Linear Algebra

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Utilisation de `R` pour produire du code LaTeX en algèbre linéaire

Initialisation de la librairie

Nous devons installer la librairie. Si vous n'avez pas la librairie `devtools`, vous devez l'installer.

```
install.packages("devtools")
```

Vous installer ensuite la librairie à l'aide de la commande suivante:

```
devtools::install_github("desautm/linalgr")
```

Vous pouvez charger la librairie:

```
library(linalgr)
```

Affichage de matrices

Nous allons définir quelques matrices:

```
m <- 5
n <- 5
A <- matrix(sample(-10:10, m*n, replace = TRUE), m, n)
B <- matrix(sample(-10:10, m, replace = TRUE), m, 1)
```

Voici l'affichage directement avec `R`:

```
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    5   -7  -10   -1   -1
## [2,]    8   -4   -7   -2   -4
## [3,]    5    0    5   -2   10
## [4,]    8    5  -10   -7    4
## [5,]   -1   10   -2    9    3
```

```
B
```

```
##      [,1]
## [1,]   -2
## [2,]    4
## [3,]    1
## [4,]   -6
## [5,]    0
```

Affichage avec la librairie `linalgr`

Pour afficher, il faut utiliser l'option `results = 'asis'` dans le bloc de code `R`.

Voici l'affichage en utilisant la librairie.

```
mat2latex(A)
mat2latex(B)
```

$$\begin{bmatrix} 5 & -7 & -10 & -1 & -1 \\ 8 & -4 & -7 & -2 & -4 \\ 5 & 0 & 5 & -2 & 10 \\ 8 & 5 & -10 & -7 & 4 \\ -1 & 10 & -2 & 9 & 3 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ 4 \\ 1 \\ -6 \\ 0 \end{bmatrix}$$

Affichage de SEL

Affichage de systèmes d'équations linéaires

```
sel2latex(A,B, variables = "xi")
```

$$\begin{array}{rcrcrcrcrcr} 5x_1 & - & 7x_2 & - & 10x_3 & - & x_4 & - & x_5 & = & -2 \\ 8x_1 & - & 4x_2 & - & 7x_3 & - & 2x_4 & - & 4x_5 & = & 4 \\ 5x_1 & & & + & 5x_3 & - & 2x_4 & + & 10x_5 & = & 1 \\ 8x_1 & + & 5x_2 & - & 10x_3 & - & 7x_4 & + & 4x_5 & = & -6 \\ -x_1 & + & 10x_2 & - & 2x_3 & + & 9x_4 & + & 3x_5 & = & 0 \end{array}$$

Affichage de systèmes matriciels

```
sel2latex(A,B, sel = FALSE, variables = "xi")
```

$$\begin{bmatrix} 5 & -7 & -10 & -1 & -1 \\ 8 & -4 & -7 & -2 & -4 \\ 5 & 0 & 5 & -2 & 10 \\ 8 & 5 & -10 & -7 & 4 \\ -1 & 10 & -2 & 9 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2 \\ 4 \\ 1 \\ -6 \\ 0 \end{bmatrix}$$

Nous pouvons changer les variables utilisées

```
sel2latex(A,B, variables = "a")
```

$$\begin{array}{rcrcrcrcrcr} 5a & - & 7b & - & 10c & - & d & - & e & = & -2 \\ 8a & - & 4b & - & 7c & - & 2d & - & 4e & = & 4 \\ 5a & & & + & 5c & - & 2d & + & 10e & = & 1 \\ 8a & + & 5b & - & 10c & - & 7d & + & 4e & = & -6 \\ -a & + & 10b & - & 2c & + & 9d & + & 3e & = & 0 \end{array}$$

ou alors…

```
sel2latex(matrix(sample(-10:10,4),2,2),matrix(sample(-10:10,4),2,1), variables = "x")
```

$$\begin{array}{rcrcr} 2x & + & 4y & = & 9 \\ -8x & - & 3y & = & 5 \end{array}$$

Affichage avec des fractions décimales et contrôle du nombre de chiffres à droite de la virgule

```
sel2latex(A/3,B, variables = "xi", digits = 3)
```

$$\begin{array}{rcrcrcrcrcr} 1.667x_1 & - & 2.333x_2 & - & 3.333x_3 & - & 0.333x_4 & - & 0.333x_5 & = & -2 \\ 2.667x_1 & - & 1.333x_2 & - & 2.333x_3 & - & 0.667x_4 & - & 1.333x_5 & = & 4 \\ 1.667x_1 & + & 0.000x_2 & + & 1.667x_3 & - & 0.667x_4 & + & 3.333x_5 & = & 1 \\ 2.667x_1 & + & 1.667x_2 & - & 3.333x_3 & - & 2.333x_4 & + & 1.333x_5 & = & -6 \\ -0.333x_1 & + & 3.333x_2 & - & 0.667x_3 & + & 3.000x_4 & + & 1.000x_5 & = & 0 \end{array}$$

Avec des fractions ordinaires

Mode en ligne

```
sel2latex(A/3,B, variables = "xi", style = "inline")
```

$$\begin{array}{rcrcrcrcrcr} 5/3x_1 & - & 7/3x_2 & - & 10/3x_3 & - & 1/3x_4 & - & 1/3x_5 & = & -2 \\ 8/3x_1 & - & 4/3x_2 & - & 7/3x_3 & - & 2/3x_4 & - & 4/3x_5 & = & 4 \\ 5/3x_1 & & & + & 5/3x_3 & - & 2/3x_4 & + & 10/3x_5 & = & 1 \\ 8/3x_1 & + & 5/3x_2 & - & 10/3x_3 & - & 7/3x_4 & + & 4/3x_5 & = & -6 \\ -1/3x_1 & + & 10/3x_2 & - & 2/3x_3 & + & 3x_4 & + & x_5 & = & 0 \end{array}$$

Mode commande frac

```
sel2latex(A/3,B, variables = "xi", style = "frac")
```

$$\begin{array}{rcrcrcrcrcr} \frac{5}{3}x_1 & - & \frac{7}{3}x_2 & - & \frac{10}{3}x_3 & - & \frac{1}{3}x_4 & - & \frac{1}{3}x_5 & = & -2 \\ \frac{8}{3}x_1 & - & \frac{4}{3}x_2 & - & \frac{7}{3}x_3 & - & \frac{2}{3}x_4 & - & \frac{4}{3}x_5 & = & 4 \\ \frac{5}{3}x_1 & & & + & \frac{5}{3}x_3 & - & \frac{2}{3}x_4 & + & \frac{10}{3}x_5 & = & 1 \\ \frac{8}{3}x_1 & + & \frac{5}{3}x_2 & - & \frac{10}{3}x_3 & - & \frac{7}{3}x_4 & + & \frac{4}{3}x_5 & = & -6 \\ -\frac{1}{3}x_1 & + & \frac{10}{3}x_2 & - & \frac{2}{3}x_3 & + & 3x_4 & + & x_5 & = & 0 \end{array}$$

Matrices creuses

```
C <- matrix(c(-2, 2, 0, 0, 0, 0, 0, 0,
              3, 4, -5, 0, 0, 0, 0, 0,
              0, 3, 1, 0, 0, 0, 0, 0,
              0, 0, -1, -1, -1, 0, 0, 0,
              0, 0, 0, 3, 4, 7, 0, 0,
```

```
              0, 0, 0, 0, -6, 6, 1, 0,
              0, 0, 0, 0, 0, 1, 1, 1,
              0, 0, 0, 0, 0, 0, -1, 4), 8, 8, byrow = TRUE)
D <- matrix(c(1,2,3,4,5,6,7,8), 8, 1)
```

Option de base

```
sel2latex(C, D, variables = "xi")
```

$$
\begin{array}{rcrcrcrcrcrcrcrcr}
-2x_1 & + & 2x_2 & & & & & & & & & & & & & = & 1 \\
3x_1 & + & 4x_2 & - & 5x_3 & & & & & & & & & & & = & 2 \\
 & & 3x_2 & + & x_3 & & & & & & & & & & & = & 3 \\
 & & & & -x_3 & - & x_4 & - & x_5 & & & & & & & = & 4 \\
 & & & & & & 3x_4 & + & 4x_5 & + & 7x_6 & & & & & = & 5 \\
 & & & & & & & & -6x_5 & + & 6x_6 & + & x_7 & & & = & 6 \\
 & & & & & & & & & & x_6 & + & x_7 & + & x_8 & = & 7 \\
 & & & & & & & & & & & & -x_7 & + & 4x_8 & = & 8
\end{array}
$$

Option `concise`

```
sel2latex(C, D, variables = "xi", concise = TRUE)
```

$$
\begin{array}{rcrcrcr}
 & & -2x_1 & + & 2x_2 & = & 1 \\
3x_1 & + & 4x_2 & - & 5x_3 & = & 2 \\
 & & 3x_2 & + & x_3 & = & 3 \\
-x_3 & - & x_4 & - & x_5 & = & 4 \\
3x_4 & + & 4x_5 & + & 7x_6 & = & 5 \\
-6x_5 & + & 6x_6 & + & x_7 & = & 6 \\
x_6 & + & x_7 & + & x_8 & = & 7 \\
 & & -x_7 & + & 4x_8 & = & 8
\end{array}
$$

Création de SEL

Solution unique (matrice échelon)

```
E <- create_sel(4,4, type = "unique")
sel2latex(E$A, E$B, variables = "xi")
rref(E$A, E$B, style = "inline", echelon = TRUE)
rref_entier(E$A, E$B, style = "inline", echelon = TRUE)
```

$$
\begin{array}{rcrcrcrcr}
x_1 & + & 5x_2 & - & 27x_3 & + & 61x_4 & = & 469 \\
x_1 & + & 6x_2 & - & 30x_3 & + & 69x_4 & = & 528 \\
4x_1 & + & 22x_2 & - & 117x_3 & + & 264x_4 & = & 2024 \\
-4x_1 & - & 19x_2 & + & 103x_3 & - & 233x_4 & = & -1794
\end{array}
$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 1 & 6 & -30 & 69 & 528 \\ 4 & 22 & -117 & 264 & 2024 \\ -4 & -19 & 103 & -233 & -1794 \end{array}\right] \begin{array}{l} L_1 \to L_3 \\ \\ L_3 \to L_1 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 4 & 22 & -117 & 264 & 2024 \\ 1 & 6 & -30 & 69 & 528 \\ 1 & 5 & -27 & 61 & 469 \\ -4 & -19 & 103 & -233 & -1794 \end{array}\right] \begin{array}{l} 1/4L_1 \to L_1 \\ \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 1 & 6 & -30 & 69 & 528 \\ 1 & 5 & -27 & 61 & 469 \\ -4 & -19 & 103 & -233 & -1794 \end{array}\right] \begin{array}{l} \\ L_2 - L_1 \to L_2 \\ L_3 - L_1 \to L_3 \\ L_4 + 4L_1 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1/2 & -3/4 & 3 & 22 \\ 0 & -1/2 & 9/4 & -5 & -37 \\ 0 & 3 & -14 & 31 & 230 \end{array}\right] \begin{array}{l} \\ L_2 \to L_4 \\ \\ L_4 \to L_2 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 3 & -14 & 31 & 230 \\ 0 & -1/2 & 9/4 & -5 & -37 \\ 0 & 1/2 & -3/4 & 3 & 22 \end{array}\right] \begin{array}{l} \\ 1/3L_2 \to L_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 230/3 \\ 0 & -1/2 & 9/4 & -5 & -37 \\ 0 & 1/2 & -3/4 & 3 & 22 \end{array}\right] \begin{array}{l} \\ \\ L_3 + 1/2L_2 \to L_3 \\ L_4 - 1/2L_2 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 230/3 \\ 0 & 0 & -1/12 & 1/6 & 4/3 \\ 0 & 0 & 19/12 & -13/6 & -49/3 \end{array}\right] \begin{array}{l} \\ \\ L_3 \to L_4 \\ L_4 \to L_3 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 230/3 \\ 0 & 0 & 19/12 & -13/6 & -49/3 \\ 0 & 0 & -1/12 & 1/6 & 4/3 \end{array}\right] \begin{array}{l} \\ \\ 12/19L_3 \to L_3 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 230/3 \\ 0 & 0 & 1 & -26/19 & -196/19 \\ 0 & 0 & -1/12 & 1/6 & 4/3 \end{array}\right] \begin{array}{l} \\ \\ \\ L_4 + 1/12L_3 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 230/3 \\ 0 & 0 & 1 & -26/19 & -196/19 \\ 0 & 0 & 0 & 1/19 & 9/19 \end{array}\right] \begin{array}{l} \\ \\ \\ 19L_4 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 230/3 \\ 0 & 0 & 1 & -26/19 & -196/19 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 11/2 & -117/4 & 66 & 506 \\ 0 & 1 & -14/3 & 31/3 & 5 \quad 230/3 \\ 0 & 0 & 1 & -26/19 & -196/19 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 1 & 6 & -30 & 69 & 528 \\ 4 & 22 & -117 & 264 & 2024 \\ -4 & -19 & 103 & -233 & -1794 \end{array}\right] \begin{array}{l} \\ L_2 - L_1 \to L_2 \\ L_3 - 4L_1 \to L_3 \\ L_4 + 4L_1 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 0 & 1 & -3 & 8 & 59 \\ 0 & 2 & -9 & 20 & 148 \\ 0 & 1 & -5 & 11 & 82 \end{array}\right] \begin{array}{l} \\ \\ L_3 - 2L_2 \to L_3 \\ L_4 - L_2 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 0 & 1 & -3 & 8 & 59 \\ 0 & 0 & -3 & 4 & 30 \\ 0 & 0 & -2 & 3 & 23 \end{array}\right] \begin{array}{l} \\ \\ \\ 3L_4 - 2L_3 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 0 & 1 & -3 & 8 & 59 \\ 0 & 0 & -3 & 4 & 30 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 0 & 1 & -3 & 8 & 59 \\ 0 & 0 & -3 & 4 & 30 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right] \begin{array}{l} L_1 \to L_1 \\ L_2 \to L_2 \\ -1/3L_3 \to L_3 \\ L_4 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & -27 & 61 & 469 \\ 0 & 1 & -3 & 8 & 59 \\ 0 & 0 & 1 & -4/3 & -10 \\ 0 & 0 & 0 & 1 & 9 \end{array}\right]$$

Solution unique

```r
E <- create_sel(4,4, type = "unique")
sel2latex(E$A, E$B, variables = "xi")
rref(E$A,E$B,style = "inline")
rref_entier(E$A,E$B,style = "inline")
```

$$\begin{array}{rcrcrcrcr} x_1 & + & 5x_2 & + & 13x_3 & + & 31x_4 & = & -208 \\ x_1 & + & 6x_2 & + & 17x_3 & + & 41x_4 & = & -280 \\ -2x_1 & - & 9x_2 & - & 23x_3 & - & 55x_4 & = & 367 \\ x_1 & & & + & 3x_3 & + & 12x_4 & = & -84 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & 13 & 31 & -208 \\ 1 & 6 & 17 & 41 & -280 \\ -2 & -9 & -23 & -55 & 367 \\ 1 & 0 & 3 & 12 & -84 \end{array}\right] \begin{array}{l} L_1 \to L_3 \\ \\ L_3 \to L_1 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} -2 & -9 & -23 & -55 & 367 \\ 1 & 6 & 17 & 41 & -280 \\ 1 & 5 & 13 & 31 & -208 \\ 1 & 0 & 3 & 12 & -84 \end{array}\right] \begin{array}{l} -1/2L_1 \to L_1 \\ \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 9/2 & 23/2 & 55/2 & -367/2 \\ 1 & 6 & 17 & 41 & -280 \\ 1 & 5 & 13 & 31 & -208 \\ 1 & 0 & 3 & 12 & -84 \end{array}\right] \begin{array}{l} \\ L_2 - L_1 \to L_2 \\ L_3 - L_1 \to L_3 \\ L_4 - L_1 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 9/2 & 23/2 & 55/2 & -367/2 \\ 0 & 3/2 & 11/2 & 27/2 & -193/2 \\ 0 & 1/2 & 3/2 & 7/2 & -49/2 \\ 0 & -9/2 & -17/2 & -31/2 & 199/2 \end{array}\right] \begin{array}{l} \\ L_2 \to L_4 \\ \\ L_4 \to L_2 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 9/2 & 23/2 & 55/2 & -367/2 \\ 0 & -9/2 & -17/2 & -31/2 & 199/2 \\ 0 & 1/2 & 3/2 & 7/2 & -49/2 \\ 0 & 3/2 & 11/2 & 27/2 & -193/2 \end{array}\right] \begin{array}{l} \\ -2/9L_2 \to L_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 9/2 & 23/2 & 55/2 & -367/2 \\ 0 & 1 & 17/9 & 31/9 & -199/9 \\ 0 & 1/2 & 3/2 & 7/2 & -49/2 \\ 0 & 3/2 & 11/2 & 27/2 & -193/2 \end{array}\right] \begin{array}{l} L_1 - 9/2L_2 \to L_1 \\ \\ L_3 - 1/2L_2 \to L_3 \\ L_4 - 3/2L_2 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 3 & 12 & -84 \\ 0 & 1 & 17/9 & 31/9 & -199/9 \\ 0 & 0 & 5/9 & 16/9 & -121/9 \\ 0 & 0 & 8/3 & 25/3 & -190/3 \end{array}\right] \begin{array}{l} \\ \\ L_3 \to L_4 \\ L_4 \to L_3 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 3 & 12 & -84 \\ 0 & 1 & 17/9 & 31/9 & -199/9 \\ 0 & 0 & 8/3 & 25/3 & -190/3 \\ 0 & 0 & 5/9 & 16/9 & -121/9 \end{array}\right] \begin{array}{l} \\ \\ 3/8L_3 \to L_3 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 3 & 12 & -84 \\ 0 & 1 & 17/9 & 31/9 & -199/9 \\ 0 & 0 & 1 & 25/8 & -95/4 \\ 0 & 0 & 5/9 & 16/9 & -121/9 \end{array}\right] \begin{array}{l} L_1 - 3L_3 \to L_1 \\ L_2 - 17/9L_3 \to L_2 \\ \\ L_4 - 5/9L_3 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 21/8 & -51/4 \\ 0 & 1 & 0 & -59/24 & 91/4 \\ 0 & 0 & 1 & 25/8 & -95/4 \\ 0 & 0 & 0 & 1/24 & -1/4 \end{array}\right] \begin{array}{l} \\ \\ \\ 24L_4 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 21/8 & -51/4 \\ 0 & 1 & 0 & -59/24 & 91/4 \\ 0 & 0 & 1 & 25/8 & -95/4 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right] \begin{array}{l} L_1 - 21/8L_4 \to L_1 \\ L_2 + 59/24L_4 \to L_2 \\ L_3 - 25/8L_4 \to L_3 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & 13 & 31 & -208 \\ 1 & 6 & 17 & 41 & -280 \\ -2 & -9 & -23 & -55 & 367 \\ 1 & 0 & 3 & 12 & -84 \end{array}\right] \begin{array}{l} \\ L_2 - L_1 \to L_2 \\ L_3 + 2L_1 \to L_3 \\ L_4 - L_1 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 5 & 13 & 31 & -208 \\ 0 & 1 & 4 & 10 & -72 \\ 0 & 1 & 3 & 7 & -49 \\ 0 & -5 & -10 & -19 & 124 \end{array}\right] \begin{array}{l} L_1 - 5L_2 \to L_1 \\ \\ L_3 - L_2 \to L_3 \\ L_4 + 5L_2 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & -7 & -19 & 152 \\ 0 & 1 & 4 & 10 & -72 \\ 0 & 0 & -1 & -3 & 23 \\ 0 & 0 & 10 & 31 & -236 \end{array}\right] \begin{array}{l} L_1 - 7L_3 \to L_1 \\ L_2 + 4L_3 \to L_2 \\ \\ L_4 + 10L_3 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 2 & -9 \\ 0 & 1 & 0 & -2 & 20 \\ 0 & 0 & -1 & -3 & 23 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right] \begin{array}{l} L_1 - 2L_4 \to L_1 \\ L_2 + 2L_4 \to L_2 \\ L_3 + 3L_4 \to L_3 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right] \begin{array}{l} L_1 \to L_1 \\ L_2 \to L_2 \\ -L_3 \to L_3 \\ L_4 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & -6 \end{array}\right]$$

Aucune solution

```
E <- create_sel(3,3, type = "aucune")
sel2latex(E$A, E$B, variables = "xi")
rref(E$A,E$B,style = "inline")
rref_entier(E$A,E$B,style = "inline")
```

$$\begin{array}{rcrcr} 5x_1 & + & 11x_2 & = & 30 \\ 5x_1 & + & 6x_2 & = & 24 \\ x_1 & + & 2x_2 & = & 6 \end{array}$$

$$\left[\begin{array}{rrr|r} 5 & 11 & 0 & 30 \\ 5 & 6 & 0 & 24 \\ 1 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} 1/5L_1 \to L_1 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & 11/5 & 0 & 6 \\ 5 & 6 & 0 & 24 \\ 1 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} \\ L_2 - 5L_1 \to L_2 \\ L_3 - L_1 \to L_3 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & 11/5 & 0 & 6 \\ 0 & -5 & 0 & -6 \\ 0 & -1/5 & 0 & 0 \end{array}\right] \begin{array}{l} \\ -1/5L_2 \to L_2 \\ \\ \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & 11/5 & 0 & 6 \\ 0 & 1 & 0 & 6/5 \\ 0 & -1/5 & 0 & 0 \end{array}\right] \begin{array}{l} L_1 - 11/5L_2 \to L_1 \\ \\ L_3 + 1/5L_2 \to L_3 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 84/25 \\ 0 & 1 & 0 & 6/5 \\ 0 & 0 & 0 & 6/25 \end{array}\right]$$

$$\left[\begin{array}{rrr|r} 5 & 11 & 0 & 30 \\ 5 & 6 & 0 & 24 \\ 1 & 2 & 0 & 6 \end{array}\right] \begin{array}{l} \\ L_2 - L_1 \to L_2 \\ 5L_3 - L_1 \to L_3 \end{array}$$

$$\left[\begin{array}{rrr|r} 5 & 11 & 0 & 30 \\ 0 & -5 & 0 & -6 \\ 0 & -1 & 0 & 0 \end{array}\right] \begin{array}{l} 5L_1 + 11L_2 \to L_1 \\ \\ 5L_3 - L_2 \to L_3 \end{array}$$

$$\left[\begin{array}{rrr|r} 25 & 0 & 0 & 84 \\ 0 & -5 & 0 & -6 \\ 0 & 0 & 0 & 6 \end{array}\right] \begin{array}{l} 1/25L_1 \to L_1 \\ -1/5L_2 \to L_2 \\ \\ \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 0 & 84/25 \\ 0 & 1 & 0 & 6/5 \\ 0 & 0 & 0 & 6 \end{array}\right]$$

Infinité de solutions

```
E <- create_sel(4,4, type = "infinite")
sel2latex(E$A, E$B, variables = "xi")
rref(E$A,E$B,style = "inline")
rref_entier(E$A,E$B,style = "inline")
```

$$\begin{array}{rcrcrcrcr} x_1 & - & 7x_2 & - & 2x_3 & - & 10x_4 & = & 49 \\ -5x_1 & + & 35x_2 & + & 11x_3 & + & 53x_4 & = & -257 \\ -3x_1 & + & 21x_2 & + & 4x_3 & + & 25x_4 & = & -130 \\ 3x_1 & - & 21x_2 & - & x_3 & - & 11x_4 & = & 59 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -2 & -10 & 49 \\ -5 & 35 & 11 & 53 & -257 \\ -3 & 21 & 4 & 25 & -130 \\ 3 & -21 & -1 & -11 & 59 \end{array}\right] \begin{array}{l} L_1 \to L_2 \\ L_2 \to L_1 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} -5 & 35 & 11 & 53 & -257 \\ 1 & -7 & -2 & -10 & 49 \\ -3 & 21 & 4 & 25 & -130 \\ 3 & -21 & -1 & -11 & 59 \end{array}\right] \begin{array}{l} -1/5L_1 \to L_1 \\ \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -11/5 & -53/5 & 257/5 \\ 1 & -7 & -2 & -10 & 49 \\ -3 & 21 & 4 & 25 & -130 \\ 3 & -21 & -1 & -11 & 59 \end{array}\right] \begin{array}{l} \\ L_2 - L_1 \to L_2 \\ L_3 + 3L_1 \to L_3 \\ L_4 - 3L_1 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -11/5 & -53/5 & 257/5 \\ 0 & 0 & 1/5 & 3/5 & -12/5 \\ 0 & 0 & -13/5 & -34/5 & 121/5 \\ 0 & 0 & 28/5 & 104/5 & -476/5 \end{array}\right] \begin{array}{l} \\ L_2 \to L_4 \\ \\ L_4 \to L_2 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -11/5 & -53/5 & 257/5 \\ 0 & 0 & 28/5 & 104/5 & -476/5 \\ 0 & 0 & -13/5 & -34/5 & 121/5 \\ 0 & 0 & 1/5 & 3/5 & -12/5 \end{array}\right] \begin{array}{l} \\ 5/28L_2 \to L_2 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -11/5 & -53/5 & 257/5 \\ 0 & 0 & 1 & 26/7 & -17 \\ 0 & 0 & -13/5 & -34/5 & 121/5 \\ 0 & 0 & 1/5 & 3/5 & -12/5 \end{array}\right] \begin{array}{l} L_1 + 11/5L_2 \to L_1 \\ \\ L_3 + 13/5L_2 \to L_3 \\ L_4 - 1/5L_2 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & 0 & -17/7 & 14 \\ 0 & 0 & 1 & 26/7 & -17 \\ 0 & 0 & 0 & 20/7 & -20 \\ 0 & 0 & 0 & -1/7 & 1 \end{array}\right] \begin{array}{l} \\ \\ 7/20L_3 \to L_3 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & 0 & -17/7 & 14 \\ 0 & 0 & 1 & 26/7 & -17 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & -1/7 & 1 \end{array}\right] \begin{array}{l} L_1 + 17/7L_3 \to L_1 \\ L_2 - 26/7L_3 \to L_2 \\ \\ L_4 + 1/7L_3 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -2 & -10 & 49 \\ -5 & 35 & 11 & 53 & -257 \\ -3 & 21 & 4 & 25 & -130 \\ 3 & -21 & -1 & -11 & 59 \end{array}\right] \begin{array}{l} \\ L_2 + 5L_1 \to L_2 \\ L_3 + 3L_1 \to L_3 \\ L_4 - 3L_1 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & -2 & -10 & 49 \\ 0 & 0 & 1 & 3 & -12 \\ 0 & 0 & -2 & -5 & 17 \\ 0 & 0 & 5 & 19 & -88 \end{array}\right] \begin{array}{l} L_1 + 2L_2 \to L_1 \\ \\ L_3 + 2L_2 \to L_3 \\ L_4 - 5L_2 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & 0 & -4 & 25 \\ 0 & 0 & 1 & 3 & -12 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 4 & -28 \end{array}\right] \begin{array}{l} L_1 + 4L_3 \to L_1 \\ L_2 - 3L_3 \to L_2 \\ \\ L_4 - 4L_3 \to L_4 \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} L_1 \to L_1 \\ L_2 \to L_2 \\ L_3 \to L_3 \\ \\ \end{array}$$

$$\left[\begin{array}{rrrr|r} 1 & -7 & 0 & 0 & -3 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$