Formules de base :

$$
\begin{array}{rlcl}
e^+ & (a) & = & a \\
e^- & (a) & = & a \\
se^+ & (a) & = & \{a\} \\
se^- & (a) & = & \{a\} \\
sc^{all} & (a) & = & \{\} \\
\\
e^+ & (a \wedge b) & = & e^+(a) \wedge e^+(b) \\
e^- & (a \wedge b) & = & e^-(a) \wedge e^-(b) \\
se^+ & (a \wedge b) & = & se^+(a) \cup se^+(b) \\
se^- & (a \wedge b) & = & e^-(a) \wedge se^-(b) \\
sc^{all} & (a \wedge b) & = & sc^{all}(a) \cup sc^{all}(b) \\
\\
e^+ & (a \rightarrow b) & = & e^-(a) \rightarrow e^+(b) \\
e^- & (a \rightarrow b) & = & \neg e^+(a) \vee e^-(b) \\
se^+ & (a \rightarrow b) & = & e^-(a) \rightarrow se^+(b) \\
se^- & (a \rightarrow b) & = & (\neg se^+(a)) \cup se^-(b) \\
sc^{all} & (a \rightarrow b) & = & sc^{all}(a) \cup (e^-(a) \rightarrow sc^{all}(b)) \\
\\
e^+ & (\forall x.\ a) & = & \forall x.\ e^+(a) \\
e^- & (\forall x.\ a) & = & \forall x.\ e^-(a) \\
se^+ & (\forall x.\ a) & = & \forall x.\ se^+(a) \\
se^- & (\forall x.\ a) & = & \{\forall x.\ e^-(a)\} \\
sc^{all} & (\forall x.\ a) & = & \forall x.\ sc^{all}(a) \\
\\
e^+ & (a\ \&\&\ b) & = & e^+(a) \wedge (e^-(a) \rightarrow e^+(b)) \\
e^- & (a\ \&\&\ b) & = & e^-(a) \wedge e^-(b) \\
se^+ & (a\ \&\&\ b) & = & se^+(a) \cup (e^-(a) \rightarrow se^+(b)) \\
se^- & (a\ \&\&\ b) & = & e^-(a) \wedge se^-(b) \\
sc^{all} & (a\ \&\&\ b) & = & sc^{all}(a) \cup (e^-(a) \rightarrow sc^{all}(b)) \\
\\
e^+ & (a\ \texttt{so}\ b) & = & e^+(a) \\
e^- & (a\ \texttt{so}\ b) & = & e^-(a) \wedge e^-(b) \\
se^+ & (a\ \texttt{so}\ b) & = & se^+(a) \\
se^- & (a\ \texttt{so}\ b) & = & e^-(a) \wedge se^-(b) \\
sc^{all} & (a\ \texttt{so}\ b) & = & sc^{all}(a) \cup (e^-(a) \rightarrow (se^+(b) \cup sc^{all}(b)))
\end{array}
$$

$$
\begin{array}{rlcl}
e^+ & (\neg a) & = & \neg e^-(a) \\
e^- & (\neg a) & = & \neg e^+(a) \\
se^+ & (\neg a) & = & \neg se^-(a) \\
se^- & (\neg a) & = & \neg se^+(a) \\
sc^{all} & (\neg a) & = & sc^{all}(a) \\
\\
e^+ & (a \vee b) & = & e^+(a) \vee e^+(b) \\
e^- & (a \vee b) & = & e^-(a) \vee e^-(b) \\
se^+ & (a \vee b) & = & e^+(a) \vee se^+(b) \\
se^- & (a \vee b) & = & se^-(a) \cup se^-(b) \\
sc^{all} & (a \vee b) & = & sc^{all}(a) \cup sc^{all}(b) \\
\\
e^+ & (a \leftrightarrow b) & = & (e^-(a) \rightarrow e^+(b)) \wedge (e^-(b) \rightarrow e^+(a)) \\
e^- & (a \leftrightarrow b) & = & (e^-(a) \wedge e^-(b)) \vee (\neg e^+(a) \wedge \neg e^+(b)) \\
se^+ & (a \leftrightarrow b) & = & (e^-(a) \rightarrow se^+(b)) \cup (e^-(b) \rightarrow se^+(a)) \\
se^- & (a \leftrightarrow b) & = & (e^-(a) \wedge se^-(b)) \cup (\neg e^+(a) \wedge \neg se^+(b)) \\
sc^{all} & (a \leftrightarrow b) & = & sc^{all}(a) \cup sc^{all}(b) \cup (e^-(a) \rightarrow sc^{all}(b)) \cup (e^-(b) \rightarrow sc^{all}(a)) \\
\\
e^+ & (\exists x.\ a) & = & \exists x.\ e^+(a) \\
e^- & (\exists x.\ a) & = & \exists x.\ e^-(a) \\
se^+ & (\exists x.\ a) & = & \{\exists x.\ e^+(a)\} \\
se^- & (\exists x.\ a) & = & \exists x.\ se^-(a) \\
sc^{all} & (\exists x.\ a) & = & \forall x.\ sc^{all}(a) \\
\\
e^+ & (a\ ||\ b) & = & e^+(a) \vee e^+(b) \\
e^- & (a\ ||\ b) & = & e^-(a) \vee (\neg e^+(a) \wedge e^-(b)) \\
se^+ & (a\ ||\ b) & = & e^+(a) \vee se^+(b) \\
se^- & (a\ ||\ b) & = & se^-(a) \cup (\neg e^+(a) \wedge se^-(b)) \\
sc^{all} & (a\ ||\ b) & = & sc^{all}(a) \cup (e^+(a) \vee sc^{all}(b)) \\
\\
e^+ & (a\ \texttt{by}\ b) & = & e^+(b) \\
e^- & (a\ \texttt{by}\ b) & = & e^-(a) \\
se^+ & (a\ \texttt{by}\ b) & = & se^+(b) \\
se^- & (a\ \texttt{by}\ b) & = & se^-(a) \\
sc^{all} & (a\ \texttt{by}\ b) & = & sc^{all}(a) \cup sc^{all}(b) \cup (e^-(b) \rightarrow se^+(a))
\end{array}
$$

Extension au `let` :

$$\begin{array}{rlcl}
e^+ & (\texttt{let } c = b \texttt{ in } a) & = & \texttt{let } c = b \texttt{ in } e^+(a) \\
e^- & (\texttt{let } c = b \texttt{ in } a) & = & \texttt{let } c = b \texttt{ in } e^-(a) \\
se^+ & (\texttt{let } c = b \texttt{ in } a) & = & \texttt{let } c = b \texttt{ in } se^+(a) \\
se^- & (\texttt{let } c = b \texttt{ in } a) & = & \texttt{let } c = b \texttt{ in } se^-(a) \\
sc^{all} & (\texttt{let } c = b \texttt{ in } a) & = & \texttt{let } c = b \texttt{ in } sc^{all}(a)
\end{array}$$

Une “compilation” du `if` dans la logique :

$$\texttt{if } a \texttt{ then } b \texttt{ else } c \equiv (a \to b) \wedge (\neg a \to c) \equiv (a \wedge b) \vee (\neg a \wedge c)$$

Extension au `if` :

$$\begin{array}{rlcl}
e^+ & (\texttt{if } a \texttt{ then } b \texttt{ else } c) & = & (e^-(a) \to e^+(b)) \wedge (\neg e^+(a) \to e^+(c)) \\
e^- & (\texttt{if } a \texttt{ then } b \texttt{ else } c) & = & (e^-(a) \wedge e^-(b)) \vee (\neg e^+(a) \wedge e^-(c)) \\
se^+ & (\texttt{if } a \texttt{ then } b \texttt{ else } c) & = & (e^-(a) \to se^+(b)) \cup (\neg e^+(a) \to se^+(c)) \\
se^- & (\texttt{if } a \texttt{ then } b \texttt{ else } c) & = & (e^-(a) \wedge se^-(b)) \cup (\neg e^+(a) \wedge se^-(c)) \\
sc^{all} & (\texttt{if } a \texttt{ then } b \texttt{ else } c) & = & sc^{all}(a) \cup (e^-(a) \to sc^{all}(b)) \cup (\neg e^+(a) \to sc^{all}(c))
\end{array}$$

Une “compilation” du `match` dans la logique :

$$\begin{array}{rcl}
M & \coloneqq & \begin{array}{l}
\texttt{match } e \texttt{ with} \\
\begin{array}{lll}
\mid\ A & \mapsto & a \\
\mid\ B(x) & \mapsto & b(x) \\
\mid\ C(x,y) & \mapsto & c(x,y) \\
\mid\ _ & \mapsto & d
\end{array} \\
\texttt{end}
\end{array} \\
& \equiv & \big(e = A \to a\big) \wedge \big(\forall x.\ e = B(x) \to b(x)\big) \wedge \big(\forall x\ y.\ e = C(x,y) \to c(x,y)\big) \wedge \big(p(e) \to d\big) \\
& \equiv & \big(e = A \wedge a\big) \vee \big(\exists x.\ e = B(x) \wedge b(x)\big) \vee \big(\exists x\ y.\ e = C(x,y) \wedge c(x,y)\big) \vee \big(p(e) \wedge d\big)
\end{array}$$

Avec

$$p(e) \coloneqq e \neq A \wedge (\forall x.\ e \neq B(x)) \wedge (\forall x\ y.\ e \neq C(x,y))$$

Extension au `match` :

$$\begin{array}{rlclclclcl}
e^+ & (M) & = & \big(e = A \to e^+(a)\big) & \wedge & \big(\forall x.\ e = B(x) \to e^+(b(x))\big) & \wedge & \big(\forall x\ y.\ e = C(x,y) \to e^+(c(x,y))\big) & \wedge & \big(p(e) \to e^+(d)\big) \\
e^- & (M) & = & \big(e = A \wedge e^-(a)\big) & \vee & \big(\exists x.\ e = B(x) \wedge e^-(b(x))\big) & \vee & \big(\exists x\ y.\ e = C(x,y) \wedge e^-(c(x,y))\big) & \vee & \big(p(e) \wedge e^-(d)\big) \\
se^+ & (M) & = & \big(e = A \to se^+(a)\big) & \cup & \big(\forall x.\ e = B(x) \to se^+(b(x))\big) & \cup & \big(\forall x\ y.\ e = C(x,y) \to se^+(c(x,y))\big) & \cup & \big(p(e) \to se^+(d)\big) \\
se^- & (M) & = & \big(e = A \wedge se^-(a)\big) & \cup & \big(\exists x.\ e = B(x) \wedge se^-(b(x))\big) & \cup & \big(\exists x\ y.\ e = C(x,y) \wedge se^-(c(x,y))\big) & \cup & \big(p(e) \wedge se^-(d)\big) \\
sc^{all} & (M) & = & \big(e = A \to sc^{all}(a)\big) & \cup & \big(\forall x.\ e = B(x) \to sc^{all}(b(x))\big) & \cup & \big(\forall x\ y.\ e = C(x,y) \to sc^{all}(c(x,y))\big) & \cup & \big(p(e) \to sc^{all}(d)\big)
\end{array}$$