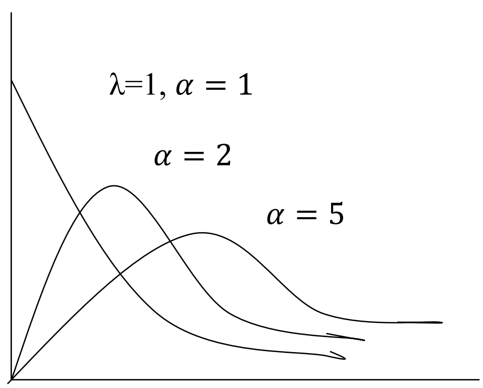
Gamma Distribution:
A crv X taking all possible non-negative values is said to follow the Gamma distribution with parameters λ, α i.e. $X \sim \Gamma(\lambda, \alpha)$ where $\lambda > 0, \alpha > 0$ if its pdf is given by

$$f(x) = \begin{cases} \dfrac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}; & x > 0, \lambda > 0, \alpha > 0 \\ 0; & \text{elsewhere} \end{cases}$$

where $\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx = (p-1)! \quad p > 0$
and $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

Now, to obtain mean and variance of Gamma distribution:
$E(X) = \int_0^{\infty} x f(x) dx = \frac{\alpha}{\lambda}$
and $V(X) = E(X^2) - (E(X))^2 = \frac{\alpha}{\lambda^2}$



When $\alpha = 1$, Gamma pdf reduces to exponential pdf with $E(X) = \frac{1}{\lambda}$ and $V(X) = \frac{1}{\lambda^2}$

Chi-square (χ^2) distribution:
A continuous rv X taking all possible non-negative values is said to follow the Chi-square (χ^2) distribution with n-degree of freedom (i.e. $X \sim \chi^2_{(n)}$), if its pdf is given by

$$f(x) = \begin{cases} \dfrac{1}{2^{\frac{n}{2}} \Gamma\left(\frac{n}{2}\right)} x^{\frac{n}{2}-1} e^{-\frac{x}{2}}; & x > 0 \\ 0; & \text{Otherwise} \end{cases}$$

Now, to obtain mean and variance of Chi-square distribution:
$E(X) = \int_0^{\infty} x f(x) dx = n$
and $V(X) = E(X^2) - (E(X))^2 = 2n$

When $\lambda = 1/2$, $\alpha = n/2$, Gamma pdf reduces to Chi-square pdf with $E(X) = n$ and $V(X) = 2n$

Problems

1. The diameter of an electric cable is normally distributed with mean 0.8 and variance 0.0004. What is the probability the diameter will exceed 0.81 units?

Solution:

If $X \sim N(\mu, \sigma^2), f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

Now $X \sim N(0.8, 0.0004)$

To find P(X > 0.81) = ?

$\therefore P(X > 0.81) = P\left(\frac{X-\mu}{\sigma} > \frac{0.81-0.8}{0.02}\right)$

= P(Z > 0.5)

$= 1 - P(Z \leq 0.5)$

$= 1 - \phi(0.5)$

= 1-0.6915 (from SN tables)

= 0.3085

2. Let $X \sim N(2, 0.16)$. Find (i) $P(X \geq 2.3)$ (ii) $P(1.8 \leq X \leq 2.1)$

Solution:

(i) $P(X \geq 2.3) = P\left(\frac{X-\mu}{\sigma} \geq \frac{2.3-2.0}{04}\right)$

= P(Z ≥ 0.75)

= 1- P(Z ≤ 0.75)

= 1- $\phi(0.75)$

= 1- 0.7734 (from SN tables)

= 0.2266

(ii) $P(1.8 \leq X \leq 2.1)$ = ?

Solve it!

3. Suppose $X \sim N(\mu, \sigma^2)$. Determine 'c' as a function of μ and σ such that $P(X \leq c) = 2P(X \geq c)$.

Solution:

Consider $P(X \leq c) = 2P(X \geq c)$

$$P\left(\frac{X-\mu}{\sigma} \leq \frac{c-\mu}{\sigma}\right) = 2P\left(\frac{X-\mu}{\sigma} \geq \frac{c-\mu}{\sigma}\right)$$

$$P\left(Z \leq \frac{c-\mu}{\sigma}\right) = 2P\left(Z \geq \frac{c-\mu}{\sigma}\right) = 2\left[1 - P\left(Z \leq \frac{c-\mu}{\sigma}\right)\right]$$

$$\phi\left(\frac{c-\mu}{\sigma}\right) = 2\left[1 - \phi\left(\frac{c-\mu}{\sigma}\right)\right]$$

$$\Rightarrow \phi\left(\frac{c-\mu}{\sigma}\right) = 2/3 = 0.667$$

$$\Rightarrow \left(\frac{c-\mu}{\sigma}\right) = 0.43 \text{ (from SN tables)}$$

$$\Rightarrow c = \mu + 0.43\,\sigma$$

4. Let $T \sim N(50,4)$. Find $P(48 < T < 53)$?

   Solve it!

5. The lifetimes of two electronic devices are normally distributed with $D_1 \sim N(40,36)$ and $D_2 \sim N(45,9)$.
   a. If the device is to be used for a 45 hour period, which device to be preferred?
   b. If the device is to be used for a 48 hour period, which device to be preferred?

   Solve it!

6. In a normal distribution 31% of the items are under 45 and 48% are over 64. Find the mean and variance of the distribution.

   Solve it!

Table 3 THE STANDARD NORMAL DISTRIBUTION

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = P(X \le x)$$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5369
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9131	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

The Standard Normal Distribution (Continued)

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9989	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000