Linear Programming Problem:

Mathematical Formulation of a LPP:

The mathematical formulation of a lpp consists of the following steps:

(i)Identify the objective of the problem
(ii) Write down the decision variables of the problem
(iii)Formulate the objective function which is to be optimized
(iv)Formulate the constraints of the problem
(v)Add the non-negative constraints

The objective function, the set of constraints and the non-negative constraints together form a linear programming problem.

Example 1: A manufacturer makes two types of furniture, say, chairs and tables. The processing of these products is done on two machines A and B. One chair requires 2 hours work on machine A and 6 hours work on machine B. One table requires 5 hours work on machine A and 3 hours work on machine B. There are 16 hours of time available on machine A and 20 hours work available on machine B. Profit gained by the manufacturer from a chair and a table are 5 dollars and 8 dollars respectively. Formulate this as a lpp in order to maximize the profit for the manufacturer.

Solution:

Step (i): Identifying the objective – Maximizing the profit coming out of selling Chairs and Tables.

Step (ii): The decision variables of the problem – The number of chairs x_1 , The number of tables x_2.

Step (iii): Objective function – Maximize $z = 5x_1 + 8x_2$ -------- (1)

Step (iv): Constraints (limitations) of the problem

$2 x_1 + 5 x_2 \leq 16$ (Machine A availability) -------- (2)

$6 x_1 + 3 x_2 \leq 20$ (Machine B availability) -------- (3)

Step (v): Non-negative constraints

$x_1 , x_2 \geq 0$ --------- (4)

Equations (1) – (4) represent the lp formulation of the given problem.

Example 2: S Bank has to formulate the loan policy for the financial year 2021 – 22. the total loan amount available is 100bn rupees. The bank grants loan to different type of clientele to maximize the return. The following table provides the interest rate charged by the bank for various types of loans and the probability of bad debts as estimated from the past records. Bad debts are assumed to be unrecoverable and also do not produce any revenue.

Type of loan	Interest rate	Likely proportion of bad debts
Car loan	8%	0.1
Housing loan	7%	0.04
Industry loan	6%	0.09
Agricultural loan	4%	0.4

As per the Bank guidelines, at least 15% of funds must be loaned to agricultural sector. Industry and agricultural sector loan should be more than 50%. Housing loan should not exceed 30% of the available amount. The overall bad debt on all loans should not exceed 0.05% of the total fund available. Formulate the above bank loan policy into a lpp to maximize the total interest earned.

Solution:

(i)Objective of the problem - To maximize the interest earned from various types of loans

(ii)Decision Variables - let x_1 be the amount allotted for car loan,
let x_2 be the amount allotted for housing loan,
let x_3 be the amount allotted for industry loan,
let x_4 be the amount allotted for agriculture loan.

(iii)objective function – Maximize $z = 0.08x_1 + 0.07x_2 = 0.06x_3 + 0.04x_4$ ---------- (1)

(iv) subject to the constraints: $x_4 \geq 15$ bn ---------- (2)

$x_3 + x_4 \geq 50$ bn ---------- (3)

$x_2 \leq 30$ bn ---------- (4)

$0.1x_1 + 0.04x_2 + 0{,}09x_3 + 0.4x_4 \leq 5$ bn ---------- (5)

(v) non-negative constraints: $x_1 , x_2 , x_3 , x_4 \geq 0$ ---------- (6)

Equations (1) – (6) represent the lpp of the given problem.

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Assignment 1: Create any simple problem and model it as a lpp (avoid copying from the book).