5- pixels and filters

image types:
↳ binary = pixels are either black (0) or white (1)
↳ grayscale = pixels are shade of gray black [0, 255] white
↳ color = have multiple channel colors RGB, LAB, HSV . 3d tensor [0, 255] in RGB

image histograms = measure the frequency of brightness within the image
↳ how many times a particular pixel value appear in an image

images as functions = $f: [a,b] \times [c,d] \rightarrow [0, 255]$ → grayscale
$g: [x, y] = \begin{bmatrix} r[x,y] \\ g[x,y] \\ b[x,y] \end{bmatrix}$ → color

resolution = dots per inch (DPI)

filters/linear systems
↳ used to extract useful information or to adjust the visual properties of the image
↳ they are example of systems $f[m,n] \xrightarrow{S} g[m,n]$

example filters
↳ moving average = sets the value of a pixel to be the average of its neighbors
  ↳ smooth out the sharper edges $g[m,n] = \frac{1}{9} \sum_i \sum_j f[m-i]\, g[n-j]$
  ↳ blurred effect
↳ image segmentation = sets the value of a pixel either too high or too low value depending on the treshold
  ↳ $g[m,n] = \begin{cases} 255 & f[m,n] \geq 0 \\ 0 & ow \end{cases}$ divides pixel into binary classification of bright and dark regions

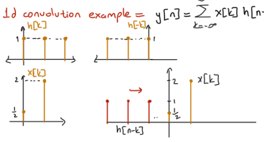
linear system = system that satisfies the property of superposition → no powers of $x(t)$
$x(t^2)$, $t\,x(t)$ → both linear

properties of systems
↳ homogeneity = $S[\alpha f(x)] = \alpha S[f(x)]$
↳ superposition = $S[f(x) + f(y)] = S[f(x)] + S[f(y)]$
↳ stability = if system's output for all possible inputs is bounded
↳ invertibility = if distinct inputs ($x(t)$ not t) leads to distinct outputs $|\overbrace{x(t)}^{2,-2}| = \overbrace{y(t)}^{2}$ ×
↳ causality = if the output depends on past or present values of an input
  ↳ if $h(\cdot) = 0$ for all $n < n_0$ → causal $y(t) = x(3t)$ × $y(1) = x(3)$ future ×
↳ shift time-invariance = if shifting input in time causes a identical shift in the output as well
  $y(t) = h(x(t)) \longrightarrow y(t+t_0) = h(x(t+t_0))$ no t outside $x(t)$ $2t + x(t)$ ×
↳ memory = if output depend on past or future values of an input
  ↳ memoryless if output depends only the present value

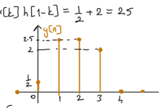
ex: $y[n] = n\,x[n]$
- linear = ✓ no powers of $x[n]$
- shift/ time invariant = × n outside the $x[n]$
- causal = memoryless → depends only the present so causal ✓
- stable = × $x[n] \rightarrow \infty$ $y[n] \rightarrow \infty$ not bounded

convolution = uses information from neighboring pixels to filter the target pixel
↳ $f[n,m] * g[n,m]$: function being multiplied by a shifted impulse response
↳ $\delta[n] = \begin{cases} 1 & n=0 \\ 0 & \text{else} \end{cases} \longrightarrow x[n] = \sum_k x[k]\,\delta[n-k]$ → any signal
↳ $y[n] = \sum_k x[k]\,h[n-k]$ → convolution of the signal x with impulse response h
↳ $y[n,m] = \sum_i \sum_j x[i,j]\,h[n-i, m-j]$ (2d)

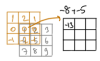
1d convolution example = $y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = x[n] * h[n]$



$y[-1] = 0$
$y[0] = \sum_k x[k]\,h[-k] = \frac{1}{2} \cdot 1 = \frac{1}{2}$
$y[1] = \sum_k x[k]\,h[1-k] = \frac{1}{2} + 2 = 2.5$
$y[2] = 2.5$
$y[3] = 2$
$y[4] = 0$



2d convolution example = $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} * \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} =$
input   kernel



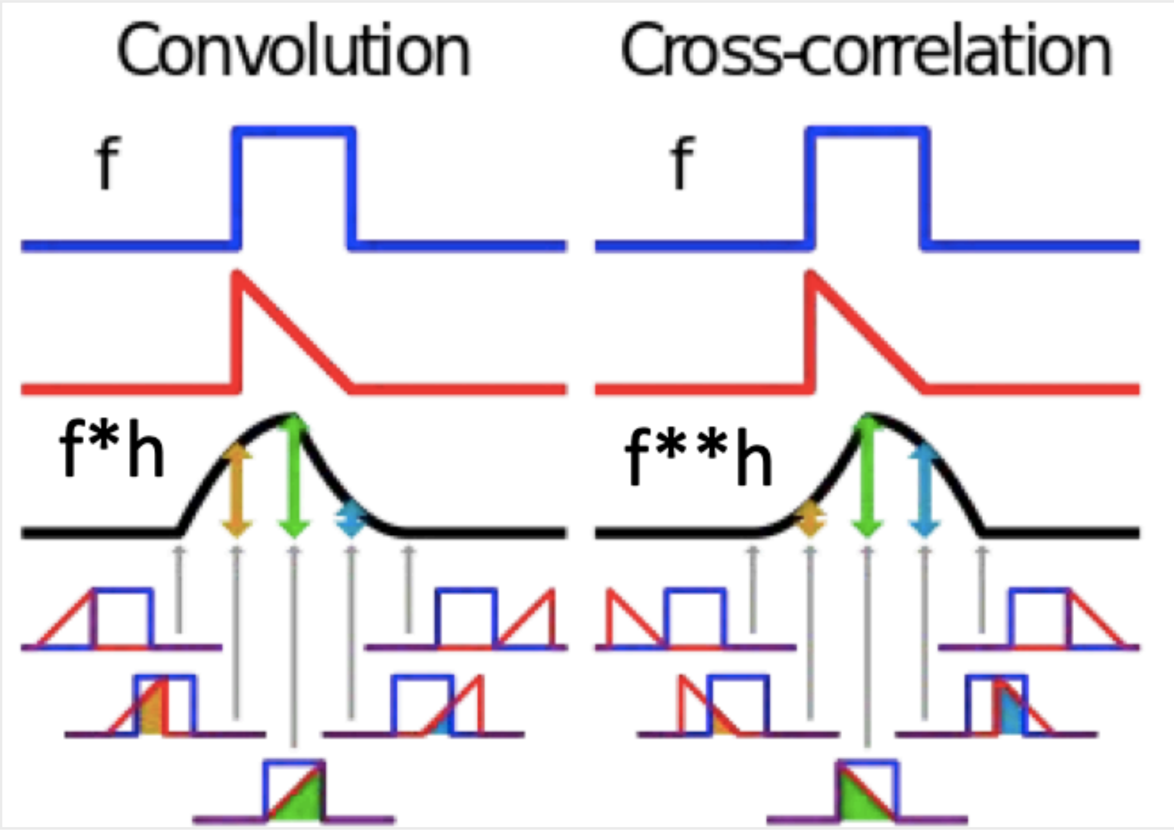
some filters:
- $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ no change
- $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ shifted 1 pixel right
- $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ blur/average
- $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ − with avg filter   sharper

shape after convolution = $\underbrace{(N1 \times M1)}_{\text{image}} * \underbrace{(N2 \times M2)}_{\text{filter}} = (N1 - N2 + 1) \times (M1 - M2 + 1)$
↳ if no padding is added

cross correlation = used to find known features by using a kernel that contains target features
↳ $r[k,l] = \sum_m \sum_n f[m+k, n+l]\, g[m,n] = f[k,l] ** g[k,l]$

$f * g$ convolution  filtering, amount of overlap as shifted over another

$f ** g$ cross correlation  similarity between two data max when they match
↳ not commutative
↳ not associative
} not much used


Convolution
Cross-correlation
f
f
f*h
f**h