generating functions

dizinin fonksiyon olarak yazılmasıdır

↳ generating function katsayıları bir a_n dizisinin terimleri olan **polinomdur**

$$G(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

- $a_n = 3n + 1$ $a_0 \to 1$ $a_1 \to 4$ $a_2 \to 7$ $a_3 \to 10$ $\to$ generating function: $1 + 4x + 7x^2 + 10x^3 + \dots$

- $1 + x^3 + x^5$ $\to$ $1, 0, 0, 1, 0, 1, 0, \dots$

- $(0, 2, 0, 0, 3, -4, 0, \dots) \to 2x + 3x^4 - 4x^5$

example

$a_0 = 0$

$a_n = 2a_{n-1} + 1$ $\qquad\qquad$ $G(x) = \sum_{n=0}^{\infty} a_n x^n$

$a_n x^n = 2 x^n a_{n-1} + x^n$ $\qquad\qquad\qquad\qquad$ $G(x) = a_0 + a_1 x + a_2 x^2$

$\sum_{n=1}^{\infty} a_n x^n = 2x \sum_{n=1}^{\infty} a_{n-1} x^{n-1} + \sum_{n=1}^{\infty} x^n$

$G(x) - \underset{0}{\underline{a_0}} = 2x \left(G(x) \right) + \frac{1}{1-x} \cdot x$

$G(x)(1 - 2x) = \frac{x}{1-x}$ $\qquad\qquad$ $G(x) = \frac{x}{(1-x)(1-2x)} = \frac{A}{\underset{(1-2x)}{1-x}} + \frac{B}{\underset{(1-x)}{1-2x}}$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $A + B = 0$ $\qquad$ $a = -1 \; B = 1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $-2A - B = 1$

$G(x) = \frac{1}{1-2x} - \frac{1}{1-x}$

$$\boxed{a^k - 1}$$