

Verilog Size Checking

Equivalence-Based Typing Rules

The statement $\Gamma \mid n \vdash e : \tau, X$ mean:

In the variable context Γ and the size context n , the size of the expression e is the value of the variable τ , with X the size-tagged set of equivalence classes of size variables.

So we have:

- $\Gamma : \mathcal{O} \rightarrow \mathbb{N}$ a mapping from operand to their size, with \mathcal{O} the set of operands,
- $n \in \mathbb{N} \cup \{*\}$ a size context (with $*$ for the empty one),
- $e \in \mathcal{E}$, with \mathcal{E} the set of expressions,
- $\tau \in \mathcal{V}$, with \mathcal{V} the set of size variable,
- $X = (S, f)$ with $S \in 2^{\mathcal{V}}$ the set of equivalence classes and $f : S \rightarrow \mathbb{N}$ a size valuation for each equivalence class.

We use the following notations:

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$$\bowtie : \begin{cases} \mathbb{N} \times (\mathbb{N} \cup \{*\}) & \rightarrow & \mathbb{N} \\ (m, *) & \mapsto & m \\ (m, n) & \mapsto & \max(m, n) \end{cases}$$

- $\Delta_X : \mathcal{V} \rightarrow \mathbb{N}$ the function that given a size variable x returns its size in the environment $X = (S, f)$. We have $\Delta_X(x) = f([x]_S)$ with $[x]_S$ the equivalence class of x in S .
- $\{v := s\}$: the declaration of a *fresh* size variable v .

$$\begin{aligned} \{v := s\} &= (S, f) \\ S &= \{\{v\}\} \\ f &= \begin{cases} S & \rightarrow & \mathbb{N} \\ \{v\} & \mapsto & s \end{cases} \end{aligned}$$

- $X/\alpha \sim \beta$ the operation that combines two classes. For the newly created class, the valuation function gives the maximum of the previous classes:

$$\begin{aligned} (S, f)/\alpha \sim \beta &= (S', f') \\ S' &= \{[\alpha]_S \sqcup [\beta]_S\} \sqcup S \setminus \{[\alpha]_S, [\beta]_S\} \\ f' &= \begin{cases} S' & \rightarrow & \mathbb{N} \\ c & \mapsto & f(c) & \text{if } c \in S \setminus \{[\alpha]_S, [\beta]_S\} \\ c & \mapsto & \max(f([\alpha]_S), f([\beta]_S)) & \text{if } c = [\alpha]_S \text{ or } c = [\beta]_S \end{cases} \end{aligned}$$

Base case

$$\frac{\Gamma(x) = m}{\Gamma \mid n \vdash x : v, \{v := m \bowtie n\}}$$

Operators

$$\oplus \in \{+, -, *, /, \%, \&, |, \wedge, \sim, \sim\sim\}$$

$$\frac{\Gamma \mid n \vdash a : \alpha, A \quad \Gamma \mid n \vdash b : \beta, B}{\Gamma \mid n \vdash a \oplus b : \alpha, A \sqcup B / \alpha \sim \beta}$$

$$\oplus \in \{+, -, ++, --, \sim\}$$

$$\frac{\Gamma \mid n \vdash e : \varepsilon, E}{\Gamma \mid n \vdash \oplus e : \varepsilon, E}$$

$$\oplus \in \{++, --\}$$

$$\frac{\Gamma \mid n \vdash e : \varepsilon, E}{\Gamma \mid n \vdash e \oplus : \varepsilon, E}$$

$$\oplus \in \{\$signed, \$unsigned, signed', unsigned'\}$$

$$\frac{\Gamma \mid * \vdash e : \varepsilon, E}{\Gamma \mid n \vdash \oplus(e) : v, \{v := \Delta_E(\varepsilon) \bowtie n\}}$$

$$\oplus \in \{==, !=, ==?, !=?, ==, !=, >, >=, <, <=\}$$

$$\frac{\Gamma \mid * \vdash a : \alpha, A \quad \Gamma \mid * \vdash b : \beta, B \quad A \sqcup B / \alpha \sim \beta}{\Gamma \mid n \vdash a \oplus b : v, \{v := 1 \bowtie n\}}$$

$$\oplus \in \{\&\&, ||, ->, <->\}$$

$$\frac{\Gamma \mid * \vdash a : \alpha, A \quad \Gamma \mid * \vdash b : \beta, B}{\Gamma \mid n \vdash a \oplus b : v, \{v := 1 \bowtie n\}}$$

$$\oplus \in \{\&, \sim\&, |, \sim|, \wedge, \sim\wedge, \sim\sim, !\}$$

$$\frac{\Gamma \mid * \vdash e : \varepsilon, E}{\Gamma \mid n \vdash \oplus e : \varepsilon, \{v := 1 \bowtie n\}}$$

$$\oplus \in \{>>, <<, **, >>>, <<<\}$$

$$\frac{\Gamma \mid n \vdash a : \alpha, A \quad \Gamma \mid * \vdash b : \beta, B}{\Gamma \mid n \vdash a \oplus b : \alpha, A}$$

$$\oplus \in \{=, +=, -=, *=, /=, \%=, \&=, |=, \wedge=\}$$

$$\frac{\Gamma \mid * \vdash a : \alpha, A \quad \Gamma \mid \Delta_A(\alpha) \vdash b : \beta, B}{\Gamma \mid n \vdash a \oplus b : v, \{v := \Delta_A(\alpha) \bowtie n\}}$$

$$\oplus \in \{<<=, >>=, <<<=, >>>=\}$$

$$\frac{\Gamma \mid * \vdash a : \alpha, A \quad \Gamma \mid c * \vdash b : \beta, B}{\Gamma \mid n \vdash a \oplus b : v, \{v := \Delta_A(\alpha) \bowtie n\}}$$

$$\frac{\Gamma \mid * \vdash e : \varepsilon, E \quad \Gamma \mid n \vdash a : \alpha, A \quad \Gamma \mid n \vdash b : \beta, B}{\Gamma \mid n \vdash e?a:b : \alpha, A \sqcup B / \alpha \sim \beta}$$

$$\frac{\Gamma \mid * \vdash e_1 : \varepsilon_1, E_1 \quad \dots \quad \Gamma \mid * \vdash e_n : \varepsilon_n, E_n}{\Gamma \mid n \vdash \{e_1, \dots, e_n\} : v, \{v := (\Delta_{E_1}(\varepsilon_1) + \dots + \Delta_{E_n}(\varepsilon_n)) \bowtie n\}}$$

$$\frac{i \in \mathbb{N} \quad \Gamma \mid * \vdash e_1 : \varepsilon_1, E_1 \quad \dots \quad \Gamma \mid * \vdash e_k : \varepsilon_k, E_k}{\Gamma \mid n \vdash \{i\{e_1, \dots, e_k\}\} : v, \{v := i \times (\Delta_{E_1}(\varepsilon_1) + \dots + \Delta_{E_k}(\varepsilon_k)) \bowtie n\}}$$