1 - intoduction - direction fields

introduction

relations → equations
rates → derivatives

differential equations: equations containing derivatives

example

air resistance

$F = mg - \gamma v$
$ma = mg - \gamma v$
$m.\frac{dv}{dt} = mg - \gamma v$

→ m, g, γ are constants
to find a function
$v = v(t)$

→ $\frac{dv}{dt} = g - \frac{\gamma v}{m}$

ex: $\frac{dv}{dt} = 10 - \frac{v}{2}$

☆ when we are given a certain value of v, we can find the slope of the function

t = independent variable
v = dependent variable

☆ solutions of a differential equation are functions, not numbers

System of differential equations = several equations containing several variables interrelated with each other.

ex: $\frac{dx}{dt} + y = t$ $\frac{dy}{dt} + xy = \sin(t)$

solution ⇒ $(x(t), y(t))$
↳ pair of functions

ordinary differential equations (ODE) = when there is only one independent variable (others are partial differential equations)

order of the equation = the highest derivative appering in diff. equ.

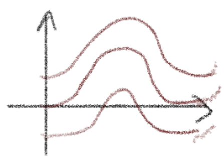
solution of first order ODE's

a function $y(t)$ that satisfies the equation at all points t in an open interval (a,b) → must be differentiable (so contin.)

example:

$\frac{dy}{dt} = \sin(t)$ → $\int \sin(t).dt$ → $-\cos(t) + \boxed{C}$

infinitely many solutions



example:

$\frac{dy}{dt} = y \longrightarrow \int y\,dt \longrightarrow$ we do not know how to integrate it, bc it is a func. with respect to t

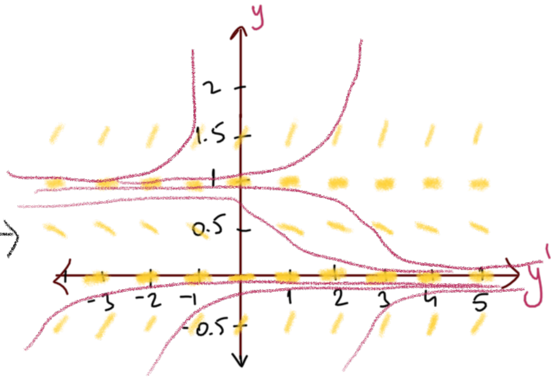
initial condition = $y(t_0) = y_0$ the solution becomes unique

direction fields

$\frac{dy}{dt} = y(y-1) \longrightarrow$

y	$f(t,y) = y'$
-0.5	0.75
0	0
0.5	-0.25
1	0
1.5	0.75

$\longrightarrow$



$y(0) < 0$	$t \to \infty \quad \to 0$ $t \to -\infty \quad \to -\infty$	increasing
$0 < y(0) < 1$	$t \to \infty \quad \to 0$ $t \to -\infty \quad \to 1$	decreasing
$1 < y(0)$	$t \to \infty \quad \to \infty$ $t \to -\infty \quad \to 1$	increasing

(stable) equilibrium solution

$y' = 0$

example

$\alpha^2 \underbrace{u_{xx}}_{\text{with respect to x two times derivative}} = \underbrace{u_t}_{\text{with respect to t, one time derivative}}$ verify that $u_1(x,t) = e^{-\alpha^2 t} \sin x$ is one of its solution

$\alpha^2 . e^{-\alpha^2 t} . (\sin x)'' \stackrel{?}{=} \sin x \left(e^{-\alpha^2 t}\right)'$

$\alpha^2 e^{-\alpha^2 t} (-\sin x) \stackrel{?}{=} \sin x (-\alpha^2)\, e^{-\alpha^2 t}$ ✓