

## MA844: Advanced Data Science

### Assignment - 3 (20 Marks)

**Answer all the questions neatly and in detail. Each question carries 5 Marks.**

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1. Describe in detail how the SVD is used to solve the Discrete Optimization problem of finding a Max-Cut in a directed graph.
2. Give a detailed proof of the Fundamental theorem of Markov Chains.
3. Let  $\sum \sigma_i u_i v_i^T$  be the SVD of a rank 'r' matrix A. Let  $A_k = \sum_{i=1}^k \sigma_i u_i v_i^T$  be a rank-'k' approximation to A for some  $k < r$ .

Express the following quantities in terms of the singular values  $\{\sigma_i: 1 \leq i \leq r\}$ :

- a)  $\|A_k\|_F^2$
  - b)  $\|A_k\|_2^2$
  - c)  $\|A - A_k\|_F^2$
  - d)  $\|A - A_k\|_2^2$
4. We proved the following Lemma in the class: "With probability atleast  $\frac{2}{3} - \frac{d}{M}$ , we have  $\frac{d}{6} \leq \frac{M}{\min} \leq 6d$ , where  $\min$ =smallest element of S by showing that  $\frac{M}{\min} > 6d$  with probability atleast  $\frac{1}{6} + \frac{d}{M}$  and  $\frac{M}{\min} < \frac{d}{6}$  with probability atleast  $\frac{1}{6}$ . Give all the details of how the Lemma follows from this.

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