CENG 384 - Signals and Systems for Computer Engineers
Spring 2022
Homework 1

ALAN, Şeyma
e2380053@ceng.metu.edu.tr

TINMAZ, Derya
e2380947@ceng.metu.edu.tr

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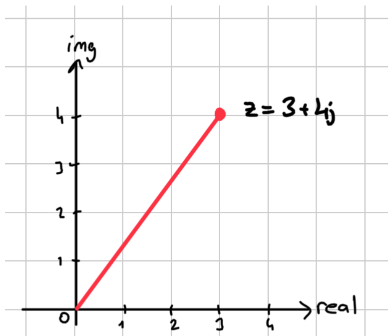
1. (a) i.
$z = x + jy$ and $\bar{z} = x - jy$
$2(x + jy) - 9 = 4j - (x - jy)$
$(4 - y)j + 3x - 9 = 0$
$y = 4, x = 3$
$z = 3 + 4j$
Since $|z|^2 = x^2 + y^2$
$|z|^2 = 3^2 + 4^2 = 25$
ii.



(b) If $z^3 = -27j$, then $z = 3j$ due to $j^3 = -j$. Given $z = a + jb$ rectangular form, we can find the polar form by $z = \sqrt{a^2 + b^2}e^{j\arctan{(b/a)}}$. For our question, $a = 0, b = 3$. So the polar form is $z = 3e^{j\pi/2}$.

(c) Given $z = \frac{(1+j)(\sqrt{3}-j)}{(\sqrt{3}+j)}$

$= \frac{(1+j)(\sqrt{3}-j)(\sqrt{3}-j)}{4}$ (multiplied both sides with $(\sqrt{3} - j)$)

$= \frac{(1+j)(3-2\sqrt{3}j-1)}{4}$ (made calculations)

$= \frac{(1+j)2(1-\sqrt{3}j)}{4}$ (made calculations)

$= \frac{(1-\sqrt{3}j+j+\sqrt{3})}{2}$ (made calculations)

Then, for $z = a + jb$, $a = \frac{1+\sqrt{3}}{2}$ and $b = \frac{1-\sqrt{3}}{2}$
Magnitude can be found by $|z| = \sqrt{a^2 + b^2}$. So, our magnitude is $\sqrt{\frac{1+2\sqrt{3}+3+1-2\sqrt{3}+3}{4}} = \sqrt{2}$.
Angle can be calculated by $\angle z = \arctan(b/a)$. So the angle is $\arctan(\frac{1-\sqrt{3}}{1+\sqrt{3}}) = \arctan(\frac{1-2\sqrt{3}+3}{-2})$ $\arctan(-2 + \sqrt{3})$ $= -15$ degrees.

(d) Since $e^{j\frac{\pi}{2}} = cos(\frac{\pi}{2}) + jsin(\frac{\pi}{2}) = 0 + j * 1 = j$, given equation can be written as $-(1 + j)^8 j$.
$-((1 + j)^2)^4 j$.
$= -(2j)^4 j$
$= -2^4 j^4 j$
$= -16j$
For $z = a + jb$, a value of the last equation is 0 and b value is -16.
We can find the polar form by $z = \sqrt{a^2 + b^2}e^{j\arctan{(b/a)}}$.
Then, using the a, b values and the above equation, polar form of $z = 16e^{-j\frac{\pi}{2}}$

2. (a) Power of a signal for discrete time is

$P = \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |nu[n]|^2$

$= \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=-N}^{0} |nu[n]|^2 + \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |nu[n]|^2$

$u[n]$ is the unit step function and its value is 0 for n values under 0 and for the $n >= 0$, $u[n] = 1$.

$= 0 + \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |n|^2$

$= \lim_{N\to+\infty} \frac{1}{2N+1} \frac{N.(N+1).(2N+1)}{6}$

$= \lim_{N\to+\infty} \frac{N.(N+1)}{6}$

Which goes to infinity, so the power is ∞.

Energy of a signal for discrete time is

$E = \sum_{n=-\infty}^{+\infty} |x[n]|^2 = \sum_{n=-\infty}^{+\infty} |nu[n]|^2$

$= \sum_{n=-\infty}^{0} |nu[n]|^2 + \sum_{n=0}^{\infty} |nu[n]|^2$

$u[n]$ is the unit step function and its value is 0 for n values under 0 and for the $n >= 0$, $u[n] = 1$.

n $= 0 + \sum_{n=0}^{\infty} |n|^2$

$= \sum_{n=0}^{\infty} |n|^2 = 0 + 1^2 + 2^2 + 3^2 + ...$

Which goes to infinity, so the energy is ∞.

Since E_x is ∞ and P_x is ∞, then x is neither an energy signal nor a power signal.

(b) Power of a signal for continuous time is

$P_X = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt.$

For the given equation in the question,

$P_X = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} (e^{-2t}u(t))^2 dt.$

$u(t)$ is the unit step function and its value is 0 for t values under 0 and for the $t >= 0$, $u(t) = 1$.

Therefore, power can be rewritten as,

$P_X = \lim_{T\to\infty} \frac{1}{2T} \int_{0}^{T} (e^{-2t})^2 dt.$

$= \lim_{T\to\infty} \frac{1}{2T} \frac{-1}{4}(e^{-4T} - 1) = 0$. So the power of the signal is 0.

Energy of a signal for continuous time is

$E_X = \int_{-\infty}^{\infty} |x(t)|^2 dt.$

Energy of the given signal is,

$E_X = \int_{-\infty}^{\infty} (e^{-2t}u(t))^2 dt.$

$u(t)$ is the unit step function and its value is 0 for t values under 0 and for the $t >= 0$, $u(t) = 1$.

Therefore, energy can be rewritten as,

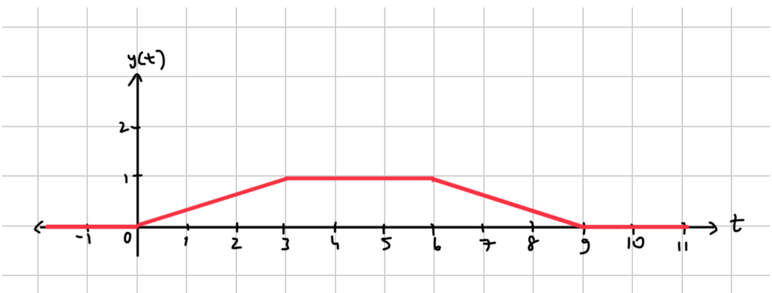
$E_X = \int_{0}^{\infty} (e^{-2t})^2 dt = \frac{1}{4}.$

So the energy of the signal is $\frac{1}{4}$.

Since $E_x < \infty$ and $P_x = 0$, then x is an energy signal.

3. For the given $y(t) = \frac{1}{2}x(\frac{-1}{3}t + 2)$ signal, $y(t)$ can be written as:

$$\frac{1}{2}x(\frac{-1}{3}t + 2) = \left\{ \begin{array}{ll} 0, & \text{for } t \leq 0 \\ \frac{1}{3}t, & \text{for } 0 < t \leq 3 \\ 1, & \text{for } 3 \leq t \leq 6 \\ \frac{-1}{3}t + 3, & \text{for } 6 \leq t \leq 9 \\ 0, & \text{for } 9 \leq t \end{array} \right\} = y(t)$$



4. (a) when

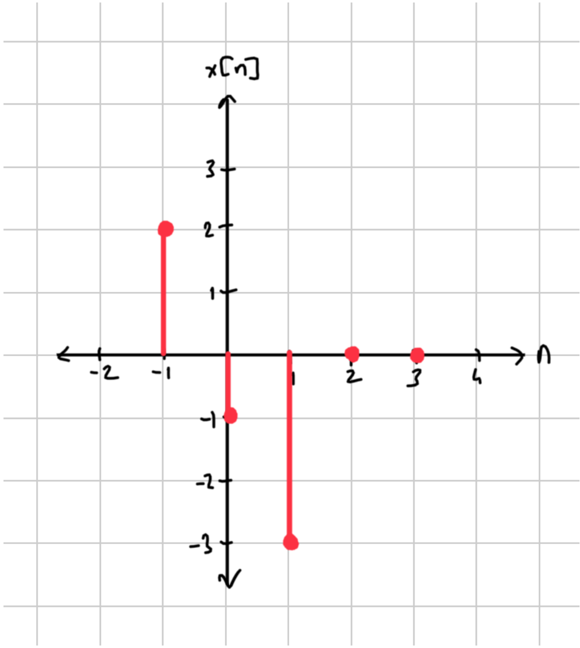
$n = -1, x[2] + x[-3] = 1 + 1 = 2 = x[-1]$

$n = 0, x[0] + x[-2] = 0 - 1 = -1 = x[0]$

$n = 1, x[-2] + x[-1] = -1 - 2 = -3 = x[1]$

$n = 2, x[-4] + x[0] = 0 + 0 = 0 = x[2]$

$n = 3, x[-6] + x[1] = 0 + 0 = 0 = x[3]$



(b) $x[n] = 2\delta[n+1] - \delta[n] - 3\delta[n-1]$

5. (a) $\frac{e^{j3t}}{-j} = \frac{e^{j3(t+T_0)}}{-j}$
$e^{j3t} - e^{j3(t+T_0)} = 0$
$e^{j3t}(1 - e^{j3T_0}) = 0$ to always satisfy this equation $(1 - e^{j3T_0})$ must be zero, since the value of e^{j3t} changes by t
Then, $e^{j3T_0} = 1$
Since $e^{j\theta} = cos\theta + jsin\theta$, when $\theta = 2\pi k$, $e^{j\theta} = 1$
$3T_0 = 2\pi k$
The smallest positive integer k value is 3.
Therefore, the fundamental period is $T_0 = 2\pi$

(b) For the first part of the signal:
$\frac{1}{2}sin[\frac{7\pi}{8}n] = \frac{1}{2}sin[\frac{7\pi}{8}(n + N_0)]$
Since the period of sin function is 2π
$\frac{7\pi N_0}{8} = 2\pi k_1$
$N_0 = \frac{16k_1}{7}$
For the second part of the signal:
$4cos[\frac{3\pi}{4}n - \frac{\pi}{2}] = 4cos[\frac{3\pi}{4}(n + N_0) - \frac{\pi}{2}]$
Since the period of cos function is 2π
$\frac{3\pi N_0}{4} = 2\pi k_2$
$N_0 = \frac{8k_2}{3}$
Then $N_0 = \frac{16k_1}{7} = \frac{8k_2}{3}$
For smallest positive integers of $k_1 = 7$ and $k_2 = 6$, The fundamental period is $N_0 = 16$

6. (a) If $x(t) = x(-t)$, then $x(t)$ is an even signal. If $x(t) = -x(-t)$, then $x(t)$ is an odd signal. However, the given signal $x(t)$ is neither even nor odd signal. To show that, for example,
$x(1) = 2, x(-1) = 0$. So $x(t)$ is neither even nor odd.

(b) Firstly, let us write $x(t)$.

$$x(t) = \begin{Bmatrix} 0, & \text{for } t \leq -1 \\ 2+2t, & \text{for } -1 \leq t \leq 0 \\ 2, & \text{for } 0 \leq t \leq 1 \\ 4-2t, & \text{for } 1 \leq t \leq 2 \\ 0, & \text{for } 2 \leq t \end{Bmatrix}$$

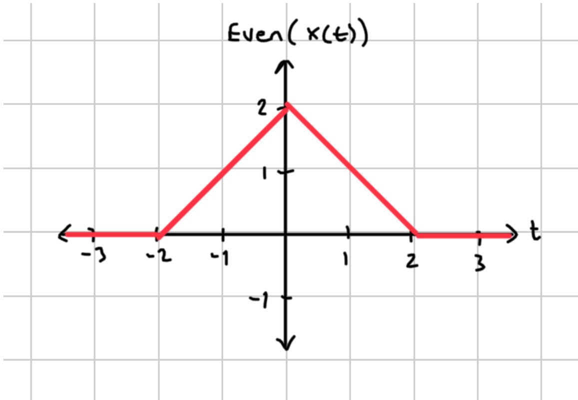
Secondly, find and write the $x(-t)$.

$$x(-t) = \begin{Bmatrix} 0, & \text{for } 1 \leq t \\ 2-2t, & \text{for } 0 \leq t \leq 1 \\ 2, & \text{for } -1 \leq t \leq 0 \\ 4+2t, & \text{for } -2 \leq t \leq -1 \\ 0, & \text{for } t \leq -2 \end{Bmatrix}$$

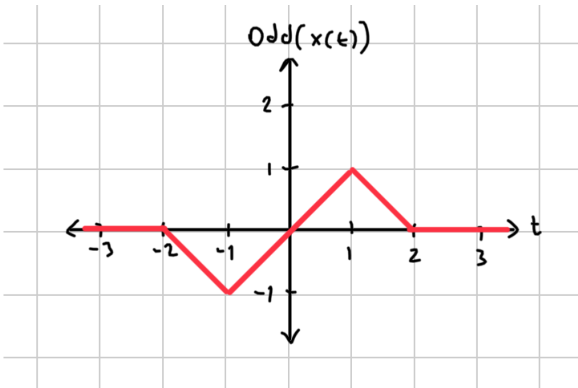
Any signal can be represented by its even and odd components, as follows;
$x(t) = Odd\{x(t)\} + Even\{x(t)\}$, where
$Odd\{x(t)\} = \frac{1}{2}(x(t) - x(-t))$,
$Even\{x(t)\} = \frac{1}{2}(x(t) + x(-t))$.

So, even and odd parts of the signal can be written as,

$$\frac{x(t)+x(-t)}{2} = \left\{\begin{array}{ll} 0, & \text{for } t \leq -2 \\ 2+t, & \text{for } -2 \leq t \leq 0 \\ 2-t, & \text{for } 0 \leq t \leq 2 \\ 0, & \text{for } 2 \leq t \end{array}\right\} = Even\{x(t)\}$$

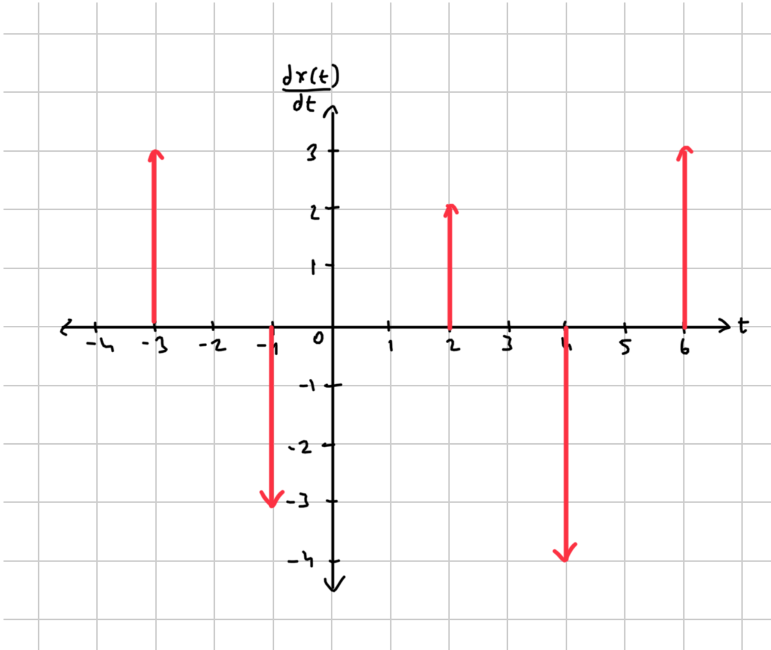


$$\frac{x(t)-x(-t)}{2} = \left\{\begin{array}{ll} 0, & \text{for } t \leq -2 \\ -2-t, & \text{for } -2 \leq t \leq -1 \\ t, & \text{for } -1 \leq t \leq 1 \\ 2-t, & \text{for } 1 \leq t \leq 2 \\ 0, & \text{for } 2 \leq t \end{array}\right\} = Odd\{x(t)\}$$



7. (a) $x(t) = 3u(t+3) - 3u(t+1) + 2u(t-2) - 4u(t-4) + 3u(t-6)$

   (b) $\frac{dx(t)}{dt} = 3\delta(t+3) - 3\delta(t+1) + 2\delta(t-2) - 4\delta(t-4) + 3\delta(t-6)$



8. (a) Memory:
For $n = 0$, $y[0] = x[-2]->$ past, for $n = 3$, $y[3] = x[4]->$ future
The present value of the output depends on past and future values of the input, so the system has memory.
Stability:
For a definite bounded input, we can get a bounded output. For example if we put $x[2n-2] = 3$, $y[n] = 3$ which is bounded in nature. Therefore the system is stable.
Causality:
For $n = 3$, $y[3] = x[4]->$ future
The present value of the output depends on future values of the input, so the system is not causal.
Linearity:
$x_1[n] \rightarrow x_1[2n-2] = y_1[n]$
$x_2[n] \rightarrow x_2[2n-2] = y_2[n]$
Let's write the superposition of inputs: $a_1x_1[n] + a_2x_2[n] \rightarrow a_1x_1[2n-2] + a_2x_2[2n-2]$.
Super position of outputs $a_1y_1[n] + a_2y_2[n] = a_1x_1[2n-2] + a_2x_2[2n-2]$. Since they are equal, the system is linear.
Invertibility:
Since there are no two inputs which produce the same output for the given system, the system is invertible.
Time In-variance:
If the signal is first passed through the system and then through the delay, the output will be $x[2n-2-n_0]$
If it is passed through the time delay first and then through the system, the output will be $x[2(n-n_0)-2] = x[2n-2-2n_0]$
Since the outputs are not same, the system is time variant.

(b) Memory:
For $t = -4$, $y(-4) = -4x(-3)->$ future, for $t = 4$, $y(4) = 4x(3)->$ past
The present value of the output depends on past and future values of the input, so the system has memory.
Stability:
For a finite input, we cannot expect a finite output. For example if we put $x(\frac{t}{2}-1) = 2->y(t) = 2t$. This is not a finite value, because we do not know the value of t. It can be ranged from anywhere. Therefore, the system is unstable.
Causality:
For $t = -4$, $y(-4) = -4x(-3)->$ future
The present value of the output depends on future values of the input, so the system is not causal.
Linearity:
$x_1(t) \rightarrow x_1(\frac{t}{2}-1) = y_1(t)$
$x_2(t) \rightarrow x_2(\frac{t}{2}-1) = y_2(t)$
Let's write the superposition of inputs: $a_1x_1(t) + a_2x_2(t) \rightarrow ta_1x_1(\frac{t}{2}-1) + ta_2x_2(\frac{t}{2}-1)$.
Super position of outputs $a_1y_1(t) + a_2y_2(t) \rightarrow a_1tx_1(\frac{t}{2}-1) + a_2tx_2(\frac{t}{2}-1)$. Since superposition of outputs and inputs are equal, the system is linear.
Invertibility:
When $t = 0$, inverse $\frac{y(t)}{t}$ is undefined. So when $t \neq 0$, since there are no two inputs which produce the same output for the given system, the system is invertible.

Time In-variance:

If the signal is first passed through the system and then through the delay, the output will be $(t-t_0)x(\frac{t}{2}-1-t_0)$
If it is passed through the time delay first and then through the system, the output will be $(t-t_0)x(\frac{(t-t_0)}{2}-1) = (t-t_0)x(\frac{t}{2}-1-\frac{t_0}{2})$
Since the outputs are not same, the system is time variant.