State occupation probabilities

The unconditional probabilities with which a Markov Chain occupies its various states are called state occupation probabilities. Let S= {1,2,3,….,m}. Then we define, $p_j^{(n)}$ = P[X_n = j] (i.e absolute probabilities at time 'n') and $p^{(n)}$ = $\{p_1^{(n)}, p_2^{(n)}, \dots \dots, p_m^{(n)}\}$ is the absolute probability vector at time n. In particular $p^{(0)}$ is called the initial probability vector or initial probability distribution.

Result: It can be shown that $p^{(n)} = p^{(0)}. P^{(n)}$ or ≡ $p_j^{(n)} = \sum_{i=1}^{m} p_i^{(0)} p_{ij}^{(n)}$

Problems

1) Let $$P = \begin{array}{c c} & \begin{array}{ccc} j=1 & j=2 & j=3 \end{array} \\ \begin{array}{c} i=1 \\ i=2 \\ i=3 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1/4 & 1/2 & 1/4 \\ 5/8 & 1/8 & 1/4 \end{bmatrix} \end{array}$$ be the tpm of a Markov Chain $\{X_n\}$

Find: (i) p_{13} (ii) P[X_1=2/ X_0=2] (iii) $p_{12}^{(3)}$

Solution:

(i) p_{13} = 0

(ii) P[X_1=2/ X_0=2] = p_{22} = ½

(iii) $p_{12}^{(3)} \Longrightarrow$ It is the $(1,2)^{th}$ element of $P^{(3)}$ and we have $P^{(3)} = P^3$. Thus obtain P^3 as $P^3 = P^2.P$ and obtain $p_{12}^{(3)}$ = 17/32

2) A particle performs a random walk with absorbing barriers at 0 & 4. Whenever it is at any position 'r' (0 < r < 4) it moves to r+1 with probability 'p' or to r-1 with probability 'q', so that p+q =1. As soon as it reaches 0 or 4 it remains there itself. Let X_n be the position of the particle after 'n' moves. Write the tpm of {Xn}.

Solve it!

3) Let M be a Markov Chain with tpm P = $\begin{bmatrix} 1/4 & 3/4 \\ 0 & 1 \end{bmatrix}$ and $p^{(0)}$ = {1/3,2/3}, where $p^{(0)}$ is the initial probability distribution. Find (1) $P^{(2)}$ and (2) $p^{(2)}$

Solution:

(1) $P^{(2)} = P^2 = P.P = \begin{bmatrix} 1/16 & 15/16 \\ 0 & 1 \end{bmatrix}$

(2) $p^{(2)} = \{p_1^{(2)}, p_2^{(2)}\}$

Where $p_1^{(2)} = p_1^{(0)} p_{11}^{(2)} + p_2^{(0)} p_{21}^{(2)}$ = (1/3)(1/16) +(2/3)(0) = (1/48)

$p_2^{(2)} = p_1^{(0)} p_{12}^{(2)} + p_2^{(0)} p_{22}^{(2)}$ = (1/3)(15/16) +(2/3)(1) = (47/48)

∴ $p^{(2)} = \{p_1^{(2)}, p_2^{(2)}\}$ = {1/48, 47/48}

4) Consider a set of coin tossing experiments where the outcomes of the n^{th} trial are denoted by 1 for a Head and 0 for a tail. Let X_n be the rv denoting the outcome of the n^{th} trial and $S_n = X_1 + X_2 + X_3 + \ldots\ldots + X_n$ be the partial sum. The possible values of S_n are 0,1,2,3,…..,n. (i.e the states of S_n). S_n is a Markov Chain find its tpm.

Solve it!

5) Suppose that it has rained for the past two days, then it will rain tomorrow with a probability 0.7. If it rained today but not yesterday then it will rain tomorrow with a probability 0.5. If it rained yesterday but not today then it will rain tomorrow with a probability 0.4. If it has not rained for the past two days then it will rain tomorrow with a probability 0.2.
   (i). Write the tpm for the weather conditions.
   (ii). Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday?
   (iii). Suppose it did not rain yesterday, nor the day before yesterday, what is the probability that it will rain tomorrow?

Solve it!

6) Customers in a city are continuously switching the brand of soap they use. If a customer is using brand A, the probability that he continues to use it the next week is 0.5 and the probabilities that he switches to brands B and C are 0.3 and 0.2 respectively. On the other hand, if he now uses brand B, the probability that he continues to use it is 0.6 while the probability that he switches to brand C is 0.4. Moreover, if he now uses brand C, the probability that he stays with it is 0.4 and the probabilities of switching to brands A and B are 0.2 and 0.4 respectively.
   (i). Write the tpm of the process
   (ii). If a customer is using brand A now, what is the probability that he will still be using it for two weeks?
   (iii). What brand of soap is most likely to be in use in two weeks?

Solve it!

7) A monkey is being trained to read the word ‘BANANA’. A series of words are flashed on a screen. If it pushes the button when ‘BANANA’, appears he receives a banana, if he doesn’t, he receives a shock (for an incorrect response). Suppose that the probability that a correct response is followed by an incorrect one is 0.1 while the probability that an incorrect response followed by a correct one is 0.3. If the probability that the first response is correct is 0.1, find the probability that
   (i). the second response is correct
   (ii). the third response is correct
   (iii). the second and the third responses are correct

Solve it!