Reliability Theory

Reliability is an old concept and a new discipline. For ages things and people have been called reliable if they had lived upto certain expectations and unreliable otherwise. A reliable person would never (or hardly ever) fail to deliver what he had promised.

The types of expectation to judge reliability have all been related to the performance of some function. The reliability of a device has been considered high if it had repeatedly performed its function with success and low if it had tended to fail in repeated trails. Reliability theory deals with the general methods and procedures to be followed during the process of planning, preparation, acceptance, etc. of manufactured items so as to ensure maximum effectiveness in their usage.

It develops general methods of evaluation of the quality of a system from the known qualities of components used in the system. The theory introduces quantitative indices to the quality of the product.

The history reliability engineering goes back to World War II. Of late, the subject has assumed greater importance with recent advances in the field of electronics, nuclear engineering, computers, aeronautic engineering, etc.

There is a common misunderstanding that quality and reliability are not different, in fact quality and reliability do not imply the same thing. Quality control no doubt contributes significantly to the improvement of the reliability of a product. Quality control is a management function it aims at preventing the manufacture of defectives by exercising control over various factors affecting the manufacturing process. The traditional concept of quality is not time dependent. A product is accepted if it meets certain specifications otherwise it is rejected.

While reliability is usually concerned with failures in time domain.

Definition:

The reliability of a system is the probability that it performs its intended function adequately for a specified time interval under the given operating conditions. Reliability of a system is also defined as the probability of its failure free operation during the period it is intended to be in use.

The failure of a system can be visualized as an event associated with the deviation in the operating characteristics of the system from its permissible limits.

In the above definition, we note four important factors or elements which contribute to reliability of the system.

1. Reliability of a device is expressed in terms of probability.
2. Device is required to perform adequately.
3. Duration of its performance is to be specified.
4. Operating or environmental conditions are to be prescribed.

The basic quantity of interest is the time to failure of the system. It is the time that has elapsed from the start of the operation of the system till its failure for the first time. This failure time is a random variable and is governed by a probability distribution.

Let $T(0 \le T < \infty)$ be the time to failure of a system which started operating at some time origin. Let f(t) be the pdf of T i.e.

$f(t)dt = P\{t \le T \le t + dt\}$

= Probability that failure occurs in the time interval of length dt.

Let F(t) be the cdf of T

$F(t) = P(T \le t) = \text{Probability that failure takes place at time } \le t$

i.e. F(t) denotes the probability that the system which started at T=0 will fail before t.

Let R(t)=P{System operates without failure till time t}

R(t) = P{T>t}

= 1-F(t)

= 1-$\int_0^t f(x)dx$

R(t) = $\int_t^\infty f(x)dx$

From the properties of the pdf we have, R(0)=1 and $R(\infty) = 0$

Also, we have, R(t)=1-F(t) from which we get f(t) = $\frac{-d}{dt}[R(t)]$

Failure rate:

This is also called hazard rate, instantaneous failure rate or age specific failure rate. The failure rate h(t) may be defined in terms of reliability R(t) and time to failure f(t) as follows:

Let h(t)dt be the conditional probability of time to failure of a system in the interval [t, t+dt] given that the system has not failed at time T=t.

$h(t)dt = P\{(t \le T \le t + dt)/(T > t)\}$

$= \frac{P\{(t \le T \le t+dt) \cap (T>t)\}}{P(T>t)}$

$= \frac{P(t \le T \le t+dt)}{R(t)}$

$= \frac{f(t)dt}{R(t)}$

$\therefore$ h(t) = $\frac{f(t)}{R(t)}$

This quantity h(t), the failure rate is also referred to as the hazard function or instantaneous hazard rate.

One can establish following relationship between h(t), f(t) and R(t)

$$h(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{1-F(t)}$$

$$= \frac{d[-\log(1-F(t))]}{dt}$$

$$\frac{d[\log(1-F(t)]}{dt} = -h(t)$$

$$\log(1 - F(t)) = \int_0^t -h(x)dx$$

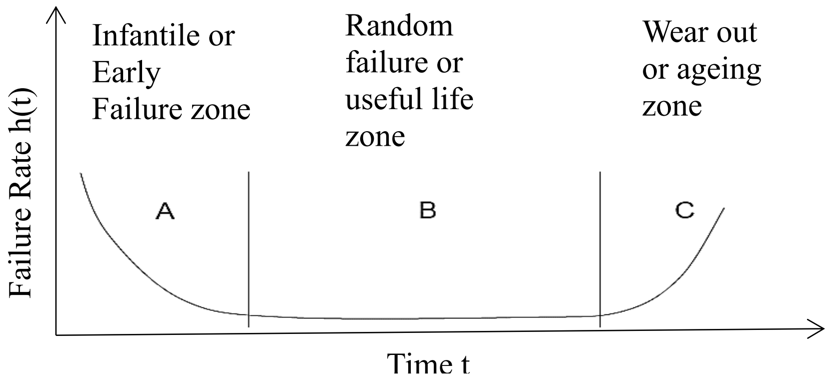
$$1 - F(t) = e^{-\int_0^t h(x)dx}$$

$$R(t) = e^{-\int_0^t h(x)dx}$$

$$f(t) = h(t)e^{-\int_0^t h(x)dx}$$

Note: When $h(t) = \lambda$, a constant, then $f(t) = \lambda e^{-\int_0^t \lambda\, dx} = \lambda e^{-\lambda t}$ i.e. a constant failure rate leads to an exponential distribution.

Failure Pattern:



The behavior of h(t), the failure rate, is quite revealing with respect to the causes of failure. Unless a system has redundant components, it will invariably have the general characteristic of a “Bath tub curve” as shown.

At the very beginning the possibility of defective design or manufacturing or assembly or poor quality of component parts etc. produce a very high hazard on the system and is the significant cause of failures. Thus h(t) has a very high value at the beginning. This region is referred to as infantile or early failure zone. h(t) decreases rapidly as the items with such above mentioned defects are eliminated. Those which survive the infant mortality attain a constant failure rate, where failures due to chance is predominant. This persists for some time. This region is referred to as useful life zone and failures during this period of time are random failures. Possible causes may be due to external loading of the system or its components. In the final stage h(t) starts rising as items fail due to aging and wear and tear. This region is referred as ageing zone.

It should be noted that no single failure distribution has a failure rate which satisfies the bathtub model but one of the three phases may be predominant for a particular class of system.