# 5 - continuous distributions

↳ time, temperature, length, weight on $(a,b)$, $(-\infty,a)$, $(b,+\infty)$ ..

$\boxed{\text{probability density}}$

☆ for all continuous variables, the probability mass function(pmf) is always equal to zero.

$P(x)=0$ for all $x$ $\left(\frac{1}{\infty}\right)$ → so we use probability density function(pdf)

→ cumulative distribution function $F(x)$ - cdf is again non-decreasing without jumps (unlike discrete one) continuous function $[0,1]$

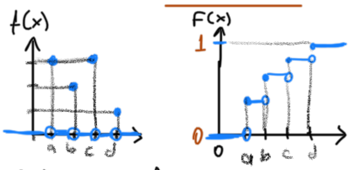
probability density function(pdf) = derivative of cdf, $f(x) = F'(x) = F'_X(x)$
↳ distribution is called continuous if it has a density

$$pdf = f(x) = F'(x)$$

$$P\{a<x<b\} = \int_a^b f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)dx = 1$$

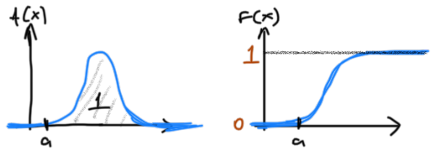
discrete



$f(x)=P(x)=pmf$ $\quad$ $F(x)=cdf$

$CDF = \sum PDF$

continuous case



$f(x)=pdf$, $P(a<x<b) = \int_a^b f(x)$ $\quad$ $F(x)=cdf$

$CDF = \int PDF$

expected value $= \sum x P(x)$ $\qquad \mu = E(x)$ $\qquad \int x f(x)\,dx$

$$\text{variance} = \int x^2 f(x)\,dx - \mu^2$$

example

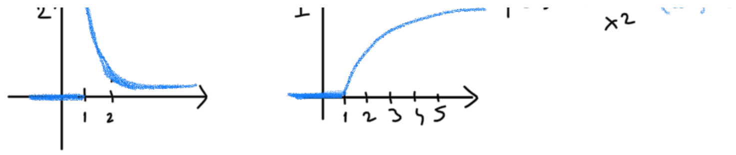
$$f(x)=\begin{cases} \frac{k}{x^3} & x\geq 1 \\ 0 & x<1 \end{cases}$$

• $\int_1^\infty F(x)\,dx = 1 = \int_1^\infty \frac{k}{x^3}dx = \frac{k.x^{-2}}{-2}\Big|_1^\infty = \frac{k}{-2}\left(\frac{1}{x^2}\right) = 0 + \frac{k}{2} = 1$

$k=2$

pdf is like: $\frac{2}{x^3}$ $\qquad$ cdf is like: $\int \frac{2}{x^3} = C + \frac{-1}{x^2}$ $\quad f(1)=0 \rightarrow C-1=0 \quad C=1$

$F(x) = 1 - \frac{1}{\ldots}$ $(1,\infty)$




• $P(X>5) = 1 - CDF(5) = 1 - \left(1 - \frac{1}{25}\right) = \frac{1}{25} = \underline{0.04}$

expected value $= \int_1^\infty x \underbrace{f(x)}_{pdf} = \frac{x \cdot 2}{x^3} = \int_1^\infty \frac{2}{x^2} = -\frac{2}{x}\Big|_1^\infty = 0 + 2 = 2$

variance $= \int_1^\infty x^2 \frac{2}{x^3} - 4 = 2\ln x\Big|_1^\infty + 4 = \infty$