2) A telephone company is planning to install telephone booths in a new airport. It has established policy that a person should not have to wait for more than 10 percent of the times he tries to use a phone. The demand for use is estimated to be Poisson with an average of 30 per hour. The average phone call has an exponential distribution with a mean time of 5 minutes. How many phone booths should be installed?

Solution:

Here λ = 30 per hour, $\mu = \frac{60}{5}$ = 12 per hour ($\because \frac{1}{\mu} = 5$ minutes)

Now $\rho = \frac{\lambda}{s\mu}$

If s = 1 $\Rightarrow$ $\rho = \frac{\lambda}{\mu} = \frac{30}{12} = 2.5 \nless 1$

If $s = 2 \Rightarrow \rho = \frac{5}{4} \nless 1$ $\therefore$ queue explodes

If $s = 3 \Rightarrow \rho = \frac{5}{6} < 1$ $\therefore$ minimum value of s is 3

Let us assume that the company decides to install two telephone booths,

i.e. s= 2, then $s\mu = 24$, i.e. $s\mu < \lambda$, the arrival rate for s = 2. Thus the company must have atleast three telephones to meet the demand for service.

Note: Our problem is to decide on s keeping in mind the company planning that probability that a customer will have to wait is < 10% = 0.10.

Now consider s = 5, then

$$\pi_0 = \left(\sum_{n=0}^{5-1} \frac{(s\rho)^n}{n!} + \frac{(s\rho)^s}{s!}\frac{s\mu}{s\mu-\lambda}\right)^{-1} = 0.080$$

Now Probability that a customer on arrival has to wait

$$P(N \geq s) = \frac{\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)}\,\pi_0 \quad \text{i.e. } P(N \geq 5) = \frac{12.(2.5)^5}{4!\,(5\times 12-30)}(0.080) = 0.13$$

Thus we have for the given problem,

Probability that a customer has to wait = 0.13 > 0.10

Hence installing 5 booths will not meet the company's policy.

Now, if s = 6, then we have $\pi_0 = 0.0816$ and $P(N \geq 6) = 0.047 < 0.10$
Thus an installation of Six phones would meet the company's policy.

3) Given an average arrival rate of 20 per hour, is it better for a customer to get service at a single channel with mean service rate of 22 customers and at one of two channels in parallel with mean service rate of 11 customers for each of the two channels? Assume that both queues are of (M | M | s) type.

Solution:

For a single channel λ = 20 arrival per hour & μ = 22 customers

Here we compare W_q for s = 1 & s = 2

$\pi_0 = 1 - \rho = 1 - \frac{\lambda}{\mu} = 0.09$

$L_s = \frac{\lambda}{(\mu - \lambda)} = 10$ and $W_q = \frac{\lambda}{\mu(\mu - \lambda)} = 0.45$

Now when there are 2 parallel channels, we have λ = 20, μ = 11 & s = 2

Here $\pi_0 = \left(\sum_{n=0}^{2-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{2!}\left(\frac{\lambda}{\mu}\right)^2 \frac{2\mu}{2\mu-\lambda}\right)^{-1} = 0.16$

and $L_s = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)^2}\,\pi_0 + \frac{\lambda}{\mu} = 30.9$

Similarly, $L_q = L_s - s\,\rho = 290.09$

$W_q = \frac{\mu\left(\frac{\lambda}{\mu}\right)^s}{(s-1)!\,(s\mu-\lambda)^2}\,\pi_0 = \frac{1}{\lambda}\,L_q = 1.45$

On comparison we see that it is better for a customer to get service at single channel, since in the case of single channel he has to wait for 0.45 hours whereas in the two channels he will have to wait for 1.45 hours.

4) Patients arrive at an OPD of a hospital in accordance with a Poisson process at mean rate of 12/hour and the distribution of time for examination by an attending physician is exponential with a mean of 10 minutes.
   (a). What is minimum number of physicians to be posted for ensuring steady state distribution?
   (b). For this number find W_q.
   (c). Find Ls.
   (d). How many physicians will remain idle?

Solve it!

5) A supermarket has 2 girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes and if people arrive in a Poisson fashion at the counter at the rate of 10/hour.

   (a). What is the probability of having to wait for service?
   (b). What is the expected percentage of idle time for each girl?

   Solve it!

6) A bank has 2 tellers working on savings account, the first teller handles withdrawals only while the second teller handles deposits only. It has been found that service time distributions for both deposits and withdrawals are exponential with mean service times 3 minutes per customers. Depositors are found to arrive in Poisson fashion throughout the day at a rate 16/hour. Withdrawals also arrive in Poisson fashion with rate 14/hour. What would be effect on average waiting time for depositors and withdrawers if each teller would handle both withdrawals and deposits? What would be the effect if this could be accomplished by increasing the service time to 3.5 minutes?

   Solve it!

7) Ships arrive at a port at the rate of 1 in every 4 hours with an exponential distribution of inter-arrival times. The time a ship occupies a berth for unloading has an exponential distribution with mean of 10 hours. If the average delay of ships waiting for a berth is to be kept below 14 hours, how many berths should be provided at the port?

   Solve it!

8) A library wants to improve its service facilities in terms of the waiting time of its borrowers. The library has 2 counters at present and borrowers arrive at a rate of 1 every 6 minutes and the service time is 10 minutes per borrower. The library has relaxed its membership rules and a substantial increase in the number of borrowers is expected. Find the number of additional counters to be provided if the arrival rate is expected to be twice the present value and the average waiting time of borrower must be limited to half the present value.

   Solve it!

Non – Poisson Queues

M | G | 1: FIFO | ∞ | ∞ Model

This queueing model has single server and Poisson input process with mean arrival rate λ. However, there is a general distribution for the service time whose mean and variance are $1/\mu$ and σ^2 respectively.

For this system the steady state condition is $\rho = \frac{\lambda}{\mu} < 1$.

The characteristics of this model are given below:

(i) $\pi_0 = 1 - \rho$

(ii) $L_q = \frac{\lambda^2\sigma^2+\rho^2}{2(1-\rho)}$

(iii) $L_s = L_q + \rho$

(iv) $W_q = \frac{L_q}{\lambda}$

(v) $W_s = W_q + \frac{1}{\mu}$