3 - linear equations; method of integrating factor

linear equation $\Rightarrow$ $y' + p(t)y = q(t)$ integrating factor $= \mu = e^{\int p(t)dt}$

$$\mu.(y' + p(t)y) = \mu\, q(t) \rightarrow \int (\mu.y)' = \int \mu.q(t) \rightarrow \mu y = \int \mu.q(t).dt$$

examples:

▲ $y' - y - te^t = 0 \rightarrow y' - y = te^t$ $\mu = e^{-\int 1.dt} \rightarrow e^{-t}$

$p(t) = -1$

$q(t) = te^t$

$\int (e^{-t}.y)' = \int (te^t).e^{-t} \rightarrow e^{-t}.y = \frac{t^2}{2} + C$

$\Rightarrow y = \frac{e^t.t^2}{2} + Ce^t$

▲ $ty' + 3y = \frac{2}{t} \rightarrow y' + \frac{3}{t}y = \frac{2}{t^2}$ $\int (t^3.y)' = \int 2t$ $y = \frac{1}{t} + \frac{C}{t^3}$

$y(1) = 2$ $\mu = e^{3\ln t} = t^3$ $t^3 y = t^2 + C$ $2 = 1 + C$

▲ $\frac{dy}{dx} = \frac{y}{x + y^2} \rightarrow \frac{dx}{dy} = \frac{x + y^2}{y}$ $\mu = e^{-\int \frac{1}{y}.dy}$ $\frac{x}{y} = y + c$

$\mu = e^{-\ln|y|}$

$\mu = y^{-1}$ $x = y^2 + yc$

$x'y - x = y^2$

$x' - \frac{1}{y}.x = y$

$\int (\frac{x}{y})' = \int 1\, dy$

exponential growth and decay

$y' + a y = q(t) \longrightarrow e^{at}.y = \int e^{at} q(t).dt$

when this is constant $y = e^{-at} \int e^{at} q(t).dt$

when $a > 0$ and b constant $(q(t)) \Rightarrow e^{at}.y = \int e^{at}.b\, dt$

(same for) $-at$

$\lim_{t \to \infty} y = \frac{b}{a}$ (all solution) $\leftarrow$ $y = \frac{b}{a} + ce$

when $a > 0$, c, d are constants

$y(t) = d + ce^{at}$ $\longrightarrow$ said to be growing exponentially

$y(t) = d + ce^{-at}$ $\longrightarrow$ decaying exponentially