**Problems:**

1) What is the probability that a leap year selected at random contain 53 Sundays?

**Solution:** A leap year has 366 days i.e. 52 complete weeks and 2 days over and above.
via (S,M),(M,T),(T, W),(W, Thu),(Thu, F),(F, Sat),(Sat, S)
Let event A={A leap year contains 53 Sundays}
Total No. of cases= n=7 and favorable No. of cases= m=2
Therefore, P(A)= m/n= 2/7.

2) Suppose that A, B & C are events, Such that $P(A)= P(B)=P(C)=1/4$, $P(A\cap B)=P(B\cap C)=0$ and $P(A\cap C)=1/8$. Find the probability that atleast one of the events A, B or C occurs.

**Solution:** $P(AUBUC)= P(A)+P(B)+P(C)-P(A\cap C)-P(A\cap B)-P(B\cap C)+P(A\cap B\cap C)$
$= 1/4+1/4+1/4-0-1/8-0+0$
$= (6-1)/8$
$= 5/8.$

3) Suppose that A & B are events. Given that $P(A)=x$, $P(B)=y$, $P(A\cap B)=z$.
Find, i) $P(\bar{A}U\bar{B})$ ii) $P(\bar{A}\cap B)$ iii) $P(\bar{A}UB)$ iv)$P(\bar{A}\cap\bar{B})$

**Solution:**

i) $P(\bar{A}U\bar{B})=P(\overline{(A \cap B)})=1-P(A\cap B)=1-z$
ii) $P(\bar{A} \cap B)=P(B)-P(A\cap B)= y-z$
iii) $P(\bar{A}UB)=P(\bar{A})+P(B)-P(\bar{A}\cap B)$
$=1-x+y-y+z$
$=1-x+z$
iv) $P(\bar{A}\cap\bar{B})=P(\overline{(A \cup B)})=1-P(AUB)=1-P(A)-P(B)+P(A\cap B)$
$=1-x-y+z$

4) A class contains 10 boys and 20 girls, of which half the boys and half the girls have brown eyes. A person is chosen at random from this group. What is the probability that the chosen person is a boy or has brown eyes?

**Solution:** Define the events A : {The person chosen is a boy}
B : {The person chosen has brown eyes}
$\therefore$ Required probability = $P(A\cup B)$ = ? = $P(A)+P(B)-P(A\cap B)$

Now $P(A) = \frac{\binom{10}{1}}{\binom{30}{1}} = \frac{10}{30} = \frac{1}{3}$

$P(B) = \frac{\binom{15}{1}}{\binom{30}{1}} = \frac{15}{30} = \frac{1}{2}$

$P(A \cap B) = \frac{\binom{5}{1}}{\binom{30}{1}} = \frac{5}{30} = \frac{1}{6}$

$\therefore P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$

5) A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. An article is chosen at random. Find the probability that
   a) it has no defects
   b) it has no major defects
   c) it is either good or has major defects

   Solve it!

**Finite Sample Space:**

A sample space consisting of a finite or countably infinite number of elements is referred to as **Finite sample space.**

Ex: $S=\{a_1,a_2\ldots.a_k\}$

In order to characterize P(A), consider an event consisting of single outcome, say, $A=\{a_i\}$ we assign a number $p_i$ is called the probability of $\{a_i\}$ satisfying

i) $p_i \geq 0$, $i=1,2,\ldots.k$

ii) $p_1+p_2+\ldots.+p_k=1$

Suppose that an event A consists of r outcomes, $1 \leq r \leq k$, so that $A=\{a_{j1},a_{j2},\ldots a_{jr}\}$ where j1,j2…jr are any indices from 1,2,3….k. Then $P(A)=p_{j1}+p_{j2}+\ldots+p_{jr}$.

Thus, by assigning probabilities $p_i$ to each elementary event $\{a_i\}$ subject to condition (i) and (ii) above, one can uniquely determine P(A) for each $A \subset S$.

To evaluate $p_i$'s and hence P(A), some assumptions such as equally likely outcomes concerning the individual outcome must be made.

**Note 1**: In most of the experiments we are concerned with choosing at random one or more objects from a given collection of objects. Suppose we have N objects say $a_1, a_2 \ldots a_N$.

a) **To choose one object at random from N objects** means each object has the same probability of selection. i.e. P(choosing any $a_i$)=1/N, i=1,2….N.

b) **To choose 2 objects at random from N objects** means each pair of objects has the same probability of being chosen as any other pair. Thus if there are k such pairs, then P(choosing any pair)=1/k.

c) **To choose n objects at random from N objects** means that each n-tuple, say ($a_i$, $a_2 \ldots a_n$) is as likely to be chosen as any other n tuple. If there are k such groups of n objects then P(choosing any group of n objects)=1/k.

**Note 2:** There are several ways in which samples may be selected from a population. Here we consider, for example:

i) the samples drawn sequentially (one after another)
ii) the samples drawn simultaneously (together)

Let Z denote the set of balls in the urn. If the balls are drawn sequentially then we may describe the outcome of the game by the ordered k-tuple, $(z_1, z_2 \ldots z_k)$ of elements of Z, where $z_1$ denotes the first ball drawn, $z_2$ denotes the $2^{nd}$ and so on and $k^{th}$ the total no. of balls drawn. Thus we shall refer to $(z_1, z_2 \ldots z_k)$ as an *ordered sample* of size k.

If the balls are drawn simultaneously, it no longer makes sense to speak of a first ball or $2^{nd}$ ball etc., we may describe the outcome of our sampling only by the subset $\{z_1, z_2 \ldots z_k\}$ as an *unordered sample of size k.*

1) The number of ways of choosing an unordered sample of k objects out of n objects is $\binom{n}{k}$ i.e. ${}^nC_k$
2) The no. of ways of choosing ordered sample with replacement (WR) is $n^k$
3) The no. of ways of choosing ordered samples without replacement (WOR) is ${}^nP_k = \dfrac{n!}{(n-k)!}$

**Problems:**

1) A lot consists of 10 good articles, 4 with minor defects and 2 with major defects. Two articles are chosen at random without replacement. Find the probability that
   a) both are good
   b) both have major defects
   c) atleast one is good
   d) atmost one is good

   Solve it!

2) Ten persons are wearing badges marked 1 through 10. Three persons are chosen at random and asked to leave the room simultaneously with their badge no. being noted.
   a) What is the probability that the smallest badge no. is 5?
   b) What is the probability that the largest badge no. is 5?

   Solve it!

3) A box contains tags marked 1,2,….n. Two tags are chosen at random. Find the probability that the no.s on the tag will be consecutive integers if
   a) the tags are chosen without replacement
   b) the tags are chosen with replacement

   Solve it!