Examining the geometry of steering gears

I define a set of data for plotting, but the idea is to construct a geometrical relation that works on any arrangement.

```
data = {r1 → 6, r2 → 8, r3x → -12, r0x → 19, r0y → 11};
```

In[ ]:=
```
r3 = {r3x, 0};
r0 = {r0x, r0y};
```

Firstly, let us look at the state, when the wheels are pointing straight ahead. In this state, I will determine the position of the rods.

The left wheel is in the origo.
The endpoint of the rod fixed to the wheel may only move on a circle:

In[ ]:=
```
k1 = r1 {Cos[φ], Sin[φ]};
% // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} r1\,\mathrm{Cos}[\varphi] \\ r1\,\mathrm{Sin}[\varphi] \end{pmatrix}$$

One of the next rod’s endpoints is set on the horizontal rod, the other may only move on this circle:

In[ ]:=
```
k2 = r0 + r3 + r2 {-Cos[γ], -Sin[γ]};
% // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} r3x + r0x - r2\,\mathrm{Cos}[\gamma] \\ r0y - r2\,\mathrm{Sin}[\gamma] \end{pmatrix}$$

There must be two mutual points of these circles. If there was only one, it could not be moved, and if none, then the rods would not reach each-other

The two angle-pairs belonging to these points:

In[∘]:=
```
mo = Solve[k1 == k2, {φ, γ}] // FullSimplify
```

Out[∘]=
```
{{φ →
    ConditionalExpression[ArcTan[(-r2^3 (r3x + rOx) + r2 (r3x + rOx) (r1^2 + (r3x + rOx)^2 + rOy^2) +
          Sqrt[(-r2^2 rOy^2 (r1^4 + (-r2^2 + (r3x + rOx)^2 + rOy^2)^2 - 2 r1^2 (r2^2 + (r3x + rOx)^2 + rOy^2)))]) /
        (r1 r2 ((r3x + rOx)^2 + rOy^2)), (r1^2 r2 rOy^2 - r2^3 rOy^2 - (r3x + rOx)
          Sqrt[(-r2^2 rOy^2 ((r1 - r2)^2 - (r3x + rOx)^2 - rOy^2) ((r1 + r2)^2 - (r3x + rOx)^2 - rOy^2))] +
          r2 rOy^2 ((r3x + rOx)^2 + rOy^2)) / (r1 r2 rOy ((r3x + rOx)^2 + rOy^2))] +
      2 π C[2], C[2] ∈ ℤ], γ → ConditionalExpression[
     ArcTan[(r2^3 (r3x + rOx) + r2 (r3x + rOx) (-r1^2 + (r3x + rOx)^2 + rOy^2) -
          Sqrt[(-r2^2 rOy^2 (r1^4 + (-r2^2 + (r3x + rOx)^2 + rOy^2)^2 - 2 r1^2 (r2^2 + (r3x + rOx)^2 + rOy^2)))]) /
        (r2^2 ((r3x + rOx)^2 + rOy^2)), (-r1^2 r2 rOy^2 + r2^3 rOy^2 + (r3x + rOx)
          Sqrt[(-r2^2 rOy^2 ((r1 - r2)^2 - (r3x + rOx)^2 - rOy^2) ((r1 + r2)^2 - (r3x + rOx)^2 - rOy^2))] +
          r2 rOy^2 ((r3x + rOx)^2 + rOy^2)) / (r2^2 rOy ((r3x + rOx)^2 + rOy^2))] +
      2 π C[1], C[1] ∈ ℤ]}, {φ → ConditionalExpression[
     ArcTan[-((r2^3 (r3x + rOx) - r2 (r3x + rOx) (r1^2 + (r3x + rOx)^2 + rOy^2) +
            Sqrt[(-r2^2 rOy^2 (r1^4 + (-r2^2 + (r3x + rOx)^2 + rOy^2)^2 - 2 r1^2 (r2^2 + (r3x + rOx)^2 + rOy^2)))]) /
          (r1 r2 ((r3x + rOx)^2 + rOy^2))), (r1^2 r2 rOy^2 - r2^3 rOy^2 + (r3x + rOx)
          Sqrt[(-r2^2 rOy^2 ((r1 - r2)^2 - (r3x + rOx)^2 - rOy^2) ((r1 + r2)^2 - (r3x + rOx)^2 - rOy^2))] +
          r2 rOy^2 ((r3x + rOx)^2 + rOy^2)) / (r1 r2 rOy ((r3x + rOx)^2 + rOy^2))] +
      2 π C[2], C[2] ∈ ℤ], γ → ConditionalExpression[
     ArcTan[(r2^3 (r3x + rOx) + r2 (r3x + rOx) (-r1^2 + (r3x + rOx)^2 + rOy^2) +
          Sqrt[(-r2^2 rOy^2 (r1^4 + (-r2^2 + (r3x + rOx)^2 + rOy^2)^2 - 2 r1^2 (r2^2 + (r3x + rOx)^2 + rOy^2)))]) /
        (r2^2 ((r3x + rOx)^2 + rOy^2)), (-r1^2 r2 rOy^2 + r2^3 rOy^2 - (r3x + rOx)
          Sqrt[(-r2^2 rOy^2 ((r1 - r2)^2 - (r3x + rOx)^2 - rOy^2) ((r1 + r2)^2 - (r3x + rOx)^2 - rOy^2))] +
          r2 rOy^2 ((r3x + rOx)^2 + rOy^2)) / (r2^2 rOy ((r3x + rOx)^2 + rOy^2))] + 2 π C[1], C[1] ∈ ℤ]}}
```

after substitution:

In[◦]:= `mo /. data`

Out[◦]= $\left\{\left\{\varphi \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{137456 - 1232\sqrt{1079}}{11\left(7952 + 176\sqrt{1079}\right)}\right] + 2\pi\, C[2],\ C[2] \in \mathbb{Z}\right],\right.\right.$

$\left.\gamma \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{191664 + 1232\sqrt{1079}}{11\left(11088 - 176\sqrt{1079}\right)}\right] + 2\pi\, C[1],\ C[1] \in \mathbb{Z}\right]\right\},$

$\left\{\varphi \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{137456 + 1232\sqrt{1079}}{11\left(7952 - 176\sqrt{1079}\right)}\right] + 2\pi\, C[2],\ C[2] \in \mathbb{Z}\right],\right.$

$\left.\left.\gamma \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{191664 - 1232\sqrt{1079}}{11\left(11088 + 176\sqrt{1079}\right)}\right] + 2\pi\, C[1],\ C[1] \in \mathbb{Z}\right]\right\}\right\}$

The first solution in radians and then in angles:

In[◦]:= `φ1 = mo[[1, 1, 2, 1, 1]] /. data // N;`

In[◦]:= `γ1 = mo[[1, 2, 2, 1, 1]] /. data // N;`

In[◦]:= `{φ1, γ1} // MatrixForm`

Out[◦]//MatrixForm=

$$\begin{pmatrix} 0.570744 \\ 1.32444 \end{pmatrix}$$

In[◦]:= $\left\{\varphi 1 \frac{180}{2\pi},\ \gamma 1 \frac{180}{2\pi}\right\}$ `// MatrixForm`

Out[◦]//MatrixForm=

$$\begin{pmatrix} 16.3506 \\ 37.9423 \end{pmatrix}$$

The second solution in radians and then in angles:

In[◦]:= `φ2 = mo[[2, 1, 2, 1, 1]] /. data // N;`

In[◦]:= `γ2 = mo[[2, 2, 2, 1, 1]] /. data // N;`

In[◦]:= `{φ2, γ2} // MatrixForm`

Out[◦]//MatrixForm=

$$\begin{pmatrix} 1.43739 \\ 0.683698 \end{pmatrix}$$

In[◦]:= $\left\{\varphi 2 \frac{180}{2\pi},\ \gamma 2 \frac{180}{2\pi}\right\}$ `// MatrixForm`

Out[◦]//MatrixForm=

$$\begin{pmatrix} 41.1782 \\ 19.5865 \end{pmatrix}$$

The plot of the first solution:

In[∘]:=
```
p11 = r1 {Cos[φ], Sin[φ]} /. data /. {φ → φ1, γ → γ1};
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 5.049 \\ 3.24155 \end{pmatrix}$$

In[∘]:=
```
p11ellenőrzés = r0 + r3 + r2 {-Cos[γ], -Sin[γ]} /. data /. {φ → φ1, γ → γ1};
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 5.049 \\ 3.24155 \end{pmatrix}$$

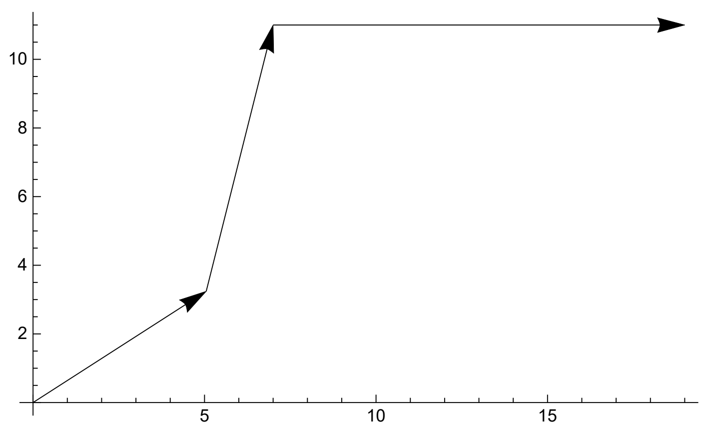
In[∘]:=
```
p21 = r0 + r3 /. data;
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

In[∘]:=
```
Graphics[{Arrow[{{0, 0}, p11}], Arrow[{p11, p21}], Arrow[{p21, r0 /. data}]}, Axes → True]
```

Out[∘]=



The plot of the second solution:

In[∘]:=
```
p12 = r1 {Cos[φ], Sin[φ]} /. data /. {φ → φ2, γ → γ2};
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0.798062 \\ 5.94669 \end{pmatrix}$$

In[∘]:=
```
p12ellenőrzés = r0 + r3 + r2 {-Cos[γ], -Sin[γ]} /. data /. {φ → φ2, γ → γ2};
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 0.798062 \\ 5.94669 \end{pmatrix}$$

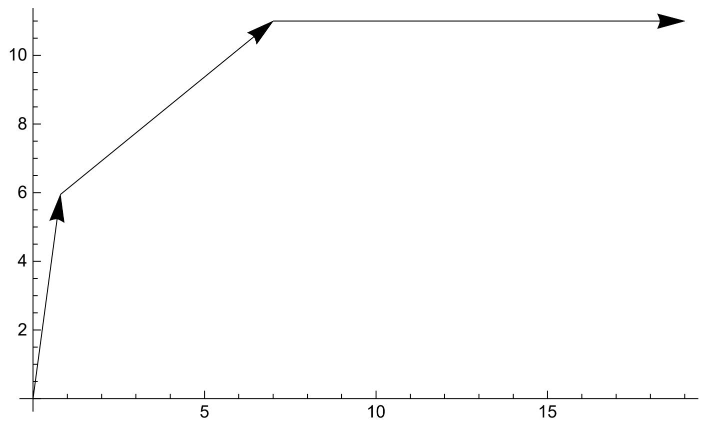
In[∘]:=
```
p22 = r0 + r3 /. data;
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 7 \\ 11 \end{pmatrix}$$

In[∘]:=
```
Graphics[{Arrow[{{0, 0}, p12}], Arrow[{p12, p22}], Arrow[{p21, r0 /. data}]}, Axes → True]
```

Out[∘]=



I will use the equations belonging to this plot in jupyter notebook, because this is the one that contains the relevant case for us.

Examining the other side

Like above, let us look at the state, when the wheels are pointing straight ahead. In this state, I will determine the position of the rods.

The endpoint of the rod fixed to the right-side wheel may only move on a circle: (th is the distance between the front wheels)

In[∘]:=
```
k1s = {th - r1 Cos[α], r1 Sin[α]};
% // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} th - r1 \cos[\alpha] \\ r1 \sin[\alpha] \end{pmatrix}$$

One of the next rod’s endpoints is set on the horizontal rod, the other may only move on this circle:

In[ ]:=
```
k2s = {rOx - r3x + r2 Cos[β], rOy - r2 Sin[β]};
% // MatrixForm
```

Out[ ]//MatrixForm=

$$\begin{pmatrix} -r3x + rOx + r2\,Cos[\beta] \\ rOy - r2\,Sin[\beta] \end{pmatrix}$$

There must be two mutual points of these circles. If there was only one, it could not be moved, and if none, then the rods would not reach each-other

The two angle-pairs belonging to these points:

In[ ]:=
```
mos = Solve[k1s == k2s, {α, β}] // FullSimplify
```

Out[ ]=

$$
\begin{aligned}
\{\{\alpha \to \mathrm{ConditionalExpression}\Big[&\mathrm{ArcTan}\Big[\Big(-\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\mathrm{r1}\,(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\left(\mathrm{r1}^2-\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\\
&\left(\mathrm{r1}^2\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right),\ \Big(\mathrm{r1}^3\,\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\\
&\mathrm{r1}\,\mathrm{rOy}^2\left(-\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\left(\mathrm{r1}^2\,\mathrm{rOy}\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right)\Big]+2\pi\,C[1],\ C[1]\in\mathbb{Z}\Big],\\
\beta \to \mathrm{ConditionalExpression}\Big[&\mathrm{ArcTan}\Big[\Big(\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\\
&\mathrm{r1}\,(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\left(-\mathrm{r1}^2+\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\left(\mathrm{r1}\,\mathrm{r2}\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right),\\
&\Big(-\mathrm{r1}^3\,\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\mathrm{r1}\,\mathrm{rOy}^2\left(\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\\
&\left(\mathrm{r1}\,\mathrm{r2}\,\mathrm{rOy}\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right)\Big]+2\pi\,C[2],\ C[2]\in\mathbb{Z}\Big]\},\\
\{\alpha \to \mathrm{ConditionalExpression}\Big[&\mathrm{ArcTan}\Big[\Big(\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\mathrm{r1}\,(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\\
&\left(\mathrm{r1}^2-\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\left(\mathrm{r1}^2\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right),\\
&\Big(\mathrm{r1}^3\,\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\mathrm{r1}\,\mathrm{rOy}^2\left(-\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\\
&\left(\mathrm{r1}^2\,\mathrm{rOy}\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right)\Big]+2\pi\,C[1],\ C[1]\in\mathbb{Z}\Big],\\
\beta \to \mathrm{ConditionalExpression}\Big[&\mathrm{ArcTan}\Big[\Big(-\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\mathrm{r1}\,(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\\
&\left(-\mathrm{r1}^2+\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\left(\mathrm{r1}\,\mathrm{r2}\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right),\\
&\Big(-\mathrm{r1}^3\,\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})\sqrt{-\mathrm{r1}^2\,\mathrm{rOy}^2\left((\mathrm{r1}-\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\left((\mathrm{r1}+\mathrm{r2})^2-\mathrm{rOy}^2-(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)}+\mathrm{r1}\,\mathrm{rOy}^2\left(\mathrm{r2}^2+\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\Big)\Big/\\
&\left(\mathrm{r1}\,\mathrm{r2}\,\mathrm{rOy}\left(\mathrm{rOy}^2+(\mathrm{r3x}-\mathrm{rOx}+\mathrm{th})^2\right)\right)\Big]+2\pi\,C[2],\ C[2]\in\mathbb{Z}\Big]\}\}
\end{aligned}
$$

after substitution:

In[∘]:= `mos /. th → 2 rOx /. data`

Out[∘]=
$$\left\{\left\{\alpha \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{103092+924\sqrt{1079}}{11\left(5964-132\sqrt{1079}\right)}\right]+2\pi C[1],\ C[1]\in\mathbb{Z}\right],\right.\right.$$
$$\left.\beta \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{143748-924\sqrt{1079}}{11\left(8316+132\sqrt{1079}\right)}\right]+2\pi C[2],\ C[2]\in\mathbb{Z}\right]\right\},$$
$$\left\{\alpha \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{103092-924\sqrt{1079}}{11\left(5964+132\sqrt{1079}\right)}\right]+2\pi C[1],\ C[1]\in\mathbb{Z}\right],\right.$$
$$\left.\left.\beta \to \text{ConditionalExpression}\left[\text{ArcTan}\left[\frac{143748+924\sqrt{1079}}{11\left(8316-132\sqrt{1079}\right)}\right]+2\pi C[2],\ C[2]\in\mathbb{Z}\right]\right\}\right\}$$

The first solution in radians and then in angles:

In[∘]:= `α1 = mos[[1, 1, 2, 1, 1]] /. th → 2 rOx /. data // N;`

In[∘]:= `β1 = mos[[1, 2, 2, 1, 1]] /. th → 2 rOx /. data // N;`

In[∘]:= `{α1, β1} // MatrixForm`

Out[∘]//MatrixForm=
$$\begin{pmatrix} 1.43739 \\ 0.683698 \end{pmatrix}$$

In[∘]:= $\left\{\alpha 1 \frac{180}{2\pi}, \beta 1 \frac{180}{2\pi}\right\}$ `// MatrixForm`

Out[∘]//MatrixForm=
$$\begin{pmatrix} 41.1782 \\ 19.5865 \end{pmatrix}$$

The second solution in radians and then in angles:

In[∘]:= `α2 = mos[[2, 1, 2, 1, 1]] /. th → 2 rOx /. data // N;`

In[∘]:= `β2 = mos[[2, 2, 2, 1, 1]] /. th → 2 rOx /. data // N;`

In[∘]:= `{α2, β2} // MatrixForm`

Out[∘]//MatrixForm=
$$\begin{pmatrix} 0.570744 \\ 1.32444 \end{pmatrix}$$

In[∘]:= $\left\{\alpha 2 \frac{180}{2\pi}, \beta 2 \frac{180}{2\pi}\right\}$ `// MatrixForm`

Out[∘]//MatrixForm=
$$\begin{pmatrix} 16.3506 \\ 37.9423 \end{pmatrix}$$

Which pair of solutions do we need?
The one that equals the chosen above.

```
In[]:= {φ2, γ2} // MatrixForm
```

Out[]//MatrixForm=

$$\begin{pmatrix} 1.43739 \\ 0.683698 \end{pmatrix}$$

```
In[]:= {α1, β1} // MatrixForm
```

Out[]//MatrixForm=

$$\begin{pmatrix} 1.43739 \\ 0.683698 \end{pmatrix}$$

So the first.

The data defined had no effect on the points of the calculation until now. I must use concrete numerical values for the plot.
The angles depending on only r_{Ox}.
(r_{Ox} is the location of the midpoint of the rack.)

r_{Ox} reaches its maximum, when the first two rods on the left hand side relocate to form one line segment. Using the pitagorean theorem:

In[]:= rOxmax = $-\text{r3x} + \sqrt{(\text{r1} + \text{r2})^2 - \text{rOy}^2}$ /. data // N

Out[]= 20.6603

The difference between the minimal and the maximal r_{Ox} :

```
In[]:= ΔrOxmax = rOxmax - rOx /. data
```

Out[]= 1.66025

Finally the plot:

```
In[]:= fi = mo[[2, 1, 2, 1, 1]] /. {r1 → 6, r2 → 8, r3x → -12, rOy → 11} // N;
gam = mo[[2, 2, 2, 1, 1]] /. {r1 → 6, r2 → 8, r3x → -12, rOy → 11} // N;
alf = mos[[1, 1, 2, 1, 1]] /. {r1 → 6, r2 → 8, r3x → -12, rOy → 11, th → 2 rOx /. data} // N;
bet = mos[[1, 2, 2, 1, 1]] /. {r1 → 6, r2 → 8, r3x → -12, rOy → 11, th → 2 rOx /. data} // N;
```

In[ ]:=
```
Plot[{fi, gam, alf, bet}, {rOx, (rOx /. data) - ΔrOxmax, (rOx /. data) + ΔrOxmax},
 PlotLegends → "Expressions", AxesOrigin → {(rOx /. data) - ΔrOxmax, 0},
 PlotRange → Full, AspectRatio → Full]
```

Out[ ]=