Quiz 5

MAT 332
Fall 2017

First   Last

Student Number

- What is a tree?

  A tree is a connected acyclic graph.

- Is the following statement True or False? (Provide a proof or give a counter example.)

  Every graph with fewer edges than vertices has a component that is a tree.

  True.

  Since the number of vertices or edges in a graph is the sum of the number in each component and G has fewer edges than vertices, there must be a component H with fewer edges than vertices. Let H have n vertices and k edges. By assumption, $k < n$. But (by Proposition 1.2.11) the number components of H is at least $(n - k)$. That is,

  $$\text{The number of components of } H = 1 \geq (n - k) > 0.$$

  Which means $(n - k) = 1$ or $k = (n - 1)$.

  That is, H is connected and has $(n - 1)$ edges. Therefore, (by Theorem 2.1.4) it is a tree.