

**MA859: Selected Topics in Graph Theory**  
**Assignment 2**

Date: March 23, 2021

Maximum marks: 50

1. Justify whether the following statement is TRUE or FALSE:  
Every graph with fewer edges than vertices has a component that is a tree.
2. Prove that a graph  $G$  is a tree if and only if for all  $x, y \in V(G)$ , adding a copy of  $xy$  as an edge creates exactly one cycle.
3. If  $x$  and  $y$  are adjacent vertices in a connected graph  $G$ , then show that  $|d(x, z) - d(y, z)| \leq 1$  for any vertex  $z$  in  $G$ .
4. The square of a simple graph  $G$  is a graph  $G'$  where two vertices  $x$  and  $y$  are adjacent in  $G'$  if and only if  $d_G(x, y) \leq 2$ . Show that the square of a connected graph  $G$  has diameter  $\left\lceil \frac{\text{diam}(G)}{2} \right\rceil$ .
5. Prove that if an  $n$ -vertex graph  $G$  has  $n - 1$  edges and no cycles, then it is connected.
6. Show that every non-trivial tree has at least two maximal independent sets, with equality only for star graphs.
7. Show that among the trees with  $n$  vertices, the star graph has the most independent sets.
8. Show that an edge of a connected graph  $G$  is a cut-edge (bridge) if and only if it belongs to every spanning tree.
9. Show that every tree on even number of vertices has exactly one subgraph in which every vertex has odd degree.
10. Show that a connected graph with  $n$  vertices has exactly one cycle if and only if it has exactly  $n$  edges.