DEPARTMENT OF MATHEMATICAL AND COMPUTATIONAL SCIENCES, NITK SURATHKAL

MA724: PROBABILITY, STATISTICS AND STOCHASTIC PROCESSES

ASSIGNMENT No. 2

Date: 23/12/2020 **Max. Marks: 20**
Due Date: 31/12/2020

Answer ALL Questions

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1. If X and Y are independent Poisson variates with parameters λ and μ respectively, then show that the conditional distribution of $X = r$ given $X + Y = n$ is Binomial with parameters $\left(n, \frac{\lambda}{\lambda+\mu}\right)$.

2. a) A factory has two machines (identical), but on any given day not more than one is in use. This machine has a constant probability, p, of failure and if it fails the breakdown occurs at the end of the day's work. A single repairman is employed. It takes him two days to repair a machine and he works on only one machine at a time. Write down the TPM of the Markov Chain $\{X_n\}$, which describes the working of the factory, where X_n is the number of days that would be needed to get both the machines back in working order and X_n is recorded at the end of day n. (Hint: Write down the states and their description).

   b) Three tanks A, B and C fight a duel. Tank A hits its target with probability $\frac{2}{5}$, Tank B with $\frac{2}{3}$ and Tank C with probability $\frac{3}{5}$. Shots are fixed simultaneously and once a Tank is hit it is out of action. When all the Tanks are in action, each one picks up its target at random. Let the set of Tanks still in action be the states of the system. Obtain the TPM of the Markov chain.

3. Is the following TPM of a Markov chain Regular? If so, find the limiting distribution.

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & 0 & 1-a \end{bmatrix} \end{array}; \ 0 < a < 1$$

4. Let {N(t)} be a Poisson process. For s<t, find P[N(s)=k/N(t)=n]

5. Customers arrive at random at a checkout facility at an average rate of 12/hr. The service time has an exponential distribution with parameter μ. If the queing time of atleast 90% of the customers should be less than 4 minutes, show that μ must exceed μ_0, where μ_0 satisfies $\mu_0 e^{4\mu_0} = 2e^{4/5}$.