# 3 - combinational circuits - analysis

combinational circuit = input variables + logic gates + output variables

☆ storage elements of previous inputs and feedback paths are in sequential circuits

analysis

literal analysis

from the schema and the truth table → we assign values to the inputs
↳ trace the results
↳ repeat until reach output that matches with the output in the truth table

symbolic analysis

from the schema → truth table
→ logic expression
→ we assign expression instead of values
↳ we get the output function and expression
↳ we construct the truth table as we go

☆ symbolic analysis is more work but gives us complete information.

standard (canonical) expression forms

I. sum of products:
sum form
disjunctive normal form
or of and terms

$$F[A,B,C] = \underbrace{(\bar{A}.\bar{B}.\bar{C}) + (\bar{A}.B.C) + (A.B.\bar{C}) + (A.B.C)}_{\text{minterms}}$$

each minterm → one '1' in the truth table

II. product of sums:
product form
conjunctive normal form
and of or terms

$$F[A,B,C] = \underbrace{(A+B+\bar{C}).(A+\bar{B}+C).(\bar{A}+B+C).(\bar{A}+B+\bar{C})}_{\text{maxterms}}$$

each maxterm → one '0' in the truth table

☆ in both forms, each 1st level operator corresponds to one row of truth table
☆ each variable appears exactly once

example = $F(A,B,C) = ?$

| | F | using minterms (m) | | using maxterms (M) | |
|---|---|---|---|---|---|
| 0 | 1 | $\bar{A}.\bar{B}.\bar{C}$ | $m_0$ | | |
| 1 | 0 | | | $A+B+\bar{C}$ | $M_1$ |
| 2 | 0 | | | $A+\bar{B}+C$ | $M_2$ |
| 3 | 1 | $\bar{A}BC$ | $m_3$ | | |
| 4 | 0 | | | $\bar{A}+B+C$ | $M_4$ |
| 5 | 0 | | | $\bar{A}+B+\bar{C}$ | $M_5$ |
| 6 | 1 | $AB\bar{C}$ | $m_6$ | | |
| 7 | 1 | $+ABC$ | $m_7$ | | |

$$F(A,B,C) = \underbrace{(\bar{A}.\bar{B}.\bar{C})+(\bar{A}BC)+(AB\bar{C})+(ABC)}_{m_0+m_3+m_6+m_7} = \underbrace{(A+B+\bar{C}).(A+\bar{B}+C).(\bar{A}+B+C).(\bar{A}+B\ldots}_{M_1.M_2.M_4.M_5}$$

$$m_0+m_3+m_6+m_7 = M_1.M_2.M_4.M_5$$

$$\sum m(0,3,6,7) = \prod M(1,2,4,5)$$

## Karnaugh map (k-map) minimization

☆ application of adjacency

☆ guarantees a minimal expression

adjacent terms = differ in one variable

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | | |
| 01 | | | | |
| 11 | | | | |
| 10 | | | | |

ABCD

- group size is a power of 2, and they are rectangular (1,2,4,8)

☆ the inputs are always arranged in Gray code sequence

☆ it is simply rearranged truth table

| | 1's are grouped | 0's are grouped |
|---|---|---|
| expression form: | $\boxed{abc}+\boxed{ab\bar{c}}+\boxed{a\bar{b}c}$ | $\boxed{(m+n+k)}.\boxed{(m+\bar{n}+k)}.\boxed{(\bar{m}+\bar{n}+k)}$ |
| variable value is 1 : | $A$ | $\bar{A}$ |
| variable value is 0 : | $\bar{B}$ | $B$ |

☆ if variable changes within the group do not include in both

☆ they both will give the same answer.

implicants = single cells or groups that could be part of a larger group.

prime implicant = a group that is as large as possible

| | |
|---|---|
| 1 | 1 |
| 0 | 0 |

essential prime implicant = there is at least single 1 which cannot be combined in any other way

secondary / nonessential prime implicant = a prime implicant that is not essential

☆ the smallest set of prime implicants that covers all values forms a minimal expression for the desired function

↳ there may be more than one minimal set.

## examples

| $b_1b_0$ \ $b_3b_2$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 0 | 1 |
| 01 | 0 | 1 | 0 | 1 |
| 11 | 0 | 1 | 0 | 0 |
| 10 | 0 | 1 | 0 | 0 |

→ $b_3\bar{b}_2\bar{b}_1$

→ $\bar{b}_3 b_2 b_0$

→ $\bar{b}_3 b_2 b_1$

$= b_3\bar{b}_2\bar{b}_1 + \bar{b}_3 b_2 b_0 + \bar{b}_3 b_2 b_1$

secondary prime implicants

essential prime implicants

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

→ $A\bar{C}$

→ $BCD / ABD$

→ $\bar{A}CD$

$= A\bar{C} + \bar{A}CD + BCD$

$= A\bar{C} + \bar{A}CD + ABD$

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | 1 | 1 |
| 01 | 0 | 0 | 1 | 1 |
| 11 | 1 | 1 | 1 | 0 |
| 10 | 0 | 0 | 0 | 0 |

→ $A+C$

→ $\bar{A}+B+\bar{C}$

→ $\bar{C}+D$

$= (A+C).(\bar{A}+B+\bar{C}) + (\bar{C}+D)$

minimize $F(a,b,c,d) = \sum m(0,2,8,10)$

| | a b c d |
|---|---|
| 0 → | 0000 |
| 2 → | 0010 |
| 8 → | 1000 |
| 10 → | 1010 |

| cd \ ab | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | 0 | 0 | 1 |
| 01 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 0 | 0 |
| 10 | 1 | 0 | 0 | 1 |

→ $\bar{b}\bar{d}$

minimize $F(x,y,z) = xyz + x'y + xyz' + x'y'z'$

111 ; $x'yz + x'yz'$ ; 011 010 ; 110 ; 000

| z \ xy | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |

→ $y$

→ $\bar{x}\bar{z}$

$= y + \bar{x}\bar{z}$

don't cares = - / x can be assigned 0 or 1

| CD \ AB | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 0 | 0 | x | 1 |

→ A

| | | | | |
|---|---|---|---|---|
| 01 | 0 | 1 | x | 1 |
| 11 | 0 | 1 | x | x |
| 10 | 0 | 1 | x | x |

$BD$

$BC$

$= A + BD + BC$