Assignment - I

Name:	**Gavali Deshabhakt Nagnath**
Subject:	**Machine Learning**
Specialization :	**Computational and Data Sciences**
Department:	**Mathematical and Computational Sciences**
Course Instructor:	**Dr. Jidesh P.**
Data:	**14/04/2021**

Q1. Write codes to perform, LU, LDU, QR, and SV Decomposition.

A. LU - Decomposition:

Program:

```
import numpy as np
import copy

def inputMatrix():                  # Function to take matrix input from user
    print("Enter the size of matrix: ")
    n= int(input())

    A= np.zeros((n,n),dtype=float)
    print("Now enter elements of matrix 'A':")
    for i in range(n):
        print("Enter elements for row:",i+1)
        for j in range(n):
            A[i][j]=int(input())
    return A

def printMatrix(V):                 # Function to print Matrix
    for i in range(n):
        for j in range(n):
            print(f'{V[i][j]:15.08f}' ,  end="  ")
        print()
    print()

def LUDecomposition(A):     #LU-Decomposition function definition

    n = len(A)

    L= np.zeros((n,n),dtype=float)
    for i in range(len(L)):
        L[i][i]=1

    U = copy.copy(A)    # copying matrix A into U

    for i in range(0,n-1):
        for j in range(i+1,n):
            L[j][i]= (U[j][i]/U[i][i])
            U[j][:]=U[j][:]-L[j][i]*U[i][:]

    return L,U


# Default input
n = 3

A = np.array([
```

```
    [1,2,4],
    [3,8,14],
    [2,6,13]
])

# Uncomment below line to take input from user
# A = inputMatrix()

# Calling Function of matrix A
L,U = LUDecomposition(A)

# Printing Results
print("A = ")
printMatrix(A)

print("L = ")
printMatrix(L)

print("U = ")
printMatrix(U)
```

Output:

```
A =
    1.00000000    2.00000000    4.00000000
    3.00000000    8.00000000   14.00000000
    2.00000000    6.00000000   13.00000000

L =
    1.00000000    0.00000000    0.00000000
    3.00000000    1.00000000    0.00000000
    2.00000000    1.00000000    1.00000000

U =
    1.00000000    2.00000000    4.00000000
    0.00000000    2.00000000    2.00000000
    0.00000000    0.00000000    3.00000000
```

B. LDU – Decomposition

Program:

```
import numpy as np
import copy

def inputMatrix():                    # Function to take matrix input from user
    print("Enter the size of matrix: ")
    n= int(input())

    A= np.zeros((n,n),dtype=float)
    print("Now enter elements of matrix 'A':")
```

```
    for i in range(n):
        print("Enter elements for row:",i+1)
        for j in range(n):
            A[i][j]=int(input())
    return A

def printMatrix(V):                # Function to print matrix
    for i in range(n):
        for j in range(n):

            print(f'{V[i][j]:15.08f}' ,  end="  ")
        print()
    print()


def LUDecomposition(A):                # LDU – Decomposition Function Definition

    n = len(A)

    L= np.zeros((n,n),dtype=float)
    D = np.zeros((n,n),dtype=float)

    for i in range(len(L)):
        L[i][i]=1

    U = copy.copy(A)    # copying matrix A into U



    for i in range(0,n-1):
        for j in range(i+1,n):
            L[j][i]= (U[j][i]/U[i][i])
            U[j][:]=U[j][:]-L[j][i]*U[i][:]

    for i in range(n):
        if(U[i][i]!=0):
            D[i][i] = copy.copy(U[i][i])
            U[i,:] = copy.copy(U[i,:]/U[i][i])

    return L,D,U

# Default Input
n = 3

A = np.array([
    [1,2,4],
    [3,8,14],
    [2,6,13]
])
```

```
# Calling LDU decomposition function
L, D, U = LUDecomposition(A)

print("A = ")
printMatrix(A)

print("L = ")
printMatrix(L)

print("D = ")
printMatrix(D)

print("U = ")
printMatrix(U)
```

Output:

```
A =
    1.00000000     2.00000000     4.00000000
    3.00000000     8.00000000    14.00000000
    2.00000000     6.00000000    13.00000000

L =
    1.00000000     0.00000000     0.00000000
    3.00000000     1.00000000     0.00000000
    2.00000000     1.00000000     1.00000000

D =
    1.00000000     0.00000000     0.00000000
    0.00000000     2.00000000     0.00000000
    0.00000000     0.00000000     3.00000000

U =
    1.00000000     2.00000000     4.00000000
    0.00000000     1.00000000     1.00000000
    0.00000000     0.00000000     1.00000000
```

C. QR – Decomposition

Program:

```
import numpy as np

def matrixInput():
    m = int(input("Enter row size :"))
    n = int(input("Enter column size :"))

    A = np.zeros((m,n), dtype=float)
```

```
    print("Enter elements of matrix: ")
    for i in range(m):
        for j in range(n):
            A[i][j] = float(input())

    return A


def Normalize(v):
    sum = 0.0
    for i in v:
        sum+=i**2
    v=v/(sum**0.5)
    return v

def QRDecomp(A):

    n = len(A[0])   # Columns/Vectors
    m = len(A)      # Rows/Components
    q = []

    q.append(Normalize(A[:,0].reshape(m,1)))

    for i in range(1,n):

        vec = A[:,i].astype('float64').reshape(m,1)
        temp = np.zeros((m,1),dtype=float)

        for j in range(i):

            multiplier = (((vec.transpose()).dot((q[j]))))/(q[j].transpose().dot(q[j]))
            temp -= (multiplier)*q[j]

        vec = vec + temp
        normalizedvec = Normalize(vec)
        q.append(normalizedvec)

    Q = np.array(q).transpose().reshape(m,n)    # typecasting python list to numpy array and taking
np.array's transpose

    # Calculating R
    R = np.zeros((n,n))
    for i in range(n):
        for j in range(n):
            if i<=j:
                R[i][j] = A[:,j].transpose().dot(Q[:,i])

    return Q,R
```

```
def printMatrix(V):
    m = len(V)
    n = len(V[0])
    for i in range(m):
        for j in range(n):
                print(f'{V[i][j]:10.05f}' ,  end="  ")
        print()
    print()

# Default Input

A = np.array(((
    (1, -1, 4),
    (1, 4, -2),
    (1, 4, 2),
    (1, -1, 0)
)))

# Uncomment following lines for custom input
# A = matrixInput()

# Calling QR-decomposition function
Q,R = QRDecomp(A)

print("A = ")
printMatrix(A)

print("Q =")
printMatrix(Q)

print("R =")
printMatrix(R)
```

Output:

```
A =
   1.00000    -1.00000     4.00000
   1.00000     4.00000    -2.00000
   1.00000     4.00000     2.00000
   1.00000    -1.00000     0.00000

Q =
   0.50000    -0.50000     0.50000
   0.50000     0.50000    -0.50000
   0.50000     0.50000     0.50000
   0.50000    -0.50000    -0.50000

R =
   2.00000     3.00000     2.00000
```

```
0.00000    5.00000   -2.00000
0.00000    0.00000    4.00000
```

D. SVD

Program:

```
# # SVD Implementation
# ## Importing libraries

# In[1]:

import numpy as np
import copy

# ## SVD function Definition

# In[2]:


def printMatrix(V):
    m = len(V)
    n = len(V[0])

    for i in range(m):
        for j in range(n):
            print(f'{V[i][j]:10.05f}' ,  end="  ")
        print()
    print()


def SVD(A):
    m = len(A)
    n = len(A[0])

    At = A.transpose()

    AtA = np.matmul(At,A)

    AAt = np.matmul(A,At)

    # Finding Eigen Values and Vectors of AAt and AtA
    eigValuesAAt, eigVectorsAAt = np.linalg.eig(AAt)

    eigValuesAtA, eigVectorsAtA = np.linalg.eig(AtA)

    # Forming U, D and VT
    U = eigVectorsAAt
```

```
# Sorting eigen values in descending order and also changing position of corresponding eigen
vectors
    idx = eigValuesAAt.argsort()[::-1]
    eigValuesAAt = eigValuesAAt[idx]
    eigVectorsAAt = eigVectorsAAt[:,idx]
    eigVectorsAtA = eigVectorsAtA[:,idx]


    D = np.zeros((m,n))
    for i in range(m):
        for j in range(n):
            if i==j:
                D[i][j] = (eigValuesAAt[i])**(1/2)
            else:
                D[i][j] = 0

    Vt = eigVectorsAtA.transpose()

    return U,D,Vt


# In[3]:


A = np.array(((
    (1,2,3),
    (4,5,6),
    (7,8,9)
)))

# A = np.array(((
#     (1, -1, 4),
#     (1, 4, -2),
#     (1, 4, 2),
#     (1, -1, 0)
# )))

# Calling SVD-decomposition function
U,D,Vt = SVD(A)


# In[4]:

print("A = ")
printMatrix(A)
print("U = ")
printMatrix(U)
print("D = ")
printMatrix(D)
```

```
print("VT = ")
printMatrix(Vt)
```

Output:

```
A =
   1.00000     2.00000     3.00000
   4.00000     5.00000     6.00000
   7.00000     8.00000     9.00000

U =
  -0.21484    -0.88723     0.40825
  -0.52059    -0.24964    -0.81650
  -0.82634     0.38794     0.40825

D =
  16.84810     0.00000     0.00000
   0.00000     1.06837     0.00000
   0.00000     0.00000     0.00000

VT =
  -0.47967    -0.57237    -0.66506
  -0.77669    -0.07569     0.62532
   0.40825    -0.81650     0.40825
```

Q2.1 PCA of Yale Face Database

Importing Libraries

In [1]:
```
import numpy as np
from matplotlib.image import imread
import matplotlib.pyplot as plt
import scipy.io
import copy

plt.rcParams['figure.figsize'] = [8,4]
```

Importing Yale Faces Database from .mat file

scipy.io.loadmat() function imports .mat file as dictonary

In [2]:
```
data = scipy.io.loadmat('./Yale_64x64.mat')
print(type(data))
```

```
<class 'dict'>
```

Dictonary to Numpy Array

In [3]:
```
A = np.array(data['fea']).T
```

In [4]:
```
print(A.shape)
```

```
(4096, 165)
```

Sample Image/Face from database

In [5]:
```
img = plt.imshow(A[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

Forming Covariance-Matrix

In [6]:
```
Amean = A.mean(axis=1,keepdims=True)
Am = A - Amean
```

In [7]:
```
img = plt.imshow(Am[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

Calculating SVD

In [8]:
```
U,D,Vt = np.linalg.svd(Am)         # Complete SVD i.e. calculation corresponding to ze

# U,D,Vt = np.linalg.svd(Am, full_matrices=False)    # Economy SVD i.e. Calculations c

D = np.diag(D)
```

In [9]:
```
print(U.shape,D.shape,Vt.shape)
```

```
(4096, 4096) (165, 165) (165, 165)
```

Finding number of eigen values with least significance

In [10]:
```
i = 0
n = len(D)
while(i<n):
    if abs(D[i][i]) <10:
        break
    i += 1
eig_vals_with_least_significance = n - i
```

In [11]:
```
print(eig_vals_with_least_significance)
```

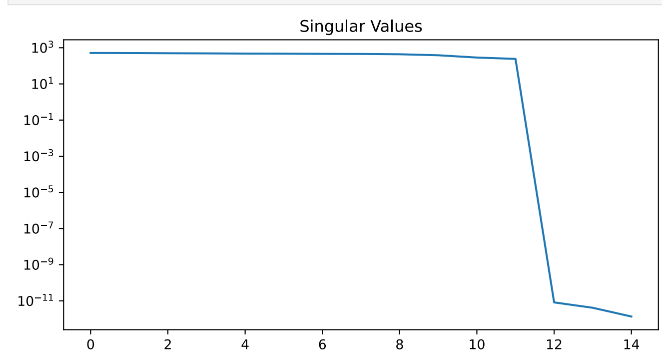
```
3
```

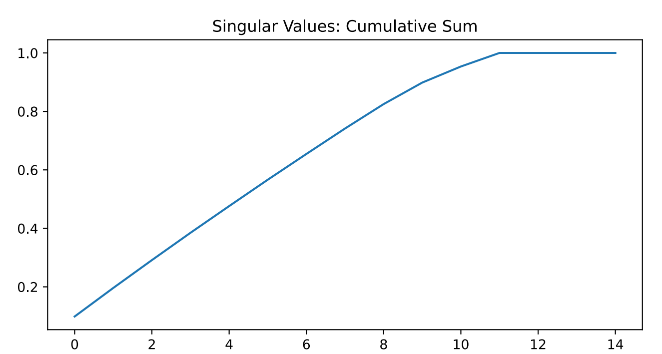
Visualizing singular values by plotting graph

1. Singular values vs Count

2. (Cumulative sum/Total sum) vs Count

In [12]:
```
d = D[150:,150:]

plt.figure(1)
plt.semilogy(np.diag(d))
plt.title('Singular Values')
plt.show()

plt.figure(2)
plt.plot(np.cumsum(np.diag(d))/np.sum(np.diag(d)))
plt.title('Singular Values: Cumulative Sum')
plt.show()
```

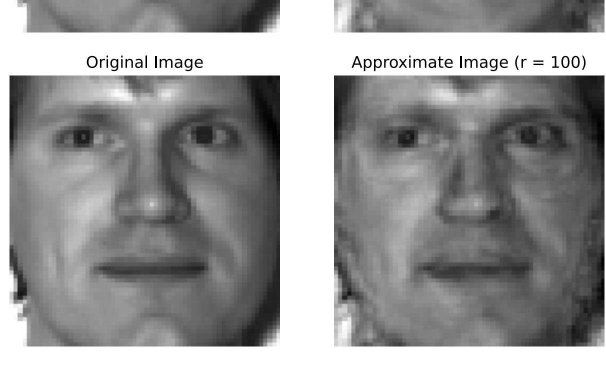


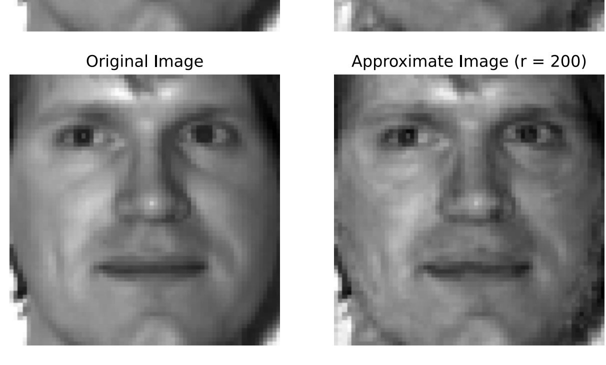


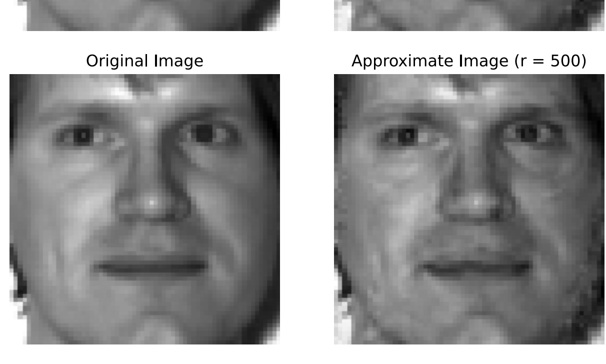
In Sample Projection and Prediction

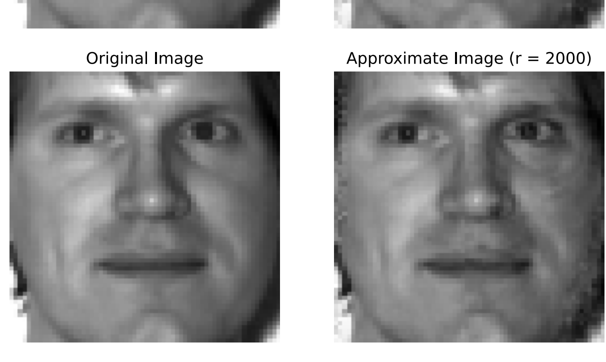
In [13]:
```
sample_size = 150

def InSampleProjectionAndReconstruction(image_number):

    j = 0

    for r in (50, 100, 200, 500, 800, 2000, 4096, 4096-eig_vals_with_least_significanc

        # Construct approximate image
        u = U[:,:r]

        # Projection
        A_train_model = np.matmul(u.T,A[:,:sample_size])

        # Reconstruction
        A_train_pred = np.matmul(u,A_train_model)
        Fimg = A_train_pred

        plt.figure(j+1)
        j += 1

        plot1 = plt.subplot(121)
        img =  plt.imshow(A[:,image_number].reshape(64,64).transpose())
        img.set_cmap('gray')
        plt.title(f'Original Image')
        plt.axis('off')

        plot2 = plt.subplot(122)
        img2 = plt.imshow(Fimg[:,image_number].reshape(64,64).transpose())
        img2.set_cmap('gray')
        plt.axis('off')
        plt.title(f'Approximate Image (r = {r})')
        plt.show()
```

In [14]:
```
InSampleProjectionAndReconstruction(0)
```





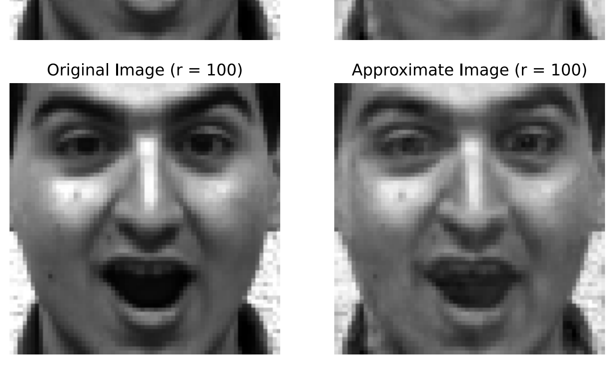


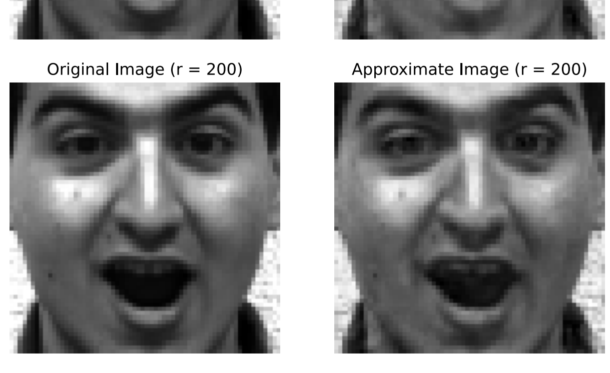


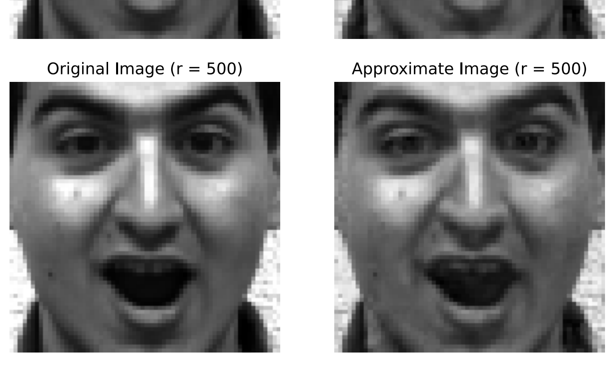
Out off Sample Projection and Prediction

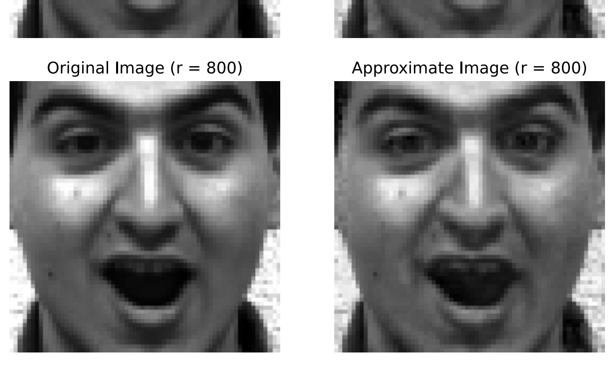
In [15]:
```
def outOffSampleProjectionAndReconstruction(image_number):
    if(image_number>=sample_size):
        j = 0
        for r in (50, 100, 200, 500, 800, 2000,4096, 4096-eig_vals_with_least_signific

            # Construct approximate image
            u = U[:,:r]

            # Projection
            A_test_model = np.matmul(u.T,A[:,image_number])

            # Reconstruction
            A_test_pred = np.matmul(u,A_test_model)
            Fimg = A_test_pred

            plt.figure(j+1)
            j += 1

            plt.subplot(121)
            img =  plt.imshow(A[:,image_number].reshape(64,64).transpose() )

            img.set_cmap('gray')
            plt.axis('off')
            plt.title(f'Original Image (r = {r})')

            plt.subplot(122)
            img2 = plt.imshow(Fimg.reshape(64,64).transpose())
            img2.set_cmap('gray')
            plt.axis('off')
            plt.title(f'Approximate Image (r = {r})')
            plt.show()

    else:
            print("Object Belongs to Sample")
```

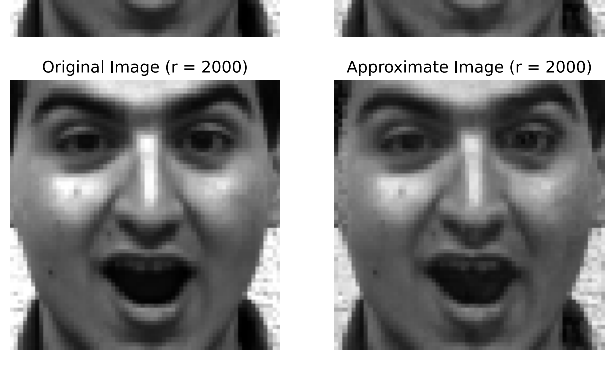
In [16]:
```
outOffSampleProjectionAndReconstruction(155)
```

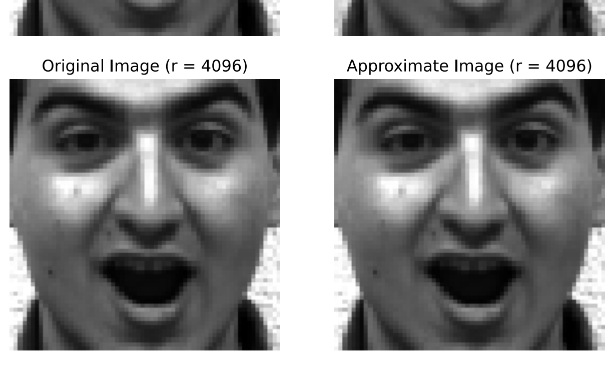


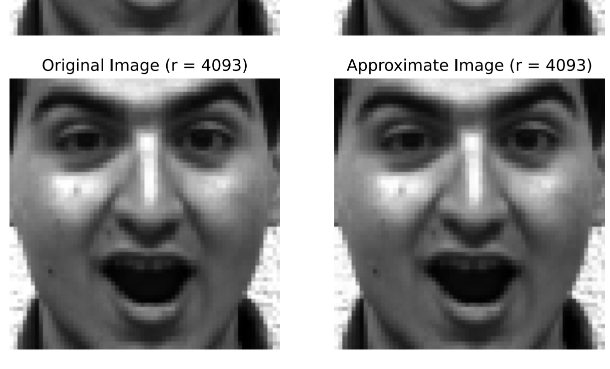












Q2.2 Dual PCA of Yale Face Database

Importing Libraries

In [35]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.io

plt.rcParams['figure.figsize'] = [10,5]
```

Importing Yale Face Dababase

In [36]:
```
data = scipy.io.loadmat('./YaleFaceDataBase/Yale_64x64.mat')
```

In Dual PCA if A has dimensions n by t then n >> t

Taking only t-number of samples for Analysis

In [37]:
```
t = 100
X = np.array(data['fea'])[:t,:].T
```

In [38]:
```
print(X.shape)
```

```
(4096, 100)
```

Visualizing one of the sample image

In [39]:
```
img = plt.imshow(X[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

Calculating At*A

In [40]:
```
XtX = np.matmul(X.T,X)
```

In [41]:
```
print(XtX.shape)
```

```
(100, 100)
```

Calculating Eigen values of At*A

In [42]:
```
eigValues, eigVectors = np.linalg.eigh(XtX)
```

In [43]:
```
print(eigValues.shape)
```

```
(100,)
```

In [44]:
```
print(eigValues)
```

```
[-1.44213332e+03 -1.32446410e+03 -1.30623412e+03 -1.24429096e+03
 -1.21678564e+03 -1.19797082e+03 -1.11729236e+03 -1.06525344e+03
 -1.05588651e+03 -1.04058313e+03 -9.76025720e+02 -9.18680023e+02
 -8.97465034e+02 -8.84722094e+02 -8.75651691e+02 -8.37038678e+02
 -8.14724711e+02 -7.79495263e+02 -7.38948305e+02 -7.15844200e+02
 -6.91368810e+02 -6.71493309e+02 -6.58297084e+02 -6.00216821e+02
 -5.79281157e+02 -5.36942209e+02 -5.10119884e+02 -5.03721547e+02
 -4.50460504e+02 -4.40144652e+02 -4.31936255e+02 -4.05144336e+02
 -3.76509059e+02 -3.47656317e+02 -3.10738072e+02 -2.98757430e+02
 -2.82611833e+02 -2.22261329e+02 -2.11557931e+02 -1.90942705e+02
 -1.68230865e+02 -1.63777417e+02 -1.15412219e+02 -8.18540193e+01
 -6.54047420e+01 -4.83431968e+01 -3.63863311e+01 -2.96248105e+01
  4.63655445e-14  1.78288084e-13  1.11059645e+01  4.91114620e+01
  6.64342907e+01  9.25994971e+01  1.01413404e+02  1.48782785e+02
  1.87128456e+02  2.21946170e+02  2.34283584e+02  2.62349975e+02
  2.80922163e+02  2.92709212e+02  2.99399922e+02  3.39613235e+02
  3.58319602e+02  4.04102718e+02  4.23277169e+02  4.38690758e+02
  4.72931541e+02  5.07492373e+02  5.11577217e+02  5.40440401e+02
  5.62420730e+02  5.90376424e+02  6.19289307e+02  6.23520739e+02
  6.62120820e+02  6.80666083e+02  6.87894811e+02  7.19088198e+02
  7.34998271e+02  7.85210254e+02  7.99554795e+02  8.40891654e+02
  8.59706496e+02  9.04948515e+02  9.23296090e+02  9.70877545e+02
  9.86367520e+02  1.01280301e+03  1.07439711e+03  1.10354195e+03
  1.13845770e+03  1.21809367e+03  1.27040551e+03  1.30376543e+03
  1.39149840e+03  1.40632623e+03  1.49057913e+03  1.28139567e+04]
```

Sorting eigen values in descending values and changing order of

eigen vectors correspondingly

In [45]:
```
idx = eigValues.argsort()[::-1]
eigValues = eigValues[idx]
eigVectors = eigVectors[:,idx]
```

In [46]:
```
print(eigValues)
```

```
[ 1.28139567e+04  1.49057913e+03  1.40632623e+03  1.39149840e+03
  1.30376543e+03  1.27040551e+03  1.21809367e+03  1.13845770e+03
  1.10354195e+03  1.07439711e+03  1.01280301e+03  9.86367520e+02
  9.70877545e+02  9.23296090e+02  9.04948515e+02  8.59706496e+02
  8.40891654e+02  7.99554795e+02  7.85210254e+02  7.34998271e+02
  7.19088198e+02  6.87894811e+02  6.80666083e+02  6.62120820e+02
  6.23520739e+02  6.19289307e+02  5.90376424e+02  5.62420730e+02
  5.40440401e+02  5.11577217e+02  5.07492373e+02  4.72931541e+02
  4.38690758e+02  4.23277169e+02  4.04102718e+02  3.58319602e+02
  3.39613235e+02  2.99399922e+02  2.92709212e+02  2.80922163e+02
  2.62349975e+02  2.34283584e+02  2.21946170e+02  1.87128456e+02
  1.48782785e+02  1.01413404e+02  9.25994971e+01  6.64342907e+01
  4.91114620e+01  1.11059645e+01  1.78288084e-13  4.63655445e-14
 -2.96248105e+01 -3.63863311e+01 -4.83431968e+01 -6.54047420e+01
 -8.18540193e+01 -1.15412219e+02 -1.63777417e+02 -1.68230865e+02
 -1.90942705e+02 -2.11557931e+02 -2.22261329e+02 -2.82611833e+02
 -2.98757430e+02 -3.10738072e+02 -3.47656317e+02 -3.76509059e+02
 -4.05144336e+02 -4.31936255e+02 -4.40144652e+02 -4.50460504e+02
 -5.03721547e+02 -5.10119884e+02 -5.36942209e+02 -5.79281157e+02
 -6.00216821e+02 -6.58297084e+02 -6.71493309e+02 -6.91368810e+02
 -7.15844200e+02 -7.38948305e+02 -7.79495263e+02 -8.14724711e+02
 -8.37038678e+02 -8.75651691e+02 -8.84722094e+02 -8.97465034e+02
 -9.18680023e+02 -9.76025720e+02 -1.04058313e+03 -1.05588651e+03
 -1.06525344e+03 -1.11729236e+03 -1.19797082e+03 -1.21678564e+03
 -1.24429096e+03 -1.30623412e+03 -1.32446410e+03 -1.44213332e+03]
```

In [47]:
```
eigVals = eigValues.copy()
```

Finding out number of least significant eigen Values

In [48]:
```
r = 0
index_of_small_eig_values = []
while(r<len(eigValues)):
        if eigValues[r]<1:
            index_of_small_eig_values.append(eigValues[r])
        r += 1
```

Here that number turns out to be 50

In [49]:
```
small_eig_vals = len(index_of_small_eig_values)
print(small_eig_vals)
```

```
50
```

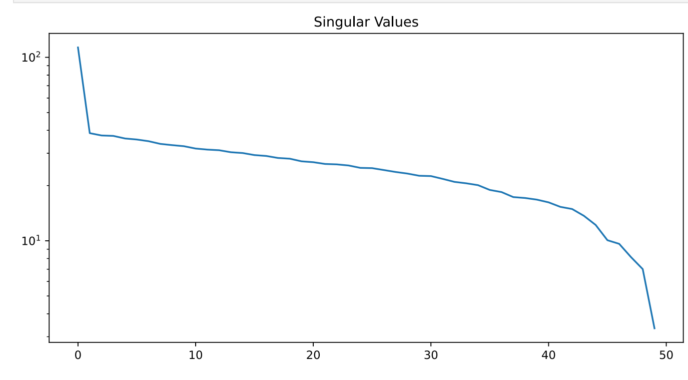
In [50]:
```
eigVals = np.array(eigVals)
```

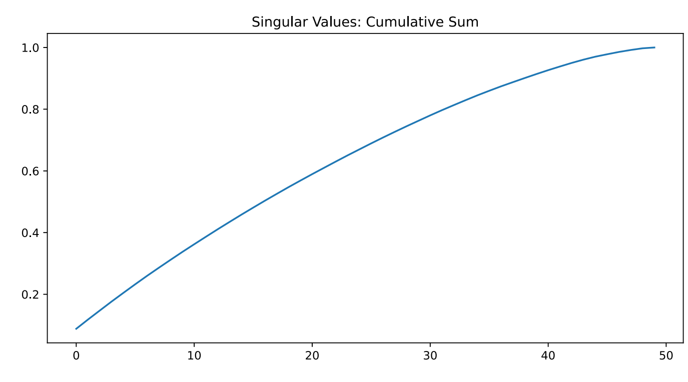
Creating Singular value matrix

In [51]:
```
D = eigVals[:-small_eig_vals]**(1/2)
```

Visualizing Singular values matrix pattern

In [52]:
```
plt.figure(1)
plt.semilogy(D)
plt.title('Singular Values')
plt.show()

plt.figure(2)
plt.plot(np.cumsum(D)/np.sum(D))
plt.title('Singular Values: Cumulative Sum')
plt.show()
```





Formint V.transpose() Matrix

In [53]:
```
Vt = eigVectors.copy().T
```

Reconstruction of Training data

xcap = X**V**Vt

In [54]:
```
Xcap = (X.dot(Vt.T)).dot(Vt)
```

In [55]:
```
print(Xcap.shape)
```

```
(4096, 100)
```

Visualizing Reconstructed Data

In [56]:
```
plt.figure(figsize=(16,20))
for i in range(1,81):
    plt.subplot(10,8,i,xticks=[],yticks=[])
    img = plt.imshow(Xcap[:,i-1].reshape(64,64).T.astype('uint8'),cmap='gray')
    plt.plot()
```

Reconstruction of Test Data

ycap = X**V**(Σ^−2)**Vt**Xtx

In [57]:
```
D_temp = np.zeros((len(D),len(D)))
for i in range(len(D)):
    D_temp[i][i] = D[i]

D = D_temp
```

In [58]:
```
for i in range(len(D)):
    print(D[i][i])
```

```
113.19874858014401
38.608019036836296
37.50101635278476
37.302793485663805
36.10769217838276
35.64274829061269
34.90119875071728
33.741038839022025
33.21960183939981
32.777997404141246
31.824566061490245
31.40648850212336
31.158907953977785
30.385787625614764
30.0823621968586
29.320751967869843
28.998131908246993
28.276399959960777
28.021603345223525
27.11085153717597
26.81581991334121
26.227748880385747
26.089578056453934
25.731708462098233
24.970397256256746
24.885524047988394
24.297662938058185
23.715411233093413
23.24737407555087
22.618072787836233
22.527591375113797
21.746989243648752
20.944945890945384
20.573700896829767
20.102306292414788
18.929331786546687
18.428598304297516
17.303176645659565
17.108746638795072
16.76073275974999
16.19722121171101
15.306324969058053
14.897857891115656
13.679490338037773
12.197654899409132
10.07042222643836
9.622863246118387
8.150723322988473
7.007957045914438
3.3325612515534706
```

In [59]:
```
invD_sq = np.linalg.inv(np.matmul(D,D))
```

In [60]:
```
print(invD_sq)
```

```
[[7.80399080e-05 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 6.70880182e-04 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 7.11072567e-04 ... 0.00000000e+00
  0.00000000e+00 0.00000000e+00]
 ...
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 1.50524675e-02
  0.00000000e+00 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  2.03618455e-02 0.00000000e+00]
 [0.00000000e+00 0.00000000e+00 0.00000000e+00 ... 0.00000000e+00
  0.00000000e+00 9.00417069e-02]]
```

In [61]:
```
print(X.shape)
print(Vt.T.shape)
print(invD_sq.shape)
print(Vt.shape)
print(X.T.shape)
print(X.shape)
```

```
(4096, 100)
(100, 100)
(50, 50)
(100, 100)
(100, 4096)
(4096, 100)
```

In [62]:
```
X_approx = X[:,:t-small_eig_vals]
Vt_approx = Vt[:t-small_eig_vals,:t-small_eig_vals]
```

In [63]:
```
print(X_approx.shape)
print(Vt_approx.T.shape)
print(invD_sq.shape)
print(Vt_approx.shape)
print(X_approx.T.shape)
print(X_approx.shape)
```

```
(4096, 50)
(50, 50)
(50, 50)
(50, 50)
(50, 4096)
(4096, 50)
```

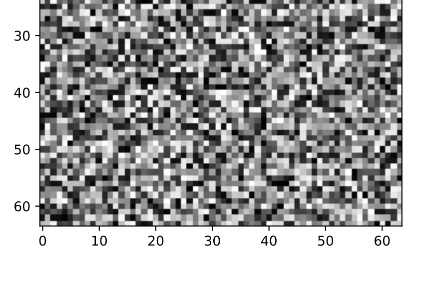
In [64]:
```
UUt = np.matmul(X_approx[:,:],np.matmul(Vt_approx.T,np.matmul(invD_sq,np.matmul(Vt_app
```

In [65]:
```
y = np.matmul(UUt,X[:,0])
```

In [66]:
```
print(y.shape)
```

```
(4096,)
```

In [67]:
```
img = plt.imshow(y.reshape(64,64).astype('uint8').T)
img.set_cmap('gray')
```



In Dual PCA, in most cases reconstruction of test data

i.e. out of sample reconstruction is not possible

Q3.1 Linear Least Square Fitting

Importing Libraries

In [1]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from prettytable import PrettyTable as ptbl
```

Importing Database

In [2]:
```python
data = pd.read_csv('Salary_Data.csv')
```

In [3]:
```python
data.describe()
```

Out[3]:

	YearsExperience	Salary
count	30.000000	30.000000
mean	5.313333	76003.000000
std	2.837888	27414.429785
min	1.100000	37731.000000
25%	3.200000	56720.750000
50%	4.700000	65237.000000
75%	7.700000	100544.750000
max	10.500000	122391.000000

Extracting Dependent and independent data from database int X and y variables

In [4]:
```python
X = data.iloc[:,0].values
y = data.iloc[:,-1].values
```

Function for Linear Least Square Fitting

y = mx + b

In [5]:
```python
def linearfitting(x,y):
    n = len(x)
    x_sq_sum = sum(x**2)
    x_sum = sum(x)
    yx_sum = sum(x*y)
    y_sum = sum(y)

    A = np.array([
        [x_sq_sum,x_sum],
        [x_sum,n]
        ])

    b = np.array([
        [yx_sum],
        [y_sum]
        ])

    invA = np.linalg.inv(A)
    M = np.matmul(invA,b)

    return M
```

Calling Linear Least Square fitting function on given database

In [6]:
```python
M = linearfitting(X,y)
m = M[0][0]
b = M[1][0]
```

Visualizing Calculated Coefficient and constant

In [7]:
```python
print("m = ",m,"\tb = ",b)
```

```
m =  9449.962321455096   b =  25792.200198668637
```

Calculating Approximate Values

In [8]:
```python
y_pred = m*X + b
```

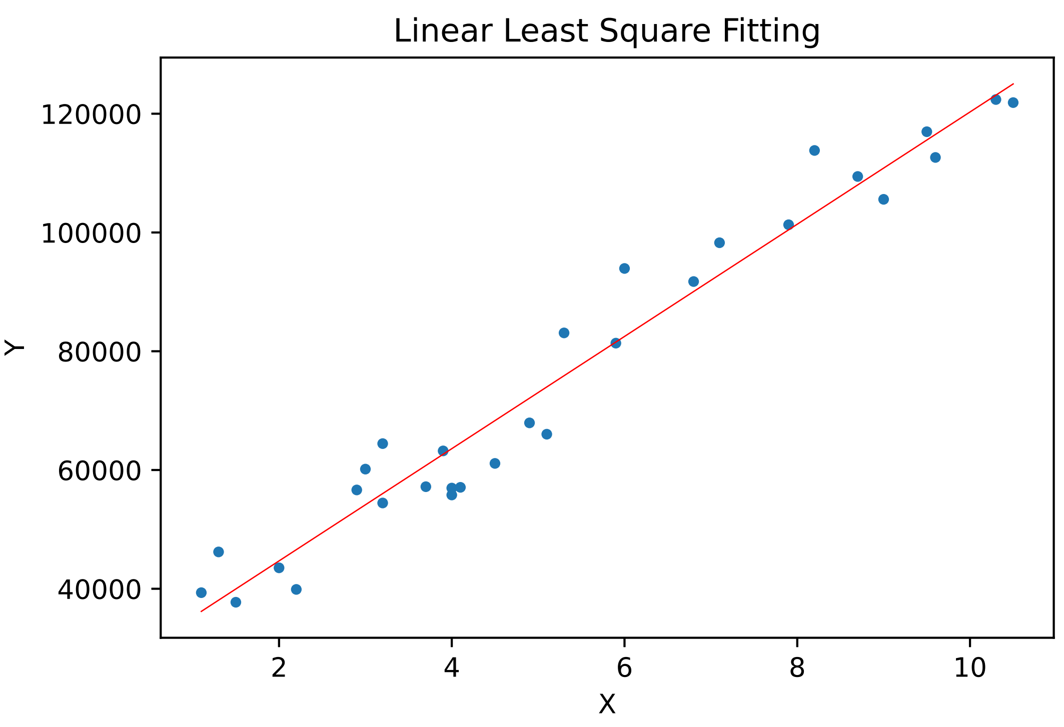
Table of actual values and predicted values

In [9]:
```python
table = ptbl(['X','y','y-predicted'])
for i in range(len(X)):
    table.add_row([X[i],y[i],y_pred[i]])
print(table)
```

```
+------+----------+--------------------+
|  X   |    y     |    y-predicted     |
+------+----------+--------------------+
| 1.1  | 39343.0  | 36187.15875226924  |
| 1.3  | 46205.0  | 38077.15121656026  |
| 1.5  | 37731.0  | 39967.14368085128  |
| 2.0  | 43525.0  | 44692.12484157883  |
| 2.2  | 39891.0  | 46582.11730586985  |
| 2.9  | 56642.0  | 53197.090930888415 |
| 3.0  | 60150.0  | 54142.087163033924 |
| 3.2  | 54445.0  | 56032.07962732494  |
| 3.2  | 64445.0  | 56032.07962732494  |
| 3.7  | 57189.0  | 60757.06078805249  |
| 3.9  | 63218.0  | 62647.05325234351  |
| 4.0  | 55794.0  | 63592.04948448902  |
| 4.0  | 56957.0  | 63592.04948448902  |
| 4.1  | 57081.0  | 64537.04571663453  |
| 4.5  | 61111.0  | 68317.03064521657  |
| 4.9  | 67938.0  |  72097.0155737986  |
| 5.1  | 66029.0  | 73987.00803808963  |
| 5.3  | 83088.0  | 75877.00050238064  |
| 5.9  | 81363.0  |  81546.9778952537  |
| 6.0  | 93940.0  | 82491.97412739921  |
| 6.8  | 91738.0  | 90051.94398456329  |
| 7.1  | 98273.0  | 92886.93268099982  |
| 7.9  | 101302.0 | 100446.9025381639  |
| 8.2  | 113812.0 | 103281.89123460042 |
| 8.7  | 109431.0 | 108006.87239532797 |
| 9.0  | 105582.0 | 110841.8610917645  |
| 9.5  | 116969.0 | 115566.84225249205 |
| 9.6  | 112635.0 | 116511.83848463756 |
| 10.3 | 122391.0 | 123126.81210965612 |
| 10.5 | 121872.0 | 125016.8045739714  |
+------+----------+--------------------+
```

Visualizing Best Fit Line

In [10]:
```python
plt.scatter(X,y, marker = '.')
plt.plot(X,y_pred,color = 'red',linewidth = 0.5)
plt.title('Linear Least Square Fitting')
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



Evaluating Error in reconstruction

In [11]:
```python
max_error = max(abs(y-y_pred)/y)
print(max_error)
```

```
0.17590842513666785
```

Q3.2 Quadratic Least Square Fitting

Importing Libraries

In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from prettytable import PrettyTable as ptbl
```

Importing database

In [2]:
```python
data = pd.read_csv('Quadratic_curve_fitting_dataset.csv')
```

Visualizing database

In [3]:
```python
data.head()
```

Out[3]:

	x	y
0	2	5
1	5	140
2	8	455
3	11	950
4	14	1625

Extracing Dependent and independent variables from database in X and y variables respectively

In [4]:
```python
X = data.iloc[:,0].values
y = data.iloc[:,1].values
```

Quadratic Least square fitting function

y = c1*x^2 + c2*x + c3

In [5]:
```python
def QuadraticFitting(x,y):

    x_four_sum = sum(x**4)
    x_three_sum = sum(x**3)
    x_sq_sum = sum(x**2)
    x_sum = sum(x)
    n = len(x)

    y_xsq_sum = sum(y*(x**2))
    yx_sum = sum(x*y)
    y_sum = sum(y)

    A = np.array([
        [x_four_sum,  x_three_sum,  x_sq_sum],
        [x_three_sum, x_sq_sum,     x_sum],
        [x_sq_sum,    x_sum,        n],
        ])

    b = np.array([
        [y_xsq_sum],
        [yx_sum],
        [y_sum]
        ])

    invA = np.linalg.inv(A)
    M = np.matmul(invA,b)

    return M
```

Calling Quadratic least square fitting function on given database

In [6]:
```python
c1, c2, c3 = QuadraticFitting(X,y)
```

Visualizing coefficients and constants

In [7]:
```python
print(f"c1 = {c1}\tc2 = {c2}\tc3 = {c3}")
```

```
c1 = [10.]      c2 = [-25.]     c3 = [15.]
```

Calculating Approximate Values

In [8]:
```python
y_pred = c1*(X**2) + c2*X + c3
```

Table of actual values and predicted values

In [9]:
```python
table = ptbl(['X','y','y-predicted'])
for i in range(len(X)):
    table.add_row([X[i],y[i],y_pred[i]])
print(table)
```

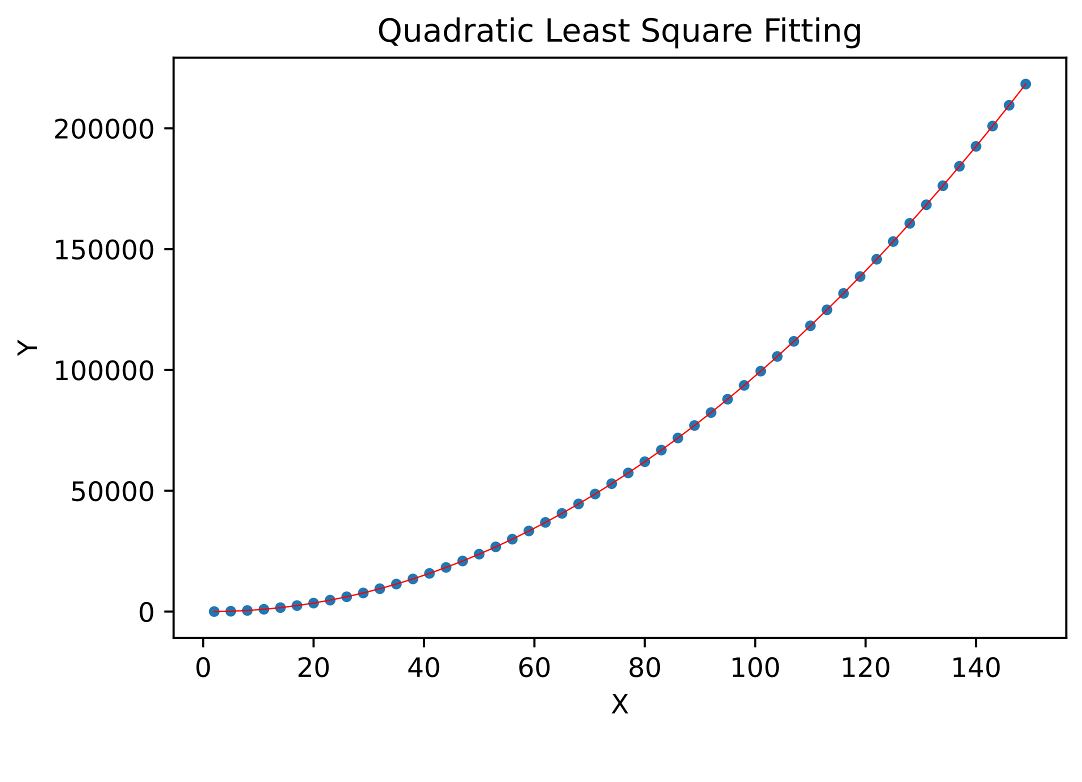
```
+-----+--------+--------------------+
|  X  |   y    |    y-predicted     |
+-----+--------+--------------------+
|  2  |   5    | 5.000000000204238  |
|  5  |  140   | 140.00000000016325 |
|  8  |  455   | 455.000000001246   |
|  11 |  950   | 950.000000000882   |
|  14 |  1625  | 1625.0000000000541 |
|  17 |  2480  | 2480.0000000000223 |
|  20 |  3515  | 3514.9999999999927 |
|  23 |  4730  | 4729.999999999965  |
|  26 |  6125  | 6124.999999999941  |
|  29 |  7700  | 7699.999999999918  |
|  32 |  9455  | 9454.999999999898  |
|  35 | 11390  | 11389.99999999988  |
|  38 | 13505  | 13504.999999999865 |
|  41 | 15800  | 15799.99999999985  |
|  44 | 18275  | 18274.99999999984  |
|  47 | 20930  | 20929.999999999833 |
|  50 | 23765  | 23764.999999999825 |
|  53 | 26780  | 26779.99999999982  |
|  56 | 29975  | 29974.999999999818 |
|  59 | 33350  | 33349.99999999982  |
|  62 | 36905  | 36904.999999999825 |
|  65 | 40640  | 40639.999999999825 |
|  68 | 44555  | 44554.99999999983  |
|  71 | 48650  | 48649.99999999985  |
|  74 | 52925  | 52924.999999999854 |
|  77 | 57380  | 57379.99999999987  |
|  80 | 62015  | 62014.99999999988  |
|  83 | 66830  | 66829.99999999991  |
|  86 | 71825  | 71824.99999999993  |
|  89 | 77000  | 76999.99999999996  |
|  92 | 82355  | 82354.99999999997  |
|  95 | 87890  |      87890.0       |
|  98 | 93605  | 93605.00000000003  |
| 101 | 99500  | 99500.00000000007  |
| 104 | 105575 | 105575.0000000001  |
| 107 | 111830 | 111830.00000000015 |
| 110 | 118265 | 118265.00000000017 |
| 113 | 124880 | 124880.00000000022 |
| 116 | 131675 | 131675.00000000026 |
| 119 | 138650 | 138650.0000000003  |
| 122 | 145805 | 145805.00000000035 |
| 125 | 153140 | 153140.0000000004  |
| 128 | 160655 | 160655.00000000047 |
| 131 | 168350 | 168350.00000000052 |
| 134 | 176225 | 176225.00000000058 |
| 137 | 184280 | 184280.00000000064 |
| 140 | 192515 | 192515.0000000007  |
| 143 | 200930 | 200930.00000000076 |
| 146 | 209525 | 209525.00000000084 |
| 149 | 218300 | 218300.00000000093 |
+-----+--------+--------------------+
```

Visualizing Best Fit Curve

Note: The database used here was generated by me using Microsoft EXCEL

that's why the actual points are perfectly overlapping with approximate line

In [10]:
```python
plt.scatter(X,y, marker = '.')
plt.plot(X,y_pred,color = 'red',linewidth = 0.5)
plt.title('Quadratic Least Square Fitting')
plt.xlabel('X')
plt.ylabel('Y')
plt.show()
```



Evaluating Error in reconstruction

In [11]:
```python
max_error = max(abs(y-y_pred)/y)
print(max_error)
```

```
4.084768079337664e-11
```

Error is less because the data was generate using excel

In [12]:
```python
for i in range(5):
    print(f"y[{i}] = {y[i]}\ty_predict[{i}] = {y_pred[i]}")
```

```
y[0] = 5        y_predict[0] = 5.000000000204238
y[1] = 140      y_predict[1] = 140.00000000016325
y[2] = 455      y_predict[2] = 455.000000001246
y[3] = 950      y_predict[3] = 950.000000000882
y[4] = 1625     y_predict[4] = 1625.0000000000541
```

Q4. Denoising Using L2-Regularisation

In [1]:

```
from matplotlib.image import imread
import matplotlib.pyplot as plt
import numpy as np
import os
plt.rcParams['figure.figsize'] = [12,6]
```

Importing and Visualizing input image

In [2]:

```
Oimg = imread('dog.jpg')
print(Oimg.shape)
img = plt.imshow(Oimg)
plt.axis('off')
img.set_cmap('gray')
plt.title("Original Image")
```

```
(2000, 1500, 3)
```

Out[2]:

```
Text(0.5, 1.0, 'Original Image')
```

Original Image

In [3]:

```
Oimg = np.mean(Oimg,-1)                  # Converting to Grayscale
```

Adding Gaussian Noise

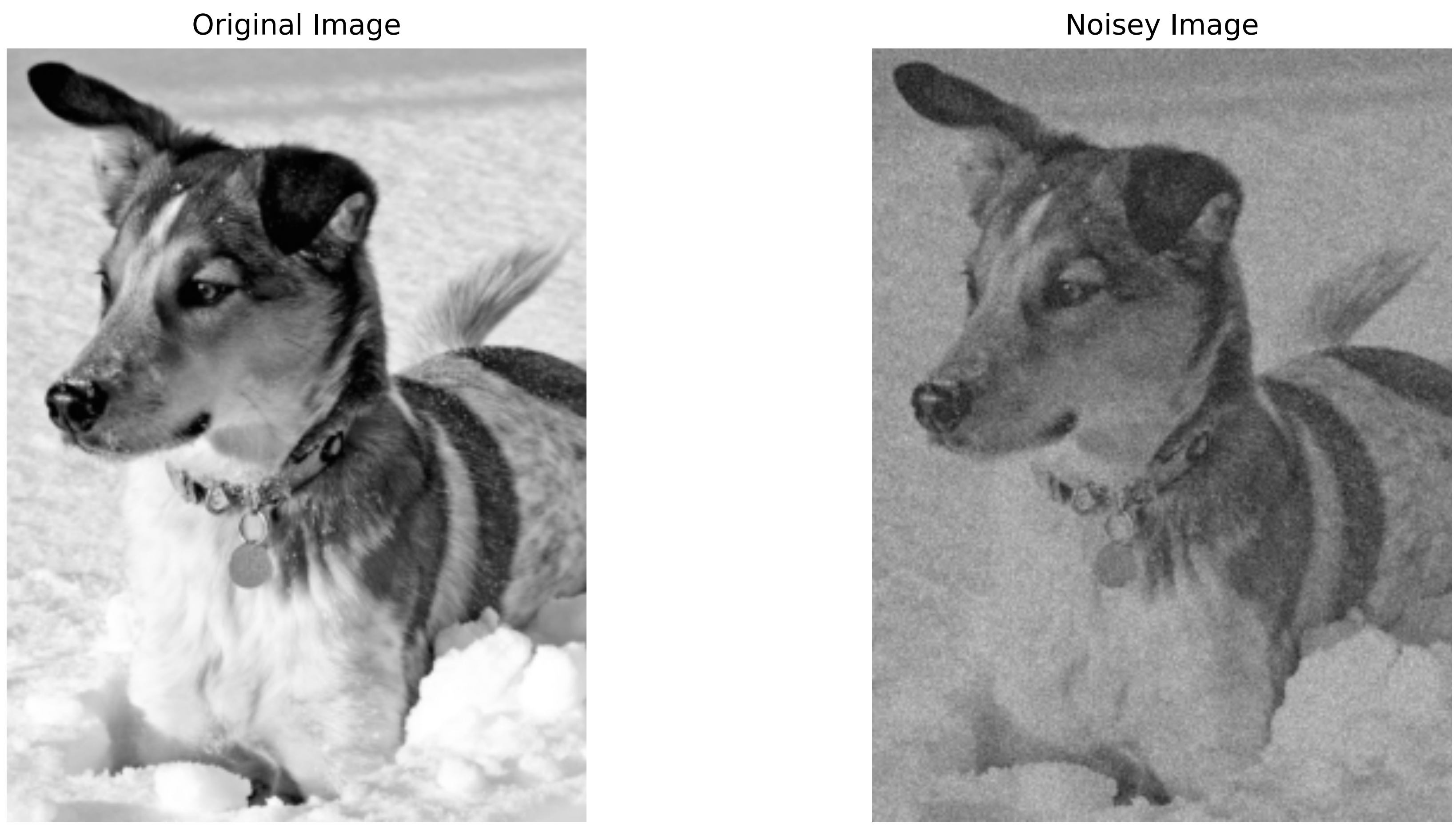
In [4]:

```
mean = 0
sigma = 3

Noise = np.random.normal(mean, sigma, (Oimg.shape[0],Oimg.shape[1])).astype('uint8')
OimgNoisey = Oimg + Noise            # Add some noise
```

Visualizing Noise and original image

In [5]:

```
plt.figure(1)
plt.subplot(121)
img = plt.imshow(Oimg)
plt.axis('off')
img.set_cmap('gray')
plt.title("Original Image")

plt.subplot(122)
img2 = plt.imshow(OimgNoisey)
plt.axis('off')
img2.set_cmap('gray')
plt.title("Noisey Image")
plt.show()
```



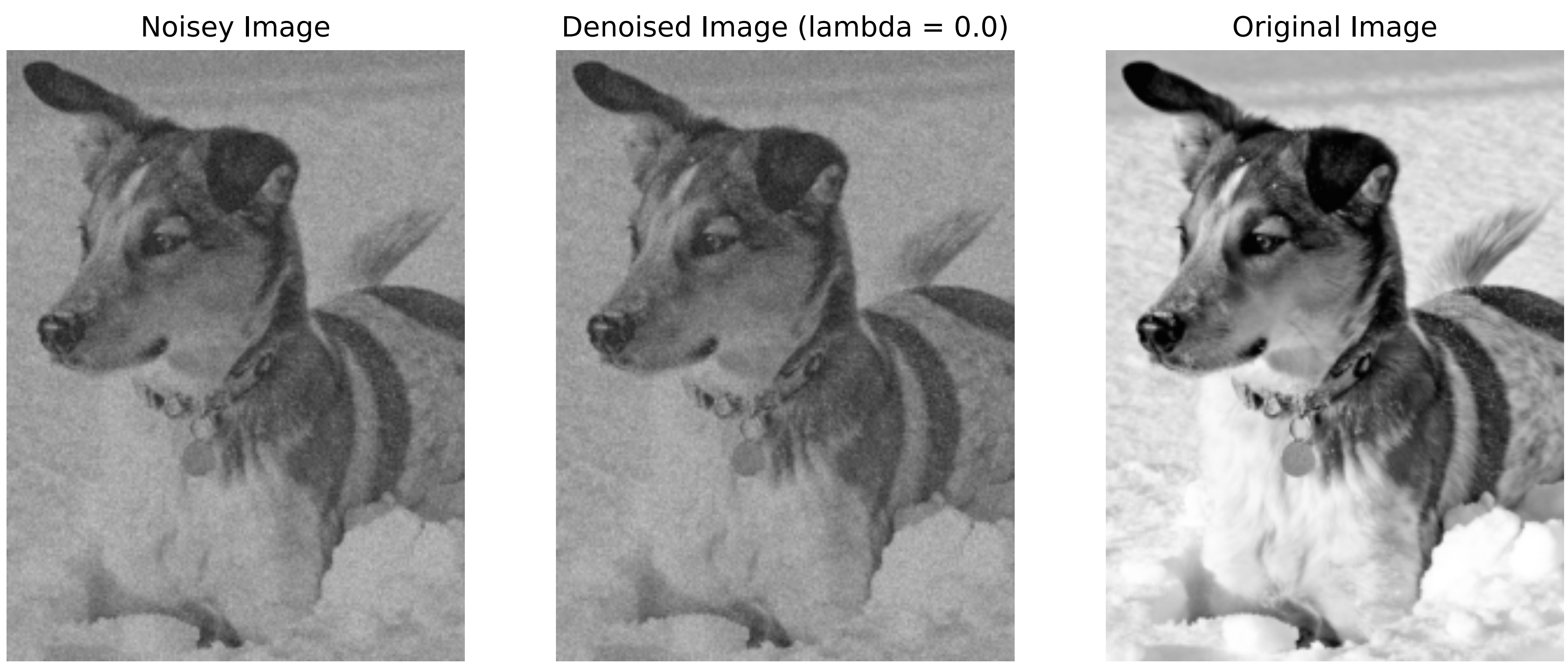
L2-regularisation Function

In [6]:

```
def L2Regularisation(NoiseyInput, ExpectedOutput,factor):

    n = len(ExpectedOutput)
    I = np.identity(n)
    A = I
    At = A.T
    AtA = np.matmul(At,A)
    M = (AtA - factor*I)

    T = np.matmul(np.linalg.inv(M),At)
    pred = np.matmul(T,NoiseyInput)

    plt.figure()
    plt.subplot(131)
    img1 = plt.imshow(NoiseyInput)
    img1.set_cmap('gray')
    plt.axis('off')
    plt.title(f'Noisey Image')

    plt.subplot(132)
    img2 = plt.imshow(pred)
    img2.set_cmap('gray')
    plt.axis('off')
    plt.title(f'Denoised Image (lambda = {factor})')

    plt.subplot(133)
    img3 = plt.imshow(ExpectedOutput)
    img3.set_cmap('gray')
    plt.axis('off')
    plt.title('Original Image')
    plt.show()
```

In [7]:

```
fact = np.arange(0,1,0.2)
for i in fact:
    L2Regularisation(OimgNoisey,Oimg,i)
```




Noisey Image
Denoised Image (lambda = 0.2)
Original Image
Noisey Image
Denoised Image (lambda = 0.4)
Original Image
Noisey Image
Denoised Image (lambda = 0.6000000000000001)
Original Image

Q5 Linear Discriminant Analysis of IRIS database

Iris dataset contains 4-features (i.e. Sepal length, Sepal width, Petal length, Petal Width)

of flowers of 3 - species (i.e. Setosa, Versicolor, Virginica)

Importing Libraries

In [1]:
```python
import math
import copy
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.preprocessing import LabelEncoder
```

Importing IRIS Dataset

In [2]:
```python
dataset = pd.read_csv('Iris.csv')
X = dataset.iloc[:,1:-1].values
y = dataset.iloc[:,-1].values
```

Encoding Class Labels using sklearn's labe encoder

In [3]:
```python
y_prev = copy.copy(y)

le = LabelEncoder()
le_y = le.fit(y)
y = le_y.transform(y) + 1

y_prev_vs_y_encoded = np.hstack((y_prev.reshape(len(y_prev),1), y.reshape(len(y),1)))
print(y_prev_vs_y_encoded)
```

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[['Iris-setosa' 1]
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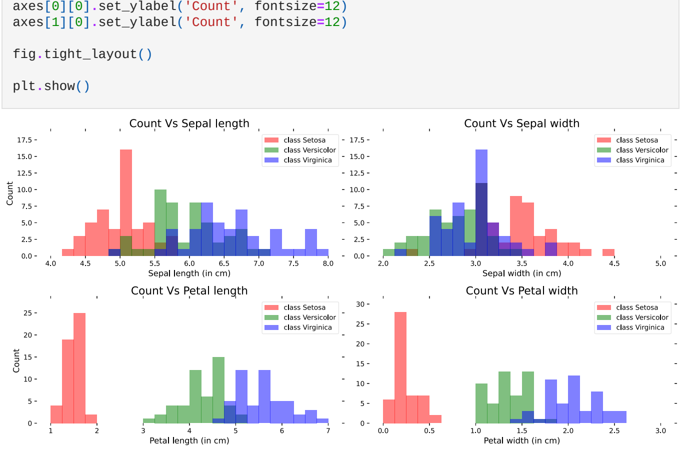
```

Creating Feature and Class Label Dictonaries

In [4]:
```python
label_dict = {1: 'Setosa', 2: 'Versicolor', 3:'Virginica'}

feature_dict = {i:label for i,label in zip(
                range(4),
                  ('Sepal length',
                  'Sepal width',
                  'Petal length',
                  'Petal width', ))}
```

Visualizing Dataset using histogram

In [5]:
```python
fig, axes = plt.subplots(nrows=2, ncols=2, figsize=(14, 7))

for ax,cnt in zip(axes.ravel(), range(4)):

    # set bin sizes
    min_b = math.floor(np.min(X[:,cnt]))
    max_b = math.ceil(np.max(X[:,cnt]))
    bins = np.linspace(min_b, max_b, 25)

    # plottling the histograms
    for lab,col in zip(range(1,4), ('red', 'green', 'blue')):
        ax.hist(X[y==lab, cnt],
                   color=col,
                   label=f'class {label_dict[lab]}',
                   bins=bins,
                   alpha=0.5,)
    ylims = ax.get_ylim()

    # plot annotation
    leg = ax.legend(loc='upper right', fancybox=True, fontsize=10)
    leg.get_frame().set_alpha(0.5)
    ax.set_ylim([0, max(ylims)+2])
    ax.set_xlabel(feature_dict[cnt]+' (in cm)', fontsize=12)
    ax.set_title(f'Count Vs {feature_dict[cnt]}', fontsize=16)

    # hide axis ticks
    ax.tick_params(axis="both", which="both", bottom="off", top="off",
            labelbottom="on", left="off", right="off", labelleft="on")

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

axes[0][0].set_ylabel('Count', fontsize=12)
axes[1][0].set_ylabel('Count', fontsize=12)

fig.tight_layout()

plt.show()
```



Peforming Linear Discriminant Analysis

Step 1: Computing the d-dimensional mean vectors

In [6]:
```python
np.set_printoptions(precision = 4)

mean_vectors = []
for col in range(1,4):
    mean_vectors.append(np.mean(X[y==col], axis=0))
    print(f'Mean Vector class {col}: {mean_vectors[col-1]}\n')
```

```
Mean Vector class 1: [5.006 3.418 1.464 0.244]

Mean Vector class 2: [5.936 2.77  4.26  1.326]

Mean Vector class 3: [6.588 2.974 5.552 2.026]
```

Step 2: Computing the Scatter Matrices

2.1 Within-class scatter matrix S_W

In [7]:
```python
S_W = np.zeros((4,4))
for cl,mv in zip(range(1,4), mean_vectors):
    class_sc_mat = np.zeros((4,4))                  # scatter matrix for every class
    for row in X[y == cl]:
        row, mv = row.reshape(4,1), mv.reshape(4,1) # make column vectors
        class_sc_mat += (row-mv).dot((row-mv).T)
    S_W += class_sc_mat                             # sum class scatter matrices
print('Within-class Scatter Matrix:\n', S_W)
```

```
Within-class Scatter Matrix:
 [[38.9562 13.683  24.614   5.6556]
 [13.683  17.035   8.12    4.9132]
 [24.614   8.12   27.22    6.2536]
 [ 5.6556  4.9132  6.2536  6.1756]]
```

2.2 Between-class scatter matrix S_B

In [8]:
```python
overall_mean = np.mean(X, axis=0)

S_B = np.zeros((4,4))
for i,mean_vec in enumerate(mean_vectors):
    n = X[y==i+1,:].shape[0]
    mean_vec = mean_vec.reshape(4,1) # make column vector
    overall_mean = overall_mean.reshape(4,1) # make column vector
    S_B += n * (mean_vec - overall_mean).dot((mean_vec - overall_mean).T)

print('between-class Scatter Matrix:\n', S_B)
```

```
between-class Scatter Matrix:
 [[ 63.2121 -19.534  165.1647  71.3631]
 [-19.534   10.9776 -56.0552 -22.4924]
 [165.1647 -56.0552 436.6437 186.9081]
 [ 71.3631 -22.4924 186.9081  80.6041]]
```

Step 3: Solving the generalized eigenvalue problem for the matrix (S_W^-1)*(S_B)

In [9]:
```python
eig_vals, eig_vecs = np.linalg.eig(np.linalg.inv(S_W).dot(S_B))

for i in range(len(eig_vals)):
    eigvec_sc = eig_vecs[:,i].reshape(4,1)
    print('\nEigenvector {}: \n{}'.format(i+1, eigvec_sc.real))
    print('Eigenvalue {:}: {:.2e}'.format(i+1, eig_vals[i].real))
```

```
Eigenvector 1:
[[ 0.2049]
 [ 0.3871]
 [-0.5465]
 [-0.7138]]
Eigenvalue 1: 3.23e+01

Eigenvector 2:
[[-0.009 ]
 [-0.589 ]
 [ 0.2543]
 [-0.767 ]]
Eigenvalue 2: 2.78e-01

Eigenvector 3:
[[-0.8379]
 [ 0.1696]
 [ 0.1229]
 [ 0.5041]]
Eigenvalue 3: -4.13e-15

Eigenvector 4:
[[ 0.2   ]
 [-0.3949]
 [-0.4567]
 [ 0.7717]]
Eigenvalue 4: 1.20e-14
```

Checking Eigen Value and Eigen Vector Calculations

In [10]:
```python
for i in range(len(eig_vals)):
    eigvec = eig_vecs[:,i].reshape(4,1)
    np.testing.assert_array_almost_equal(np.linalg.inv(S_W).dot(S_B).dot(eigvec),
                                         eig_vals[i] * eigvec,
                                         decimal=6, err_msg='', verbose=True)
print('ok')
```

```
ok
```

Step 4: Selecting linear discriminants for the new feature subspace

4.1. Sorting the eigenvectors by decreasing eigenvalues

In [11]:
```python
# Make a list of (eigenvalue, eigenvector) tuples
eig_pairs = [(np.abs(eig_vals[i]), eig_vecs[:,i]) for i in range(len(eig_vals))]

# Sort the (eigenvalue, eigenvector) tuples from high to low
eig_pairs = sorted(eig_pairs, key=lambda k: k[0], reverse=True)

# Visually confirm that the list is correctly sorted by decreasing eigenvalues

print('Eigenvalues in decreasing order:\n')
for i in eig_pairs:
    print(i[0])
```

```
Eigenvalues in decreasing order:

32.27195779972981
0.27756686384004264
1.1953730364935478e-14
4.1311796919088535e-15
```

In [12]:
```python
print('Variance Cotained by Eigen Values (in percentage):\n')
eigv_sum = sum(eig_vals)
for i,j in enumerate(eig_pairs):
    print('eigenvalue {0:}: {1:.2%}'.format(i+1, (j[0]/eigv_sum).real))
```

```
Variance Cotained by Eigen Values (in percentage):

eigenvalue 1: 99.15%
eigenvalue 2: 0.85%
eigenvalue 3: 0.00%
eigenvalue 4: 0.00%
```

4.2. Choosing k eigenvectors with the largest eigenvalues

In [13]:
```python
W = np.hstack((eig_pairs[0][1].reshape(4,1), eig_pairs[1][1].reshape(4,1)))
print('Matrix W:\n', W.real)
```

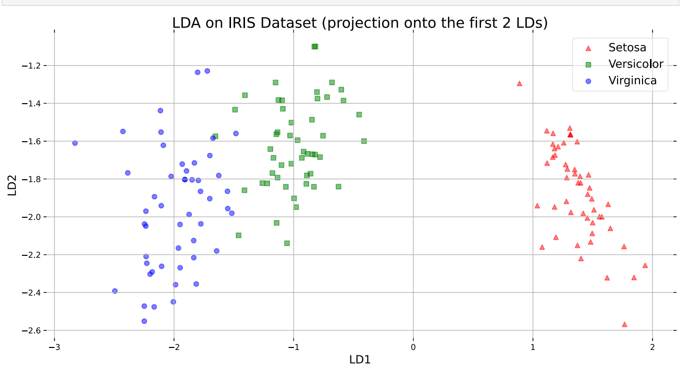
```
Matrix W:
 [[ 0.2049 -0.009 ]
 [ 0.3871 -0.589 ]
 [-0.5465  0.2543]
 [-0.7138 -0.767 ]]
```

Step 5: Transforming the samples onto the new subspace

In [14]:
```python
X_lda = X.dot(W)
```

In [15]:
```python
def lda_plot():
    plt.rcParams['figure.figsize'] = [14,7]
    ax = plt.subplot(111)
    for label,marker,color in zip(
        range(1,4),('^', 's', 'o'),('red', 'green', 'blue')):

        plt.scatter(x=X_lda[:,0].real[y == label],
                y=X_lda[:,1].real[y == label],
                marker=marker,
                color=color,
                alpha=0.5,
                label=label_dict[label]
                )

    plt.xlabel('LD1', fontsize=14)
    plt.ylabel('LD2', fontsize=14)

    leg = plt.legend(loc='upper right', fancybox=True, fontsize=14)
    leg.get_frame().set_alpha(0.5)
    plt.title('LDA on IRIS Dataset (projection onto the first 2 LDs)', fontsize=18)

    # hide axis ticks
    plt.tick_params(axis="both", which="both", bottom="off", top="off", labelbottom="o

    # remove axis spines
    ax.spines["top"].set_visible(False)
    ax.spines["right"].set_visible(False)
    ax.spines["bottom"].set_visible(False)
    ax.spines["left"].set_visible(False)

    plt.grid()
    plt.tight_layout
    plt.show()
```

In [16]:
```python
lda_plot()
```



We get perfect separation/Classification of data of all 3-classes.