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BLIND DECONVOLUTION FOR SISO FIR CHANNELS BASED ON AUTOCORRELATION FUNCTION

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ABSTRACT

A new algorithm based on autocorrelation function of single input for blind deconvolution is proposed. We deduce a LMS iterative algorithm for blind deconvolution by minimizing the squared sum of received signal autocorrelation function over a region excluding the zero delay point. In this algorithm, only second order moment is adopted, which makes it widely applicable for Gaussian signals as well as non-Gaussian signals. Computer simulation results agree well to the theoretical analysis.

1. INTRODUCTION

In the past ten years, more and more people thought much of blind deconvolution technology. Blind deconvolution can find many important applications in channel estimation, system identification, data transmission and so on. People are also getting more and more research interest in underwater acoustics channel estimate and passive target localization.

Blind digital deconvolution focuses on the problem of recovering a source signal $s(n)$ distorted by a linear channel with impulse response $\mathbf{h}$ from observations of the channel output $x(n)$, without knowledge about its impulse response $\mathbf{h}$ and temporal characteristics of the source. In vector notation, the linear channel input/output model writes:

$$x(n) = \mathbf{h}^{\mathrm{T}}\mathbf{s}(n) + e(n) \qquad (1)$$

where $\mathbf{s}$ is a vector containing the input samples: $\mathbf{s} = [s(n), s(n-1), \cdots s(n-\ell+1)]^{\mathrm{T}}$ with ℓ being the number of tap-weights in $\mathbf{h}$, $\mathbf{h} = [h_1, h_2, \cdots, h_\ell]^{\mathrm{T}}$ and $e(n)$ being an additive noise that originates by many simultaneous effects, as additive external disturbance, measurement errors, sampling and round off errors.

A filter described by its impulse response $\mathbf{w}$ is the inverse filter or channel equalizer if $\mathbf{w}$ cancels the effects of $\mathbf{h}$ on the source signal so that $y(n) \sim s(n)$. Where "$\sim$" denotes similarity in waveform. The output of the filter writes:

$$y(n) = \mathbf{w}^{\mathrm{T}}\mathbf{x}(n) \qquad (2)$$

where $\mathbf{w} = [w_1, w_2, \cdots, w_L]^{\mathrm{T}}$ with L being the number of tap-weights in $\mathbf{w}$ and $\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-L+1)]^{\mathrm{T}}$ being the channel output vector.

Since $\mathbf{h}$ and $s(n)$ are unknown, the equalizer $\mathbf{w}$ has to be blindly found usually by means of an iterative algorithm.

If there is no noise

$$\mathbf{x} = \mathbf{s} \otimes \mathbf{h}, \quad \mathbf{y} = \mathbf{x} \otimes \mathbf{w} = \mathbf{s} \otimes (\mathbf{h} \otimes \mathbf{w}) \sim \mathbf{s}$$

so

$$\mathbf{h} \otimes \mathbf{w} = \delta(n-\Delta) \qquad (3)$$

where "$\otimes$" denotes convolution.

When $\mathbf{h}$ represents a non-minimum phase system, its inversion cannot be performed by means of a FIR filter. Therefore every time a FIR equalizer is used an approximation error occurs. We get:

$$y(n) = cs(n-\Delta) + N(n) \qquad (4)$$

where $N(n)$ is the so-called deconvolution noise, c is an amplitude factor and Δ is an finite delay.

A fundamental signal processing problem encountered in sonar, radar, and communications appearing in numerous applications is multipath. This phenomenon can cause a signal emanating from a source to a receiver via two or more different paths. By ignoring multipath the delay estimation is degraded and results in large biases of target bearing estimation. Blind deconvolution is very useful in equalize the multipath signal.

In recent years, several neural networks based [1] and higher order cumulant based [2] blind deconvolution algorithms have been developed. One of the most known algorithms is the "Bussgang" one in [3], which based on a memoryless Bayesian estimation of source data under the hypothesis of IID (Independent Identically Distribution) source sequences. Unfortunately, both the "Bussgang" one and its modified version [4], together with the higher order cumulant based algorithms depend on the statistics of the deconvolution noise and the source sequence. All of them are valid to non-Gaussian signals while invalid for Gaussian signals that is very popular in domains such as passive sonar, noise source localization and radio monitoring. Accordingly, we should develop a blind deconvolution algorithm that is widely applicable, not only for non-Gaussian signal but also for Gaussian signal.

In the present paper we propose a blind deconvolution algorithm based on autocorrelation function of single input. Only second-order moment is adopted in this algorithm, which make it effective for arbitrary statistics of random signal except the IID hypothesis has to be justified.

2. ALGORITHM OF BLIND DECONVOLUTION

Multipath channel can be modeled by convolution. The problem of equalizing the multipath channel is just the same as deconvolution. In multipath channel, we model the received signal as the following [5]:

$$x(n)=\sum_{i=0}^{N-1} g_i s(n-n_i)+e(n) \qquad (5)$$

where $x(n)$ is received signal, $s(n)$ is a transmitted signal, n_i is the time delay of multipath arrivals, g_i is the attenuation factors of each delayed replicas of $s(n)$, and $e(n)$ is the noise, N is the number of paths. Eq.(5) constitute multipath propagation problem that is modeled as a tap delay line. To describe the multipath model to the convolution form as Eq.(1), we get:

$$\mathbf{h}=\left[g_0,\underbrace{0,\cdots,0}_{n_1-1},g_1,\underbrace{0,\cdots,0}_{n_2-n_1-1},g_2,0,\cdots,g_i,\cdots,0,g_{N-1}\right]^{\mathrm{T}} \qquad (6)$$

In multipath channel, the convolution filters generally contained some zero values just as in Eq.(6).

The ensemble averaged autocorrelation function of the received signal $\mathbf{x}$ is given by:

$$\begin{aligned}\Re_{xx}(m)&=E[x(n)x(n-m)]\\&=\Re_{ee}(m)+\left(\sum_{i=0}^{N-1}g_i^2\right)\Re_{ss}(m)+\sum_{i=1}^{N-1}g_i\Re_{ss}(m-n_i)\\&\quad+\sum_{i=1}^{N-2}\sum_{j=i+1}^{N-1}g_ig_j\Re_{ss}(m+n_i-n_j)\end{aligned} \qquad (7)$$

If transmitted signal $s(n)$ is white noise with IID, the extremum of the autocorrelation $\Re_{ss}(m)$ for transmitted signal occurs at the time $m=0$. From Eq.(7), the extrema of the autocorrelation $\Re_{xx}(m)$ for the received signal occur at the time delays and the differences between each two time delays, that is $m=0$, $n_i(i=1,\cdots,N-1)$, n_j-n_i $(i=1,\cdots N-2$, $j=i+1,\cdots N-1)$. From Eq.(4), the autocorrelation $\Re_{yy}(m)$ for the equalizer output $\mathbf{y}$ should resemble $\Re_{ss}(m)$, that is the extremum of $\Re_{yy}(m)$ occurs only when $m=0$. For this notion, we can deduce the blind deconvolution algorithm by minimizing the squared sum of received signal autocorrelation function over a region excluding zero delay point.

Clearly, Some constraint on the value of $\mathbf{w}$ must be established in order to prevent the trivial solution $\|\mathbf{w}\|=0$. Constant gain is often adopted which keep the norm of $\mathbf{w}$ at a fixed value. Thus

$$\mathbf{w}=\arg\left(\min_{\mathbf{w}^{\mathrm{T}}\mathbf{w}=\kappa^2}\sum_{m=\Delta}^{M}\Re_{yy}^2(m)\right) \quad (\Delta\geq 1) \qquad (8)$$

where $\kappa>0$ is an arbitrarily-chosen constant. This can be implemented by a simple iterative re-normalization of the vector $\mathbf{w}$ at each step. Ultimately, the learning rule for $\mathbf{w}$ reads:

$$\begin{cases}\mathbf{w}(n+1)=\mathbf{w}(n)-\mu\,\partial\left\{\sum_{m=\Delta}^{M}\left[\Re_{yy}^2(m)\right]\right\}\Big/\partial\mathbf{w}\\[2ex] \mathbf{w}=\kappa\mathbf{w}/\|\mathbf{w}\|\end{cases} \qquad (9)$$

where μ is learning stepsize.

Adopt the following cost function, which is minimized to obtain the expected deconvolution filter:

$$r(n)=\sum_{m=\Delta}^{M}[z_m(n)]^2 \qquad (10)$$

where $z_m(n)=\sum_{k=0}^{K}y(n-k)y(n-m-k)$ is the estimate of received signal autocorrelation function.

$$\frac{\partial r}{\partial\mathbf{w}}=2\sum_{m=\Delta}^{M}\left(z_m(n)\frac{\partial z_m(n)}{\partial\mathbf{w}}\right) \qquad (11)$$

for

$$\frac{\partial z_m(n)}{\partial\mathbf{w}}=\sum_{k=0}^{K}\left(y(n-k)\frac{\partial y(n-m-k)}{\partial\mathbf{w}}+y(n-m-k)\frac{\partial y(n-k)}{\partial\mathbf{w}}\right)$$

and $\dfrac{\partial y(n)}{\partial\mathbf{w}}=\mathbf{x}(n)$ so

$$\frac{\partial r}{\partial\mathbf{w}}=2\sum_{m=\Delta}^{M}\left\{z_m(n)\sum_{k=0}^{K}\left[y(n-k)\mathbf{x}(n-m-k)+y(n-m-k)\mathbf{x}(n-k)\right]\right\} \qquad (12)$$

Using the LMS learning rule the adaptive algorithm for $\mathbf{w}$ reads:

$$\begin{cases} \mathbf{w}(n+1) = \mathbf{w}(n) - 2\mu \sum_{m=\Delta}^{M} \left\{ z_m(n) \sum_{k=0}^{K} \left[y(n-k)\mathbf{x}(n-m-k) + y(n-m-k)\mathbf{x}(n-k) \right] \right\} \\ \mathbf{w} = \kappa \mathbf{w} / \|\mathbf{w}\| \end{cases} \tag{13}$$

where μ is learning stepsize.

3. COMPUTER SIMULATION AND RESULTS

In support of the new deconvolution algorithm, as an experimental case we present simulations performed under the following conditions:

For vector $\mathbf{h}$, we take the sampled impulse response the same as used in the reference [1], in which $\mathbf{h} = [1,0,0,0,0.8]^T$ and $\ell = 5$. The histogram of its (truncated, with $L = 25$) inverse $\mathbf{w}_{ideal}$ obtained by the use of discrete-time Fourier transform is shown in Fig.1. As source signal, a Gaussian random process with zero mean and unit variance has been taken. The other variant is set as follows: $M = 30$, $K = 40$, $\kappa = 0.5$, $\mu = 0.00025$.

The deconvolution filter $\mathbf{w}$ produced by our algorithm is shown in Fig.2. While Fig.3 show the convolution of $\mathbf{w}$ and $\mathbf{h}$, which should be a delta-function (ie: consist of only a single high value at some position) if $\mathbf{w}$ correctly inverts $\mathbf{h}$.

Compare Fig.1 and Fig.2, the inverting filter $\mathbf{w}$ resembles the ideal one $\mathbf{w}_{ideal}$ very much, though the resemblance tails off towards the right since we are really learning an optimal filter of finite length, not a truncated infinite filter. The resultant convolution in Fig.3 should be recognized very satisfactory.

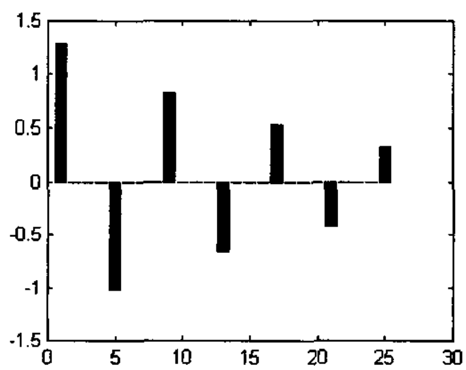

Fig.1. Ideal deconvolution filter $\mathbf{w}_{ideal}$

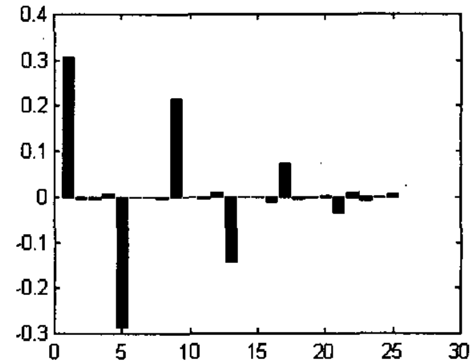

Fig.2. Deconvolution filter $\mathbf{w}$ from our algorithm

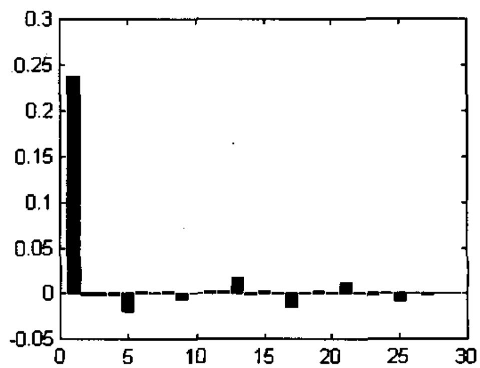

Fig.3. Convolution of $\mathbf{W}$ and $\mathbf{h}$

The waveform of Waveform of source signal $s(n)$, received signal $x(n)$ and equalizer output $y(n)$ are shown in Fig.4.The received signal distorted from the source signal, while the equalizer can recover the source signal. The equalizer output $y(n)$ resembles source signal $s(n)$ very much. Additional simulation results show that the blind deconvolution algorithm proposed here is indeed a good algorithm.

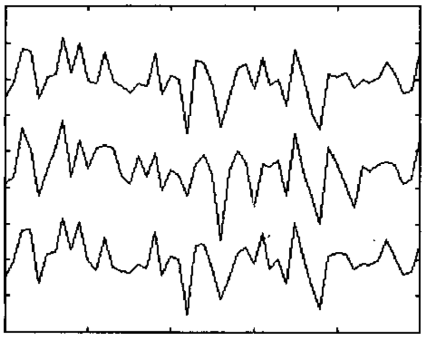

Fig.4. Waveform of $s(n), x(n)$ and $y(n)$

4. CONCLUSION

A new blind deconvolution algorithm based on autocorrelation function of single input is proposed. It can equalize the multipath channel quite well. By minimizing the squared sum of received signal autocorrelation function over a region excluding zero delay point, a LMS iterative algorithm for blind deconvolution is deduced. Only second order moment is adopted while without other knowledge about impulse response and the source, the algorithm is valid for arbitrary statistics of the signal only if the IID property is justified that makes the algorithm widely applicable no matter how the signal received is Gaussian or not. Those are verified by computer simulation results. The blind deconvolution proposed here is a good algorithm to equalize multipath channel.

5. REFERENCES

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