MILP Formulation for ECU Optimization

Decision Variables

$$x_{s,m} \in \mathbb{Z}_{\geq 0} \quad \text{Number of ECU model } m \text{ assigned to shelter } s$$
$$e_s \in \mathbb{R}_{\geq 0} \quad \text{Excess BTU capacity for shelter } s$$

Parameters

$$\text{CoolingCapacity}_m : \text{Cooling capacity (BTU/hr) of model } m$$
$$\text{HeatingCapacity}_m : \text{Heating capacity (BTU/hr) of model } m$$
$$\text{CoolingPower}_m : \text{Cooling power consumption (kW) of model } m$$
$$\text{HeatingPower}_m : \text{Heating power consumption (kW) of model } m$$
$$\text{Cost}_m : \text{Cost (USD) of model } m$$
$$\text{Weight}_m : \text{Weight (lbs) of model } m$$
$$\text{Size}_m : \text{Size (ft}^3\text{) of model } m$$
$$\text{TargetBTU}_s : \text{BTU requirement of shelter } s$$
$$w_{\text{cost}}, w_{\text{power}}, w_{\text{weight}}, w_{\text{size}} : \text{User-defined weights for each metric}$$
$$p : \text{User-defined penalty weight for excess BTU capacity}$$
$$\text{Window}_m : \begin{cases} 1 & \text{if model } m \text{ is window-mounted} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{Compat}_s : \begin{cases} 1 & \text{if shelter } s \text{ accepts window-mounted ECUs} \\ 0 & \text{otherwise} \end{cases}$$

Normalization

For use *only in the objective function*, each attribute is normalized by dividing by its maximum value across all ECU models:

$$\hat{Value}_m = \frac{Value_m}{\max_j(Value_j)} \quad \in [0, 1]$$

This normalization is applied to cost, power, weight, and size. Cooling and heating capacities and power are normalized separately.

Objective Function

$$\min \quad \sum_m x_{s,m} \Big(w_{\text{cost}} \cdot \hat{\text{Cost}}_m + w_{\text{power}} \cdot \hat{\text{Power}}_m + w_{\text{weight}} \cdot \hat{\text{Weight}}_m + w_{\text{size}} \cdot \hat{\text{Size}}_m \Big) + p \cdot e_s \quad \forall s$$

Where:

- Cooling mode: $\hat{\text{Power}}_m$ and $\hat{\text{Capacity}}_m$ are based on **cooling** power and capacity.
- Heating mode: $\hat{\text{Power}}_m$ and $\hat{\text{Capacity}}_m$ are based on **heating** power and capacity.

Constraints

$$\sum_m \text{CoolingCapacityBTU}_m \cdot x_{s,m} \geq \text{TargetBTU}_s \quad \forall s \quad \text{(Cooling mode: Meet cooling demand)}$$

$$\sum_m \text{HeatingCapacityBTU}_m \cdot x_{s,m} \geq \text{TargetBTU}_s \quad \forall s \quad \text{(Heating mode: Meet heating demand)}$$

$$\sum_m \text{CoolingCapacityBTU}_m \cdot x_{s,m} - e_s \leq \text{TargetBTU}_s \quad \forall s \quad \text{(Cooling mode: Define excess cooling)}$$

$$\sum_m \text{HeatingCapacityBTU}_m \cdot x_{s,m} - e_s \leq \text{TargetBTU}_s \quad \forall s \quad \text{(Heating mode: Define excess heating)}$$

$$x_{s,m} = 0 \qquad \forall s, m \text{ when } \text{Compatible}_s{=}0 \text{ and } \text{Window}_m{=}1$$

$$x_{s,m} \in \mathbb{Z}_{\geq 0} \qquad \forall s, m$$

$$e_s \in \mathbb{R}_{\geq 0} \qquad \forall s$$