Problems:

1) Urn 1 contains x white balls and y red balls. Urn 2 contains z white balls and v red balls. A ball is chosen at random from Urn1 and put into Urn2. Then a ball is chosen from Urn2 at random. What is the probability that the ball is white?

Solution: Define A= {First selection is white} => $\bar{A}$ = {First selection is red}
B= {Second selection is white}
To find P(B)=?
By total probability theorem, we have $P(B)= P(B/A).P(A)+ P(B/\bar{A}).P(\bar{A})$
where $P(A)=x/(x+y)$, $P(\bar{A})=y/(x+y)$,
$P(B/A)=(z+1)/(z+v+1)$,
$P(B/ \bar{A})= z/(z+v+1)$.
$\therefore P(B)= [(z+1)/(z+v+1)]. [x/(x+y)] + [z/(z+v+1)]. [y/(x+y)]$

2) 2 two defective tubes get mixed up with 2 good ones. The tubes are tested one by one until both the defectives are found. What is the probability that the last defective tube is obtained **a)** on the 2nd test **b)** on the 3rd test **c)** on the 4th test

Solution:
a) Define the events A_1:{The 1st tube tested is defective}
A_2:{The 2nd tube tested is defective}
D :{ the last defective tube is obtained on the 2nd test}
$P(D)= P(A_1 \cap A_2)= P(A_1)P(A_2/A_1)= (2/4) . (1/3) = 1/6$
b) Define A_3:{3rd tube tested is defective}
D :{last defective tube is obtained in the 3rd test}
$P(D) =P(A_1 \cap \bar{A}_2 \cap A_3)+P(\bar{A}_1 \cap A_2 \cap A_3)$
$= P(A_1)P(\bar{A}_2/A_1). P(A_3/\bar{A}_2,A_1)+ P(\bar{A}_1)P(A_2/\bar{A}_1). P(A_3/\bar{A}_1,A_2)$
$=(1/2)(2/3)(1/2)+(1/2)(2/3)(1/2)=1/6+1/6=1/3.$
c) P{the last defective tube is obtained on the 4th test}
=1-{P(getting the last defective either in the 2nd test or the 3rd test)}
=1-(1/6+1/3)
=1/2.

3) A box contains 4 bad and 6 good tubes. Two are drawn out together. One of them is tested and found to be good. What is the probability that other one is also good?

Solve it!

4) Suppose that A and B are independent events associated with an experiment E. If the probability that A or B occurs is 0.6. If the probability that A occurs is 0.4, determine the probability that B occurs.

Solution: Given : P(AUB)=0.6, P(A)=0.4
$P(AUB)=P(A)+P(B)-P(A \cap B)$
$=P(A)+P(B)-P(A).P(B)$
$0.6= 0.4+P(B)(1-0.4)= 0.4+0.6P(B)$
$P(B)=0.2/0.6=1/3$

5) Let A and B be 2 events associated with an experiment. Suppose that $P(A)=0.4$ while $P(AUB)$ is 0.7. Let $P(B)$ be p. For what choice of p are the events A and B
i) mutually exclusive? ii) independent?

Solution: $P(A)=0.4$ $P(AUB)=0.7$ $P(B)=p$.

i) $P(AUB)=P(A)+P(B)$ (here $P(A\cap B)=0$)

$0.7 = 0.4+P(B)$

$\therefore p =P(B)=0.3$

ii) $P(AUB)=P(A)+P(B)-P(A\cap B)$

$= P(A)+P(B)-P(A).P(B)$

$0.7= 0.4+ P(B)0.6$

$\therefore p =P(B)=0.3/0.6=0.5$

6) An electrical assembly has 2 subsystems A and B. Given P(A fails)=0.2, P(B fails alone)=0.15, P(both A and B fail)=0.15. Evaluate i)P(A fails given B has failed) ii)P(A fails alone)

Solve it!

7) A vacuum tube may come from any of the 3 manufacturers with probabilities $p_1=0.25$, $p_2=0.25$, $p_3=0.5$. The probabilities that the tube will function properly during a specified period of time equal 0.1, 0.2 and 0.4 respectively for the three manufacturers. Compute the probability that a randomly chosen tube will function for the specified period of time.

Solution: Let A:{tube comes from 1st manufacturer}

B:{ tube comes from 2nd manufacturer}

C:{tube comes from 3rd manufacturer}

Let X={Tube will function for the specified period of time}

$P(X)=?$

$P(X)= P(X\cap A)+ P(X\cap B)+ P(X\cap C)$ {A,B,C are disjoint}

$= P(X/A)P(A)+ P(X/B)P(B)+ P(X/C)P(C)$

$= (0.1).(0.25)+ (0.2).(0.25)+ (0.4).(0.5)= 0.275$

8) Three newspapers A,B,C are published in a city and a recent survey of readers indicates the following :20%read A , 16%read B, 14% read C, 8% read A and B, 5% read A and C, 4% read B and C and 2% read A,B & C. For One adult chosen at random, compute the probability that **a)** he reads none of the papers **b)** he reads exactly one of the papers **c)** he reads at least one of A and B, if it is known that he reads least one of the papers published.

Solve it!

9) Given a) $P(\overline{A})=0.4$ $P(B/A)=0.5$ $P(AUB)=0.95$ find $P(B)$?

b) $P(AUB)=0.7$ $P(\overline{B}/\overline{A})=0.5$ find $P(A)$?

Solve it!

10) A,B,C,D,E are mutually independent events with P(A)=1/2, P(B)=3/4, P(C)=5/6, P(D)=1/8 and P(E)=2/3. Find P(AUBUCUDUE)?

Solution: $P(A\cup B\cup C\cup D\cup E)=1-P(\overline{A\cup B\cup C\cup D\cup E}) =1-[P(\bar{A}\cap\bar{B}\cap\bar{C}\cap\bar{D}\cap\bar{E})]$

$=1-[P(\bar{A})P(\bar{B})P(\bar{C})P(\bar{D})P(\bar{E})]$

$=1-[(1/2)(1/4)(1/6)(7/8)(1/3)]$

$=1145/1152.$

11) It is found in manufacturing certain articles defects of Type 1 occur with probability 0.1 and defects of Type 2 occur with probability 0.05. (Assume independence with type of defects). What is the probability that :

i) an article does not have both types of defects?
ii) an article is defective?
iii) an article has one type of defect, given that it is defective?

Solve it!

12) What is the probability that in a group of n people at least 2 of them have same birthday (the same day, month, but other year) [classical birthday problem]?

Solve it!

13) Three dice are rolled independently. Let A:{sum of the digits shown is 6}, and B:{all three digits are different}. Are A and B independent?

Solve it!

14) In a bolt factory, machines A, B, and C manufacture 25%, 35%, and 40% of the total output, respectively. Of their outputs, 5%, 4%, and 2% respectively, are defective bolts. A bolt is chosen at random and found to be defective. What is the probability that the bolt came from machine A? B? C?

Solution:

Let E:{The bolt is defective}

A:{The bolt selected at random is manufactured by M/c A}

B:{The bolt selected at random is manufactured by M/c B}

C:{The bolt selected at random is manufactured by M/c C}

Given, P(A)=0.25, P(B)=0.35, P(C)=0.4

P(selected bolt is defective given it is manufactured by M/c A)=P(E/A)=0.05

Similarly, P(E/B)= 0.04, P(E/C)=0.02

Now the required probability is,

P(selected bolt is manufactured by M/c A given that bolt is defective)

=P(A/E)

=P(E/A).P(A)/ **(** P(E/A).P(A)+ P(E/B).P(B)+ P(E/C).P(C) **)**

=0.05(0.25)/**(**0.05(0.25)+0.04(0.35)+0.02(0.4)**)**

=0.362

Similarly we can obtain P(B/E)=0.406 and P(C/E)=0.232.