MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

CENG 384

Spring 2022
Homework 3

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Submission:** You need to submit a pdf file named 'hw3.pdf' to the odtuclass page of the course. You need to use the given template 'hw3.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, 15 May, 2022 (Sunday).
- **Late Submission:** Not allowed.

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1. (20 pts) Find the spectral coefficients of the Fourier series representation for the following signals.
   (a) (10 pts) $x(t) = \sin(\frac{\pi}{5}t) + \cos(\frac{\pi}{4}t)$
   (b) (10 pts) $x[n] = \frac{1}{2} + e^{j\pi n} + \sin 4\pi n + \cos(2\pi n)$

2. (20 pts) $x[n]$ is a real valued periodic signal with fundamental period $N = 7$. The nonzero Fourier series coefficients are given below.

$$a_1 = a_{-1}^* = 2j, a_2 = a_{-2} = 2, a_3 = a_{-3}^* = 2j$$

   Express $x[n]$ in the form,

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k)$$

3. (20 pts) Consider the following periodic signals,

$$x(t) = \sin(\frac{\pi}{8}t)$$
$$y(t) = \cos(\frac{\pi}{8}t)$$
$$z(t) = x(t)y(t)$$

   (a) (5 pts) Determine Fourier series coefficients for $x(t)$.
   (b) (5 pts) Determine Fourier series coefficients for $y(t)$.
   (c) (10 pts) Determine Fourier series coefficients for $z(t)$ using results of parts a and b (use multiplication property).

4. (20 pts) Specify the signal $x(t)$, that satisfies the following conditions:
   i) $x(t)$ is real and odd periodic signal with $T = 4$,
   ii) $a_k = 0$ for $|k| > 2$,
   iii) $a_2 = 3j$,
   iv) $\frac{1}{4}\int_0^4 |x(t)|^2 dt = 18$.

5. (20 pts) Consider the following periodic input output pair of a discrete time LTI system:

$$x[n] = \begin{cases} 1, & \text{for } 0 \le n \le 4 \\ 0, & \text{for } 5 \le n \le 8 \end{cases}$$

$$y[n] = \begin{cases} 1, & \text{for } 0 \le n \le 3 \\ 0, & \text{for } 4 \le n \le 8 \end{cases}$$

   where $x[n] = x[n+9]$ and $y[n] = y[n+9]$.
   (a) (6 pts) Find the spectral coefficients of $x[n]$.
   (b) (6 pts) Find the spectral coefficients of $y[n]$.
   (c) (8 pts) Find the frequency response of this system.