10 - confidence intervals

$P\{a \le \theta \le b\} = 1-\alpha$ → confidence interval $[a,b]$ with probability $(1-\alpha)$ for θ paramet

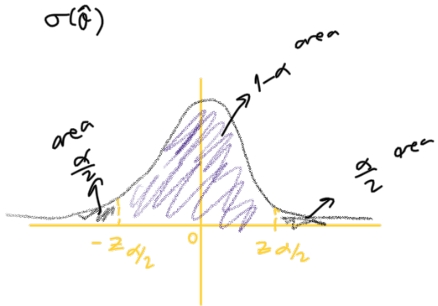
$1-\alpha$ = coverage probability = confidence level

↳ parameter is constant, it either belongs to the interval or not.

- standardize the parameter → $z = \dfrac{\hat{\theta} - E(\hat{\theta})}{\sigma(\hat{\theta})} = \dfrac{\hat{\theta} - \theta}{\sigma(\hat{\theta})}$ unbiased

$$P\left\{-z_{\frac{\alpha}{2}} \le \frac{\hat{\theta}-\theta}{\sigma(\hat{\theta})} \le z_{\frac{\alpha}{2}}\right\} = 1-\alpha$$

$$\hookrightarrow P\{\underbrace{\hat{\theta} - z_{\frac{\alpha}{2}}\cdot\sigma(\hat{\theta})}_{a} \le \theta \le \underbrace{\hat{\theta} + z_{\frac{\alpha}{2}}\cdot\sigma(\hat{\theta})}_{b}\}$$



$\hat{\theta}$ = center of the interval

$z_{\frac{\alpha}{2}} \cdot \sigma(\hat{\theta})$ = margin

$\boxed{\text{confidence interval for the population mean}}$

$\theta = \mu = E(x)$

$\hat{\theta} = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$E(\bar{x}) = \mu$ (unbiased)

$\sigma(\bar{x}) = \frac{\sigma}{\sqrt{n}}$

$\left[\bar{x} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$ is a $(1-\alpha)$10(confidence interva for μ.

- If sample comes from normal distribution
- if sample comes from any distribution, but the sample size (n) is large (central limit theorem)

↳ then $(1-\alpha)$ 100% confidence interval

- method cannot be used only when the sample size is small and the distribution of data is not Normal (then student's)