15. The characteristic equation is $\lambda^2 + (1/(RC)\lambda + 1/(LC) = 0$ which has solutions

$$\frac{-1/(RC) \pm \sqrt{1/(R^2C^2) - 4/(LC)}}{2}$$

Complex solutions exist if $1/(R^2C^2) - 4/LC < 0$. This leads to $R^2 > L/(4C)$.

PROBLEMS 6.1

1. (**a**) H has eigenvalue 1, of multiplicity 10.

(**b**) The characteristic equation for $H + E$ is

$$(1-\lambda)^{10} - \frac{1}{2^{10}} = 0 \qquad \lambda = \frac{1}{2}$$

satisfies the equation.

(**c**) $\|E\|_F = (\Sigma|e_{ij}|^2)^{1/2} = \left(\left(\frac{1}{2^{10}}\right)^2\right)^{1/2}$

Note that $\|E\|_F = 1/2^{10}$, but $|1 - \frac{1}{2}|$ is not less than $\|E\|_F$.

(**d**) H and E are not symmetric.

3. Consider a matrix A as a vector $\tilde{A} = (a_{11}, \dots, a_{1n}, a_{21}, \dots, a_{2n}, \dots, a_{n1}, \dots, a_{nn})$ in E^{n^2}. Now $\|A\|_F$ is just the standrad form of $\tilde{A}$ in E^{n^2}. Reasoning this way $\|A + B\|_F = \|\tilde{A} + \widetilde{B}\|$ and $\|\tilde{A} + \widetilde{B}\| \leq \|\tilde{A}\| + \|\widetilde{B}\|$ by the triangle inequality. Thus $\|A + B\|_F \leq \|A\|_F + \|B\|_F$ for any two $n \times n$ matrices. Thus $\|D\|_F + \|I\|_F \geq \|D + I\|_F$.

5. $\|A\|_1 = 5, \|A\|_F = \sqrt{4 + 4 + 1} = 3$. The 1 norm is larger.

PROBLEMS 6.2

1. After the step with A^6 calculated the approximate dominant eigenpair is

$$\left(9.09, \begin{pmatrix} 1 \\ 1.62 \end{pmatrix}\right)$$

3. No dominant eigenpair. A has all complex eigenvalues.

5. After the step with A^6 calculated the approximate dominant eigenpair is

$$\left(7.16, \begin{pmatrix} 1 \\ 1.39 \\ 0 \end{pmatrix}\right)$$

7. As described in this section, the power method will not find complex eigenvalues. However the method can be modified. For example consider

$$A = \begin{pmatrix} 0 & 2 \\ 1 & 3i \end{pmatrix}$$

The dominant eigenpair is

$$\left(2i, \begin{pmatrix} -i \\ 1 \end{pmatrix}\right)$$

Choose

$$X_0 = \begin{pmatrix} i \\ 0 \end{pmatrix}$$

and compute $AX_0, A^2X_0, \ldots$ as before. We find, using the trick of scaling that the scaled versions of A^6X_0 and A^7X_0 are

$$\begin{pmatrix} -.984i \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -.992i \\ 1 \end{pmatrix}$$

so we can choose

$$\begin{pmatrix} -.992i \\ 1 \end{pmatrix}$$

as an approximate eigenvector.

Then we calculate

$$A\begin{pmatrix} -.992i \\ 1 \end{pmatrix}$$

and see if it is nearly a multiple of

$$\begin{pmatrix} -.992i \\ 1 \end{pmatrix}$$

We have

$$A\begin{pmatrix} -.992i \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2.008i \end{pmatrix}$$

Now $2/.992i = 2.016i$; $2.008i/1 = 2.008i$. These ratios are nearly equal. We could use their average $2.012i$ as an approximate eigenvalue.

So we see that a power rule can work for matrices with complex eigenvalues. However if A has all real entries and complex eigenvalues, they must occur in conjugate pairs which is trouble as far as dominance is concerned. In any case complex arithmetic must be used on the computer.

9. (1) $\left(-2.09, \begin{pmatrix} -1.611 \\ 1 \end{pmatrix}\right)$

(2)

$$(\lambda_2, X_2) = \left(2, K\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$$

(λ_3, X_3) are not found; lack of symmetry causes this difficulty.

(5) $(\lambda_2, X_2) = \left(6, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right)$ $\quad (\lambda_3, X_3) = \left(.841, \begin{pmatrix} -1.41 \\ 1 \\ 0 \end{pmatrix}\right)$

PROBLEMS 6.3

1. $QR = \begin{pmatrix} -.1960 & -.9806 \\ -.9806 & .1960 \end{pmatrix}\begin{pmatrix} -5.099 & -6.864 \\ 0 & -3.727 \end{pmatrix}$

3. $QR = \begin{pmatrix} -.7072 & .7072 \\ .7072 & .7071 \end{pmatrix}\begin{pmatrix} -2.8288 & -1.4144 \\ 0 & 2.8287 \end{pmatrix}$

5. $QR = \begin{pmatrix} -.7072 & -.7072 & 0 \\ -.7072 & .7071 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -4.2432 & -5.6576 & 0 \\ 0 & 1.4139 & 0 \\ 0 & 0 & 6 \end{pmatrix}$