9 - eigen values and eigen vectors

$\boxed{\text{eigen values and eigen vectors}}$

A = n×n matrix
v = vector $\neq 0$
λ = some number

$$A.v = \underbrace{\lambda}_{\text{eigen value}} \overbrace{v}^{\text{eigen vector}}$$

* $\lambda = 0$ is also valid, but we do not consider because ambiguos

ex: $v = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $= ?$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 \\ 5 \end{bmatrix} = \lambda . \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$
$$\begin{bmatrix} 15 \\ 15 \end{bmatrix} = \lambda \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$\Rightarrow$ $\lambda = 3$ is eigen value
$\begin{bmatrix} 5 \\ 5 \end{bmatrix}$ is eigen vector

ex: $w = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $= ?$

$$\left.\begin{aligned} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} &= \lambda \begin{bmatrix} 1 \\ 4 \end{bmatrix} \\ \begin{bmatrix} 6 \\ 9 \end{bmatrix} &\neq \lambda \begin{bmatrix} 1 \\ 4 \end{bmatrix} \end{aligned}\right\}$$

w is not an eigen vector

how to find them

$A v = \lambda v$
$(A - \lambda I) v = 0$
$\det(A - \lambda I) = 0$

ex: $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1-\lambda & -2 \\ 3 & -4-\lambda \end{bmatrix} \rightarrow (1-\lambda).(-4-\lambda) - (-2).3$

characteristic equation

$= \lambda^2 + 3\lambda + 2$ $\lambda_1 = -1$, $\lambda_2 = -2$

$\begin{bmatrix} 2 & -2 \\ 3 & -3 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$ $2a - 2b = 0$, $a = b$ $v = \begin{bmatrix} v_1 \\ v_1 \end{bmatrix} \rightarrow \lambda_1 = -1$

$\begin{bmatrix} 3 & -2 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = 0$ $3a - 2b = 0$, $a = \frac{2b}{3}$ $v = \begin{bmatrix} \frac{2v_2}{3} \\ v_2 \end{bmatrix} \rightarrow \lambda_2 = -2$

complex case:

$A = \begin{bmatrix} 1 & -3 \\ 5 & -3 \end{bmatrix} \rightarrow \begin{bmatrix} 1-\lambda & -3 \\ 5 & -3-\lambda \end{bmatrix} = (\lambda^2 + 2\lambda - 3 + 15) = (\lambda + 1)^2 = -11$

$\lambda_{1,2} = \mp\sqrt{11}\, i - 1$

$$= \left[\begin{array}{cc|c} 2-\sqrt{11}i & -3 & 0 \\ 5 & -2-\sqrt{11} & 0 \end{array}\right] = \left[\begin{array}{cc|c} 1 & -\frac{2}{5} - \frac{i\sqrt{11}}{5} & 0 \\ 0 & 0 & 0 \end{array}\right] \quad v = \begin{bmatrix} \left(\frac{2}{5} \overset{\mp}{+} \frac{i\sqrt{11}}{5}\right) v_2 \\ v_2 \end{bmatrix}$$

eigen value$_1$ × eigen value$_2$ ×.. = det (A)