Verilog Size Checking

Set-Based Typing Rules

The statement $\Gamma, \xi \vdash e : n$ mean:

In the variable context Γ and the size context ξ, the size of the expression e is n.

So we have:

- $\Gamma : \mathcal{O} \to \mathbb{N}$ a mapping from operand to their size, with $\mathcal{O}$ the set of operands,
- $\xi \in \mathbb{N} \cup \{*\}$ a size context (with $*$ for the empty one).
- $e \in \mathcal{E}$, with $\mathcal{E}$ the set of expressions,
- $n \in \mathcal{N}$, the size of the expression.

We use the following notations:

-
$$
\begin{array}{ccccl}
\text{ctx} & \mathcal{E} & \to & \mathbb{N} \cup \{*\} & \\
 & x & \mapsto & \Gamma(x) & \\
 & a \oplus b & \mapsto & \max(\text{ctx}(a), \text{ctx}(b)) & \oplus \in \{\texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{\%}, \texttt{\&}, \texttt{|}, \texttt{\textasciicircum}, \texttt{\textasciicircum\textasciitilde}, \texttt{\textasciitilde\textasciicircum}\} \\
 & \oplus a & \mapsto & \text{ctx}(a) & \oplus \in \{\texttt{+}, \texttt{-}, \texttt{++}, \texttt{--}, \texttt{\textasciitilde}\} \\
 & a \oplus & \mapsto & \text{ctx}(a) & \oplus \in \{\texttt{++}, \texttt{--}\} \\
 & \oplus(a) & \mapsto & ? & \oplus \in \{\texttt{\$signed}, \texttt{\$unsigned}, \texttt{signed'}, \texttt{unsigned'}\} \\
 & a \oplus b & \mapsto & * & \oplus \in \{\texttt{===}, \texttt{!==}, \texttt{==?}, \texttt{!=?}, \texttt{==}, \texttt{!=}, \texttt{>}, \texttt{>=}, \texttt{<}, \texttt{<=}\} \\
 & a \oplus b & \mapsto & * & \oplus \in \{\texttt{\&\&}, \texttt{||}, \texttt{->}, \texttt{<->}\} \\
 & \oplus a & \mapsto & * & \oplus \in \{\texttt{\&}, \texttt{\textasciitilde\&}, \texttt{|}, \texttt{\textasciitilde|}, \texttt{\textasciicircum}, \texttt{\textasciitilde\textasciicircum}, \texttt{\textasciicircum\textasciitilde}, \texttt{!}\} \\
 & a \oplus b & \mapsto & \text{ctx}(a) & \oplus \in \{\texttt{>>}, \texttt{<<}, \texttt{**}, \texttt{>>>}, \texttt{<<<}\} \\
 & a \oplus b & \mapsto & \Gamma(a) & \oplus \in \{\texttt{=}, \texttt{+=}, \texttt{-=}, \texttt{*=}, \texttt{/=}, \texttt{\%=}, \texttt{\&=}, \texttt{|=}, \texttt{\textasciicircum=}\} \\
 & a \oplus b & \mapsto & \Gamma(a) & \oplus \in \{\texttt{<<=}, \texttt{>>=}, \texttt{<<<=}, \texttt{>>>=}\} \\
 & a \oplus b & \mapsto & ? & \\
 & a \oplus b & \mapsto & ? & \\
 & a \oplus b & \mapsto & ? & \\
 & a \oplus b & \mapsto & ? & \\
 & a \oplus b & \mapsto & ? &
\end{array}
$$

Base case

$$\frac{n = \Gamma(x) \bowtie \xi}{\Gamma, \xi \vdash x : n}$$

Operators

$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{\%}, \texttt{\&}, \texttt{|}, \texttt{\textasciicircum}, \texttt{\textasciicircum\textasciitilde}, \texttt{\textasciitilde\textasciicircum}\}$

$$\frac{\Gamma, \xi \vdash a : n \qquad \Gamma, \xi \vdash b : n}{\Gamma, \xi \vdash a \oplus b : n}$$

$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{++}, \texttt{--}, \texttt{\textasciitilde}\}$

$$\frac{\Gamma, \xi \vdash e : n}{\Gamma, \xi \vdash \oplus e : n}$$

$\oplus \in \{\texttt{++}, \texttt{--}\}$

$$\frac{\Gamma, \xi \vdash e : n}{\Gamma, \xi \vdash e \oplus : n}$$

$\oplus \in \{\texttt{\$signed}, \texttt{\$unsigned}, \texttt{signed'}, \texttt{unsigned'}\}$

$$\frac{\zeta = \max^{*}(\text{ctx}(e)) \qquad \Gamma, \zeta \vdash e : k \qquad n = k \bowtie \xi}{\Gamma, \xi \vdash \oplus(e) : n}$$

$\oplus \in \{\texttt{===}, \texttt{!==}, \texttt{==?}, \texttt{!=?}, \texttt{==}, \texttt{!=}, \texttt{0>}, \texttt{>=}, \texttt{<}, \texttt{<=}\}$

$$\frac{\zeta = \max^{*}(\text{ctx}(a) \cup \text{ctx}(b)) \qquad \Gamma, \zeta \vdash a : k \qquad \Gamma, \zeta \vdash b : k \qquad n = k \bowtie \xi}{\Gamma, \xi \vdash a \oplus b : n}$$

Equivalence-Based Typing Rules

The statement $\Gamma \mid n \vdash e : \tau, X$ mean:

In the variable context Γ and the size context n, the size of the expression e is the value of the variable τ, with X the size-tagged set of equivalence classes of size variables.

So we have:

- $\Gamma : \mathcal{O} \to \mathbb{N}$ a mapping from operand to their size, with $\mathcal{O}$ the set of operands,
- $n \in \mathbb{N} \cup \{*\}$ a size context (with $*$ for the empty one),
- $e \in \mathcal{E}$, with $\mathcal{E}$ the set of expressions,
- $\tau \in \mathcal{V}$, with $\mathcal{V}$ the set of size variable,
- $X = (S, f)$ with $S \in 2^{\mathcal{V}}$ the set of equivalence classes and $f : S \to \mathbb{N}$ a size valuation for each equivalence class.

We use the following notations:

- $$\bowtie\ : \begin{cases} \mathbb{N} \times (\mathbb{N} \cup \{*\}) & \to & \mathbb{N} \\ (m, *) & \mapsto & m \\ (m, n) & \mapsto & \max(m, n) \end{cases}$$
- $\Delta_X : \mathcal{V} \to \mathbb{N}$ the function that given a size variable x returns its size in the environment $X = (S, f)$. We have $\Delta_X(x) = f\left([x]_S\right)$ with $[x]_S$ the equivalence class of x in S.
- $\{v \coloneqq s\}$: the declaration of a *fresh* size variable v.

$$\{v \coloneqq s\} = (S, f)$$
$$S = \{\{v\}\}$$
$$f = \begin{cases} S & \to & \mathbb{N} \\ \{v\} & \mapsto & s \end{cases}$$

- $X / \alpha \sim \beta$ the operation that combines two classes. For the newly created class, the valuation function gives the maximum of the previous classes:

$$(S, f)/\alpha \sim \beta = (S', f')$$
$$S' = \left\{[\alpha]_S \sqcup [\beta]_S\right\} \sqcup S \setminus \left\{[\alpha]_S, [\beta]_S\right\}$$
$$f' = \begin{cases} S' & \to & \mathbb{N} & \\ c & \mapsto & f(c) & \text{if } c \in S \setminus \left\{[\alpha]_S, [\beta]_S\right\} \\ c & \mapsto & \max\left(f\left([\alpha]_S\right), f\left([\beta]_S\right)\right) & \text{if } c = [\alpha]_S \text{ or } c = [\beta]_S \end{cases}$$

Base case

$$\frac{\Gamma(x) = m}{\Gamma \mid n \vdash x : v, \{v \coloneqq m \bowtie n\}}$$

Operators

$$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{\%}, \texttt{\&}, \texttt{|}, \texttt{\^{}}, \texttt{\^{}\~{}}, \texttt{\~{}\^{}}\} \qquad \frac{\Gamma \mid n \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid n \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \alpha,\ \mathrm{A} \sqcup \mathrm{B} / \alpha \sim \beta}$$

$$\oplus \in \{\texttt{+}, \texttt{-}, \texttt{++}, \texttt{--}, \texttt{\~{}}\} \qquad \frac{\Gamma \mid n \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash \oplus e : \varepsilon,\ \mathrm{E}}$$

$$\oplus \in \{\texttt{++}, \texttt{--}\} \qquad \frac{\Gamma \mid n \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash e \oplus : \varepsilon,\ \mathrm{E}}$$

$$\oplus \in \{\texttt{\$signed}, \texttt{\$unsigned}, \texttt{signed'}, \texttt{unsigned'}\} \qquad \frac{\Gamma \mid * \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash \oplus(e) : \upsilon,\ \{\upsilon \coloneqq \Delta_{\mathrm{E}}(\varepsilon) \bowtie n\}}$$

$$\oplus \in \{\texttt{===}, \texttt{!==}, \texttt{==?}, \texttt{!=?}, \texttt{==}, \texttt{!=}, \texttt{>}, \texttt{>=}, \texttt{<}, \texttt{<=}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash b : \beta,\ \mathrm{B} \quad \mathrm{A} \sqcup \mathrm{B} / \alpha \sim \beta}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$

$$\oplus \in \{\texttt{\&\&}, \texttt{||}, \texttt{->}, \texttt{<->}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$

$$\oplus \in \{\texttt{\&}, \texttt{\~{}\&}, \texttt{|}, \texttt{\~{}|}, \texttt{\^{}}, \texttt{\~{}\^{}}, \texttt{\^{}\~{}}, \texttt{!}\} \qquad \frac{\Gamma \mid * \vdash e : \varepsilon,\ \mathrm{E}}{\Gamma \mid n \vdash \oplus e : \varepsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$

$$\oplus \in \{\texttt{>>}, \texttt{<<}, \texttt{**}, \texttt{>>>}, \texttt{<<<}\} \qquad \frac{\Gamma \mid n \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \alpha,\ \mathrm{A}}$$

$$\oplus \in \{\texttt{=}, \texttt{+=}, \texttt{-=}, \texttt{*=}, \texttt{/=}, \texttt{\%=}, \texttt{\&=}, \texttt{|=}, \texttt{\^{}=}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid \Delta_{\mathrm{A}}(\alpha) \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq \Delta_{\mathrm{A}}(\alpha) \bowtie n\}}$$

$$\oplus \in \{\texttt{<<=}, \texttt{>>=}, \texttt{<<<=}, \texttt{>>>=}\} \qquad \frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid c* \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash a \oplus b : \upsilon,\ \{\upsilon \coloneqq \Delta_{\mathrm{A}}(\alpha) \bowtie n\}}$$

$$\frac{\Gamma \mid * \vdash e : \varepsilon,\ \mathrm{E} \quad \Gamma \mid n \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid n \vdash b : \beta,\ \mathrm{B}}{\Gamma \mid n \vdash e\texttt{?}a\texttt{:}b : \alpha,\ \mathrm{A} \sqcup \mathrm{B} / \alpha \sim \beta}$$

$$\frac{\Gamma \mid * \vdash e_1 : \varepsilon_1,\ \mathrm{E}_1 \quad \dots \quad \Gamma \mid * \vdash e_n : \varepsilon_n,\ \mathrm{E}_n}{\Gamma \mid n \vdash \texttt{\{}e_1, \dots, e_n\texttt{\}} : \upsilon,\ \{\upsilon \coloneqq (\Delta_{\mathrm{E}_1}(\varepsilon_1) + \cdots + \Delta_{\mathrm{E}_n}(\varepsilon_n)) \bowtie n\}}$$

$$\frac{i \in \mathbb{N} \quad \Gamma \mid * \vdash e_1 : \varepsilon_1,\ \mathrm{E}_1 \quad \dots \quad \Gamma \mid * \vdash e_k : \varepsilon_k,\ \mathrm{E}_k}{\Gamma \mid n \vdash \texttt{\{}i\texttt{\{}e_1, \dots, e_k\texttt{\}\}} : \upsilon,\ \{\upsilon \coloneqq i \times (\Delta_{\mathrm{E}_1}(\varepsilon_1) + \cdots + \Delta_{\mathrm{E}_k}(\varepsilon_k)) \bowtie n\}}$$

$$\frac{\Gamma \mid * \vdash a : \alpha,\ \mathrm{A} \quad \Gamma \mid * \vdash e_1 : \varepsilon_1,\ \mathrm{E}_1 \quad \dots \quad \Gamma \mid * \vdash e_k : \varepsilon_k,\ \mathrm{E}_k \quad \mathrm{A} \sqcup \mathrm{E}_1 \sqcup \cdots \sqcup \mathrm{E}_k / \alpha \sim \varepsilon_1, \dots, \alpha \sim \varepsilon_k}{\Gamma \mid n \vdash a\ \texttt{inside}\ \texttt{\{}e_1, \dots, e_k\texttt{\}} : \upsilon,\ \{\upsilon \coloneqq 1 \bowtie n\}}$$