\end{aligned}$$

starting with greatest mod

$$\begin{aligned}x &= 7k + 6 \\7k + 6 &\equiv 4 \pmod{5} \\2k + 1 &\equiv 4 \pmod{5} \\2k &\equiv 3 \pmod{5} & k \rightarrow 4 \\k &\equiv 4 \pmod{5} \\k &= 5i + 4\end{aligned}$$

$$x = 7(5i + 4) + 6 = \frac{35i + 34}{x} \quad x \equiv 34 \pmod{35}$$

$$\begin{aligned}35i + 34 &\equiv 1 \pmod{3} \\2i + 1 &\equiv 1 \pmod{3} \\2i &\equiv 0 \pmod{3} & i \rightarrow 0 \\i &\equiv 0 \pmod{3} \\i &= 3m\end{aligned}$$

$$\begin{aligned}x &= 35(3m) + 34 \\&= 105m + 34 \quad x \equiv 34 \pmod{105}\end{aligned}$$

$$\begin{aligned}x &\equiv 2 \pmod{3} \\x &\equiv 3 \pmod{5} \\x &\equiv 2 \pmod{7}\end{aligned}$$

$$\begin{aligned}x &= 7k + 2 \\7k + 2 &\equiv 3 \pmod{5} \\2k &\equiv 1 \pmod{5} \\k &\equiv 3 \pmod{5} \\k &= 5i + 3 \\x &= 7(5i + 3) + 2 \\x &= 35i + 23 \quad x \equiv 23 \pmod{35}\end{aligned}$$

$$\begin{aligned}35i + 23 &\equiv 2 \pmod{3} \\2i + 2 &\equiv 2 \pmod{3} \\2i &\equiv 0 \pmod{3} \\i &\equiv 0 \pmod{3} \\i &= 3m\end{aligned}$$

$$\begin{aligned}35(3m) + 23 &= x \\105m + 23 &= x \quad x \equiv 23 \pmod{105}\end{aligned}$$