

Q1-Answer:

a) Taking required accuracy = 0.00001

1) For  $w = 0.9$ 

Solution: Iteration = 31

 $x_1 = 1.299979$      $x_2 = -0.300015$      $x_3 = 4.199998$      $x_4 = 4.200009$      $x_5 = -0.299984$      $x_6 = 1.300015$ 2) For  $w = 1.0$ 

Solution: Iteration = 26

 $x_1 = 1.299985$      $x_2 = -0.300011$      $x_3 = 4.199999$      $x_4 = 4.200007$      $x_5 = -0.299989$      $x_6 = 1.300010$ 3) For  $w = 1.2$ 

Solution: Iteration = 21

 $x_1 = 1.299997$      $x_2 = -0.300002$      $x_3 = 4.200002$      $x_4 = 4.200001$      $x_5 = -0.300001$      $x_6 = 1.300001$ 

b)

1) Table for  $w = 0.9$ 

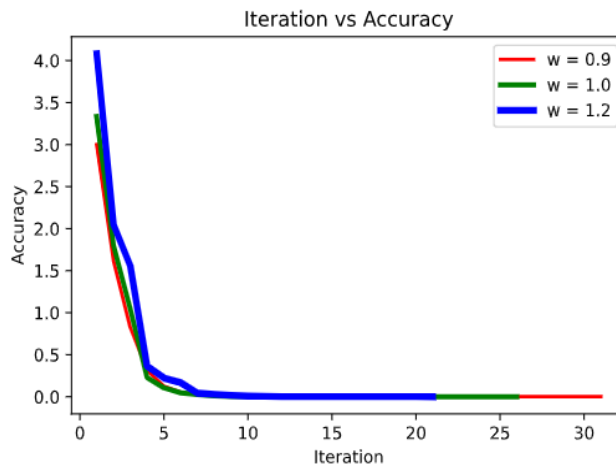
Iteration	x1	x2	x3	x4	x5	x6	Max Error
0	3.0	-0.45	1.3725	2.968875	-0.7728187499999999	0.74564625	3.0
1	2.394556125	-0.817821618750003	2.9903331121875003	3.5610084209531254	-0.6121122233132812	0.9206279272260935	1.6178331121875003
2	1.5754743294259215	-0.6283957078634135	3.6519525582337766	3.908276743452933	-0.509893446128366	1.1410014031819333	0.8190817955740783
3	1.3426238197998308	-0.4946424679481279	3.9639663457244607	4.0497297187359305	-0.40705276753780156	1.252162418234784	0.3120137874906841
...	...	...	...	...	...	...	...
29	1.2999715563278311	-0.3000209872709061	4.199997398227232	4.200012179341543	-0.29997824932567524	1.300020756859582	1.0296520730035397e-05
30	1.2999715563278311	-0.3000209872709061	4.199997398227232	4.200012179341543	-0.29997824932567524	1.300020756859582	1.0296520730035397e-05
31	1.2999791167441952	-0.3000154073417953	4.199998092053442	4.200008943520148	-0.2999840304737136	1.3000152393224271	7.560416364071898e-06

2) Table for  $w = 1.0$ 

Iteration	x1	x2	x3	x4	x5	x6	Max Error
0	3.3333333333333335	-0.3333333333333326	1.5	3.333333333333333	-0.7083333333333335	0.6388888888888893	3.3333333333333335
1	2.3981481481481484	-0.8842592592592591	3.3020833333333335	3.8582175925925926	-0.5769675925925926	0.8632330246913581	1.8020833333333335
2	1.3553883744855968	-0.6358185442386832	3.9165943287037033	4.112959747942386	-0.4910521958590537	1.1831824952846366	1.0427597736625516
...	...	...	...	...	...	...	...
24	1.299977174981797	-0.30001579026567593	4.1999992373789885	4.200010271527113	-0.29998330751026536	1.30001531282352	1.1059001014892619e-05
25	1.2999846230887133	-0.30001063764792724	4.1999994880449805	4.200006921473459	-0.29998875486275534	1.300010315237439	7.448106916330133e-06
26	1.2999846230887133	-0.30001063764792724	4.1999994880449805	4.200006921473459	-0.29998875486275534	1.300010315237439	7.448106916330133e-06

3) Table for  $w = 1.2$ 

Iteration	x1	x2	x3	x4	x5	x6	Max Error
0	4.0	0.0	1.7999999999999998	4.08	-0.4919999999999999	0.3744	4.08
1	2.3302400000000003	-1.1458559999999995	3.8490144	4.429645440000001	-0.5738770559999995	0.7620641791999997	2.0490144000000003
2	0.7728357683200004	-0.7266123310079997	4.392329371699201	4.3945067469619215	-0.5089808446126077	1.3734654823571453	1.5574042316799999
...	...	...	...	...	...	...	...
19	1.3000009044578946	-0.30000435536232173	4.200000149399344	4.200002755408012	-0.3000016301743165	1.2999982679501294	1.1774026023481099e-05
20	1.2999969678787746	-0.3000016851830683	4.20000201094376	4.200000702549945	-0.3000008949482202	1.3000005622799102	3.936579120011885e-06
21	1.2999969678787746	-0.3000016851830683	4.20000201094376	4.200000702549945	-0.3000008949482202	1.3000005622799102	3.936579120011885e-06



- i) From tables we can see that, for  $w = 1.0$  the number of iterations taken to find solution of given system with desired accuracy are 26 and for  $w = 1.2$  it is just 21.
- ii) As we know that for  $w = 1.0$ , the successive iteration method behaves as Gauss Siedel Method.
- iii) Hence in this case, over relaxation gives better convergence than Gauss Siedel method.

c)

- i) Given Matrix is not diagonally dominant.
- ii) Although the given matrix is not diagonally dominant, the Gauss Siedel and SOR method converge for given matrix.
- ii) As the matrix  $G$  satisfies the necessary and sufficient condition for convergence i.e. spectral radius of  $G$  less than 1, (where Spectral radius of  $G$  is  $= \max(\text{absolute}(\text{eigen Values}))$ ). Hence system not being diagonally dominant does not contradict with convergence of Gauss Siedel and SOR method.

Matrix  $G$ :

$$G = \begin{bmatrix} 0.00000000 & 0.66666667 & -0.33333333 & 0.00000000 & 0.00000000 & -0.33333333 \\ 0.00000000 & 0.33333333 & 0.33333333 & -0.25000000 & 0.00000000 & -0.16666667 \\ 0.00000000 & 0.00000000 & 0.25000000 & 0.37500000 & -0.25000000 & 0.00000000 \\ 0.00000000 & -0.08333333 & 0.04166667 & 0.25000000 & 0.37500000 & -0.20833333 \\ 0.00000000 & -0.04166667 & -0.04166667 & 0.03125000 & 0.25000000 & 0.39583333 \\ 0.00000000 & -0.22222222 & 0.06944444 & -0.06250000 & 0.04166667 & 0.44444444 \end{bmatrix}$$

Eigen Values of  $G$  are

Eigen Values are:

0j

$(-0.2392452174491157+0j)$

$(0.3195662365909472+0.29595646601563347j)$

$(0.3195662365909472-0.29595646601563347j)$

$(0.6736262499035985+0j)$

$(0.45426427214140114+0j)$