

MILP Formulation for ECU Optimization

Decision Variables

$$\begin{aligned} x_{s,m} &\in \mathbb{Z}_{\geq 0} && \text{Number of ECU model } m \text{ assigned to shelter } s \\ e_s &\in \mathbb{R}_{\geq 0} && \text{Excess BTU capacity for shelter } s \end{aligned}$$

Parameters

$$\begin{aligned} \text{CapacityBTU}_m &: \text{Cooling capacity (BTU/hr) of model } m \\ \text{Power}_m &: \text{Power consumption (kW) of model } m \\ \text{Cost}_m &: \text{Cost (USD) of model } m \\ \text{Weight}_m &: \text{Weight (kg) of model } m \\ \text{Size}_m &: \text{Size (volume units) of model } m \\ \text{TargetBTU}_s &: \text{BTU requirement of shelter } s \\ w_{\text{cost}}, w_{\text{power}}, w_{\text{weight}}, w_{\text{size}} &: \text{User-defined weights for each metric} \\ p_{\text{btu}} &: \text{Penalty weight for excess BTU capacity} \\ \text{Compatible}_s &: \begin{cases} 1 & \text{if shelter } s \text{ accepts window-mounted ECUs} \\ 0 & \text{otherwise} \end{cases} \\ \text{Window}_m &: \begin{cases} 1 & \text{if ECU model } m \text{ is window-mounted} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Normalization

For use *only in the objective function*, each attribute is normalized by dividing by its maximum value across all ECU models:

$$\hat{Value}_m = \frac{Value_m}{\max_j(Value_j)} \in [0, 1]$$

This normalization is applied to cost, power, weight, and size. CapacityBTU is *not normalized*, since it appears only in the constraints.

Objective Function

$$\min \sum_m x_{s,m} \left(\hat{w}_{\text{cost}} \cdot \hat{\text{Cost}}_m + \hat{w}_{\text{power}} \cdot \hat{\text{Power}}_m + \hat{w}_{\text{weight}} \cdot \hat{\text{Weight}}_m + \hat{w}_{\text{size}} \cdot \hat{\text{Size}}_m \right) + p_{\text{btu}} \cdot e_s \quad \forall s$$

Constraints

$$\begin{aligned}
 \sum_m \text{CapacityBTU}_m \cdot x_{s,m} &\geq \text{TargetBTU}_s \quad \forall s \quad (\text{Meet shelter demand}) \\
 \sum_m \text{CapacityBTU}_m \cdot x_{s,m} - e_s &\leq \text{TargetBTU}_s \quad \forall s \quad (\text{Define excess}) \\
 x_{s,m} &= 0 \quad \forall s, m \text{ s.t. } \text{Compatible}_{s,m} = 0 \quad (\text{Window-unit compatibility}) \\
 x_{s,m} &\in \mathbb{Z}_{\geq 0} \quad \forall s, m \\
 e_s &\in \mathbb{R}_{\geq 0} \quad \forall s
 \end{aligned}$$