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Q.1 Solⁿ:-

let, x_1 = No. of transistors

x_2 = no. of resistors

x_3 = No. of tubes.

Hence, Max $z = 10x_1 + 6x_2 + 4x_3$

subjected to,

$$x_1 + x_2 + x_3 \le 100 \qquad x_1, x_2, x_3 \ge 0$$
$$10x_1 + 4x_2 + 5x_3 \le 600$$
$$2x_1 + 2x_2 + 6x_3 \le 300$$

⇒ Writing in std form by adding artifical variables

Max, $z = 10x_1 + 6x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3$

Subjected to,

$$x_1 + x_2 + x_3 + S_1 = 100$$
$$10x_1 + 4x_2 + 5x_3 + S_2 = 600$$
$$2x_1 + 2x_2 + 6x_3 + S_3 = 300$$
$$x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$$

Iteration 1:-

		C_j	10	6	4	0	0	0	
C_B	X_B	X_{Bi}	x_1	x_2	x_3	S_1	S_2	S_3	Ratio
0	S_1	100	1	1	1	1	0	0	$\frac{100}{1} = 100$
0	S_2	600	10*	4	5	0	1	0	$\frac{600}{10} = 60$ ←
0	S_3	300	2	2	6	0	0	1	$\frac{300}{2} = 150$
		Z_j	0	0	0	0	0	0	
		$Z_j - C_j$	-10	-6	-4	0	0	0	
			↑ key col.						

entering variable = x_1, leaving variable = S_2,

key element = 10

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Iteration 2:

		C_j	10	6	4	0	0	0	0
C_B	X_B	X_{Bi}	x_1	x_2	x_3	s_1	s_2	s_3	Ratio
0	s_1	40	0	$(3/5)$	$(1/2)$	1	$(-1/10)$	0	$\frac{40}{3/5} = 66.67$ ← KR
10	x_1	60	1	$(2/5)$	$(1/2)$	0	$(1/10)$	0	$\frac{60}{2/5} = 150$
0	s_3	180	0	$6/5$	5	0	$-1/5$	1	$\frac{180}{6/5} = 150$
		Z_j	10	4	5	0	1	0	
		$Z_j - C_j$	0	-2	1	0	1	0	

↑ key column

⇒ entering variable = x_2, leaving = s_1
key element = $3/5$

Iteration 3:

		C_j	10	6	4	0	0	0
C_B	X_B	X_{Bi}	x_1	x_2	x_3	s_1	s_2	s_3
6	x_2	$200/3$	0	1	$5/6$	$5/3$	$-1/6$	0
10	x_1	$100/3$	1	0	$1/6$	$-2/3$	$1/6$	0
0	s_3	100	0	0	4	-2	0	1
		Z_j	10	6	$20/3$	$10/3$	$2/3$	0
		$Z_j - C_j$	0	0	$8/3$	$\frac{10}{3}$	$\frac{2}{3}$	0

Here all $Z_j - C_j \geq 0$ ⇒ optimal solⁿ Reached.

optimal solⁿ is $x_1 = \frac{100}{3}$, $x_2 = \frac{200}{3}$, $x_3 = 0$

& optimal value is Max, $Z = \frac{2200}{3}$ \$

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Q.2 Given LPP, (in converting standard form)

$$\text{Min} \Rightarrow Z = 200x_1 + 300x_2$$

$$\Rightarrow \text{Max} \Rightarrow W = -Z = -200x_1 - 300x_2$$

& constraints are,

$$2x_1 + 3x_2 \geq 1200$$

$$x_1 + x_2 \leq 400$$

$$2x_1 + \frac{3}{2}x_2 \geq 900$$

$$x_1, x_2 \geq 0$$

After adding artificial variables.

$$2x_1 + 3x_2 - S_1 + A_1 = 1200$$

$$x_1 + x_2 + S_2 = 400$$

$$2x_1 + \frac{3}{2}x_2 - S_3 + A_3 = 900$$

$$x_1, x_2, x_3, S_1, S_2, S_3, A_1, A_3 \geq 0$$

& Max, $W = -200x_1 - 300x_2 + 0S_1 + 0S_2 + 0S_3 - MA_1 - MA_3$

Iteration 1 :-

		c_j	-200	-300	0	0	0	-M	-M		
C_B	X_B	X_{Bi}	x_1	x_2	S_1	S_2	S_3	A_1	A_3	$\frac{X_B}{x_2}$	
-M	A_1	1200	2	3#	-1	0	0	1	0	$\frac{1200}{3} = 400$	← KR
0	S_2	400	1	1	0	+1	0	0	0	$\frac{400}{1} = 400$	
-M	A_3	900	2	3/2	0	0	-1	0	1	$\frac{900}{3/2} = 600$	
		Z_j	-4M	$\frac{-9M}{2}$	M	0	+M	-M	-M		
		$Z_j - C_j$	-4M +200	$\frac{-9M}{2}$ +300	M	0	M	0	0		
				↑ key column							

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Here, $z_j - c_j$ is $-\frac{9M}{2} + 300$ is negative minimum,
so key column is column corr. to x_2 & key row is row I i.e. corr. to A_1.
so entering variable $= x_2$, leaving variable $= A_1$
& key element is $\underline{\underline{3}}$

Iteration 2:-

		c_j	-200	-300	0	0	0	-M	-M	
C_B	X_B	X_{Bi}	x_1	x_2	S_1	S_2	S_3	A_1	A_3	$\frac{X_B}{X_1}$
-300	x_2	400	$2/3$	1	$-1/3$	0	0	$1/3$	0	600
0	S_2	0	$1/3$	0	$1/3$	1	0	$-1/3$	0	0 ← KR
-M	A_3	300	1	0	$1/2$	0	-1	$-1/2$	1	300
		$z_j - c_j$	$-M-200$	-300	$-\frac{M}{2}+100$	0	M	$-M$	$\frac{M}{2}+100$	
		$z_j - c_j$	$-M$	0	$-\frac{M}{2}+100$	0	M	0	$\frac{M}{2}+100$	
			↑							

key column is corr. to x_1 ⇒ entering variable
key Row ⇒ corr. to s_2 ⇒ leaving variable.
key element is $2/3$

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Iteration 3 :-

C_B	X_B	X_{Bi}	x_1	x_2	S_1	S_2	S_3	A_1	A_2
		C_j	-200	-300	0	0	0	$-M$	$-M$
-300	x_2	400	0	1	-1	-2	0	1	0
-200	x_1	0	1	0	1	3	0	-1	0
$-M$	A_2	300	0	0	$-1/2$	-3	-1	$1/2$	1
		Z_j	-200	-300	$\frac{M}{2}+100$	$3M$	M	$-\frac{M}{2}-100$	$-M$
		$Z_j - C_j$	0	0	$\frac{M}{2}+100$	$3M$	M	$\frac{M}{2}-100$	0

Here all $Z_j - C_j$ are +ve Hence optimal soln reached,

optimal soln is, $x_1 = 0$ & $x_2 = 400$

& optimal value is Max $W = -120000$

$\Rightarrow$ Min $Z = 120000$

But for $x_1 = 0$ & $x_2 = 400$, the 3rd constraint is violated Hence this is not feasible. solution. & Also Artificial variable A_2 appears in basis with +ve value 300

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Q.3 Dual simplex method,

Minimize $z = 6x_1 + 7x_2 + 3x_3 + 5x_4$

Subject to $5x_1 + 6x_2 - 3x_3 + x_4 \geq 12$

$x_2 + 5x_3 - 6x_4 \geq 10$

$2x_1 + 5x_2 + x_3 + x_4 \geq 8$

$x_1, x_2, x_3, x_4 \geq 0$

converting to standard form.

Max. $W = -z = -6x_1 - 7x_2 - 3x_3 - 5x_4 + 0s_1 + 0s_2 + 0s_3$

subject to,

$-5x_1 - 6x_2 - 3x_3 - x_4 + s_1 = -12$

$-x_2 - 5x_3 - 6x_4 + s_2 = -10$

$-2x_1 - 5x_2 - x_3 - x_4 + s_3 = -8$

Iteration 1

		c_j	-6	-7	-3	-5	0	0	0
C_B	x_B	x_{Bi}	x_1	x_2	x_3	x_4	s_1	s_2	s_3
0	s_1	-12	-5	$-6^{\#}$	3	-1	1	0	0
0	s_2	-10	0	-1	-5	-6	0	1	0
0	s_3	-8	-2	-5	-1	-1	0	0	1
		z_j	0	0	0	0	0	0	0
		$z_j - c_j$	6	7	3	5	0	0	0
		Ratio $= \frac{z_j - c_j}{s_{1,j}}$ & $s_{1j} < 0$	-1.2	-1.1667 ↑	-	-5	-	-	-

entering variable :- x_2

leaving vari :- s_1

key element = -6.

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Iteration 2:-

C_B	X_B	X_{Bi}	x_1	x_2	x_3	x_4	s_1	s_2	s_3
-7	x_2	2	5/6	1	-1/2	1/6	-1/6	0	0
0	s_2	-8	5/6	0	-11/2 #	37/6	-1/6	1	0
0	s_3	2	13/6	0	-7/2	-1/6	-5/6	0	1
		z_j	-35/6	-7	7/2	-7/6	7/6	0	0
		$z_j - c_j$	1/6	0	13/2	23/6	7/6	0	0
		$k = \frac{z_j - c_j}{s_{2,j}}$ & $s_{2j} < 0$	-	-	-1.1818 ↑ KC	-	-7	-	-

Here, entering variable, x_3,
leaving variable, s_2.
key element = $-11/2$.

Iteration 3:-

C_B	X_B	X_{Bi}	x_1	x_2	x_3	x_4	s_1	s_2	s_3
-7	x_2	$\frac{30}{11}$	22/53	1	0	-13/33	-5/33	-1/11	0
-3	x_3	16/11	-5/33	0	1	-37/33	1/33	-2/11	0
0	s_3	78/11	18/11	0	0	-45/11	-8/11	-7/11	1
		z_j	$\frac{-160}{33}$	-7	-3	$\frac{202}{33}$	$\frac{32}{33}$	13/11	0
		$z_j - c_j$	38/33	0	0	367/33	32/33	13/11	0

as all $z_j - c_j \geq 0$ & $X_{Bi} \geq 0 \Rightarrow$ optimal soln reached
optimal soln is $x_1 = 0$, $x_2 = \frac{30}{11}$, $x_3 = \frac{16}{11}$ & $x_4 = 0$
optimal value $\Rightarrow$ Max $z = \frac{-258}{11}$ $\Rightarrow$ Min $z = \frac{258}{11}$ ← Ans