

Graphical Procedure: (for 2 variables only)

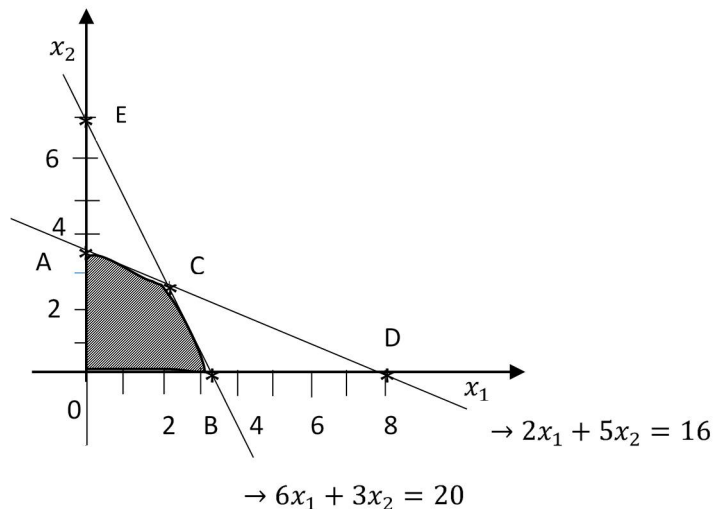
Example: Maximize  $z = 5x_1 + 8x_2$

$$\text{s. t} \quad 2x_1 + 5x_2 \leq 16$$

$$6x_1 + 3x_2 \leq 20$$

$$x_1, x_2 \geq 0$$

Solution:



$$2x_1 + 5x_2 = 16$$

Two points on this line  $\left(0, \frac{16}{5}\right), (8, 0)$

$$6x_1 + 3x_2 \leq 20$$

Two points on this line  $\left(0, \frac{20}{3}\right), \left(\frac{10}{3}, 0\right)$

AOBC  $\rightarrow$  Feasible region

Pts. O(0,0), A  $\left(0, \frac{16}{5}\right)$ , C  $\left(\frac{13}{6}, \frac{7}{3}\right)$ , B  $\left(\frac{10}{3}, 0\right)$  are Basic Feasible solutions.

Any one of BFS will be optimum

Here,  $z = 0$  at O(0,0)

$$z = \frac{128}{5} \quad \text{at } A\left(0, \frac{16}{5}\right)$$

$$z = \frac{177}{6} \quad \text{at } C\left(\frac{13}{6}, \frac{7}{3}\right)$$

$$z = \frac{50}{3} \quad \text{at } B\left(\frac{10}{3}, 0\right)$$

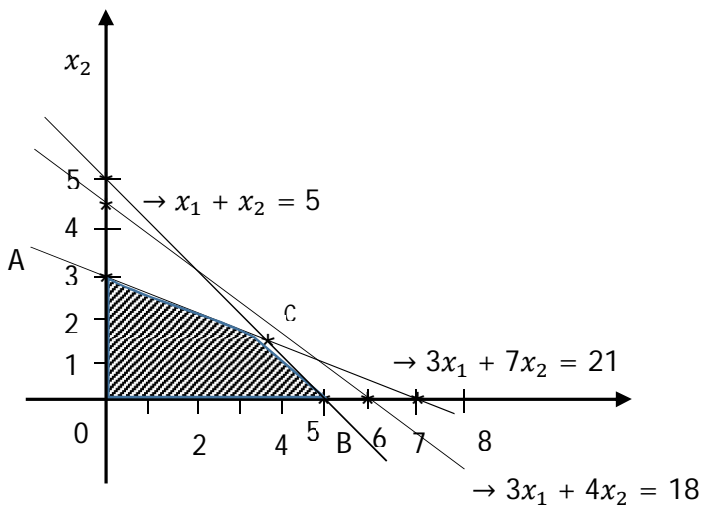
$$z = \frac{177}{6} \text{ is Maximum.}$$

The solution is  $x_1 = \frac{13}{6}, x_2 = \frac{7}{3}$

Is the optimal solution.

$$\begin{aligned}
 &2) \text{ Maximize} && z = 10x_1 + 20x_2 \\
 &\text{s. t} && 3x_1 + 4x_2 \leq 18 \\
 &&& 3x_1 + 7x_2 \leq 21 \\
 &&& x_1 + x_2 \leq 5 \\
 &&& x_1, x_2 \geq 0
 \end{aligned}$$

Solution:



BFS  $O(0,0)$ ,  $A(0,3)$ ,  $C\left(\frac{7}{2}, \frac{3}{2}\right)$ ,  $B(5,0)$

$z = 0$  at  $(0,0)$

$z = 60$  at  $(0,3)$

$z = 65$  at  $\left(\frac{7}{2}, \frac{3}{2}\right)$

$z = 50$  at  $(5,0)$

Maximum at  $\left(\frac{7}{2}, \frac{3}{2}\right)$

Optimal Solution is  $x_1 = \frac{7}{2}, x_2 = \frac{3}{2}$ .

Optimal Value is  $z=65$

Infeasible Solution

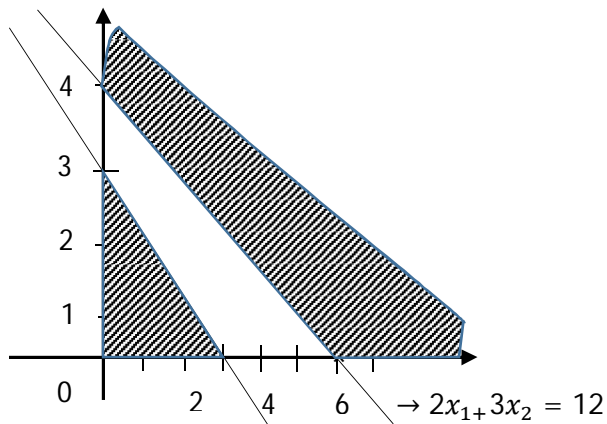
When there is no feasible solution to a lpp, it is said to have infeasible solution.

Example: Max.  $z = 4x_1 + 5x_2$

s.t.  $x_1 + x_2 \leq 3$

$$2x_1 + 3x_2 \geq 12$$

$$x_1, x_2 \geq 0$$



No feasible solution  $\rightarrow x_1 + x_2 = 3$

### Unbounded Solution

When the feasible region of a lpp is an unbounded it is said to have an unbounded solution.

Example: Max  $z = 4x_1 + 5x_2$

s.t.  $x_1 + x_2 \geq 3$

$$2x_1 + 3x_2 \geq 12$$

