CENG 280
Formal Languages and Abstract Machines
Spring 2021-2022
Homework 1

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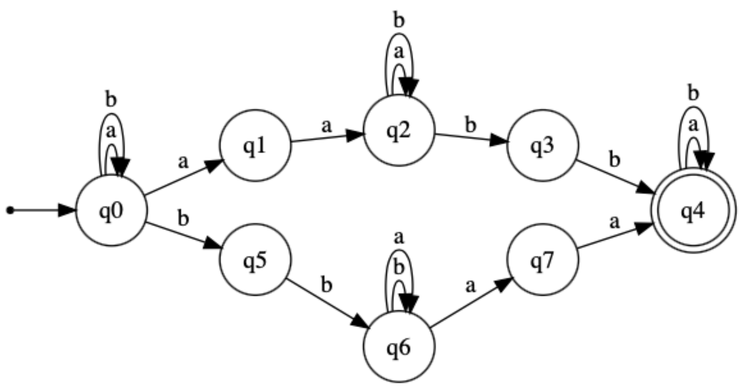
1. (a) $((a \cup b)^* aa (a \cup b)^* bb (a \cup b)^*) \cup ((a \cup b)^* bb (a \cup b)^* aa (a \cup b)^*)$
or $(a \cup b)^*((aa(a \cup b)^* bb) \cup (bb(a \cup b)^* aa))(a \cup b)^*$

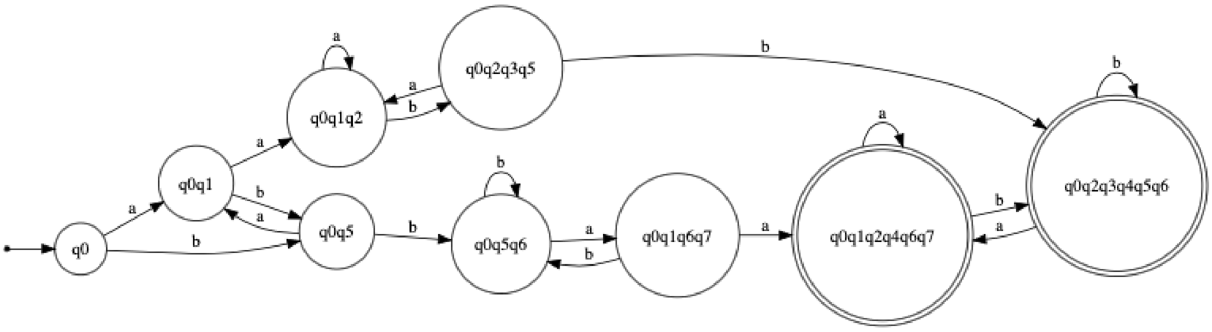
(b) $M = (K, \sum, \Delta, s, F)$
$K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$
$\sum = \{a, b\}$
$s = q_0$
$F = \{q_4\}$
$\Delta = \{(q_0, a, q_0), (q_0, b, q_0), (q_0, a, q_1), (q_1, a, q_2), (q_2, a, q_2), (q_2, b, q_2), (q_2, b, q_3), (q_3, b, q_4),$
$(q_4, a, q_4), (q_4, b, q_4), (q_0, b, q_5), (q_5, b, q_6), (q_6, a, q_6), (q_6, b, q_6), (q_6, a, q_7), (q_7, a, q_4)\}$



(c) $M' = (K', \sum, \delta, s', F')$
$K' = \{\{q_0\}, \{q_0, q_1\}, \{q_0, q_5\}, \{q_0, q_1, q_2\}, \{q_0, q_5, q_6\}, \{q_0, q_2, q_3, q_5\},$
$\{q_0, q_1, q_6, q_7\}, \{q_0, q_2, q_3, q_4, q_5, q_6\}, \{q_0, q_1, q_2, q_4, q_6, q_7\}\}$
$\sum = \{a, b\}$
$s' = q_0$
$F' = \{\{q_0, q_2, q_3, q_4, q_5, q_6\}, \{q_0, q_1, q_2, q_4, q_6, q_7\}\}$ (the ones that include q_4)

$E(q_0) = \{q_0\}, E(q_1) = \{q_1\}, E(q_2) = \{q_2\}, E(q_3) = \{q_3\}, E(q_4) = \{q_4\}, E(q_5) = \{q_5\}, E(q_6) = \{q_6\}, E(q_7) = \{q_7\},$
$\delta(\{q_0\}, a) = E(q_0) \cup E(q_1) = \{q_0, q_1\}$
$\delta(\{q_0\}, b) = E(q_0) \cup E(q_5) = \{q_0, q_5\}$
$\delta(\{q_0, q_1\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) = \{q_0, q_1, q_2\}$
$\delta(\{q_0, q_1\}, b) = E(q_0) \cup E(q_5) \cup \{\} = \{q_0, q_5\}$
$\delta(\{q_0, q_5\}, a) = E(q_0) \cup E(q_1) \cup \{\} = \{q_0, q_1\}$
$\delta(\{q_0, q_5\}, b) = E(q_0) \cup E(q_5) \cup E(q_6) = \{q_0, q_5, q_6\}$
$\delta(\{q_0, q_1, q_2\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup E(q_2) = \{q_0, q_1, q_2\}$
$\delta(\{q_0, q_1, q_2\}, b) = E(q_0) \cup E(q_5) \cup \{\} \cup E(q_2) \cup E(q_3) = \{q_0, q_2, q_3, q_5\}$
$\delta(\{q_0, q_5, q_6\}, a) = E(q_0) \cup E(q_1) \cup \{\} \cup E(q_6) \cup E(q_7) = \{q_0, q_1, q_6, q_7\}$
$\delta(\{q_0, q_5, q_6\}, b) = E(q_0) \cup E(q_5) \cup E(q_6) \cup E(q_6) = \{q_0, q_5, q_6\}$
$\delta(\{q_0, q_2, q_3, q_5\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup \{\} \cup \{\} = \{q_0, q_1, q_2\}$
$\delta(\{q_0, q_2, q_3, q_5\}, b) = E(q_0) \cup E(q_5) \cup E(q_2) \cup E(q_3) \cup E(q_4) \cup E(q_6) = \{q_0, q_2, q_3, q_4, q_5, q_6\}$
$\delta(\{q_0, q_1, q_6, q_7\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup E(q_6) \cup E(q_7) \cup E(q_4) = \{q_0, q_1, q_2, q_4, q_6, q_7\}$
$\delta(\{q_0, q_1, q_6, q_7\}, b) = E(q_0) \cup E(q_5) \cup \{\} \cup E(q_6) \cup \{\} = \{q_0, q_5, q_6\}$
$\delta(\{q_0, q_2, q_3, q_4, q_5, q_6\}, a) = E(q_0) \cup E(q_1) \cup E(q_2) \cup \{\} \cup E(q_4) \cup \{\} \cup E(q_6) \cup E(q_7) = \{q_0, q_1, q_2, q_4, q_6, q_7\}$
$\delta(\{q_0, q_2, q_3, q_4, q_5, q_6\}, b) = E(q_0) \cup E(q_5) \cup E(q_2) \cup E(q_3) \cup E(q_4) \cup E(q_4) \cup E(q_6) \cup E(q_6) = \{q_0, q_2, q_3, q_4, q_5, q_6\}$

$\delta(\{q_0,q_1,q_2,q_4,q_6,q_7\},a) = E(q_0)\cup E(q_1)\cup E(q_2)\cup E(q_2)\cup E(q_4)\cup E(q_6)\cup E(q_7)\cup E(q_4) = \{q_0,q_1,q_2,q_4,q_6,q_7\}$
$\delta(\{q_0,q_1,q_2,q_4,q_6,q_7\},b) = E(q_0)\cup E(q_5)\cup \{\}\cup E(q_2)\cup E(q_3)\cup E(q_4)\cup E(q_6)\cup \{\} = \{q_0,q_2,q_3,q_4,q_5,q_6\}$



(d) For NFA:
$(\{q_0\}, bbabb) \vdash_M (\{q_0\}, babb) \vdash_M (\{q_0\}, abb) \vdash_M (\{q_0\}, bb) \vdash_M (\{q_0\}, b) \vdash_M (\{q_0\}, e)$ (q_0 is not a final state)
$(\{q_0\}, bbabb) \vdash_M (\{q_0\}, babb) \vdash_M (\{q_0\}, abb) \vdash_M (\{q_0\}, bb) \vdash_M (\{q_0\}, b) \vdash_M (\{q_5\}, e)$ (q_5 is not a final state)
$(\{q_0\}, bbabb) \vdash_M (\{q_0\}, babb) \vdash_M (\{q_0\}, abb) \vdash_M (\{q_0\}, bb) \vdash_M (\{q_5\}, b) \vdash_M (\{q_6\}, e)$ (q_6 is not a final state)
$(\{q_0\}, bbabb) \vdash_M (\{q_0\}, babb) \vdash_M (\{q_0\}, abb) \vdash_M (\{q_1\}, bb)$ (q_1 does not accept b)
$(\{q_0\}, bbabb) \vdash_M (\{q_0\}, babb) \vdash_M (\{q_5\}, abb)$ (q_5 does not accept a)
$(\{q_0\}, bbabb) \vdash_M (\{q_5\}, babb) \vdash_M (\{q_6\}, abb) \vdash_M (\{q_6\}, bb) \vdash_M (\{q_6\}, b) \vdash_M (\{q_6\}, e)$ (q_6 is not a final state)
$(\{q_0\}, bbabb) \vdash_M (\{q_5\}, babb) \vdash_M (\{q_6\}, abb) \vdash_M (\{q_7\}, bb)$ (q_7 does not accept b)
Since none of them reaches the final state, the w' is not accepted.
For DFA:
$(\{q_0\}, bbabb) \vdash_{M'} (\{q_0,q_5\}, babb) \vdash_{M'} (\{q_0,q_5,q_6\}, abb) \vdash_{M'} (\{q_0,q_1,q_6,q_7\}, bb) \vdash_{M'} (\{q_0,q_5,q_6\}, b) \vdash_{M'} (\{q_0,q_5,q_6\}, e)$
Since $\{q_0,q_5,q_6\}$ is not a final state, the w' is not accepted.

2. (a) First let check if L_1 is regular.
Let L_1 be a regular language. Then there exists an integer $n \geq 1$ such that every string w in L_1 of length at least n can be written as $w = xyz$ with satisfying the following conditions:
1.$|y| \geq 1$
2.$|xy| \leq n$
3.$xy^iz \in L_1$ for $\forall i \geq 0$
Let $m = n+1$ then $w = a^{n+1}b^n$ where $|w| = (2n+1) \geq n$
Find xyz such that $|xy| \leq n$
$w = aaaa..aabb...b$ then $x = a^\alpha, y = a^\beta, z = a^{n+1-\alpha-\beta}b^n$ where $\beta \geq 1$ and $\alpha+\beta \leq n$
xy^iz must be in the language for all $i \geq 0$
If we take $i = 0$, we get $xz = a^\alpha a^{n+1-\alpha-\beta}b^n = a^{n+1-\beta}b^n$. Since $(n+1-\beta) \not> n$ this does not satisfy the $a^mb^n, m > n$ rule of the language, by contradiction L_1 is not a regular language.
L_2 is the complement of the L_1. Complement of a non-regular language is always a non-regular language. Then L_2 also is a non-regular language. Proof of this by contradiction:
Let L be a non regular language, and let $\overline{L}$ be a regular language. Since $\overline{L}$ is regular $\overline{\overline{L}} = L$ also is regular. Which contradicts our assumption that $\overline{L}$ is non regular. L is regular if and only if $\overline{L}$ is regular.
Therefore, L_2 is not a regular language.

(b) L_4 is a language that consist of strings which starts with n a's followed by same number of n b's. Such as $\{ab, aabb, aaabbb, ...\}$
L_5 is a language that consist of strings which starts with some number of a's followed by some number of b's. Such as $\{e, a, b, ab, aa, bb, aaa, aab, abb, bbb, aaaa, aaab, aabb, abbb, bbbb, ..\}$
Therefore, set of strings that are accepted by L_5 includes the set of strings that are accepted by L_4.(Since L_5 includes all number of a's followed by b's, and L_5 includes only equal number of a's followed by b's)
We can say that L_4 is a subset of L_5. Therefore their union is L_5 language. $L_4 \cup L_5 = L_5$
We can express L_5 as regular expression a^*b^*.
$L_5 \cup L_6 = a^*b^* \cup b^*a(ab^*a)^*$
Since a regular language is a language that can be expressed with a regular expression, and we can express $L_4 \cup L_5 \cup L_6$ as $a^*b^* \cup b^*a(ab^*a)^*$, their union is a regular language.