INTRODUCTION TO PROBABILITY

Whenever we use mathematics in order to study some observational phenomena we build the mathematical model for the phenomena. There are two types of mathematical models:

1) Deterministic models
2) Non-deterministic or Probabilistic or Stochastic or Random models.

Deterministic Models:

A model which stipulates that the condition under which an experiment is performed, determine the outcome of the experiment. It is an experiment in which the outcomes can be predicted with certainty.

Ex: 1. If a particle is released with initial velocity u and acceleration a, several times, then in each release the distance traveled by the particle in time t is $ut+(1/2)at^2$

2. If we insert a battery into a simple circuit the mathematical model which gives the flow of current is $I=\frac{V}{R}$, Ohms Law. If the experiment is repeated a number of times using the same circuit i.e. keeping V & R fixed, the value of I would remain the same.

Non-deterministic Models:

Suppose we have an experiment such that the collection of all possible outcomes of the experiment is the same whenever it is conducted under identical condition, but its outcome cannot be predicted with certainty in any treat of the experiment is the experiment require a different mathematical model for their investigation.

Ex: 1) Tossing a coin
2) Drawing a card at random from a pack of cards.

Random or Stochastic Experiment:

The real-world phenomena which involve randomness and for which the non-deterministic models are appropriate is referred to as a *Random or Stochastic Experiment*. The following are the features of a random experiment.

1) Each experiment is capable of being repeated indefinitely under essentially the same condition.
2) Though it will not be possible to predict a particular outcome, it is possible to describe the 'set of all possible outcomes' of the experiment.
3) As the experiment is performed repeatedly the individual outcomes seem to occur in a haphazard manner. However, as the experiment is repeated a large number of times, a definite pattern or regularity appears.

Note: 1) Each performance in a random experiment is called a **Trial.**
2) All the trails are conducted under the same set of conditions in a random experiment.
3) The result of a trial in a random experiment is called an **Outcome.**

Ex: 1) Toss a die and observe the number that shows on top.
2) Toss a coin 4 times and observe the total number of heads obtained.
3) A tube light is tested for its life length by recording the time elapsed until it burns out.
4) From an urn containing only black balls, a ball is chosen and its colour is noted.

Sample Space:

The totality of all possible outcomes of a random experiment E is called a **sample space** S. A sample space S is said to be finite, if the no. of elements in S is finite.

Ex: 1) S={1,2,3,4,5,6}
2) S={0,1,2,3,4}
3) S={t: t>=0}
4) S={black balls}

Events:

Every subset A of S which is a disjoint union of a single element subsets of the sample space S of a random experiment E is called an **event** i.e. an event A is simply a set of possible outcomes.

Ex: 1) A={2,4,6} is an even no. occurs.
2) A={2} is 2 heads occur.
3) A={t/t<3} is tube glows less than 3 hrs.

Mutually Exclusive Events:

Two events A & B are said to be mutually exclusive if they cannot occur together i.e. $A \cap B = \phi$.

Ex: An electronic device is tested and its total time of service, t, is recorded. Let S={t/t>=0}. Consider the events A={t/t<100} B={t/50<=t<=200} C={t/t>150}. Then

A∪B={t/t ≤ 200}

A∩B={t/50≤t≤100}

B∪C={t/t≥50}

B∩C={t/150≤t≤200}

A∩C=ϕ A∪C={t/t<100 or t>150}

$\overline{A}$ ={t/t≥100}

$\overline{C}$ ={t/t≤150}

Equally Likely Events:

Two or more events are said to be equally likely or equiprobable if they have equal chance of occurrence i.e. there is no reason to expect one in preference to the other.

Independent Events:

Two events are said to be independent if the occurrence of one event does not effect the occurrence or the non-occurrence of the other.

Classical Definition of Probability: (Due to **Laplace**)

If a trial results in n mutually exclusive and equally likely events or outcomes and m of them are favorable to the happening of an event A, then, the probability of happening of A is

$$P(A) = \frac{\text{Favourable no.of cases}}{\text{Total no.of cases}} = \frac{m}{n}$$

Ex: 1) Probability of head appearing when a coin is tossed is P(A)=1/2.

Note: 1) If m=n, then P(A)=1 i.e the event A is a certain event.
2) If m=0, then P(A)=0 i.e. A is an impossible event.

Limitations: It fails when the various outcomes of the trail are not equally likely or equally probable.

Relative Frequency: (Due to **Richard Von Mises)**

Let an experiment E be repeated n times essentially under the same conditions and let A be the event associated with E. Let m be the No. of times the event A occurs. Then f=m/n is called the relative frequency of the event A in the n repetitions of E.

It can be shown that, $f \to P(A)$ as $n \to \infty$. However, this definition has its own limitations

Geometrical Definition

The probability of an event is the status of the area favorable for an event of the total area of events. (It is the generalization of the classical definition). This definition also has its own limitations.

Axiomatic Definition of Probability: (Due to **A. N Kolmogorov, 1933**).

Let E be an experiment. Given a sample space S associated with E, **Probability** is a function which assigns a non-negative real number to every event A denoted by P(A), and is called the probability of the event A, satisfying the following axioms:

i) $0 \leq P(A) \leq 1$

ii) $P(S)=1$

iii) If A & B are mutually exclusive events, then $P(A \cup B)=P(A)+P(B)$

Note: If $A_1, A_2, \ldots A_n \ldots$ are pairwise mutually exclusive events then

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$