Q2.2 Dual PCA of Yale Face Database

Importing Libraries

In [35]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.io

plt.rcParams['figure.figsize'] = [10,5]
```

Importing Yale Face Dababase

In [36]:
```
data = scipy.io.loadmat('./YaleFaceDataBase/Yale_64x64.mat')
```

In Dual PCA if A has dimensions n by t then n >> t

Taking only t-number of samples for Analysis

In [37]:
```
t = 100
X = np.array(data['fea'])[:t,:].T
```

In [38]:
```
print(X.shape)
```

(4096, 100)

Visualizing one of the sample image

In [39]:
```
img = plt.imshow(X[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

Calculating At*A

In [40]:
```
XtX = np.matmul(X.T,X)
```

In [41]:
```
print(XtX.shape)
```

(100, 100)

Calculating Eigen values of At*A

In [42]:
```
eigValues, eigVectors = np.linalg.eigh(XtX)
```

In [43]:
```
print(eigValues.shape)
```

(100,)

In [44]:
```
print(eigValues)
```

```
[-1.44213332e+03 -1.32446410e+03 -1.30623412e+03 -1.24429096e+03
 -1.21678564e+03 -1.19797082e+03 -1.11729236e+03 -1.06525344e+03
 -1.05588651e+03 -1.04058313e+03 -9.76025720e+02 -9.18680023e+02
 -8.97465034e+02 -8.84722094e+02 -8.75651691e+02 -8.37038678e+02
 -8.14724711e+02 -7.79495263e+02 -7.38948305e+02 -7.15844200e+02
 -6.91368810e+02 -6.71493309e+02 -6.58297084e+02 -6.00216821e+02
 -5.79281157e+02 -5.36942209e+02 -5.10119884e+02 -5.03721547e+02
 -4.50460504e+02 -4.40144652e+02 -4.31936255e+02 -4.05144336e+02
 -3.76509059e+02 -3.47656317e+02 -3.10738072e+02 -2.98574308e+02
 -2.82611833e+02 -2.22261329e+02 -2.11557931e+02 -1.90942705e+02
 -1.68230865e+02 -1.63777417e+02 -1.15412219e+02 -8.18540193e+01
 -6.54047420e+01 -4.83431968e+01 -3.63863311e+01 -2.96248105e+01
  4.63655445e-14  1.78288084e-13  1.11059645e+01  4.91114620e+01
  6.64342907e+01  9.25994971e+01  1.01413404e+02  1.48782785e+02
  1.87128456e+02  2.21946170e+02  2.34283584e+02  2.62349975e+02
  2.80922163e+02  2.92709212e+02  2.99399922e+02  3.39613235e+02
  3.58319602e+02  4.04102718e+02  4.23277169e+02  4.38690758e+02
  4.72931541e+02  5.07492373e+02  5.11577217e+02  5.40440401e+02
  5.62420730e+02  5.90376424e+02  6.19289307e+02  6.23520739e+02
  6.62120820e+02  6.80666083e+02  6.87894811e+02  7.19088198e+02
  7.34998271e+02  7.85210254e+02  7.99554795e+02  8.40891654e+02
  8.59706496e+02  9.04948515e+02  9.23296090e+02  9.70877545e+02
  9.86367520e+02  1.01280301e+03  1.07439711e+03  1.10354195e+03
  1.13845770e+03  1.21809367e+03  1.27040551e+03  1.30376543e+03
  1.39149840e+03  1.40632623e+03  1.49057913e+03  1.28139567e+04]
```

Sorting eigen values in descending values and changing order of

eigen vectors correspondingly

In [45]:
```
idx = eigValues.argsort()[::-1]
eigValues = eigValues[idx]
eigVectors = eigVectors[:,idx]
```

In [46]:
```
print(eigValues)
```

```
[ 1.28139567e+04  1.49057913e+03  1.40632623e+03  1.39149840e+03
  1.30376543e+03  1.27040551e+03  1.21809367e+03  1.13845770e+03
  1.10354195e+03  1.07439711e+03  1.01280301e+03  9.86367520e+02
  9.70877545e+02  9.23296090e+02  9.04948515e+02  8.59706496e+02
  8.40891654e+02  7.99554795e+02  7.85210254e+02  7.34998271e+02
  7.19088198e+02  6.87894811e+02  6.80666083e+02  6.62120820e+02
  6.23520739e+02  6.19289307e+02  5.90376424e+02  5.62420730e+02
  5.40440401e+02  5.11577217e+02  5.07492373e+02  4.72931541e+02
  4.38690758e+02  4.23277169e+02  4.04102718e+02  3.58319602e+02
  3.39613235e+02  2.99399922e+02  2.92709212e+02  2.80922163e+02
  2.62349975e+02  2.34283584e+02  2.21946170e+02  1.87128456e+02
  1.48782785e+02  1.01413404e+02  9.25994971e+01  6.64342907e+01
  4.91114620e+01  1.11059645e+01  1.78288084e-13  4.63655445e-14
 -2.96248105e+01 -3.63863311e+01 -4.83431968e+01 -6.54047420e+01
 -8.18540193e+01 -1.15412219e+02 -1.63777417e+02 -1.68230865e+02
 -1.90942705e+02 -2.11557931e+02 -2.22261329e+02 -2.82611833e+02
 -2.98574308e+02 -3.10738072e+02 -3.47656317e+02 -3.76509059e+02
 -4.05144336e+02 -4.31936255e+02 -4.40144652e+02 -4.50460504e+02
 -5.03721547e+02 -5.10119884e+02 -5.36942209e+02 -5.79281157e+02
 -6.00216821e+02 -6.58297084e+02 -6.71493309e+02 -6.91368810e+02
 -7.15844200e+02 -7.38948305e+02 -7.79495263e+02 -8.14724711e+02
 -8.37038678e+02 -8.75651691e+02 -8.84722094e+02 -8.97465034e+02
 -9.18680023e+02 -9.76025720e+02 -1.04058313e+03 -1.05588651e+03
 -1.06525344e+03 -1.11729236e+03 -1.19797082e+03 -1.21678564e+03
 -1.24429096e+03 -1.30623412e+03 -1.32446410e+03 -1.44213332e+03]
```

In [47]:
```
eigVals = eigValues.copy()
```

Finding out number of least significant eigen Values

In [48]:
```
r = 0
index_of_small_eig_values = []
while(r<len(eigValues)):
        if eigValues[r]<1:
            index_of_small_eig_values.append(eigValues[r])
        r += 1
```

Here that number turns out to be 50

In [49]:
```
small_eig_vals = len(index_of_small_eig_values)
print(small_eig_vals)
```

50

In [50]:
```
eigVals = np.array(eigVals)
```

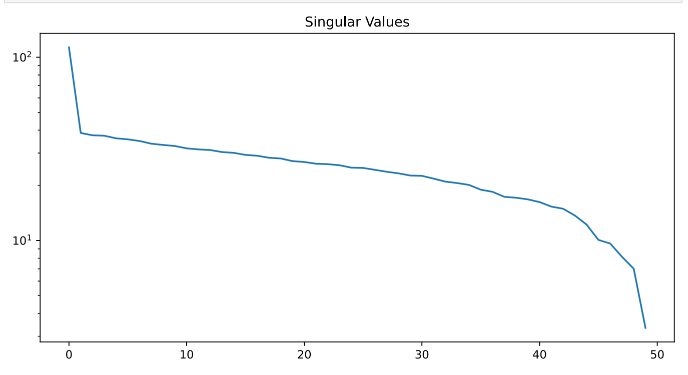
Creating Singular value matrix

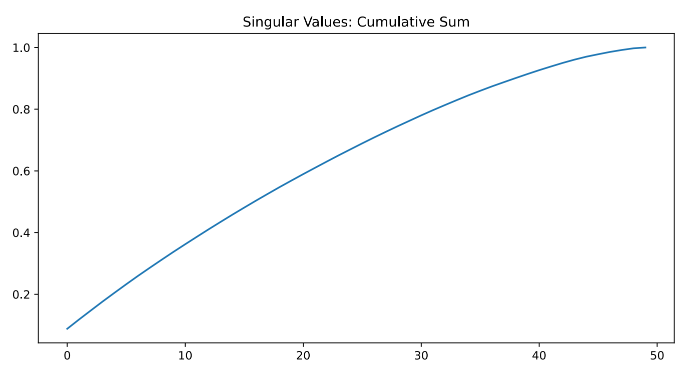
In [51]:
```
D = eigVals[:-small_eig_vals]**(1/2)
```

Visualizing Singular values matrix pattern

In [52]:
```
plt.figure(1)
plt.semilogy(D)
plt.title('Singular Values')
plt.show()

plt.figure(2)
plt.plot(np.cumsum(D)/np.sum(D))
plt.title('Singular Values: Cumulative Sum')
plt.show()
```





Formint V.transpose() Matrix

In [53]:
```
Vt = eigVectors.copy().T
```

Reconstruction of Training data

xcap = XVVt

In [54]:
```
xcap = X.dot(Vt.T).dot(Vt)
```

In [55]:
```
print(xcap.shape)
```

(4096, 100)

Visualizing Reconstructed Data

In [56]:
```
plt.figure(figsize=(16,20))
for i in range(1,51):
    plt.subplot(10,5,i),plt.xticks([]),plt.yticks([])
    img = plt.imshow(xcap[:,i].reshape(64,64).astype('uint8').T,cmap='gray')
plt.plot()
```

Reconstruction of Test Data

ycap = XV(Σ^-2)VtXtx

In [57]:
```
D_temp = np.zeros((len(D),len(D)))
for i in range(len(D)):
    D_temp[i][i] = D[i]
D = D_temp
```

In [58]:
```
for i in range(0,50):
    print(D[i][i])
```

[illegible]

In [59]:
```
invD_sq = np.linalg.inv(np.matmul(D,D))
```

In [60]:
```
print(invD_sq)
```

[illegible]

In [61]:
```
print(X.shape)
print(Vt.T.shape)
print(invD_sq.shape)
print(Vt.shape)
print(X.T.shape)
```

[illegible]

In [62]:
```
X_approx = X[:,:-small_eig_vals]
Vt_approx = Vt[:-small_eig_vals,:-small_eig_vals]
```

In [63]:
```
print(X_approx.shape)
print(Vt_approx.T.shape)
print(invD_sq.shape)
print(Vt_approx.shape)
print(X_approx.T.shape)
print(X_approx.shape)
```

```
(4096, 50)
(50, 50)
(50, 50)
(50, 50)
(50, 4096)
(4096, 50)
```

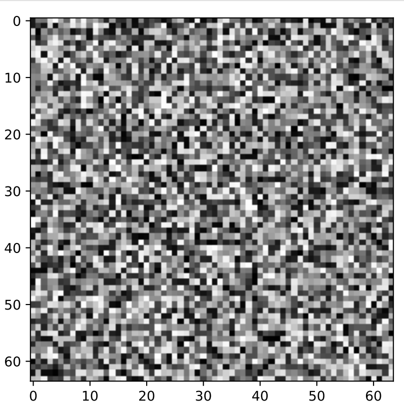
In [64]:
```
UUt = np.matmul(X_approx[:,:],np.matmul(Vt_approx.T,np.matmul(invD_sq,np.matmul(Vt_app
```

In [65]:
```
y = np.matmul(UUt,X[:,0])
```

In [66]:
```
print(y.shape)
```

(4096,)

In [67]:
```
img = plt.imshow(y.reshape(64,64).astype('uint8').T)
img.set_cmap('gray')
```



In Dual PCA, in most cases reconstruction of test data

i.e. out of sample reconstruction is not possible