Classification of states of Markov Chain:

If $p_{ij}^{(n)} > 0$ and $n \geq 0$ then we say that state j can be reached from state i [state j is accessible from state i ($i \overset{@}{\rightarrow} j$)]. If $i \overset{@}{\rightarrow} j$ and in addition $j \overset{@}{\rightarrow} i$, then states i and j are said to communicate

($j \overset{©}{\leftrightarrow} i$).

Reflexivity: Any state communicates with itself. i.e. $p_{ij}^{(n)} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases}$

Symmetry: If $i \overset{©}{\leftrightarrow} j$ then $j \overset{©}{\leftrightarrow} i$

Transitivity: If $i \overset{©}{\leftrightarrow} j$ and $j \overset{©}{\leftrightarrow} k$ then $i \overset{©}{\leftrightarrow} k$

If all the states communicate then the Markov chain is said to be Irreducible. i.e. every state of a Markov chain can be reached from every other state, after a finite number of transitions

The corresponding TPM is said to be an Irreducible TPM.

Let C be a set of states such that no state outside can be reached from any state in C. Then the set C is said to be closed. i.e. once the system is in one of the states of C it will continue to remain in C indefinitely.

If C is a closed set and if $i \in C$ and $j \notin C$, then $p_{ij}^{(n)} = 0$.

A special case of a closed set is single state j with transition probability $p_{jj} = 1$. Then, we call state j as an Absorbing State. If a state is an absorbing state, the process will never leave it once it enters it. All states of an irreducible Markov Chain must form a closed set.

Chains which are not irreducible are said to be non-irreducible or reducible or decomposable Markov Chains. Every Markov Chain must contain at least one closed set. If the number of closed sets is 2 or more, then the chain is said to be reducible.

The set of all states of a Markov Chain that communicate with each other are grouped into a class called the Equivalence class. A Markov Chain may have one or more such equivalence classes. If there are more than one equivalence class, then it is not possible to have communicating states in different equivalence classes. However, it is possible to have states in one class that are accessible from another class.

If a Markov Chain has all its states belonging to one equivalence class it is then said to be irreducible.

Problems:

1) Let P = $\begin{array}{c c} & \begin{array}{cc} j=1 & j=2 \end{array} \\ \begin{array}{c} i=1 \\ i=2 \end{array} & \begin{bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{bmatrix} \end{array}$

Solution:

Irreducible - since both states communicate

2) Are the following tpms irreducible or reducible? Justify your answer.

a) P = $\begin{array}{c c} & \begin{array}{ccccc} j=1 & j=2 & j=3 & j=4 & j=5 \end{array} \\ \begin{array}{c} i=1 \\ i=2 \\ i=3 \\ i=4 \\ i=5 \end{array} & \begin{bmatrix} 0.2 & 0.7 & 0.07 & 0.02 & 0.01 \\ 0.2 & 0.7 & 0.07 & 0.02 & 0.01 \\ 0 & 0.2 & 0.7 & 0.07 & 0.03 \\ 0 & 0 & 0.2 & 0.7 & 0.1 \\ 0 & 0 & 0 & 0.2 & 0.8 \end{bmatrix} \end{array}$

Solve it!

b) P = $\begin{array}{c c} & \begin{array}{ccccc} j=1 & j=2 & j=3 & j=4 & j=5 \end{array} \\ \begin{array}{c} i=1 \\ i=2 \\ i=3 \\ i=4 \\ i=5 \end{array} & \begin{bmatrix} 0.6 & 0.1 & 0 & 0.3 & 0 \\ 0.2 & 0.5 & 0.1 & 0.2 & 0 \\ 0.2 & 0.2 & 0.4 & 0.1 & 0.1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$

Solve it!