# 8 - review of matrices

$$A = \begin{bmatrix} 1 & 2+9i & 0 \\ -1 & 3 & 7-4i \end{bmatrix}$$

transpose $A^T$ $= \begin{bmatrix} 1 & -1 \\ 2+9i & 3 \\ 0 & 7-4i \end{bmatrix}$

conjugate $\bar{A}$ $= \begin{bmatrix} 1 & 2-9i & 0 \\ -1 & 3 & 7+4i \end{bmatrix}$

adjoint $A^* = \bar{A}^T$ $= \begin{bmatrix} 1 & -1 \\ 2-9i & 3 \\ 0 & 7+4i \end{bmatrix}$

matrix product $= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 10 & 11 \\ 20 & 21 \\ 30 & 31 \end{bmatrix} = \begin{bmatrix} 1\times10+2\times20+3\times30 & 1\times11+2\times21+3\times31 \\ 4\times10+5\times20+6\times30 & 4\times11+5\times21+6\times31 \end{bmatrix} = \begin{bmatrix} 140 & 146 \\ 320 & 335 \end{bmatrix}$

$(2\times3)$ $(3\times2)$ $(2\times2)$

☆ $A(BC) = (AB)C$ but $AB \neq BA$

identity matrix $(I) = \begin{bmatrix} 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & & \ddots & 1 \end{bmatrix}_{(n\times n)}$

- $I.A = A$
- $A.I = A$

$\boxed{\text{determinants}}$

det (A) of $n\times n$ matrix: $|A|$

- $n=1 \rightarrow$ $A=[a]$ $=$ $\det(A) = a$
- $n=2 \rightarrow$ $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $=$ $\det(A) = a.d - b.c$
- $n=3 \rightarrow$ $A=\begin{bmatrix} a & b & c \\ d & e & f \\ k & m & l \end{bmatrix}$ $=$ $\det(A) = \begin{vmatrix} a & b & c \\ d & e & f \\ k & m & l \\ a & b & c \\ d & e & f \end{vmatrix}$ $= (ael + dmc + kbf) - (cek + fma + lbd)$

## invertibility

$A \times A^{-1} = I$ $\qquad$ $A^{-1}$ = inverse of A $\left.\right\}$ unique
(n×n)

☆ inverse of a matrix exists iff $\det(A) \neq 0$

- $n=1 \rightarrow$ $A=[a]$ $= A^{-1} = \frac{1}{a}$
- $n=2 \rightarrow$ $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $= A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{1}{a.d - b.c} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
- $n=3 \rightarrow$ $A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$ $\longrightarrow [A \vdots I] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 0 \\ -6 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$

$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -3 & -1 \\ 0 & 1 & 0 & -3 & -3 & -1 \\ 0 & 0 & 1 & -2 & -4 & -1 \end{array}\right] \rightarrow [I \mid A^{-1}]$

$\rightarrow A^{-1}$

☆ $(c.A)^{-1} = \frac{1}{c}.A^{-1}$

$(A.B)^{-1} = B^{-1}.A^{-1}$

## standard inner product of matrices

$(x,y) = xy^*$ $\qquad$ $x \rightarrow (1 \times n)$, $y \rightarrow (n \times 1)$ $\Rightarrow (1\times1)$ result, number $= x_1.y_1 + x_2y_2 + x_3y_3 + \ldots +$

$\hookrightarrow |x|.|y|.\cos\theta$

orthogonal = if $(x,y)=0$ $\qquad\qquad$ 9 -1 -i -3,

## row echelon form

$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

- start with 1
- below 1, they are 0

$\cancel{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$

## effect of elementary row operations on determinants

1. $cR_i \rightarrow R_i$ $\qquad \det(A) \rightarrow c.\det(A)$
2. $R_i \leftrightarrow R_j$ $\qquad \det(A) \rightarrow -\det(A)$
3. $aR_i + R_j \rightarrow R_j$ $\qquad \det(A) \rightarrow \det(A)$ (same)

☆☆ if $\det(A) \neq 0$ then $\rightarrow \det(A) \neq 0$
$\det(A) = 0$ then $\rightarrow \det(A) = 0$ $\left.\right\}$ elementary row operations do not change the invertibility of A.

→ system $Ax = b$ is homogeneous when $b = 0$

- if $A$ is invertible → one solution ($x = 0$)
- if $A$ is not invertible → infinitely many solutions (including $x = 0$)