2 - boolean algebra

boolean algebra = george boole, in 1854, systemized the logical thinking's principles expressed them in inequalities

switching algebra = claude shannon, in 1938, based on boolean algebra
↳ use of boolen algebra on electrical switching circuits
↳ systemized logical thinking for computer and communication systems

$\boxed{\text{boolean / switching algebra}}$

we do not prove them, accept the as basis, then bi the rest to the

elements	operators	unproved axioms or postulates
[0, 1] (logic literals)	two binary + (or) . (and)	huntington's postulates (1924)

huntington's postulates

1. closure ⇒ elements and results of operations are in the set (0,1)
2. identity ⇒ $x+0=x$ (or) $x.0=0$ (and)
3. commutativity ⇒ $x+y=y+x$ $xy=y.x$
4. distributive ⇒ $x.(y+z)=x.y+x.z$ $x+(y.z)=(x+y).(x+z)$
5. complement ⇒ for every x there exists x' such that $x+x'=1$ and $x.x'=0$
6. there exists at least two elements x,y such that $x \neq y$

not operation: result of postulate 5 → complement or inversion $\bar{x}$ / $\sim x$ / x' 

and operation: conjunction . / & / ∧ 

or operation: disjunction + / | / ∨ 

precedence = not > and > or

if $a \neq b \Rightarrow 1$ if $a=b \Rightarrow 1$

buffer: $x \rightarrow x$

nand: $(x.y)'$

nor: $(x+y)'$

exclusive or (xor): $xy' + x'y = x \oplus y$

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exclusive nor: $xy + x'y' = (x \oplus y)'$

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$\boxed{\text{logic expression minimization}}$

1. algebraic
2. karnaugh map
3. quine-mcclusky (tabular) ↳ not included

algebraic minimization

- involution ⇒ $x = (\bar{\bar{x}})$

- identity ⇒ $x+1=1$ $\quad x+0=x$ $\quad x.0=0$ $\quad x.1=x$
- idempotence ⇒ $x+x=x$ $\quad x.x=x$
- associativity ⇒ $x+(y+z)=(x+y)+z$ $\quad x.(y.z)=(x.y).z$
- adjacency ⇒ $x.y+x.\bar{y}=x$ $\quad (x+y).(x+\bar{y})=x$
- absorption ⇒ $x+(x.y)=x$ $\quad x.(x+y)=x$
- simplification ⇒ $x+(\bar{x}.y)=x+y$ $\quad x.(\bar{x}+y)=x.y$

consensus ⇒ $X.Y+\bar{X}.Z+Y.Z = X.Y+\bar{X}.Z$
(general agreement) $(X+Y).(\bar{X}+Z).(Y+Z) = (X+Y).(\bar{X}+Z)$

Involution	$X = \overline{(\overline{X})}$
Identity	$X + 1 = 1$ $\quad X \cdot 0 = 0$ $X + 0 = X$ $\quad X \cdot 1 = X$
Idempotence	$X + X = X$ $\quad X \cdot X = X$
Associativity	$X + (Y + Z) = (X + Y) + Z$ $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$
Adjacency	$X \cdot Y + X \cdot \overline{Y} = X$ $(X + Y) \cdot (X + \overline{Y}) = X$
Absorption	$X + (X \cdot Y) = X$ $X \cdot (X + Y) = X$
Simplification	$X + (\overline{X} \cdot Y) = X + Y$ $X \cdot (\overline{X} + Y) = X \cdot Y$
Consensus	$X \cdot Y + \overline{X} \cdot Z + Y \cdot Z = X \cdot Y + \overline{X} \cdot Z$ $(X + Y) \cdot (\overline{X} + Z) \cdot (Y + Z) = (X + Y) \cdot (\overline{X} + Z)$
DeMorgan's	$\overline{X + Y} = \overline{X} \cdot \overline{Y}$ $\overline{X \cdot Y} = \overline{X} + \overline{Y}$