HW-3

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① **decision variables:**

x_1 = grammage taken from sweet corn flavored chips

x_2 = grammage taken from barbeque flavored chips

constraints:

protein in 1 gram

$\frac{1.5}{100}x_1 + \frac{3.5}{100}x_2 \geq 25$ → protein constraint → $0.015x_1 + 0.035x_2 \geq 25$

calorie in 1 gram

$\frac{400}{100}x_1 + \frac{600}{100}x_2 \leq 550$ → calorie constraint → $4x_1 + 6x_2 \leq 550$

$x_1 + x_2 = 100$ → mixture grammage constraint

$|x_1 - x_2| \leq 35$ → grammage difference constraint → $-35 \leq x_1 - x_2 \leq 35$ → $x_1 - x_2 \leq 35$, $x_2 - x_1 \leq 35$

$x_1, x_2 \geq 0$ → sign constraints

objective function: minimize $z = \frac{5}{10}x_1 + \frac{7}{10}x_2$

standard form:

$\min\ z = 0.5x_1 + 0.7x_2$

$0.015x_1 + 0.035x_2 - e_1 + a_1 = 25$

$4x_1 + 6x_2 + s_2 = 550$

$x_1 - x_2 + s_3 = 35$

$-x_1 + x_2 + s_4 = 35$

$x_1 + x_2 + a_5 = 100$

$\boxed{\text{two phase simplex method}}$

phase I: $\min\ w = a_1 + a_5$ → $w - a_1 - a_5 = 0$

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1				-1				-1	0	w
	0.015	0.035	-1	1					25	a_1
	4	6			1				550	s_2
	1	-1				1			35	s_3
	-1	1					1		35	s_4
	1	1						1	100	a_5

eliminating a_1 and a_5 from row0 →

①

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var	ratio test
1	1.015	1.035	-1						102.5	w	–
	0.015	0.035	-1	1					25	a_1	71.4
	4	6			1				550	s_2	91
	1	-1				1			35	s_3	–
	-1	1					1		35	s_4	35
	1	1						1	100	a_5	100

②

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var	ratio test
1	2.05		-1				-1.035		66.275	w	–
	0.05		-1	1			-0.035		1.275	a_1	25.5
	10				1		-6		340	s_2	34
						1	1		70	s_3	–
	-1	1					1		35	x_2	–
	2						-1	1	65	a_5	32.5

③

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var	ratio test
1			40	-41			0.4		14	w	–
	1		-20	20			-0.7		25.5	x_1	–
			200	-200	1		1		85	s_2	0.425
						1	1		70	s_3	–
		1	-20	20			0.3		60.5	x_2	–
			40	-40			0.4	1	14	a_5	0.35

4

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1				-1				-1		w
	1						-0.5	0.5	32.5	x_1
					1		-1	-5	15	s_2
						1	1		70	s_3
		1					0.5	0.5	67.5	x_2
			1	-1			0.01	0.025	0.35	e_1

This is an optimal Phase I tableau.
$w=0$ and $a_1, a_5 = 0$, so we can continue to phase II.

phase II: Removing a_1 and a_2 columns and replacing row 0 with $z = 0.5x_1 + 0.7x_2$

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var
1	-0.5	-0.7						z
	1					-0.5	32.5	x_1
				1		-1	15	s_2
					1	1	70	s_3
		1				0.5	67.5	x_2
			1			0.01	0.35	e_1

eliminating x_1 and x_2 from row 0 →

1

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var	ratio
1						0.1	63.5	z	–
	1					-0.5	32.5	x_1	–
				1		-1	15	s_2	–
					1	1	70	s_3	70
		1				0.5	67.5	x_2	135
			1			0.01	0.35	e_1	35

2

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var
1			-10				60	z
	1		50				50	x_1
			100	1			50	s_2
			-100		1		35	s_3
		1	-50				50	x_2
			100			1	35	s_4

This is an optimal Phase II tableau.

$$\left.\begin{array}{l} z=60 \\ x_1 = 50 \text{ and } x_2 = 50 \\ e_1 = 0,\ s_2 = 50,\ s_3 = 35,\ s_4 = 35 \end{array}\right\} \text{optimum solution}$$

2

	x1	x2			
	50	50	100	=	100
protein	0,015	0,035	2,5	>=	2,5
cal	4	6	500	<=	550
diff1	1	-1	-1,421E-14	<=	35
diff2	1	-1	-1,421E-14	>=	-35
cost	0,5	0,7	60		

3

constraints:

$\frac{1.5}{100} x_1 + \frac{3.5}{100} x_2 \geq 2.75 \rightarrow$ protein constraint $\rightarrow 0.015 x_1 + 0.035 x_2 \geq 2.75$

protein in 1 gram

standard form:

$\min\ z = 0.5 x_1 + 0.7 x_2$

$0.015 x_1 + 0.035 x_2 - e_1 + a_1 = 2.75$

$4 x_1 + 6 x_2 + s_2 = 550$

$x_1 - x_2 + s_3 = 35$

$-x_1 + x_2 + s_4 = 35$

$x_1 + x_2 + a_5 = 100$

two phase simplex method

phase I: $\min\ w = a_1 + a_5 \rightarrow w - a_1 - a_5 = 0$

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1				-1				-1	0	w
	0.015	0.035	-1	1					2.75	a_1
	4	6			1				550	s_2
	1	-1				1			35	s_3
	-1	1					1		35	s_4
	1	1						1	100	a_5

eliminating a_1 and a_5 from row0 $\rightarrow$

1

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var	ratio test
1	1.015	1.035	-1						102.75	w	–
	0.015	0.035	-1	1					2.75	a_1	78
	4	6			1				550	s_2	91
	1	-1				1			35	s_3	–
	-1	1					1		35	s_4	35
	1	1						1	100	a_5	100

2

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var	ratio test
1	2.05		-1				-1.035		66.525	w	–
	0.05		-1	1			-0.035		1.525	a_1	30.5
	10				1		-6		340	s_2	34
						1	1		70	s_3	–
	-1	1					1		35	x_2	–
	2						-1	1	65	a_5	32.5

3

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var	ratio test
1			40	-41			0.4		4	w	–
	1		-20	20			-0.7		30.5	x_1	–
			200	-200	1		1		35	s_2	0.175
						1	1		70	s_3	–
		1	-20	20			0.3		65.5	x_2	–
			40	-40			0.4	1	4	a_5	0.1

④

w	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1				-1				-1		w
	1						-0.5		32.5	x_1
					1		-1		15	s_2
						1	1		70	s_3
		1					0.5	0.5	67.5	x_2
			1	-1			0.01	0.025	0.1	e_1

This is an optimal Phase I tableau. $w=0$ and $a_1, a_5 = 0$, so we can continue to phase II.

phase II: Removing a_1 and a_2 columns and replacing row 0 with $z = 0.5x_1 + 0.7x_2$

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var
1	-0.5	-0.7						z
	1					-0.5	32.5	x_1
				1		-1	15	s_2
					1	1	70	s_3
		1				0.5	67.5	x_2
			1			0.01	0.1	e_1

eliminating x_1 and x_2 from row 0 $\rightarrow$

①

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var	ratio
1						0.1	63.5	z	–
	1					-0.5	32.5	x_1	–
				1		-1	15	s_2	–
					1	1	70	s_3	70
		1				0.5	67.5	x_2	135
			1			0.01	0.35	e_1	35

②

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var	ratio test
1						0.1	63.5	z	–
	1					-0.5	32.5	x_1	–
				1		-1	15	s_2	–
					1	1	70	s_3	70
		1				0.5	67.5	x_2	135
			1			0.01	0.1	e_1	10

③

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var
1			-10				62.5	z
	1		50				37.5	x_1
			100				25	s_2
			-100		1		60	s_3
		1	-50				62.5	x_2
			100			1	10	s_4

This is optimal Phase II tableau $z = 62.5$

$x_1 = 37.5$ and $x_2 = 62.5$

$e_1 = 0,\ s_2 = 25,\ s_3 = 60,\ s_4 = 10$

optimum solution

	x1	x2			
	37,5	62,5	100	=	100
protein	0,015	0,035	2,75	>=	2,75
cal	4	6	525	<=	550
diff1	1	-1	-25	<=	35
diff2	1	-1	-25	>=	-35
cost	0,5	0,7	62,5		

4.1

the final tableau in part 1:

z	x_1	x_2	e_1	s_2	s_3	s_4	rhs	basic var
1			-10				60	z
	1		50				50	x_1
			100	1			50	s_2
			-100		1		35	s_3
		1	-50				50	x_2
			100			1	35	s_4

e_1 = excess in in protein constraints

As we can see from the first row, if we increase protein constraint by 1, the z value will increase by 10. Thus, shadow price for protein constraint 10

We can see the shadow price of protein constraint from the sensitivity report table in the next page

5.1

Calorie and grammage difference constraints shadow price is zero, because they are not binding constraints in optimal solution. We can see their shadow prices from the first row, s_2, s_3, s_4 are all zeros. We can see their zero shadow prices from the sensitivity report table.

To find the shadow prices of all constraints, if we do not remove the arbitrary variables from the table after phase 1 and do the same operations, we can see their values.

phase 1 value of $a_1 = -1$ $\qquad -1-(-0.65) = -0.35$

phase 2 value of $a_1 = -0.65$ $\qquad$ thus the shadow price for $x_1 + x_2 = 100$ is 0.35

We can see its shadow price from the sensitivity report table as well

z	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1	-0.5	-0.7		-1				-1		z
	1						-0.5	0.5	32.5	x_1
					1		-1	-5	15	s_2
						1	1		70	s_3
		1					0.5	0.5	67.5	x_2
			1	-1			0.01	0.025	0.25	e_1

z	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1				-1			0.1	-0.4	63.5	z
	1						-0.5	0.5	32.5	x_1
					1		-1	-5	15	s_2
						1	1		70	s_3
		1					0.5	0.5	67.5	x_2
			1	-1			0.01	0.025	0.25	e_1

z	x_1	x_2	e_1	a_1	s_2	s_3	s_4	a_5	rhs	basic var
1			-10	9				-0.65	60	z
	1		50	-50				1.75	34.5	x_1
			100	-100				-2.5	50	s_2
			-100	100		1		-2.5	35	s_3
		1	-50	50				-0.75	50	x_2
			100	-100			1	2.5	35	s_4

4.2 – 5.2

Microsoft Excel 16.16 Sensitivity Report
Worksheet: [2.xlsx]Sheet1
Report Created: 28.12.2022 20:24:01

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B2	x1	50	0	0,5	0,2	1E+30
C2	x2	50	0	0,7	1E+30	0,2

Constraints

$x_1+x_2=100$ constraint shadow price

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D2		100	0,35	100	14	14
D3	protein	2,5	10	2,5	0,35	0,35
D4	cal	500	0	550	1E+30	50
D5	diff1	-1,42109E-14	0	35	1E+30	35
D6	diff2	-1,42109E-14	0	-35	35	1E+30

other constraints' shadow prices are all zeros

protein constraint shadow price