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GOODLUCK Page No. 10
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Assignment No. 4

[Modelling of LPP, Solving by Simplex Method & Performing post optimal Analysis]

Que. A poultry farm owner feed's 3-type of meals (A, B, C) to his chicken. Each meal contains 3-type of ingredients (I_1, I_2, I_3) which are necessary for growth of chickens. Type A meal contains 3 units of I_1, 2 units of I_2 & 2 units of I_3. Type B meal contains 5 units of I_1, 1 unit of I_2 & 3 units of I_3. whereas Type C meal contains 2 units of I_1, 3 units of I_2 & 4 units of I_3. Each chicken needs atmost 15 units of I_1, 12 units of I_2 & Atleast 10 units of I_3 for well nourshied growth. Chicken gains 20gram from each meal of type A, 10 grams from each meal of type B & 15 grams from each unit of type C meal. To Maximize weight of chicken how many units of each meal should the owner feed them daily?

→ Soln:- x_1 → units of type A meal

x_2 - units of type B meal & x_3 - units of type C

Here objective function is,

$$\text{Max, } Z = 20x_1 + 10x_2 + 15x_3$$

and constraints are,

$$3x_1 + 5x_2 + 2x_3 \leq 15$$

$$2x_1 + x_2 + 3x_3 \leq 12$$

$$2x_1 + 3x_2 + 4x_3 \geq 10$$

$$x_1, x_2, x_3 \geq 0$$

adding slack / surplus variables & converting constraints into equality,

$$3x_1 + 5x_2 + 2x_3 + s_1 = 15$$
$$2x_1 + x_2 + 3x_3 + s_2 = 12$$
$$2x_1 + 3x_2 + 4x_3 - s_3 + A_1 = 10$$
$$\& \; x_1, x_2, x_3, s_1, s_2, s_3, A_1 \geq 0$$

& objective function.

$$\max \; Z = 20x_1 + 10x_2 + 15x_3 + 0s_1 + 0s_2 + 0s_3 - MA_1$$

Iter: 1

	C_j		20	10	15	0	0	0	-M	
C_B	x_B	x_{B_i}	x_1	x_2	x_3	s_1	s_2	s_3	A_1	Ratio (x_B/x_3)
0	s_1	15	03	5	2	1	0	0	0	1.5
0	s_2	12	2	1	3	0	1	0	0	2/3 = 0.667
-M	A_1	10	2	3	4#	0	0	-1	1	0.5 ←
Z = -10M	Z_j	-10M	-2M	-3M	-4M	0	0	M	-M	
	$Z_j - C_j$	~~-2M -20~~	-2M -20	-3M-10	-4M-15 ↑	0	0	M	-M.	

key column

Following simplex algorithm,

Itr 2:

			20	10	15	0	0	0	-M	
C_B	X_B	x_{B_i}	x_1	x_2	x_3	S_1	S_2	S_3	A_1	$\frac{x_B}{x_1}$
0	S_1	10	$2^{\#}$	3.5	0	1	0	0.5	-0.5	5
0	S_2	4.5	0.5	-1.25	0	0	1	0.75	-0.75	9
15	x_3	2.5	0.5	0.75	1	0	0	-0.25	0.25	5
37.5	z_j	z -	7.5	11.25	15	0	0	-3.75	3.75	
z=37.5	$z_j - c_j$		-12.5	1.25	0	0	0	-3.75		

key column.

key column ⇒ column corresponding to x_1
Here, $x_B / x_1 = 5$ for 1^{st} row & for 3^{rd} row Hence we can select any one. selecting 1^{st} row as key row,
⇒ # → key element = 2.

Using simplex Algorithm.

Iter 3:

		C_j	20	10	15	0	0	0	-M	
C_B	X_B	X_{Bi}	x_1	x_2	x_3	s_1	s_2	s_3	A_1	X_B/s_3
20	x_1	~~2~~05	1	1.75	0	0.5	0	0.25	-0.25	20
0	s_2	02	0	-2.125	0	-0.25	1	0.625 #	~~-0.625~~ -0.625	3.2 ←
15	x_3	0	0	-0.125	1	-0.25	0	-0.375	0.375	—
Z=100		Z_j	20	33.125	15	6.25	0	-0.625	0.625	
		$Z_j - C_j$	0	23.125	0	6.25	0	-0.625 ↑ key column		

Key column ⇒ S_3
key row ⇒ S_2
key element ⇒ 0.625

Itr 4:

		C_j	20	10	15	0	0	0	-M
C_B	X_B	X_{Bi}	x_1	x_2	x_3	s_1	s_2	s_3	A_1
20	x_1	4.2	1	2.6	0	0.6	-0.4	0	0
0	s_3	3.2	0	-3.4	0	-0.4	1.6	1	-1
15	x_3	1.2	0	-1.4	1	-0.4	0.6	0	0
Z=102		Z_j	20	31	15	6	1	0	0
		$Z_j - C_j$	0	21	0	6	1	0	0

optimal solution reached as $z_j - c_j \geq 0$ for all variables.

optimal solution is $x_1 = 4.2$, $x_3 = 1.2$ & $x_2 = 0$

& optimal value is $z = 102$ (Max.)

Post optimal Analysis:

case (i): change of cost coefficients corr. to non basic variables

Here, x_3 - is a non basic variable.
To find new cost coe. c_3' corr. x_3 such that $z_3 - c_3' \geq 0$ & optimal sol^n remains unchanged.

$$z_3 - c_3 + c_3 - c_3' \geq 0$$

$$0 + 15 - c_3' \geq 0$$

$$\Rightarrow 15 \geq c_3'$$

$$\Rightarrow \boxed{c_3' \leq 15}$$

That is, present optimal sol^n remains unchanged even if we change coe. of of x_3 in $[-\infty, 15]$

Case (ii): change of cost coefficients corr. to basic variables

(a) Corresponding to x_1:

Using formula,

$$D_1 = c_1' - c_1 = c_1' - 20$$

$$-\min_j\{(z_j - c_j)/y_{1j},\ y_{1j} > 0\} \le D_1 \le \min_j\{(z_j - c_j)/-y_{1j},\ y_{1j} < 0\}$$

Here $j = 3, 4, 5$

$$\Rightarrow -\min_j\{(z_4 - c_4)/y_{14}\} \le D_1 \le \min_j\{(z_5 - c_5)/-y_{15}\}$$

$$\Rightarrow -\{6/0.6\} \le D_1 \le \{1/0.4\}$$

$$-10 \le D_1 \le 2.5$$

$$-10 \le c_1' - 20 \le 2.5$$

$$\boxed{10 \le c_1' \le 22.5}$$

Hence coefficient of x_1 can be changed in range [10, 22.5] & optimal solution will still be unchanged.

(b) corresponding to x_3

$$-\min_j\{(z_3 - c_3)/y_{33},\ (z_5 - c_5)/y_{35}\} \le D_2 \le \min_j\{(z_4 - c_4)/y_{34}\}$$

$$-\{(0)/1,\ (1/0.6)\} \le D_2 \le \{(6/0.4)\}$$

$$0 \le D_2 \le 15$$

$$\Rightarrow 0 \le c_2' - 10 \le 15$$

$$\boxed{10 \le c_2' \le 25}$$

Hence cost coefficient of x_2 can be changed betn $[10, 25]$ & optimal soln will be still unchanged.

Case (iii) : change in resource levels (b_i's)

We know that, basic variables are always taken as

$$x_{B_i} = B\vec{b}$$ where $B \Rightarrow$ matrix consisting of columns corr to IBFV.

Here $x_{B_i} = B\vec{b}$

$$\begin{Bmatrix} x_1 \\ s_3 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0.6 & -0.4 & 0 \\ -0.4 & 1.6 & -1 \\ -0.4 & 0.6 & 0 \end{Bmatrix} \begin{Bmatrix} 15 \\ 12 \\ 10 \end{Bmatrix}$$

$$\cancel{\begin{Bmatrix} x_1 \\ s_3 \\ x_3 \end{Bmatrix} = \{}$$

$$\underbrace{\begin{Bmatrix} x_1 \\ s_3 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 4.2 \\ 8.2 \\ 1.2 \end{Bmatrix}}_{\text{optimal solution.}}$$ & optimal value is $\boxed{z = 102}$

New resource levels. $\begin{Bmatrix} 25 \\ 20 \\ 15 \end{Bmatrix}$

then we know that.

$$\max\{-x_{Bi}/b_{ik},\ b_{ik} > 0\} \le \Delta b_k \le \min\{-x_{Bi}/b_{ik},\ b_{ik} < 0\}$$

where b_{ik} are the elements in matrix B.

This is the range of resource levels for which optimal sol^n will remain unchanged.

(a) Corresponding to b_1:

$$B = \begin{Bmatrix} 0.6 & -0.4 & 0 \\ -0.4 & 1.6 & -1 \\ -0.4 & 0.6 & 0 \end{Bmatrix} = \begin{Bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{Bmatrix}$$

then,

$$\max\{-x_{Bi}/b_{i1},\ b_{i1} > 0\} \le \Delta b_1 \le \min\{-x_{Bi}/b_{i1},\ b_{i1} < 0\}$$

$$\max\{-x_{B_1}/b_{11}\} \le \Delta b_1 \le \min\{-x_{B_2}/b_{21},\ -x_{B_3}/b_{31}\}$$

$$\max\{-4.2/0.6\} \le \Delta b_1 \le \min\{-3.2/(-0.4),\ (-1.2)/(-0.4)\}$$

$$\Rightarrow -7 \le \Delta b_1 \le \min\{8, 3\}$$

$$-7 \le \Delta b_1 \le 3$$

$$-7 \le b_1' - b_1 \le 3$$

$$-7 \le b_1' - 15 \le 3$$

$$\boxed{8 \le b_1' \le 18}$$

Hence v b_1 can be varied betn [8, 18] & optimal resource level

Soln will still be same.

(b) Similarly for b_2,

$$\max\{-x_{Bi}/b_{i2},\ b_{i2} > 0\} \le \Delta b_2 \le \min\{-x_{Bi}/b_{i2},\ b_{i2} < 0\}$$

$$\max\{-x_{B2}/b_{22}, -x_{B3}/b_{32}\} \le \Delta b_2 \le \min\{-x_{B1}/b_{12}\}$$

$$\max\{-3.2/1.6, -1.2/0.6\} \le \Delta b_2 \le \min\{-4.2/-0.4\}$$

$$\max\{-2, -2\} \le \Delta b_2 \le \min\{10.5\}$$

$$\Rightarrow \quad -2 \le \Delta b_2 \le 10.5$$

$$-2 \le b_2' - 12 \le 10.5$$

$$\boxed{10 \le b_2' \le 22.5}$$

Hence resource level b_2 can be varied betn [10, 22.5] & optimal soln will still be unchanged.

(c) Similarly for b_3:

$$\max\{-x_{Bi}/b_{i3},\ b_{i3}>0\} \le \Delta b_3 \le \min\{-x_{Bi}/b_{i3},\ b_{i3}<0\}$$

$$\Delta b_3 \le \{-x_{B2}/b_{23}\}$$

$$\Delta b_3 \le (-3.2/-1)$$

$$\Delta b_3 \le 3.2$$

Here all β_{ik} i.e. $b_{i3} \not> 0$ Hence lower bound disappears.

$$\Rightarrow \quad b_3' - b_3 \le 3.2$$

$$\boxed{b_3' \le 13.2}$$

Hence value of resource level b_3 can be changed upto 13.2 & optimal solution will still be unchanged.