

MILP Formulation for ECU Optimization

Decision Variables

$x_{s,m} \in \mathbb{Z}_{\geq 0}$	Number of ECU model m assigned to shelter s
$e_s \in \mathbb{R}_{\geq 0}$	Excess BTU capacity for shelter s

Parameters

CapacityBTU $_m$: Cooling capacity (BTU/hr) of model m
Power $_m$: Power consumption (kW) of model m
Cost $_m$: Cost (USD) of model m
Weight $_m$: Weight (kg) of model m
Size $_m$: Size (volume units) of model m
TargetBTU $_s$: BTU requirement of shelter s
$w_{\text{cost}}, w_{\text{power}}, w_{\text{weight}}, w_{\text{size}}$: User-defined weights for each metric
p_{btu}	: Penalty weight for excess BTU capacity
Compatible $_s$: $\begin{cases} 1 & \text{if shelter } s \text{ accepts window-mounted ECUs} \\ 0 & \text{otherwise} \end{cases}$
Window $_m$: $\begin{cases} 1 & \text{if ECU model } m \text{ is window-mounted} \\ 0 & \text{otherwise} \end{cases}$

Normalization

For use *only in the objective function*, each attribute is normalized by dividing by its maximum value across all ECU models:

$$\hat{Value}_m = \frac{Value_m}{\max_j(Value_j)} \in [0, 1]$$

This normalization is applied to cost, power, weight, and size. CapacityBTU is *not normalized*, since it appears only in the constraints.

Objective Function

$$\min \sum_m x_{s,m} \left(w_{\text{cost}} \cdot \hat{\text{Cost}}_m + w_{\text{power}} \cdot \hat{\text{Power}}_m + w_{\text{weight}} \cdot \hat{\text{Weight}}_m + w_{\text{size}} \cdot \hat{\text{Size}}_m \right) + p_{\text{btu}} \cdot e_s \quad \forall s$$

Constraints

$$\begin{aligned}
& \sum_m \text{CapacityBTU}_m \cdot x_{s,m} \geq \text{TargetBTU}_s \quad \forall s \quad (\text{Meet shelter demand}) \\
& \sum_m \text{CapacityBTU}_m \cdot x_{s,m} - e_s \leq \text{TargetBTU}_s \quad \forall s \quad (\text{Define excess}) \\
& \begin{aligned}
& x_{s,m} = 0 & \forall s, m \text{ s.t. } \text{Compatible}_s=0 \text{ and } \text{Window}_m=1 & (\text{Window-unit compatibility}) \\
& x_{s,m} \in \mathbb{Z}_{\geq 0} & \forall s, m \\
& e_s \in \mathbb{R}_{\geq 0} & \forall s
\end{aligned}
\end{aligned}$$