20 - review of power series

$\sum_{n=0}^{\infty} a_n (x-x_0)^n$: infinite series centered at x_0 (when there is a term that) contains x

$\boxed{ex}$ $\sum_{n=1}^{\infty} \frac{(2x+1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{2^n (x - -\frac{1}{2})^n}{n^2} \Rightarrow a_n = \frac{2^n}{n^2}$ $x_0 = -\frac{1}{2}$

oran testi

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$ yakınsar

$\lim_{n\to\infty} \left| \frac{(2x+1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(2x+1)^n} \right| = \lim_{n\to\infty} \left| \frac{(2x+1) \cdot n^2}{(n+1)^2} \right| = |2x+1| < 1$

$-1 < 2x+1 < 1$ $\quad -1 < x < 0$ $\quad x=-1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ ✓ converges

$x=0 \quad \sum_{n=1}^{\infty} \frac{1}{n^2}$ ✓ converges $p>1$

interval of converge $\rightarrow [-1, 0]$

radious $= \frac{0+1}{2} = \frac{1}{2}$

$\boxed{ex}$ $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot n \, (x+3)^n}{4^n}$ $\quad \frac{\cancel{(-1)^{n+1}} \cdot \cancel{(n+1)} \, (x+3)^{n+1} \, \cancel{4^n}}{\cancel{4^{n+1}} \cdot \cancel{(-1)^n} \cancel{(n)} \, (x+3)^n} = \lim_{n\to\infty} \left| \frac{(-1) \cdot (n+1) \, (x+3)}{4n} \right| < 1$

$\left|\frac{x+3}{4}\right| < 1 \quad -4 < x+3 < 4 \quad -7, 1 = ?$

$-7 < x < 1$

radius of converge $= \frac{1--7}{2} = 4$

(x_0) center $= -3 \rightarrow (x - -3)$

interval of converge $= (-7, 1)$

$x = -7 \quad \frac{(-1)^n \cdot n \, (-4)^n}{4^n} \Rightarrow n \rightarrow$ does not converge $\quad \lim_{n\to\infty} a_n \neq 0$

$x = 1 \quad \frac{(-1)^n \cdot n \, \cancel{4^n}}{\cancel{4^n}} \Rightarrow \sum_{n=1}^{\infty} (-1)^n \cdot n$ $\quad$ alternate series test

if $|f(x)| \rightarrow$ converges $f(x)$ converges

else $\rightarrow$ ① $a_n > 0$

② a_n is decreasing $\rightarrow$ ✗ $\quad x=1$ does not converges

③ $\lim_{n\to\infty} a_n = 0$

analytic function $= f(x)$ at point x_0

$\rightarrow$ if there exists an open neighborhood of x_0 in which the Taylor series of $f(x)$ converges to $f(x)$

then $\rightarrow$ continuous at x_0

$\rightarrow$ it has derivatives of all orders

$\boxed{\text{maclaurin series}}$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!},$$

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!},$$

$$\cos x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!},$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$\ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \cdot \frac{x^k}{k}$$