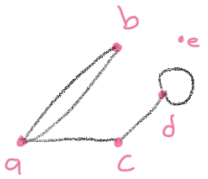
graphs

type	edges	multiple edges allowed	loops allowed
simple	undirected	-	-
multigraph	undirected	+	-
pseudograph	undirected	+	+
simple directed	directed	-	-
directed multigraph	directed	+	+
mixed	both	+	+

multigraph = can have multiple edges conneting the same two vertices



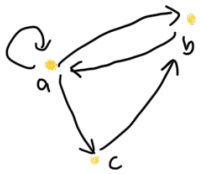
$\deg(a) = 3$ $N(a) = \{b, c\}$
$\deg(d) = 3$ $N(d) = \{c, d\}$
$\deg(e) = 0$ $N(e) = \emptyset$

- sum of all degrees = 2 x edges (undirected graph)

graph's → degree (greatest among its vertices degrees) → 3
↳ order (number of nodes) → 5 $|V(G)|$
↳ size (number of edges) → 5 $|E(G)|$ in simple graphs: $q \leq \binom{p}{2}$

pendant vertex:  degree of 1
odd vertex: odd number of degree
even vertex: even number of degree

* an undirected graph has an even number of vertices of odd degree
↳ there cannot be a graph which has 5 vertices, and each vertex have degree 3.



$\deg^-(a) = 2$
$\deg^+(a) = 3$

planar graph: graph kesişmeyen hatlardan oluşacak biçimde yazılabiliyor ise.

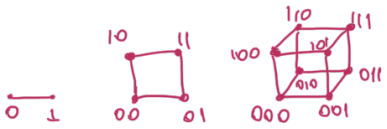
complete graph: contains exactly one edge between each pair of distinct vertices

cycle: consists of n vertices and n edges ($n \geq 3$)

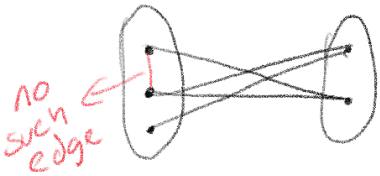
wheel: is obtained by adding an additional vertex to a cycle, and connecting this new vertex to each of the vertices

n-cubes

n-dimentional hypercube, or n-cube. is a graph with 2^n vertices representing all bit strings of length n, where there is an edge between two vertices that differ in exactly one bit position



bipartite graph:
if vertex set can be divided into two sets such that there is no edge inside the subsets.



simple path: does not contain an edge more than once.

euler path: a path that contains every edge of a graph exactly once

↓

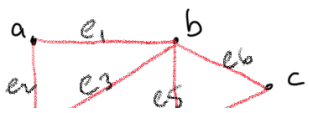
☆ a graph with euler path has exactly two vertices of odd degree (iff)

euler circuit: when euler path ends at the starting node.

hamilton path: a path that passes through each vertex exacly one.

hamilton circuit: when hamilton path ends at the starting node.

example



degrees

$\deg(a) = 2$
$\deg(b) = 4$
$\deg(c) = 2$
$\deg(d) = 3$
$\deg(e) = 3$
sum: 14

adjacency matrix

	a	b	c	d	e
a	-	1	-	-	1
b	1	-	1	1	1
c	-	1	-	1	-
d	-	1	1	-	1
e	1	1	-	1	-

number of non-zero entries: 14

(raslantı)
incidence matrix

	e_1	e_2	e_3	e_4	e_5	e_6	e_7
a	1	1	-	-	-	-	-
b	1	-	1	-	1	1	-
c	-	-	-	-	-	1	1
d	-	-	-	1	1	-	1
e	-	1	1	1	-	-	-

number of non-zero entries: 14

isomorphic graphs

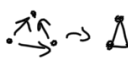
#node G_1 = #node G_2
#edge G_1 = # edge G_2
düğüm dereceleri aynı olmalı.

elemanları tek tek eşleştir.
adj matrix lei eşitse isomorfik (aynı graph yani)

definition = if there is a (one-to-one (injection) and onto) func from G_1 to G_2 (while their adjacency properties are same)

strongly connected graph = when there is a path between any two vertices (at the same time, the graph can also be weakly connec

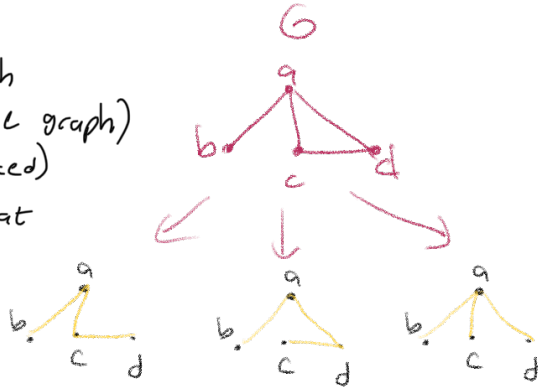
weakly connected graph = not strong but after removing directions if it is connected (underlying graph)



* the number of different paths of length r from v_i to v_j, equals to (i,j)th entry of A^r (A is the adjacency matrix)

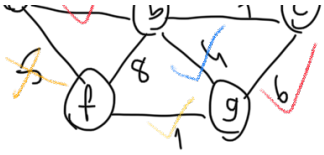
spanning tree

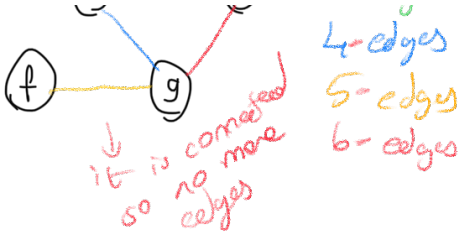
subset of G, all vertices covered with minimum possible number of edges (simple graph)
(no cycles and cannot be disconnected)
• every connected and undirected G has at least one spanning tree



to find minimum spanning tree

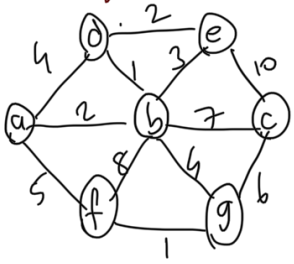
① kruskal's algorithm keep including min edges [NO CYCLES]

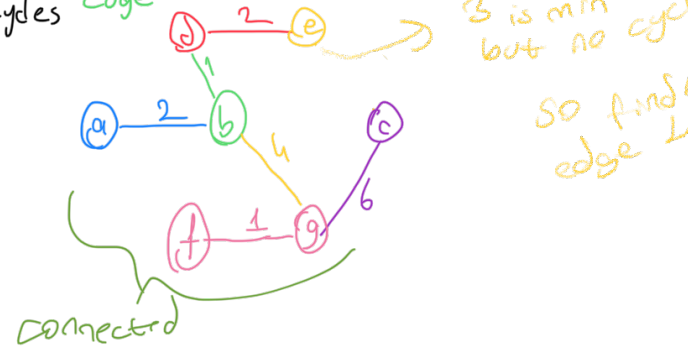




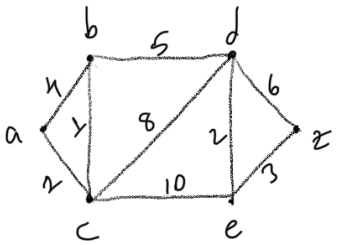
② prim's algorithm (edgeler birbirine bağlı olarak ekleniyor)

choose one ~~vertex~~ edge keep including min edges with no cycles





shortest path algorithm (dijkstra's algorithm)



a	b	c	d	e	z
0	∞	∞	∞	∞	∞
[0]	4	2	∞	∞	∞
	3	[2]	10	12	∞
	[3]		8	12	∞
			[8]	10	14
				[10]	13
					[13]

★ planar graph (haritalar) → 4 ayrı renk kullanarak çizilebilirler (en fazla)