Verilog Size Checking

Finding the Context

The statement $\vdash e : c$ mean:

The context of the expression e is c.

So we have:

- $e \in \mathcal{E}$, with $\mathcal{E}$ the set of expressions,
- $c \in \mathcal{N}$, a context identifier.

We use the following notations:

- $\text{ctx}()$ create a fresh empty context.

Base case

$$\frac{}{\vdash x : c}$$

Operators

$$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{\%}, \texttt{\&}, \texttt{|}, \texttt{\^{}}, \texttt{\^{}\~{}}, \texttt{\~{}\^{}}\} \qquad \frac{\vdash a : c \quad \vdash b : c}{\vdash a \oplus b : c}$$

$$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{++}, \texttt{--}, \texttt{\~{}}\} \qquad \frac{\vdash e : c}{\vdash \oplus e : c}$$

$$\oplus \in \{\texttt{++}, \texttt{--}\} \qquad \frac{\vdash e : c}{\vdash e \oplus : c}$$

$$\oplus \in \{\texttt{\$signed}, \texttt{\$unsigned}, \texttt{signed'}, \texttt{unsigned'}\} \qquad \frac{c' = \text{ctx}() \quad \vdash e : c'}{\vdash \oplus(e) : c}$$

$$\oplus \in \{\texttt{===}, \texttt{!==}, \texttt{==?}, \texttt{!=?}, \texttt{==}, \texttt{!=}, 0\texttt{>}, \texttt{>=}, \texttt{<}, \texttt{<=}\} \qquad \frac{c' = \text{ctx}() \quad \vdash a : c' \quad \vdash b : c'}{\vdash a \oplus b : c}$$

$$\oplus \in \{\texttt{\&\&}, \texttt{||}, \texttt{->}, \texttt{<->}\} \qquad \frac{c' = \text{ctx}() \quad \vdash a : c' \quad c'' = \text{ctx}() \quad \vdash b : c''}{\vdash a \oplus b : c}$$

$$\oplus \in \{\texttt{\&}, \texttt{\~{}\&}, \texttt{|}, \texttt{\~{}|}, \texttt{\^{}}, \texttt{\~{}\^{}}, \texttt{\^{}\~{}}, \texttt{!}\} \qquad \frac{c' = \text{ctx}() \quad \vdash e : c'}{\vdash \oplus e : c}$$

$$\oplus \in \{\texttt{>>}, \texttt{<<}, \texttt{**}, \texttt{>>>}, \texttt{<<<}\} \qquad \frac{\vdash a : c \quad c' = \text{ctx}() \quad \vdash b : c'}{\vdash a \oplus b : c}$$

$$\oplus \in \{\texttt{=}, \texttt{+=}, \texttt{-=}, \texttt{*=}, \texttt{/=}, \texttt{\%=}, \texttt{\&=}, \texttt{|=}, \texttt{\^{}=}\} \qquad \frac{c' = \text{ctx}() \quad \vdash a : c' \quad \vdash b : c'}{\vdash a \oplus b : c}$$

$$\oplus \in \{\texttt{<<=}, \texttt{>>=}, \texttt{<<<=}, \texttt{>>>=}\} \qquad \frac{c' = \text{ctx}() \quad \vdash a : c' \quad c'' = \text{ctx}() \quad \vdash b : c''}{\vdash a \oplus b : c}$$

$$\frac{c' = \text{ctx}() \quad \vdash e : c' \quad \vdash a : c \quad \vdash b : c}{\vdash e?a{:}b : c}$$

$$\frac{c_1 = \text{ctx}() \quad \vdash e_1 : c_1 \quad \ldots \quad c_k = \text{ctx}() \quad \vdash e_k : c_k}{\vdash \texttt{\{}e_1, \ldots, e_k\texttt{\}} : c}$$

$$\frac{i \in \mathbb{N} \quad c_1 = \text{ctx}() \quad \vdash e_1 : c_1 \quad \ldots \quad c_k = \text{ctx}() \quad \vdash e_k : c_k}{\vdash \texttt{\{}i\texttt{\{}e_1, \ldots, e_k\texttt{\}\}} : c}$$

$$\frac{c' = \text{ctx}() \quad \vdash a : c' \quad \vdash e_1 : c' \quad \ldots \quad \vdash e_k : c'}{\vdash a \texttt{ inside } \texttt{\{}e_1, \ldots, e_k\texttt{\}} : c}$$

Equivalence-Based Typing Rules

The statement $\Gamma \mid n \vdash e : \tau, X$ mean:

In the variable context Γ and the size context n, the size of the expression e is the value of the variable τ, with X the size-tagged set of equivalence classes of size variables.

So we have:

- $\Gamma : \mathcal{O} \to \mathbb{N}$ a mapping from operand to their size, with $\mathcal{O}$ the set of operands,
- $n \in \mathbb{N} \cup \{*\}$ a size context (with $*$ for the empty one),
- $e \in \mathcal{E}$, with $\mathcal{E}$ the set of expressions,
- $\tau \in \mathcal{V}$, with $\mathcal{V}$ the set of size variable,
- $X = (S, f)$ with $S \in 2^{\mathcal{V}}$ the set of equivalence classes and $f : S \to \mathbb{N}$ a size valuation for each equivalence class.

We use the following notations:

- $$\bowtie \; : \begin{cases} \mathbb{N} \times (\mathbb{N} \cup \{*\}) & \to & \mathbb{N} \\ (m, *) & \mapsto & m \\ (m, n) & \mapsto & \max(m, n) \end{cases}$$
- $\Delta_X : \mathcal{V} \to \mathbb{N}$ the function that given a size variable x returns its size in the environment $X = (S, f)$. We have $\Delta_X(x) = f([x]_S)$ with $[x]_S$ the equivalence class of x in S.
- $\{v \coloneqq s\}$: the declaration of a *fresh* size variable v.
$$\begin{aligned} \{v \coloneqq s\} &= (S, f) \\ S &= \{\{v\}\} \\ f &= \begin{cases} S & \to & \mathbb{N} \\ \{v\} & \mapsto & s \end{cases} \end{aligned}$$
- $X/\alpha \sim \beta$ the operation that combines two classes. For the newly created class, the valuation function gives the maximum of the previous classes:
$$\begin{aligned} (S, f)/\alpha \sim \beta &= (S', f') \\ S' &= \{[\alpha]_S \sqcup [\beta]_S\} \sqcup S \setminus \{[\alpha]_S, [\beta]_S\} \\ f' &= \begin{cases} S' & \to & \mathbb{N} & \\ c & \mapsto & f(c) & \text{if } c \in S \setminus \{[\alpha]_S, [\beta]_S\} \\ c & \mapsto & \max\left(f([\alpha]_S), f([\beta]_S)\right) & \text{if } c = [\alpha]_S \text{ or } c = [\beta]_S \end{cases} \end{aligned}$$

Base case

$$\frac{\Gamma(x) = m}{\Gamma \mid n \vdash x : v, \{v \coloneqq m \bowtie n\}}$$

Operators

$$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{\%}, \texttt{\&}, \texttt{|}, \texttt{\^{}}, \texttt{\^{}\~{}}, \texttt{\~{}\^{}}\} \qquad \frac{\Gamma \mid n \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid n \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \alpha,\ \mathrm{A} \sqcup \mathrm{B}/\alpha \sim \beta}$$

$$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{++}, \texttt{--}, \texttt{\~{}}\} \qquad \frac{\Gamma \mid n \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash \oplus e : \varepsilon,\ \mathrm{E}}$$

$$\oplus \in \{\texttt{++}, \texttt{--}\} \qquad \frac{\Gamma \mid n \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash e\oplus : \varepsilon,\ \mathrm{E}}$$

$$\oplus \in \{\texttt{\$signed}, \texttt{\$unsigned}, \texttt{signed'}, \texttt{unsigned'}\} \qquad \frac{\Gamma \mid * \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash \oplus(e) : \upsilon,\ \{\upsilon \coloneqq \Delta_{\mathrm{E}}(\varepsilon) \bowtie n\}}$$

$$\oplus \in \{\texttt{===}, \texttt{!==}, \texttt{==?}, \texttt{!=?}, \texttt{==}, \texttt{!=}, \texttt{>}, \texttt{>=}, \texttt{<}, \texttt{<=}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash b : \beta,\ \mathrm{B} \quad \mathrm{A} \sqcup \mathrm{B}/\alpha \sim \beta}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$

$$\oplus \in \{\texttt{\&\&}, \texttt{||}, \texttt{->}, \texttt{<->}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$

$$\oplus \in \{\texttt{\&}, \texttt{\~{}\&}, \texttt{|}, \texttt{\~{}|}, \texttt{\^{}}, \texttt{\~{}\^{}}, \texttt{\^{}\~{}}, \texttt{!}\} \qquad \frac{\Gamma \mid * \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash \oplus e : \varepsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$

$$\oplus \in \{\texttt{>>}, \texttt{<<}, \texttt{**}, \texttt{>>>}, \texttt{<<<}\} \qquad \frac{\Gamma \mid n \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \alpha,\ \mathrm{A}}$$

$$\oplus \in \{\texttt{=}, \texttt{+=}, \texttt{-=}, \texttt{*=}, \texttt{/=}, \texttt{\%=}, \texttt{\&=}, \texttt{|=}, \texttt{\^{}=}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid \Delta_{\mathrm{A}}(\alpha) \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq \Delta_{\mathrm{A}}(\alpha) \bowtie n\}}$$

$$\oplus \in \{\texttt{<<=}, \texttt{>>=}, \texttt{<<<=}, \texttt{>>>=}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid c* \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq \Delta_{\mathrm{A}}(\alpha) \bowtie n\}}$$

$$\frac{\Gamma \mid * \vdash e : \varepsilon,\ \mathrm{E} \quad \Gamma \mid n \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid n \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash e\texttt{?}a\texttt{:}b : \alpha,\ \mathrm{A} \sqcup \mathrm{B}/\alpha \sim \beta}$$

$$\frac{\Gamma \mid * \vdash e_1 : \varepsilon_1,\ \mathrm{E}_1 \quad \ldots \quad \Gamma \mid * \vdash e_n : \varepsilon_n,\ \mathrm{E}_n}{\Gamma \mid n \vdash \texttt{\{}e_1, \ldots, e_n\texttt{\}} : \upsilon,\ \{\upsilon \coloneqq (\Delta_{\mathrm{E}_1}(\varepsilon_1) + \cdots + \Delta_{\mathrm{E}_n}(\varepsilon_n)) \bowtie n\}}$$

$$\frac{i \in \mathbb{N} \quad \Gamma \mid * \vdash e_1 : \varepsilon_1,\ \mathrm{E}_1 \quad \ldots \quad \Gamma \mid * \vdash e_k : \varepsilon_k,\ \mathrm{E}_k}{\Gamma \mid n \vdash \texttt{\{}i\texttt{\{}e_1, \ldots, e_k\texttt{\}\}} : \upsilon,\ \{\upsilon \coloneqq i \times (\Delta_{\mathrm{E}_1}(\varepsilon_1) + \cdots + \Delta_{\mathrm{E}_k}(\varepsilon_k)) \bowtie n\}}$$

$$\frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash e_1 : \varepsilon_1,\ \mathrm{E}_1 \quad \ldots \quad \Gamma \mid * \vdash e_k : \varepsilon_k,\ \mathrm{E}_k \quad \mathrm{A} \sqcup \mathrm{E}_1 \sqcup \cdots \sqcup \mathrm{E}_k/\alpha \sim \varepsilon_1, \ldots, \alpha \sim \varepsilon_k}{\Gamma \mid n \vdash a\ \texttt{inside}\ \texttt{\{}e_1, \ldots, e_k\texttt{\}} : \upsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$