Formalization of the IEEE 1800

Determine phase

$$
\begin{aligned}
\mathsf{determine}(o) &= \Gamma(o) \quad \text{(where } o \in \mathcal{O}\text{)} \\
\mathsf{determine}(\texttt{BinaryOp}(e_1, e_2)) &= \max(\mathsf{determine}(e_1), \mathsf{determine}(e_2)) \\
\mathsf{determine}(\texttt{UnaryOp}(e)) &= \mathsf{determine}(e) \\
\mathsf{determine}(\texttt{ComparisonOp}(e_1, e_2)) &= 1 \\
\mathsf{determine}(\texttt{LogicOp}(e_1, e_2)) &= 1 \\
\mathsf{determine}(\texttt{ReductionOp}(e)) &= 1 \\
\mathsf{determine}(\texttt{ShiftOp}(e_1, e_2)) &= \mathsf{determine}(e_1) \\
\mathsf{determine}(\texttt{AssignmentOp}(l, e)) &= \phi(l) \\
\mathsf{determine}(\texttt{ConditionalOp}(e_1, e_2, e_3) &= \max(\mathsf{determine}(e_2), \mathsf{determine}(e_3)) \\
\mathsf{determine}(\texttt{Replication}(n, e)) &= n \times \mathsf{determine}(e) \\
\mathsf{determine}(\texttt{Concatenation}(e_1, \ldots, e_k)) &= \sum_{i=1}^{k} \mathsf{determine}(e_i)
\end{aligned}
$$

Propagate phase

$$\mathsf{propagate}_e([]) = \mathsf{determine}(e)$$

For $e|_p = \texttt{ReductionOp}(e')$:

$$\mathsf{propagate}_e(p \cdot 0) = \mathsf{determine}(e')$$

For $e|_p = \texttt{LogicOp}(e_1, e_2)$:

$$
\begin{aligned}
\mathsf{propagate}_e(p \cdot 0) &= \mathsf{determine}(e_1) \\
\mathsf{propagate}_e(p \cdot 1) &= \mathsf{determine}(e_2)
\end{aligned}
$$

For $e|_p = \texttt{Concatenation}(e_0, \ldots, e_k)$, with $i \in \{0, \ldots, k\}$:

$$\mathsf{propagate}_e(p \cdot i) = \mathsf{determine}(e_i)$$

For $e|_p = \texttt{Replication}(n, e')$:

$$\mathsf{propagate}_e(p \cdot 0) = \mathsf{determine}(e')$$

For $e|_p = \texttt{ComparisonOp}(e_1, e_2)$:

$$
\begin{aligned}
\mathsf{propagate}_e(p \cdot 0) &= \max(\mathsf{determine}(e_1), \mathsf{determine}(e_2)) \\
\mathsf{propagate}_e(p \cdot 1) &= \max(\mathsf{determine}(e_1), \mathsf{determine}(e_2))
\end{aligned}
$$

For $e|_p = \texttt{BinaryOp}(e_1, e_2)$:

$$
\begin{aligned}
\mathsf{propagate}_e(p \cdot 0) &= \mathsf{propagate}_e(p) \\
\mathsf{propagate}_e(p \cdot 1) &= \mathsf{propagate}_e(p)
\end{aligned}
$$

For $e|_p = \texttt{UnaryOp}(e')$:

$$\mathsf{propagate}_e(p \cdot 0) = \mathsf{propagate}_e(p)$$

For $e|_p = \texttt{ShiftOp}(e_1, e_2)$:

$$
\begin{aligned}
\mathsf{propagate}_e(p \cdot 0) &= \mathsf{propagate}_e(p) \\
\mathsf{propagate}_e(p \cdot 1) &= \mathsf{determine}(e_2)
\end{aligned}
$$

For $e|_p = \texttt{ConditionalOp}(e_1, e_2, e_3)$:

$$\begin{aligned}
\mathsf{propagate}_e(p \cdot 0) &= \mathsf{determine}(e_1) \\
\mathsf{propagate}_e(p \cdot 1) &= \mathsf{propagate}_e(p) \\
\mathsf{propagate}_e(p \cdot 2) &= \mathsf{propagate}_e(p)
\end{aligned}$$

For $e|_p = \texttt{AssignmentOp}(l, e')$:

$$\mathsf{propagate}_e(p \cdot 0) = \max(\mathsf{determine}(e'), \phi(l))$$