1) Consider the tpm P = $\begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix}$ Obtain the steady state probabilities.

Solution:

We have $P^2 = \begin{bmatrix} 0.52 & 0.48 \\ 0.36 & 0.64 \end{bmatrix}$; $P^4 = \begin{bmatrix} 0.443 & 0.557 \\ 0.417 & 0.583 \end{bmatrix}$; $P^8 = \begin{bmatrix} 0.4281 & 0.5719 \\ 0.4274 & 0.5726 \end{bmatrix}$; P^{16} =…….; P^{32}=…….;

We note that it reaches equilibrium condition. Now consider the steady state equations with i = 0 and j = 1, we have

$\pi_0 = \pi_0 p_{00} + \pi_1 p_{10}$
$\pi_1 = \pi_0 p_{01} + \pi_1 p_{11}$
$\pi_0 + \pi_1 = 1$
Solving we get $\pi_0 = 0.4286$ and $\pi_1 = 0.5714$

Also, the mean recurrence times are obtained as $\mu_{00} = 2.333$ steps and $\mu_{11} = 1.75$ steps

2) Obtain the steady state probabilities given the following tpms.

(a). $P = \begin{bmatrix} 0.080 & 0.184 & 0.368 & 0.368 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.080 & 0.184 & 0.368 & 0.368 \end{bmatrix}$

Solve it!

(b). $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 3/8 & 1/8 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

Solve it!

(c). $P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$

Solve it!

(d). $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & 0 & 1-a \end{bmatrix}$; $0 < a < 1$

Solve it!

3) Is the Markov Chain with tpm $P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ ergodic? If so find its limiting distribution.

Solve it!

4) A Markov Chain has a tpm $P = \begin{bmatrix} 1/3 & 0 & 2/3 & 0 \\ 1/4 & 0 & 3/4 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 1/2 & 0 & 1/2 \end{bmatrix}$

In which state the chain is most likely to be found in the long run?

Solve it!

5) Consider $P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$ Obtain the steady state probabilities.

Solution:

The above matrix is a doubly stochastic matrix. In such a case, the steady state probabilities are given by, $\pi_j = \frac{1}{s}$ $\forall$ j=1,2,3,…..,s. where s is the number of states.

Thus, we have, $\pi_0 = \frac{1}{3}$, $\pi_1 = \frac{1}{3}$ and $\pi_2 = \frac{1}{3}$

6) Given $P = \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix}$ Obtain the steady state probabilities.

Solve it!

Continuous Parameter Markov Process

Quite often we come across situations where a continuous parameter is required.

Let { X(t): $t \geq 0$ } be a Markov Process with discrete states 0,1,2……..m (i.e. m+1 states) and stationary transition probability function

p_{ij} = P{X(t+s)=j/X(s)=i} $\forall$ i, j=0,1,2…….

This function is assumed to be continuous at t=0 with

$$\lim_{t \to 0} p_{ij}(t) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad \forall \quad i, j = 0,1, 2\ldots.$$

This P_{ij} again satisfies the C-K equations i.e. for any state i & j and positive numbers

t & s ($0 \leq s \leq t$), we have $p_{ij} = \sum_{k=o}^{m} p_{ik}(s) p_{kj}(t-s)$

Also, classifications of states are made as earlier. i.e. state i communicates with state j if $\exists$ t_1 & t_2 $\ni$ $p_{ij}(t_1)>0$ & $p_{ji}(t_2)>0$. All states that communicate are said to form a class. If all states in a chain form a single class (irreducible chain) then $p_{ij}(t)>0$ $\forall$ t>0 & $\forall$ i & j. Further, $\lim_{t \to 0} p_{ij}(t)=\pi_j$ always exist and independent of the initial state i $\forall$ i=0, 1, 2…..m and these π_j' s satisfy $\pi_j = \sum \pi_j p_{ij}(t)$ $\forall$ j=0, 1, 2…….m and $t \geq 0$.