roc curve (receiver operating characteristic)

	True Condition	
Decision	1 (+)	0 (-)
H_1	True Positive (Correct)	False Positive (Type I Error)
H_0	False Negative (Type II Error)	True Negative (Correct)

- So far we have concentrated on Type I errors.
  - Specificity of a test:

$$\text{specificity} = \frac{\#\text{ true negatives}}{\#\text{ false positives} + \#\text{ true negatives}} = P(H_0|-) = 1-\alpha$$

  - α is the type I error rate (false positive rate)
- For Type II errors:
  - Sensitivity of a test:

$$\text{sensitivity} = \frac{\#\text{ true positives}}{\#\text{ true positives} + \#\text{ false negatives}} = P(H_1|+) = 1-\beta$$

  - β is the type II error rate (false negative rate)

Fundamental tool used in evaluating the performance of a binary classifier system as its discrimination threshold is varied. It's particularly common in fields like medicine, radiology, and machine learning. Here's an in-depth look at ROC curves:

Concept

- **Binary Classifier**: A system that classifies elements into two categories, such as positive/negative or true/false.
- **Thresholds**: The classifier's prediction is based on a threshold. Changing this threshold affects the classifier's sensitivity and specificity.

Components

1. **True Positive Rate (TPR)**: Also known as sensitivity or recall, it is the proportion of actual positives correctly identified by the classifier.
   $$\text{TPR} = \frac{\text{True Positives}}{\text{True Positives}+\text{False Negatives}}$$
2. **False Positive Rate (FPR)**: The proportion of actual negatives that are incorrectly classified as positive.
   $$\text{FPR} = \frac{\text{False Positives}}{\text{False Positives}+\text{True Negatives}}$$

Plotting the ROC Curve

- The ROC curve plots TPR against FPR at various threshold settings.
- The x-axis represents FPR, and the y-axis represents TPR.
- A point in the top left corner (high TPR, low FPR) represents a perfect classification.

Analyzing ROC Curves

- **Area Under the Curve (AUC)**: The area under the ROC curve is a measure of the classifier's ability to distinguish between the two classes. An AUC of 1 indicates perfect classification, 0.5 suggests no discriminative power (equivalent to random guessing), and 0.0 indicates complete misclassification.
- **Trade-offs**: The curve shows the trade-off between sensitivity and specificity. A higher TPR comes with a higher FPR.

Applications

- **Medical Diagnostics**: Evaluating the performance of diagnostic tests.
- **Machine Learning Models**: Assessing the performance of classification algorithms.

Comparing Classifiers

- ROC curves are used to compare the performance of different classifiers. A model whose ROC curve is closer to the top left corner is generally considered better.

Limitations

- **Imbalanced Classes**: ROC curves may present an overly optimistic view of the classifier's performance on imbalanced datasets.
- **Not Sensitive to Threshold Selection**: ROC curves can mask the effects of selecting different thresholds for decision making.

Alternatives

- **Precision-Recall Curves**: Often used in cases of highly imbalanced datasets, focusing on the positive class's perspective.

In summary, ROC curves are a powerful method for assessing and comparing the performance of binary classifiers, helping to visualize the trade-offs between true positive and false positive rates at various thresholds

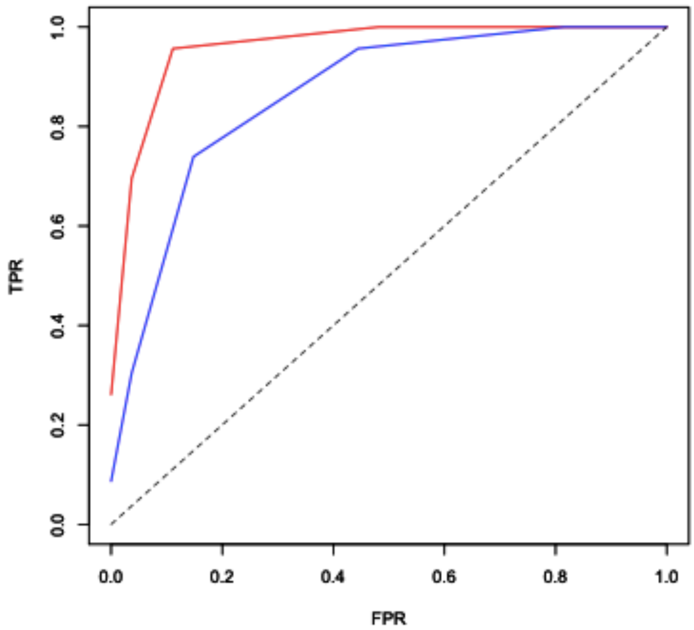
ROC Curve Example: Fish

- Using length:

Length (x)	**2**	**4**	**8**	**10**	**12**	**14**
# Bass	0	1	3	8	10	5
# Salmon	2	5	10	5	1	0

- Assume we say H_0 : Salmon
  - Test statistic is length (x)
  - Reject H_0 if $x > x*$, e.g.
    - $TPR = 2/23$, $FPR = 0/27$ if $x* = 2$
    - $TPR = 7/23$, $FPR = 1/26$ if $x* = 4$
- What about lightness?

Lightness (l)	**1**	**2**	**3**	**4**	**5**
# Bass	0	1	2	10	12
# Salmon	6	10	6	1	0



Length **Lightness**