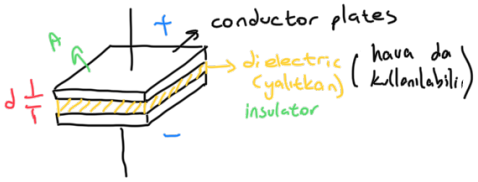
capacitors and inductors

capacitors

- static elektrik yükünü elektrik alanlarında depolarlar
- pasif devre elemanı



$$q = C \cdot V$$

(coulomb) = (farad) · (volt)

yük — kapasite — gerilim

$$C = \varepsilon \frac{A}{d}$$

ε = dielektrik sabiti

ε = permittivity C^2/Nm^2

I vs V

- $I = \frac{dq}{dt}$ · $q = CV \rightarrow \frac{dq}{dt} = C \cdot \frac{dV}{dt} \rightarrow$ $\boxed{I = C \cdot \frac{dV}{dt}}$

* gerilim değişim hızı arttığı zaman akım da artar.

→ gerilim değişimi yoksa akım sıfırdır.

$\rightarrow C_{eq}$

$C_{eq} = C_1 + C_2 + C_3$

$\rightarrow C_{eq}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$I = C \cdot \frac{dV}{dt}$

$$\int_{t_0}^{t} dV(t) = \int_{t_0}^{t} \frac{1}{C} \cdot i(t) \cdot dt$$

$$V(t) - V(t_0) = \frac{1}{C} \int_{t_0}^{t} i(t) \cdot dt$$

$$\boxed{V(t) = \frac{1}{C} \int_{t_0}^{t} i(t)\,dt + V(t_0)}$$

kapasitenin tuttuğu enerji:

$P = \frac{dE}{dt}$ $\quad P = V \cdot i = V \cdot C \cdot \frac{dV}{dt}$

$$E = \int_{-\infty}^{t} p(t) \cdot dt = \int_{V(-\infty)}^{V(t)} V \cdot C \cdot \frac{dV}{dt} dt = \frac{1}{2} C V^2 \Big|_{V(-\infty)}^{V(t)}$$

$V(-\infty) \rightarrow V(t_0)$

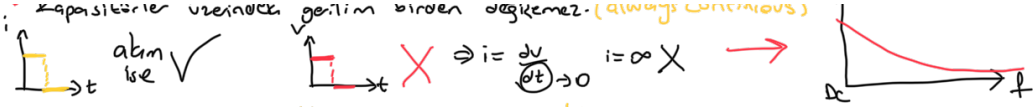
* energy stored in a capacitor is also continious

if $V(t_0) = 0$ $\quad E = \frac{1}{2} C V^2$

$= \frac{qV}{2}$ $\quad = \frac{1}{2} \frac{q^2}{C}$

- $V = DC \rightarrow i = 0$ kapasitörler DC devrelerde açık devre gibi davranır.

- kapasitörler üzerindeki gerilim birden değişemez. (always continuous)



the continuity principle

example

$C = 5\,\mu F$

$v(t) = 10\cos(6000t)\,V$

$I = ?$

$$i = C\cdot\frac{dv}{dt} = 5\mu F\cdot\frac{d}{dt}(10\cdot\cos(6000t))$$

$$= -5\cdot10^{-6}\cdot6\cdot10^{4}\cdot\sin(6000t)\,A$$

$$i(t) = -0{,}3\sin(6000t)\,A$$

example

$C = 2\,\mu F$

$i(t) = 6e^{-3000t}\,mA$

$v_0 = 0 \quad v = ?$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v_0$$

$$= \frac{1}{2\cdot10^{-6}}\cdot\int_{0}^{t} 6\cdot10^{-3}\cdot e^{-3000t}\,dt$$

$$v(t) = (1 - e^{-3000t})\,V$$

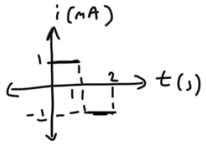
capacitor examples

① 

$C = 0.1\,mF$

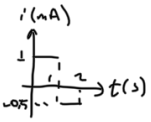
$i(t) = ?$

$$i(t) = C\cdot\frac{dv}{dt}$$



for $0 \le t \le 1$ $\quad i(t) = 0.1\cdot10^{-3}\cdot10 = 1\,mA$

for $1 \le t \le 2$ $\quad i(t) = 0.1\cdot10^{-3}\cdot(-10) = -1\,mA$

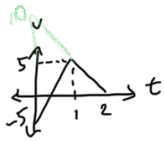
② 

$v(0) = -5V$

$C = 0.1\,mF$

$v(t) = ?$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)\,dt + v_0$$



$\frac{1}{2}\,mA$

for $0 \le t \le 1$ $\quad v(t) = 10^{4}\int_{0}^{t} i(t)\,dt - 5V = (10t - 5)\,V$

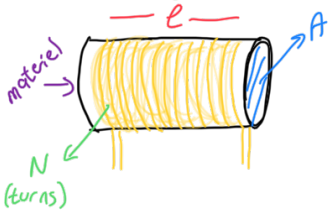
for $1 \le t \le 2$ $\quad v(t) = 10^{4}\int_{0}^{t} i(t)\,dt + 10V = -5t + 10$

$-0.5\,mA$

inductors

- elektrik enerjisini manyetik alanlarında depolarlar
- pasif devre elemanı

a inducting coil(bobin)



$$L = \frac{N^2\,\mu\cdot A}{\ell}$$

↓ inductance (henry) değeri

$$v = L\cdot\frac{di}{dt}$$

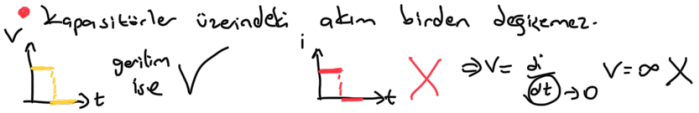
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

$$E = \frac{1}{2} L\, i^2_{(t)}$$

$$p(t) = v(t).i(t)$$

$$v = L \cdot \frac{di(t)}{dt}$$

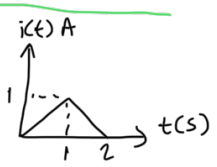
- $v = DC \rightarrow V = 0$ kapasitörler DC devrelerde kısa devre gibi davranır.
- kapasitörler üzerindeki akım birden değişemez.



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$L_{eq} = L_1 + L_2 + L_3$$

inductors examples

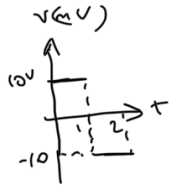
① 

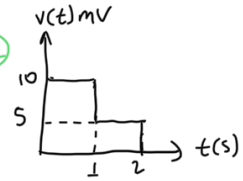
$L = 10mH$

$v(t) = ?$

$$v(t) = L \cdot \frac{di(t)}{dt}$$

$$v(t) \Rightarrow \begin{cases} 0 < t < 1 & 10.10^{-3}.1 = 10\ mV \\ 1 < t < 2 & 10.10^{-3}.-1 = -10mV \end{cases}$$



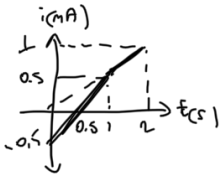
② 

$L = 10mH$

$i(0) = -0.5A$

$i(t) = ?$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$



$$i(t) \Rightarrow \begin{cases} 0 < t < 1 & \frac{10^3}{10}\int_{t_0=0}^{t} 10.dt + i(0) \rightarrow t - 0.5 \\ 1 < t < 2 & \frac{10^3}{10}\int_{t_0=1}^{t} 5dt + i(1) \rightarrow 0.5t \end{cases}$$

(mV)