Parcial I Señales y sistemas

Demostración

1-) $x(t) = 20\,sen(7t - \pi/2) - 3cos(5t) + 2cos(10t)$

Entrada: $-3.3 \to 5\,[V]$

Bits: 5

$-20cos(7t) - 3cos(5t) + 2cos(10t)$

$w_1 = 7$ $\quad T_1 = \frac{2\pi}{7} \to 10\pi \quad F_1 = 7/2\pi = 1.114\,Hz$

$w_2 = 5$ $\quad T_2 = \frac{2\pi}{5} \to 14\pi \quad F_2 = \frac{5}{2\pi} = 0.796\,Hz$

$w_3 = 10$ $\quad T_3 = \frac{2\pi}{10} \to \pi/5 \to 7\pi \quad F_3 = \frac{10}{2\pi} = 1.592\,Hz$

$\boxed{y = mx + t}$

$Amplitud_{max} = 20 + 3 + 2 = 25$

$Fmax = 2\,max\,alto$

$Fmax = 2\left(\frac{10}{2\pi}\right) = \frac{10}{\pi} = 3.184\,Hz$

cero y pendiente $= \frac{5 - (-3.3)}{25 - (-25)} = \frac{8.3}{50}$

$-3.3 = \frac{8.3}{50}(-25) + b$

Intercepto $= \frac{17}{20} = b$

$0.166\,x(t) + 0.85 = v^c(t)$

$\frac{v^c(t) - (-3.3v)}{0.2594}$

ASUNTO

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Demostracion del punto 2

Se procede a realizar el cambio de la variable del tiempo dado la siguiente ecuación.

$$t=n/Fs$$

La señal para discretizarla

$$x[n\backslash Fs]=3\cos[1000\pi(\tfrac{n}{Fs})]+5\,\text{sen}[2000\pi(n/Fs)+11\cos-$$
$$(11000\pi(n/Fs)$$

$$x[n\backslash Fs]=3\cos[1000\pi(\tfrac{n}{5000})]+5\,\text{sen}[2000\pi(\tfrac{n}{5000})]+11\cos[11000\pi(\tfrac{n}{5000})]$$

$$x[n/Fs]=3\cos[\tfrac{\pi}{5}n]+5\,\text{sen}[\tfrac{2\pi}{5}n]+11\cos[\tfrac{11\pi}{5000}n]$$

Para este ultimo coseno su frecuencia no se halla en $[-\pi,\pi]$, lo que nos indica que es un alias y hace falta hallar la frecuencia original. Entonces se resta 2π para dejarlo en el intervalo ($\Omega ori=\Omega copia\pm k2\pi$).

$$\Omega_3=\frac{11\pi}{5}\notin[-\pi,\pi]$$

$$\Omega_3-2\pi=\frac{11\pi}{5}-\frac{10\pi}{5}=\frac{\pi}{5}$$

Lo siguiente es volver a llevar esta frecuencia al coseno que consideremos, el cual quedaria con su frecuencia original de discretización.

$$x[n/Fs]=3\cos[\tfrac{\pi}{5}n]+5\sin[\tfrac{2\pi}{5}n]+11\cos[\tfrac{\pi}{5}n]$$

queda con la misma frecuencia angular que el primero se procede a sumarlos.

Se concluye que la señal obtenida en tiempo discreto por el conversor análogo digital es:

$$X[n/Fs]=13\cos[\tfrac{\pi}{5}n]+5\sin[\tfrac{2\pi}{5}n]$$

ASUNTO

Demostracion punto 3

Los intervalos de $x_2(t)$, se tiene que dividir la integral en 3 partes

$$\bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\left[\int_0^{T/4} (A\cos(w_0 t)-1)^2 dt + \int_{T/4}^{3T/4} (A\cos(w_0 t)+1)^2 dt + \int_{3T/4}^{T} (A\cos(w_0 t)-1)^2 dt \right]$$

Utilizamos factorizacion y los terminos de cada uno

$$\bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\left[\int_0^{T/4} (A^2\cos^2(w_0 t)-2A\cos(w_0 t)+1) dt + \int_{T/4}^{3T/4} (A^2\cos^2(w_0 t)+2A\cos(w_0 t)+1) dt + \int_{3T/4}^{T} (A^2\cos^2(w_0 t)-2A\cos(w_0 t)) dt \right]$$

Resolvemos primero para $0 \leq t < T/4$

$$\int_0^{T/4} (A^2\cos^2(w_0 t)-2A\cos(w_0 t)+1) dt \rightarrow \cos^2(w_0 t) = \frac{1+\cos(2w_0 t)}{2}$$

$$A^2\cos^2(w_0 t) = \frac{A^2}{A}(1+\cos(2w_0 t))$$

$$\frac{A^2}{2}\int_0^{T/4} 1\, dt + \frac{A^2}{2}\int_0^{T/4} \cos(2w_0 t) dt - 2A\int_0^{T/4} \cos(w_0 t) dt + \int_0^{T/4} 1\, dt$$

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Separamos la integral

- $\int_0^{T/4} 1\, dt = \boxed{\frac{T}{4}}$; $\cdot \int_0^{T/4} \cos(2w_0 t)\, dt = \left[\frac{\sin(2w_0 t)}{2w_0} \right]_0^{T/4}$

$= \frac{1}{2w_0}\left(\sin\left(\frac{T}{2}\cdot w_0\right) - \sin(0)\right) = \frac{1}{2w_0}\left(\sin\left(\frac{\cancel{T}}{2}\cdot\frac{2\pi}{\cancel{T}}\right) - \sin(0)\right)$

$w_0 \downarrow \frac{2\pi}{T}$

$\boxed{= 0}$

- $\int_0^{T/4} \cos(w_0 t)\, dt = \left[\frac{\sin(w_0 t)}{w_0} \right]_0^{T/4} = \frac{1}{w_0}\left(\sin\left(w_0 \frac{T}{4}\right)\right.$

$\left. - \sin(0)\right) = \frac{1}{w_0}\left(\sin\left(\frac{2\pi}{\cancel{T}}\cdot\frac{\cancel{T}}{4}\right)\right) = \frac{1}{w_0}\left(\sin\left(\frac{\pi}{2}\right)\right) = \frac{1}{w_0} \to \boxed{\frac{T}{2\pi}}$

- $\int_0^{T/4} 1\, dt = \boxed{T/4}$

Sustituimos todos los terminos

$\frac{A^2}{2}\cdot\frac{T}{4} + \frac{A^2}{2}\cdot 0 - 2A\cdot\frac{T}{2\pi} + \frac{T}{4} = \boxed{\frac{A^2 T}{8} - \frac{AT}{\pi} + \frac{T}{4}}$

Luego para $T/4 \leq t \leq 3T/4$

$\int_{T/4}^{3T/4} \left(A^2\cos^2(w_0 t) + 2A\cos(w_0 t) + 1\right) dt$

- $\int_{T/4}^{3T/4} \cos^2(w_0 t)\, dt = T/4$

- $\int_{T/4}^{3T/4} \cos(w_0 t)\, dt = \frac{1}{w_0}\left[\sin(w_0 t)\right]_{T/4}^{3T/4}$

$= \frac{1}{w_0}\left(\sin(3\pi/2) - \sin\left(\frac{\pi}{2}\right)\right)$

ASUNTO

$$\frac{T}{\omega_0}(-1-1) = -\frac{2T}{2\pi}$$

$$I_2 = A^2 \cdot \frac{T}{4} - 2A \cdot \frac{T}{\pi} + \frac{T}{2}$$

se suman los resultados

$$\bar{P}_{x_1 - x_2} = \lim_{T\to\infty} \frac{1}{T}\left[\left(\frac{A^2 T}{8} - \frac{4T}{\pi} + \frac{T}{4}\right) + \left(\frac{A^2 T}{4} - \frac{2AT}{\pi} + \frac{T}{2}\right) + \left(\frac{A^2}{4} + \frac{AT}{\pi} + \frac{T}{4}\right)\right]$$

$$\lim_{T\to\infty} \frac{\cancel{T}}{\cancel{T}}\left[\frac{A^2 \cancel{T}}{2} - \frac{4T}{\pi} + \cancel{T}\right] = \frac{A^2 T}{2} - \frac{4T}{\pi} + T$$

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4-) Demostración

$$C_n = \frac{1}{T}\int_{t_i}^{t_f} x(t)e^{-jnw_0t}\,dt \Rightarrow x(t) = \sum_n C_n e^{jnw_0t}$$

$$x'(t) = \frac{\partial}{\partial t}x(t) = \frac{\partial}{\partial t}\left\{\sum_n C_n e^{jnw_0t}\right\} = \sum_n C_n e^{jnw_0t}\,jnw_0$$

$$x''(t) = \frac{d}{dt}\left\{\sum_n C_n e^{jnw_0t}(jnw_0)\right\} = \sum_n C_n e^{jnw_0t}(jnw_0)^2$$

$$\tilde{C}_n = \frac{\langle x''(t), e^{jnw_0t}\rangle}{\| e^{jnw_0t}\|^2} = \int_{t_i}^{t_f}\frac{x''(t)e^{-jnw_0t}\,dt}{T} \;; T = t_f - t_i$$

$$\tilde{C}_n = C_n(jnw_0)^2 = \int_{t_i}^{t_f}\frac{x''(t)e^{-jnw_0t}}{T}\,dt$$

$$C_n = \frac{1}{(t_f - t_i)(jnw_0)}\int_{t_i}^{t_f} x''(t)e^{-jnw_0t}\,dt$$

$$= \frac{1}{(t_i - t_f)n^2w_0^2}\int_{t_i}^{t_f} x''(t)e^{-jnw_0t}\,dt$$

$$x(t) = a_0 + \sum_{n=1}^{N} a_n\cos(nw_0t) + b_n\,\text{sen}(nw_0t)$$

$$x'(t) = \sum_{n=1}^{n} a_n(-nw_0)\,\text{sen}(nw_0t) + b_n(nw_0)\cos(nw_0t)$$

$$x''(t) = \sum_{n=1}^{n} a_n(-nw_0)(nw_0)\cos(nw_0t) + b_n(nw_0)(-nw_0)\,\text{sen}(nw_0t)$$

ASUNTO

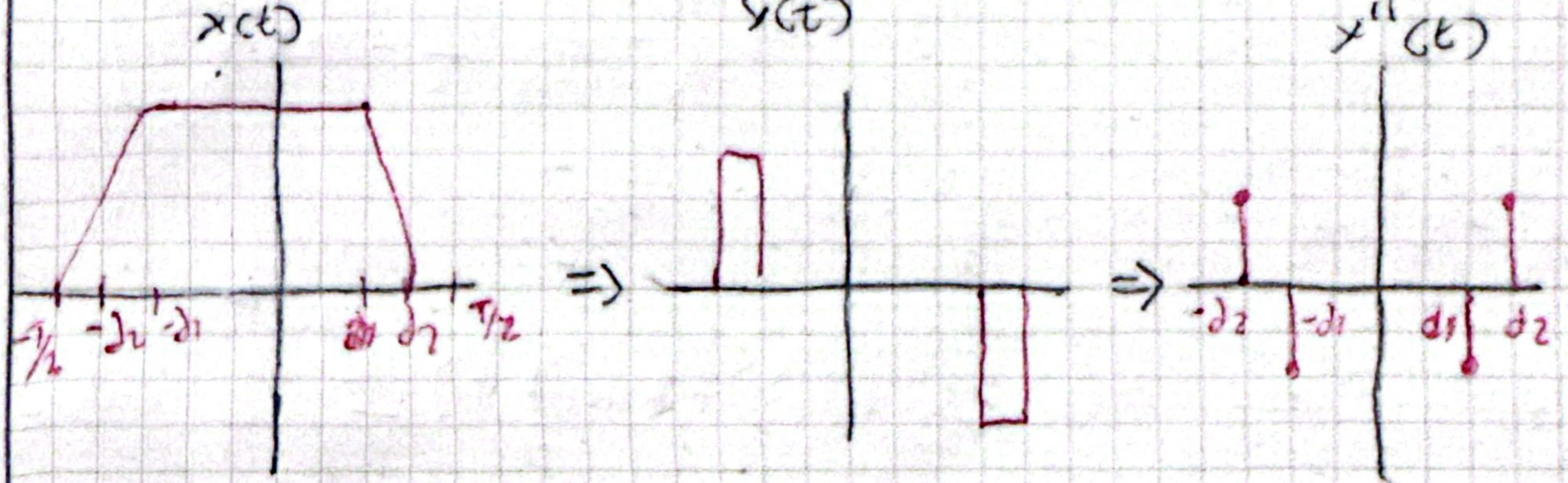
$$\tilde{a}_n = \frac{2}{T}\int_{t_i}^{t_f} x''(t)\cos(n\omega_0 t)\,dt \;;\; \tilde{b}_n = \frac{2}{T}\int_{t_i}^{t_f} x''(t)\,\mathrm{sen}(n\omega_0 t)\,dt$$

$$a_n(-n^2\omega_0^2) = \frac{2}{T}\int_{t_i}^{t_f} x''(t)\cos(n\omega_0 t)\,dt$$

$$a_n = \frac{2}{-Tn^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\cos(n\omega_0 t)\,dt$$

$$b_n(-n^2\omega_0^2) = \frac{2}{T}\int_{t_i}^{t_f} x''(t)\,\mathrm{sen}(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{-Tn^2\omega_0^2}\int_{t_i}^{t_f} x''(t)\,\mathrm{sen}(n\omega_0 t)\,dt$$



$$x''(t) = A\delta(t+d_2) - A\delta(t+d_1) - A\delta(t-d_1) + A\delta(t-d_2)$$

$$c_n = \frac{1}{-Tn^2\omega_0^2}\int_{-T/2}^{T/2} A[\delta(t+d_2) - \delta(t+d_1) - \delta(t-d_1) + \delta(t-d_1)]\,e^{-jn\omega_0 t}\,dt$$

$$c_n = -\frac{A}{Tn^2\omega_0^2}\left(e^{jn\omega_0 d_2} + e^{-jn\omega_0 d_2} - \left(e^{jn\omega_0 d_1} + e^{-jn\omega_0 d_1}\right)\right)$$

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$$C_n = -\frac{A}{T n^2 w_0^2}\left(2\cos(n w_0 d_2) - 2\cos(n w_0 d_1)\right) = -\frac{2A}{T n^2 \frac{4\pi^2}{T^2}}$$

$$\left(\cos\left(n\frac{2\pi}{T} d_2\right) - \cos\left(n\frac{2\pi}{T} d_1\right)\right)$$

$$C_n = -\frac{A}{\frac{2\pi^2 n^2}{T}}\left(\cos\left(n\frac{2\pi}{T} d_2\right) - \cos\left(n\frac{2\pi}{T} d_1\right)\right)$$

$$C_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,dt = \frac{1}{T}\int_{-d_2}^{-d_1} \frac{A}{d_2-d_1}(t+d_2)\,dt + \frac{1}{T}\int_{d_1}^{d_2} -\frac{A}{d_2-d_1}(t-d_2)\,dt$$

$$= \frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{t^2}{2}+d_2 t\right)\Big|_{-d_2}^{-d_1} + At\Big|_{-d_1}^{d_1} - \frac{A}{d_2-d_1}\left(\frac{t^2}{2}-d_2 t\right)\Big|_{d_1}^{d_2}\right]$$

$$= \frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{d_1^2}{2} - d_1 d_2 - \frac{d_2^2}{2} + d_2^2\right) + 2Ad_1\right]$$

$$\frac{1}{T}\left[\frac{A}{d_2-d_1}\left(\frac{d_1^2}{2} - d_1 d_2 - \frac{d_2^2}{2} + d_2^2\right) + A(d_1+d_1) - \frac{A}{d_2-d_1}\left(\frac{d_2^2}{2} - d_2 d_2 - \frac{d_1^2}{2} + d_1 d_2\right)\right]$$

Si: $A=1$ $d_1=1$ $d_2=2$ y $T=2d_2=4$

$$C_n = -\frac{1}{\frac{\cancel{2}\pi^2 n^2}{\cancel{4}\,2}}\left(\cos\left(n\cdot\frac{\cancel{2}\pi}{\cancel{4}}\cdot 2\right) - \cos\left(n\frac{2\pi}{4}\cdot 1\right)\right)$$

$$= -\frac{1}{\frac{\pi^2 n^2}{2}}\left(\cos(n\pi) - \cos\left(n\frac{\pi}{2}\right)\right)$$

$$C_0 = \frac{1}{4}\left[\frac{2\cdot 1}{1}\left(\frac{1}{2} - 2 - 2 + 4\right) + 2\cdot 1\cdot 1\right] = \frac{1}{4}\left[2\left(\frac{1}{2}\right)+2\right]$$

$$= \frac{1}{4}\,3 = \frac{3}{4}$$

$$P_x = \frac{1}{T}\int_{-T/2}^{T/2} (x(t))^2\,dt = \frac{2}{T}\int_{-T/2}^{0} (x(t))^2\,dt$$

$$= \frac{2}{T}\int_{-d_2}^{-d_1} \left(\frac{A}{d_2-d_1}\right)^2 (t+d_2)^2\,dt$$

$$+ \frac{2}{T}\int_{-d_1}^{0} A^2\,dt \Rightarrow P_x = \frac{2}{T}\left(\frac{A}{d_2-d_1}\right)^2 (t^2+2td_2+d_2^2)\Big|_{-d_2}^{d_1}$$

$$+ \frac{2}{T}A^2 t\Big|_{-d_1}^{0}$$

$$P_x = \frac{2}{T}\left(\frac{A}{d_2-d_1}\right)^2 (d_1^2 -$$