summary

② probability

sample space(Ω) = { HH, HT, TH, TT } (n)

event = { HH, HT } $\subset \Omega$ (2^n possible)

cardinality = set's size

disjoint = $A \cap B = \emptyset$ – mutually exclusive

exhaustive = $A \cup B = \Omega$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ (B is know to occur)

$P(A \cap B) = P(A|B) . P(B) = P(B|A) . P(A)$ → if they are independent → $P(A \cap B) = P(A) . P(B)$

* mutually independent does not mean that they are disjoint
* A and $\bar{A}$ are not independent, $A . \bar{A} \neq A \cap \bar{A}$

③ discrete random variables

$P(x)$ = probability mass function (pdf)

$F(x)$ = cumulative distribution function (cdf), always between 0 and 1, non-decreasing, ΣPDF

* (joint distribution) $P(X,Y) = P(X) . P(Y)$ (marginal distribution) → if they are independent

expectation(μ) = $\sum_x x P(x)$

variance (σ^2) = $\sum_x (x-\mu)^2 P(x) \geq 0 = E[(x-\mu)^2] = E(x^2) - \mu^2 = E(x - Ex)^2$

standard deviation (σ) = $\sqrt{Var(x)}$

covariance (σ_{xy}) = strength of a relationship between two: $E(XY) - E(X).E(Y)$

↳ if x and y are independent → $cov(x,y) = 0$ (reverse is not always true)

correlation = standardized/normalized covariance to the [-1, 1]

③.④ discrete distributions

$q = 1-p$

	P(x)	E(x)	V(x)
bernoulli = random variable with two possible outcomes	p	p	pq
binomial = number of successes in a sequence of bernoulli trials	$\binom{n}{x} p^x q^{n-x}$	np	npq
geometric = number of benoulli trials to get first success	pq^{x-1}	$\frac{1}{p}$	$\frac{1-p}{p^2}$
negative binomial = number of benoulli trials to get k successes	$\binom{x-1}{k-1} q^{x-k} p^k$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$
poisson = number of rare events occurring within a fixed time	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ

→ unbounded

→ $\sum_x P(x) = 1$ (geometric sum)

↳ poisson approximation of binomial distribution $n \geq 30$, $\boxed{p \leq 0.05}$ $\lambda \to np$

④ continuous variables

probability mass function = 0 (always) = $\frac{1}{\infty}$

$P(x)$ = probability density funcion (pdf) = derivative of cdf = $F'(x)$

$F(x)$ = cumulative distribution functio (cdf) → non-decreasing, [0,1], without jumps, continuous func. $\int$PDF

expectation(μ) = $\int x f(x) dx$

variance (σ^2) = $\int x^2 f(x) dx - \mu^2$

④.② continuous distributions

	density P(x)	E(x)	V(x)	cdf F(x)
uniform = generating a random number from a given interval	$\frac{1}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
standard uniform = [0,1], $x = [a,b] \to Y = \frac{x-a}{b-a}$	1	$\frac{1}{2}$	$\frac{1}{12}$	
exponential = the waiting time for the next event (like geometric d.)	$\frac{\lambda}{e^{\lambda x}}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$1 - \frac{1}{e^{\lambda x}}$

gamma = the total time of observing α events each with exponential waiting times ... $\Gamma(\alpha)$ e λ λ^2

normal (gaussian) = bell-shaped curve, symmetric, centered at μ (location parameter), spread controlled by σ (scale parameter)

standard normal = $Z = \frac{X-\mu}{\sigma}$ $\mu = 0$ $\sigma = 1$ pdf $\rightarrow \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ (Φ)

central limit theorem = sum of random independent variables from same distribution (any distribution)

$S_n \Rightarrow \infty$ $\frac{S_n}{n} \rightarrow \frac{\sigma^2}{n} \rightarrow 0$ $n \rightarrow \infty$ $\Rightarrow$ the standardized sum converges to standard normal variable

- as long as $n > 30$ (large), it can be applied to any distribution

$$\frac{S_n - E(S_n)}{std(S_n)} = \frac{S_n - n\mu}{\sigma \cdot \sqrt{n}} \rightarrow P\left\{ \frac{S_n - n\mu}{\sigma\sqrt{n}} \leq z \right\} \rightarrow \Phi(z)$$

normal approximation to binomial distribution = $\boxed{0.05 \leq p \leq 0.95}$ $\left.\begin{matrix} \\ n \rightarrow \text{large} \end{matrix}\right\}$ binomial(n,p)

$\Rightarrow \approx$ normal $(\mu = np, \sigma = \sqrt{n.p.(1-p)})$

↳ continuity correction = $P(a \leq X \leq b) \xrightarrow[\text{by } 0.5]{} P(a - 0.5 < x < b + 0.5)$

if $P(y < x+1) = P(y \leq x)$
$= P(y < x + 0.5)$

* exponential and geometric distributions are memoryless

summary

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