Random failures constitute the most widely used model for describing reliability phenomena. They are defined by the assumption that the rate of failure of a system is independent of its age and other characteristics of its operating history, which in turn implies a constant failure rate. The constant failure rate approximation is often quite adequate even though a system or some of its components exhibit moderate, early failures or ageing effects. Here, we shall consider the exponential distribution that is employed when constant failure rates adequately describe behavior of systems.

The constant failure rate model for continuously operating systems leads to an exponential distribution, i.e. with $h(t) = \lambda$ we have,

$f(t) = \lambda e^{-\int_0^t \lambda\, dx}$

$f(t) = \lambda e^{-\lambda t}$; $\lambda>0$ $t>0$

$\therefore F(t) = 1 - e^{-\lambda t}$

$R(t) = 1 - F(t)$

$R(t)=e^{-\lambda t}$

Mean failure time = $1/\lambda$

Since $h(t)=\lambda$, a constant, this implies that aging has no impact on the component. Thus, the probability that it will fail during some period of time in the future is independent of its age, which is the memory less property of exponential distribution.

Mean Time To Failure (MTTF):

This is just the expected value or mean value of the rv T and is given by,

$$\begin{aligned} MTTF &= \int_0^\infty t f(t) dt \\ &= \int_0^\infty t\left[-\frac{d}{dt}[R(t)]\right] dt \\ &= -tR(t)\Big|_0^\infty + \int_0^\infty R(t) dt \end{aligned}$$

Since $R(0)=1$ and $R(\infty) = 0$ the 1st term vanishes

$\therefore MTTF=\int_0^\infty R(t)dt$

Or $\equiv MTTF = \int_0^\infty e^{-\int_0^t h(x)dx}dt$

When, $h(t)=\lambda$ i.e. for an exponential distribution

$MTTF=\int_0^\infty e^{-\lambda t}\, dt$

$MTTF=1/\lambda$.

Further, more (greater value) is MTTF more reliable is the system/device.

Note: Strictly speaking, MTTF should be used in the case of simple components which are not repaired when they fail, but are replaced by good components. Similarly MTBF should be used with repairable equipment or systems.

It has become customary however, to use MTTF for both non-repairable and repairable equipments. In any case, it represents the same statistical concept of mean time at which failure occurs.

This mean must be known in order to make probabilistic calculations which are necessary for the evaluation of reliability of components and systems. The MTTF is usually considered as mean time for first failure. While MTBF is normally conceived as being mean time between n^{th} and $(n+1)^{th}$ failure in a system, where n is relatively large.

Improvement in Reliability:

Many systems generally consist of a number of components. The reliability of entire system is a function of reliability of its components. These components may be connected in series or parallel. The system configuration could be a complex one also or a combination of series and parallel structure.

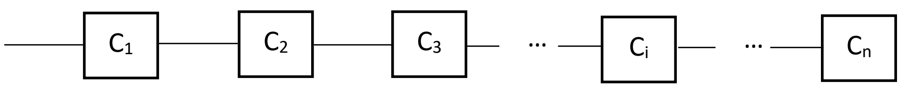
The reliability of a system can be improved in any one of the following ways.

i. Simplification of circuit design
ii. Increasing component reliability
iii. By maintenance of components and devices
iv. Carrying-out repair work of failed units.
v. By having standby units
vi. Using skilled operators

Series Configuration:

A series system is one in which all components are connected in such a way that the system fails if any one of its components fails.

Consider a system having n components connected in series as shown.



Let $R_i(t)$ be the reliability of i^{th} component i=1,2,…,n

It is assumed that each component functions independently of other components.

Then, the system reliability $R_s(t)$, is given by

$R_s(t)$ = P(System works)=P(All components work)

= P(C_1 works and C_2 works and …and C_n works)

= P(C_1 works) P(C_2 works/C_1 works)… P(C_n works/C_{n-1} works)

= P(C_1 works) P(C_2 works) … P(C_n works)(since they are independent)

= $R_1(t). R_2(t)… R_n(t)$

$$R_s(t) = \prod_{i=1}^{n} R_i(t)$$

If $R_i(t) = e^{-\alpha_i t}$ with constant failure rate α_i.

$\therefore\ R_s(t) = e^{-\alpha t}$; $\alpha = \alpha_1 + \alpha_2 + \cdots + \alpha_n = \sum_{i=1}^{n} \alpha_i$

$\Rightarrow \alpha$ is failure rate of the system

Thus the MTTF of the system is given by

$\text{MTTF}_S = \int_0^\infty R_s(t)\, dt = \frac{1}{\alpha}$

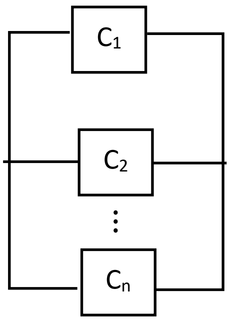
It can be shown that

$$\frac{1}{\text{MTTF}_S} = \frac{1}{(\text{MTTF})_1} + \frac{1}{(\text{MTTF})_2} + \cdots + \frac{1}{(\text{MTTF})_n}$$

Parallel Configuration:

A parallel system is one in which all components are connected in such a way that atleast one component must function for system to function.

Consider a system consisting of n components as shown



Let $R_i(t)$ be reliability of i^{th} component. It is assumed that each component functions independent of other components.

Then the system reliability $R_s(t)$ is given by,

$R_s(t)$ = P(System works) = P(atleast one component works)

= 1-P(System does not work)

= 1-P(all components fail) = 1-P(C_1 fails and C_2 fails and … and C_n fails)

= 1- P(C_1 fails) P(C_2 fails/C_1 fails)… P(C_n fails/C_{n-1} fails)

= 1- P(C_1 fails) P(C_2 fails) … P(C_n fails) (since all are independent)

= 1-[(1- R_1(t))(1-R_2(t))…(1-R_n(t))]

$R_s(t) = 1 - \prod_{i=1}^{n}(1 - R_i(t))$

If $R_i(t) = e^{-\alpha_i t}$ with constant failure rate α_i.

$R_s(t)$ = 1-[(1-$e^{-\alpha_1 t}$) (1-$e^{-\alpha_2 t}$) (1-$e^{-\alpha_3 t}$)… (1-$e^{-\alpha_n t}$)]

If $\alpha_i = \alpha\ \forall$ i then we have

$R_s(t) = 1 - (1 - e^{-\alpha t})^n$

Thus, the MTTF of the system is given by

$\text{MTTF}_S = \int_0^\infty R_S(t)dt$

$= \int_0^\infty 1 - (1 - e^{-\alpha t})^n\, dt$

$= \sum_{j=1}^{n} (-1)^{j-1} \binom{n}{j} \frac{1}{j\alpha}$

$\Rightarrow \text{MTTF}_S = \frac{1}{\alpha}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$