A queuing system is completely specified by the following **6 main characteristics**:

1. Input or Arrival time distribution / **a**

2. Output or service time distribution / **b**

3. No. of service channels / Service facility /**c**

4. Service discipline /**d**

5. Capacity of Queuing System (Queue + in Service) /**e**

6. Calling source /**f**

a/**b**/**c** are due to ⟶ Kendal (1953)

d/**e**/**f** are due to ⟶ A M Lee (1966)

The standard notations used are:

M : Poisson (Markovian) arrival or departure distribution, equivalently ,

Inter arrival or service time follow exponential distribution.

D : Constant or deterministic inter arrival or service time.

$\boldsymbol{E_k}$: Erlangian or Gamma distribution of inter arrival or service time with parameter k

$$\left(T \sim f(t) = \frac{(k\mu)^k}{(k-1)!} e^{-k\mu t} t^{k-1} , \ t \geq 0 \ , \ k \ \& \ \mu \ \text{are the parameters}\right)$$

G : General distribution of departures (service time)

G I : General independent distribution of arrivals.

Ex : M | M | 1 : GD | N |∞

Poisson arrival | Departure | Servers: Service discipline | queue capacity | calling source

Thus we have

M | M | 1 : FIFO | ∞ | ∞ , M | M | S : FIFO | N | N , M | M | 2 : GD | ∞ | N ,

M | G | S : PRI | ∞ | ∞ , etc..

We use the following **terminologies and notations**:

State of the system = No. of customers in the Queueing system.

Queue length = No. of customers waiting for service.

= (State of the system) – (No. of customers being serviced)

$N(t)$ = No. of customers in the system at time t.

$p_n(t)$ = P (there are `n' customers in the system at time t)

= P (N(t) =n) = Transient state probability ; assuming N(0) = 0

$W(t)$ = Waiting time of a customer in the system at time t.

λ = Arrival rate (No. of customers arriving per unit time)

μ = Service rate (No. of customers served per unit time)

s = No. of servers.

$\rho = \frac{\lambda}{s\mu}$ = **Utilization factor** for the service facility or the **traffic intensity**.

= Expected fraction of time the individual server in busy.

In general, if the behavior of the system depends on time then such a system is said to be in a TRANSIENT STATE. This usually occurs at the early stage of operation of the system where its behavior will depend on the initial conditions. However, after sufficient time has elapsed, the behavior of the system becomes independent of time. The system is then said to be in a STEADY STATE OR EQUILIBRIUM STATE.

Due to the complexity involved in the analysis of transient state behavior and that our interest lies in obtaining an expression for the steady state probabilities, we consider only the steady state analysis. Under the steady state conditions, we use the following notations:

π_n : P (there are n customers in the system)

L_s : Expected No. of customers in the system

L_q : Expected queue length (excludes customers being served)

W_s : Expected waiting time of a customer in the system (includes service time)

W_q : Expected waiting time of a customer in the Queue. (excludes service time)

Relationship between L_s, L_q, W_s and W_q (Little's formula) :

For most of the Queuing systems the following relationships hold good

(due to John D.C. Little)

(i) $L_s = \lambda W_s$

(ii) $L_q = \lambda W_q$

(iii) $W_s = W_q + \frac{1}{\mu}$ (where $\frac{1}{\mu}$ = mean service time)

$$\lambda W_s = \lambda W_q + \frac{\lambda}{\mu}$$

$$L_s = L_q + \rho$$

Thus, if one of the four quantities in known, then the remaining can be found.

It may be easier to calculate L_s, where $L_s = \sum_{n=0}^{\infty} n \, \pi_n$

Or $L_q = \sum_{n=s}^{\infty} (n-s) \, \pi_n$