2- separable equations - homogeneous equations

$\boxed{\text{Separable equations}}$

if $f(t,y)$ can be written in a form of $\frac{M(t)}{N(y)}$ for some functions M and N

$$\frac{dy}{dt} = \frac{M(t)}{N(y)} \longrightarrow \int N(y)\,.\,dy = \int M(t)\,.\,dt$$

examples =

- $\frac{dy}{dt} = y(y-2)t \rightarrow \int dy \frac{1}{y(y-2)} = \int t\,dt \rightarrow \frac{1}{2}\int \frac{1}{y-2} - \frac{1}{y} = \frac{t^2}{2}$

$$\downarrow \quad \frac{A}{y} + \frac{B}{y-2} \qquad \begin{matrix} A+B=0 \\ -2A=1 \end{matrix} \qquad A=-\tfrac{1}{2} \; b=\tfrac{1}{2}$$

$$= \ln\left|\frac{y-2}{y}\right| = t^2 + c \qquad \frac{y-2}{y} = e^{t^2}\cdot c \qquad 1 - \frac{2}{y} = e^{t^2} c$$

= when $y \neq 0, 2$

$$\boxed{y = \frac{2}{1-e^{t^2}\cdot c}} \rightarrow \text{so } c \neq 0$$

what about $y=0$, $y=2$ (when $c \to \infty$)

$\frac{dy}{dt} = 0(0-2)t \qquad y'=0$

$\frac{dy}{dt} = 2(2-2).t \qquad y'=0$

- $y' = \frac{2x}{y+x^2 y}$, $y(0) = -2$

$$\frac{dy}{dx} y(1+x^2) = 2x \qquad \int y\,.\,dy = \int \frac{2x}{1+x^2}\,dx$$

$$= \frac{y^2}{2} = \ln|1+x^2| + c \qquad 2 = c \qquad y = \sqrt{2\ln(1+x^2)+4}$$

$$y = -\sqrt{2\ln(1+x^2)+4}$$

$\boxed{\text{homogenenous equations}}$

they become separable after a simple substitution

$\frac{dy}{dt} = h\left(\frac{y}{t}\right)$ for some func h $\rightarrow y = \vartheta t \qquad y' = \vartheta' t + \vartheta$

check if a func homogenous

$f(at, ay) = f(t,y)$

↳ a must be canceled

ex: $\frac{dy}{dt} = t+y \quad at+ay \neq t+y$

$\frac{dy}{dt} = \frac{t+y}{t-y} \quad \frac{at+ay}{at-ay} \overset{\checkmark}{=} \frac{t+y}{t-y}$

→ when y' alone other side of the equation, must consist only terms with y and t together.

like: x, 1, y is not homo.

examples=

- $\frac{dy}{dt} = \frac{t+y}{t-y} \rightarrow \frac{dy}{dt} = \frac{1+\frac{y}{t}}{1-\frac{y}{t}} \qquad \frac{y}{t} = v \qquad y = vt$

  $y' = v't + v$

$= v't + v = \frac{1+v}{1-v} \rightarrow v't = \frac{\cancel{1}+\cancel{v}-\cancel{v}+v^2}{1-v} \rightarrow \int dv \frac{(1-v)}{1+v^2} \int = \frac{dt}{t}$

$= \arctan(v) - \frac{1}{2}(\ln|1+v^2|) = \ln|t| + c \rightarrow \arctan\left(\frac{y}{t}\right) - \frac{1}{2}\left(\ln\left|1+\frac{y^2}{t^2}\right|\right) = \ln|t| + C$

- $x^2+y^2-2xyy'=0 \rightarrow y' = \frac{x^2+y^2}{2xy} \rightarrow y' = \frac{x}{2y} + \frac{y}{2x} \qquad y = vx$

  $y' = v'x + v$

$v'x + v = \frac{1}{2v} + \frac{v}{2} \rightarrow v'x = \frac{1}{2v} - \frac{v}{2}_{(v)} \rightarrow v'x = \frac{1-v^2}{2v} \rightarrow \frac{2v}{1-v^2} \cdot dv = \frac{1}{x} dx$

$= -\ln|1-v^2| = \ln|x| + c \quad = \quad -\ln\left|1-\frac{y^2}{x^2}\right| = \ln|x| + c$

- determining which interval the result is defined with initial condition

$y' = (1-2x)/y \qquad \rightarrow \qquad y\,dy = (1-2x)\,dx \qquad \Rightarrow \qquad y = \pm\sqrt{2x-2x^2+c}$

$y(1) = -2 \qquad\qquad \frac{y^2}{2} = x - x^2 + c \qquad\qquad -2 = \pm\sqrt{\cancel{2}-\cancel{2}+c}$

$c = 4$

$y = -\sqrt{2x-2x^2+4}$