Continuous Probability Distributions

Uniform Distribution:

A crv X taking all possible values in the interval [a,b]; ($a<\infty$, $b<\infty$) is said to follow the Uniform distribution over [a,b], if its pdf is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a}; & a \le x \le b \\ 0\ ; & \text{otherwise} \end{cases}$$

Now, to obtain mean and variance of Uniform distribution

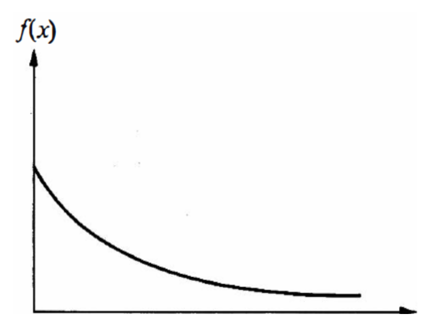
$E(X)= \int_a^b xf(x)dx = (b+a)/2$ and

$V(X) = E(X^2)-(E(X))^2=(b-a)^2/12$.

Exponential Distribution:

A continuous rv X taking all possible non-negative values is said to follow the Exponential distribution with parameter λ, if its pdf is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x}\ ; & \lambda > 0, x > 0 \\ 0 & ; \text{elsewhere} \end{cases}$$



Note: $f(x) \ge 0\ \forall x$ and $\int_0^\infty f(x)dx = \int_0^\infty \lambda e^{-\lambda x}\,dx = 1$

Hence f(x) is a pdf

Now, to obtain mean and variance of exponential distribution

$E(X) = \int_0^\infty x\lambda e^{-\lambda x}\,dx = 1/\lambda$

and $V(X) = E(X^2)-(E(X))^2 = 1/\lambda^2$

Normal Distribution:

A crv X taking all possible values in $(-\infty, \infty)$ is said to follow the Normal distribution with parameters μ and σ^2 (i.e. $X\sim N(\mu, \sigma^2)$), if its pdf is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{-(1/2)[(x-\mu)/\sigma)^2]};\ -\infty < x < \infty, -\infty < \mu < \infty \text{ and } \sigma > 0$$

Properties of Normal distribution

1) $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\ e^{-(1/2)[(x-\mu)/\sigma)^2]}$ is a pdf

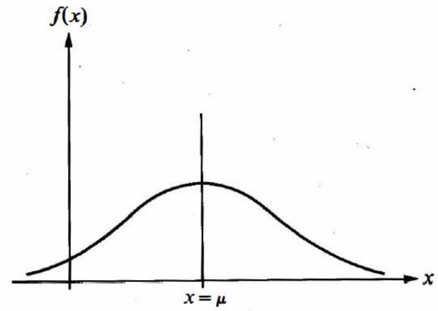
Prove it!

2) The mean and variance of normal distribution are given by

$E(X) = \int_{-\infty}^{\infty} xf(x)dx = \mu$ and $V(X)=E(X^2)-(E(X))^2=\sigma^2$

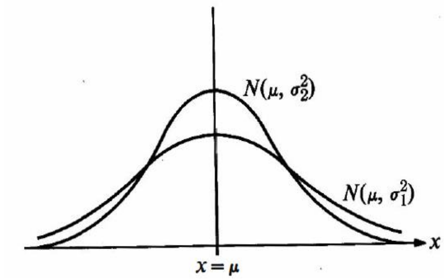
Prove it!

3) f(x) the pdf has the well-known bell shape.



4) Since f depends on x only through $(x-\mu)^2$, f(x) is symmetric about μ.

5) If the variance is large the spread will be more while if the variance is small the spread will be less, and the different values assumed by the r.v fall close to the mean.



6) $P(\mu-3\sigma \le X \le \mu+3\sigma) = 0.9973$.

Standard Normal Distribution:

If $X \sim N(\mu, \sigma^2)$, then the rv $Z = \frac{X-\mu}{\sigma}$ has a Standardized Normal Distribution (~N(0,1)) with mean=0 an variance=1 and Z is called Standardized Normal Variable (SNV). The pdf of Z is given by,

$\varphi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}; -\infty < z < \infty$

The cdf of the SNV Z denoted by $\phi(z)$, is defined as

$\phi(z) = P(Z \le z) = \int_{-\infty}^{z} \varphi(t)dt$

Now suppose $X \sim N(\mu, \sigma^2)$ then what is $P(a \le X \le b)$? We have

$P(a \le X \le b) = \int_a^b f(x)dx$

where f(x) is the normal pdf. This cannot be integrated by ordinary methods.

Thus, we consider $P(a \le X \le b) = P\left(\frac{a-\mu}{\sigma} \le \frac{x-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right)$

$= P(c_1 \le Z \le c_2)$

$= \phi(c_2) - \phi(c_1)$ $\quad (\because P(a \le X \le b) = F(b)-F(a))$

Note 1: For different values of $c_1, c_2, \ldots$, the values of $\phi(z)$ are tabulated in the standard normal tables.

Note 2: Also we have, $\phi(-z) = 1 - \phi(z)$.