

## 2 - boolean algebra

boolean algebra = george boole, in 1854, systemized the logical thinking's principles expressed then in inequalities

switching algebra = claud shannon, in 1938, based on boolean algebra

↳ use of boolean algebra on electrical switching circuits

↳ systemized logical thinking for computer and communication systems

boolean / switching algebra

We do not prove them, accept the as basis, then for the rest to the

elements

$[0, 1]$   
(logic literals)

operators

two binary  $+$  (or)  $\cdot$  (and)

unproved axioms or postulates

huntington's postulates (1924)

huntington's postulates

① closure  $\Rightarrow$  elements and results of operations are in the set  $\{0, 1\}$


② identity  $\Rightarrow x + 0 = x$  (or)  $x \cdot 0 = 0$  (and)


③ commutativity  $\Rightarrow x + y = y + x$   $xy = yx$


④ distributive  $\Rightarrow x \cdot (y + z) = x \cdot y + x \cdot z$   $x + (y \cdot z) = (x + y) \cdot (x + z)$

⑤ complement  $\Rightarrow$  for every  $x$  there exists  $x'$  such that  $x + x' = 1$  and  $x \cdot x' = 0$

⑥ there exists at least two elements  $x, y$  such that  $x \neq y$

not operation: result of postulate 5  $\rightarrow$  complement or inversion  $\bar{x} / \sim x / x'$  

and operation: conjunction  $\cdot / \& / \wedge$  


or operation: disjunction  $+$   $/$   $|$   $\vee$  

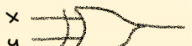
precedence = not  $>$  and  $>$  or


if  $a \neq b \Rightarrow 1$

  
buffer  
 $x \rightarrow x$

  
nand  
 $(x \cdot y)'$

  
nor  
 $(x + y)'$

  
exclusive or (xor)  
 $xy' + x'y$   
 $= x \oplus y$   
ya da

  
exclusive nor  
 $xy + x'y'$   
 $= (x \oplus y)'$   
onak ve onak  
 $\Leftrightarrow$

logic expression minimization

① algebraic

② karnaugh map

③ quine-mccluskey (tabular)  
 $\rightarrow$  not included

algebraic minimization

• involution  $\Rightarrow x = (\bar{\bar{x}})$

- identity  $\Rightarrow x + 1 = 1 \quad x + 0 = x \quad x \cdot 0 = 0 \quad x \cdot 1 = x$
- idempotence  $\Rightarrow x + x = x \quad x \cdot x = x$
- associativity  $\Rightarrow x + (y + z) = (x + y) + z \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z$
- adjacency  $\Rightarrow x \cdot y + x \cdot \bar{y} = x \quad (\bar{x} + y) \cdot (x + \bar{y}) = x$
- absorption  $\Rightarrow x + (x \cdot y) = x \quad x \cdot (x + y) = x$
- simplification  $\Rightarrow x + (\bar{x} \cdot y) = x + y \quad x \cdot (\bar{x} + y) = x \cdot y$

consensus  $\Rightarrow x \cdot y + \bar{x} \cdot z + y \cdot z = x \cdot y + \bar{x} \cdot z$   
 (general agreement)  $(x + y) \cdot (\bar{x} + z) \cdot (y + z) = (x + y) \cdot (\bar{x} + z)$

Involution	$x = \overline{\overline{x}}$
Identity	$x + 1 = 1 \quad x \cdot 0 = 0$
	$x + 0 = x \quad x \cdot 1 = x$
Idempotence	$x + x = x \quad x \cdot x = x$
$x + (y + z) = (x + y) + z$	
Associativity	$x \cdot (y \cdot z) = (x \cdot y) \cdot z$
$x \cdot y + x \cdot \bar{y} = x$	
Adjacency	$(x + y) \cdot (x + \bar{y}) = x$
$x + (x \cdot y) = x$	
Absorption	$x \cdot (x + y) = x$
$x + (\bar{x} \cdot y) = x + y$	
Simplification	$x \cdot (\bar{x} + y) = x \cdot y$
$x \cdot y + \bar{x} \cdot z + y \cdot z = x \cdot y + \bar{x} \cdot z$	
Consensus	$(x + y) \cdot (\bar{x} + z) \cdot (y + z) = (x + y) \cdot (\bar{x} + z)$
$\overline{x + y} = \bar{x} \cdot \bar{y}$	
DeMorgan's	$\overline{x \cdot y} = \bar{x} + \bar{y}$