1 Types

- Boolean type: Bool,
- Raw bit-vectors: $\text{Bit}(n, \text{r})$ with $n \geqslant 2$,
- Signed bit-vectors: $\text{Bit}(n, \text{s})$ with $n \geqslant 2$,
- Unsigned bit-vectors: $\text{Bit}(n, \text{u})$ with $n \geqslant 2$,
- User defined: κ.

2 Base Cases

$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}\ \text{var}$$

$$\frac{}{\Gamma \vdash \texttt{true} : \text{Bool}}\ \text{true} \qquad \frac{}{\Gamma \vdash \texttt{false} : \text{Bool}}\ \text{false}$$

$$\frac{d \in [\![0; 2^n - 1]\!]}{\Gamma \vdash d : \text{Bit}(n, \text{r})}\ \text{rawcst} \qquad \frac{d \in [\![0; 2^n - 1]\!]}{\Gamma \vdash d : \text{Bit}(n, \text{s})}\ \text{unsigcst} \qquad \frac{d \in [\![-2^{n-1}; 2^{n-1} - 1]\!]}{\Gamma \vdash d : \text{Bit}(n, \text{s})}\ \text{sigcst}$$

3 General operations

$$\frac{\Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash (e_1, \dots, e_n) : (\tau_1, \dots, \tau_n)}\ \text{tuple } (n \geqslant 2)$$

$$\frac{\Gamma \vdash c : \text{Bool} \quad \Gamma \vdash e : \tau \quad \Gamma \vdash e' : \tau}{\Gamma \vdash \texttt{if}\ c\ \texttt{then}\ e\ \texttt{else}\ e' : \tau}\ \text{if}$$

$$\frac{\Gamma \vdash e : \tau \quad \Gamma \vdash e' : \tau}{\Gamma \vdash e\ \texttt{fby}\ e' : \text{Bool}}\ \text{fby}$$

4 Boolean operations

$$\frac{\Gamma \vdash e : \text{Bool} \quad \Gamma \vdash e' : \text{Bool}}{\Gamma \vdash e\ \texttt{and}\ e' : \text{Bool}}\ \text{and} \qquad \frac{\Gamma \vdash e : \text{Bool} \quad \Gamma \vdash e' : \text{Bool}}{\Gamma \vdash e\ \texttt{or}\ e' : \text{Bool}}\ \text{or}$$

$$\frac{\Gamma \vdash e : \text{Bool} \quad \Gamma \vdash e' : \text{Bool}}{\Gamma \vdash e\ \texttt{xor}\ e' : \text{Bool}}\ \text{xor} \qquad \frac{\Gamma \vdash e : \text{Bool}}{\Gamma \vdash \texttt{not}\ e : \text{Bool}}\ \text{not}$$

5 Bit-vectors operations

$$\frac{\Gamma \vdash e : \text{Bit}(n, \text{r}) \quad \Gamma \vdash e' : \text{Bit}(n, \text{r})}{\Gamma \vdash e\ \ \texttt{land}\ \ e' : \text{Bit}(n, \text{r})}\ \text{bvand}$$

$$\frac{\Gamma \vdash e : \text{Bit}(n, \text{r}) \quad \Gamma \vdash e' : \text{Bit}(n, \text{r})}{\Gamma \vdash e\ \ \texttt{lor}\ \ e' : \text{Bit}(n, \text{r})}\ \text{bvor}$$

$$\frac{\Gamma \vdash e : \text{Bit}(n, \text{r}) \quad \Gamma \vdash e' : \text{Bit}(n, \text{r})}{\Gamma \vdash e\ \ \texttt{lxor}\ \ e' : \text{Bit}(n, \text{r})}\ \text{bvxor}$$