MA 723 - Introduction to Data Science ①

Assignment 1 -

Due date :- $\leq$ 02-11-2020 (Monday) 10.00 PM.

1. If $a_1, a_2, \ldots, a_n$ are n distinct odd positive integers not divisible by any prime greater than 5, show that
$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} < 2.$$

2. Let a, b, c denote the sides of a triangle, show that the quantity $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ lies between $3/2$ and 2. Can equality hold at either limit?

3. Determine the largest number in the infinite sequence
$$1, \sqrt{2}, \sqrt[3]{3}, \ldots, \sqrt[n]{n} = n^{1/n}, \ldots$$

4. If a and b are positive real numbers such that $a+b=1$, prove that

$$\left(a+\frac{1}{a}\right)^2+\left(b+\frac{1}{b}\right)^2 \geqq \frac{25}{2}$$

5. If $a_0, a_1, \ldots, a_{50}$ are the coefficients of the polynomial

$$\left(1+x+x^2\right)^{25},$$

prove that the sum

$a_0+a_2+a_4+\cdots+a_{50}$ is even.

6. Let a, b, c be real numbers with $0<a<1$, $0<b<1$, $0<c<1$ and $a+b+c=2$. Prove that

$$\frac{a}{1-a}\cdot\frac{b}{1-b}\cdot\frac{c}{1-c} \geqq 8$$

7. Prove that

$$1<\frac{1}{1001}+\frac{1}{1002}+\cdots+\frac{1}{3001}<\frac{4}{3}$$

8. If x, y and z are three real numbers such that

$x+y+z=4$ and $x^2+y^2+z^2=6$,

then show that each of x, y, z lie in the closed interval $[2/3, 2]$. Can x attain the extreme values $2/3$ and 2?

9. For positive real numbers a, b, c, d satisfying $a+b+c+d \leq 1$, prove that

$$\frac{a}{b} + \frac{b}{a} + \frac{c}{d} + \frac{d}{c} \leq \frac{1}{64abcd}.$$

10. Given positive real numbers $a_1, a_2, \ldots, a_n$, let $b_1, b_2, \ldots, b_n$ be any rearrangement (Permutation) of $a_1, a_2, \ldots, a_n$. Show that

$$\frac{a_1}{b_1} + \frac{a_2}{b_2} + \cdots + \frac{a_n}{b_n} \geq n.$$

When can equality hold?