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Assignment Number: 4

Havel Hakimi Theorem:

It states that,

‘The non-increasing sequence $(d_1,d_2,...,d_n)$ is graphic if and only if the sequence $(d_{2-1}$, d_{3-1},..., $d_{d1+1}-1$, $d_{d1}+2$, $d_{d1}+3$,..., $d_n)$ is also graphic.

The Havel-Hakimi algorithm goes as follows:

- Arrange vertices with non-increasing order of degrees.
- Pick 1^{st} vertex from the sequence and subtract 1 from degree of 2^{nd} till $(deg(1^{st})+1)^{th}$ vertex.
- Now 1^{st} vertex has been exhaust so remove it from sequence.
- Repeat steps 1 to 3 till you exhaust all the vertices.
- If all the vertices get exhausted, then the sequence has reduced to all zeroes and hence the sequence is graphic.
- If you end up with non-zero degree vertices that can't be exhausted further, then the sequence isn't graphic.

Examples:

1) D = {5,3,2,2,2,2,1}

Ans:

Using Havel-Hakimi algorithm,

Iteration 1:

Here the degree sequence is already sorted in non-increasing order, so we don’t have to sort it again.

Taking 1^{st} vertex degree from sequence. Here it is 5 so subtract 1 from next 5 vertex degrees in the sequence.

$\pi' = \{2,1,1,1,1,1\}$

Again obtained degree sequence is in sorted order so this becomes π_1

$\pi_{1\,=}$ = {2,1,1,1,1,1}

Iteration 2:

Again repeating above procedure we get,

$\pi_1' = \{0,0,1,1,1\}$

sorting π_1' in non-increasing order we get,

$\pi_{2\,=}$ = {1,1,1,0,0}

Iteration 3:

Again repeating above procedure we get,

$\pi_1' = \{0,0,1,1,1\}$

sorting π_1' in non-increasing order we get,

$\pi_{2\,=}$ = {1,1,1,0,0}

Iteration 4:

Again repeating above procedure we get,

$\pi_2' = \{0,1,0,0\}$

sorting π_1' in non-increasing order we get,

$\pi_{3\,=}$ = {1,0,0,0}

Iteration 5:

Again repeating above procedure we get,

$\pi_2' = \{-1,0,0\}$

Here we got -1 in degree sequence.

This means that the vertices with given degree sequence are **not Graphic.**

2) D = {8,8,7,7,6,6,4,3,2,1,1,1}

Ans:

Using Havel-Hakimi algorithm we get following,

D = {8,8,7,7,6,6,4,3,2,1,1,1}

Iteration 1:

$\pi' = \{7,6,6,5,5,3,2,1,1,1,1\}$

Here degree sequnce is sorted in non-increasing order. Thus,

$\pi_1 = \{7,6,6,5,5,3,2,1,1,1,1\}$

Iteration 2:

$\pi_1' = \{5,5,4,4,2,1,0,1,1,1\}$

Sorting degree sequence in non-increasing order we get,

π_2 = {6,5,5,4,4,2,1,1,1,1,0}

Iteration 3:

π_2' = {4,4,3,3,1,0,1,1,1,0}

Sorting degree sequence in non-increasing order we get,

π_3 = {4,4,3,3,1,1,1,1,0,0}

Iteration 4:

π_3' = {3,2,2,0,1,1,1,0,0}

Sorting degree sequence in non-increasing order we get,

π_4 = {3,2,2,1,1,1,0,0,0}

Iteration 5:

π_4' = {1,1,0,1,1,0,0,0}

Sorting degree sequence in non-increasing order we get,

π_5 = {1,1,1,1,0,0,0}

Iteration 6:

π_5' = {0,1,1,0,0,0}

Sorting degree sequence in non-increasing order we get,

π_6 = {1,1,0,0,0,0}

Iteration 7:

π_6' = {0,0,0,0,0}

As the degree sequence reduced to zeros, thus the it is possible to draw a graph with given vertex degree sequence (i.e. given degree sequence is **graphic**).

3) **D = {3, 3, 2, 2, 1, 1}**

Ans:

Using Havel-Hakimi Algorithm,

D = {3, 3, 2, 2, 1, 1}

Iteration 1:

π' = { 2, 1, 1, 1, 1}

Here degree sequnce is sorted in non-increasing order. Thus,

π_1 = { 2, 1, 1, 1, 1}

Iteration 2:

π_1' = { 0, 0, 1, 1}

Sorting degree sequence in non-increasing order we get,

π_2 = {1,1,0,0}

Iteration 3:

$\pi_2' = \{0,0,0\}$

As the degree sequence reduced to zeros, thus the it is possible to draw a graph with given vertex degree sequence (i.e. given degree sequence is **graphic**).

4) **D = {**4, 3, 3, 3, 1}

Ans:

Using Havel-Hakimi Algorithm,

D = **{**4, 3, 3, 3, 1}

Iteration 1:

$\pi' = \{2, 2, 2, 0\}$

Here degree sequnce is sorted in non-increasing order. Thus,

$\pi_1 = \{2, 2, 2, 0\}$

Iteration 2:

$\pi_1' = \{1, 1, 0\}$

Here degree sequnce is sorted in non-increasing order. Thus,

$\pi_2 = \{1,1,0\}$

Iteration 3:

$\pi_2' = \{0,0\}$

As the degree sequence reduced to zeros, thus the it is possible to draw a graph with given vertex degree sequence (i.e. given degree sequence is **graphic**).

5) **D = {4,4,4,4,1,1,1,1,1,1,1,1,1,1}**

Ans:

Using Havel-Hakimi algorithm we get following,

D = {4,4,4,4,1,1,1,1,1,1,1,1,1,1}

Iteration 1:

$\pi' = \{3,3,3,0,1,1,1,1,1,1,1,1,1,1\}$

Sorting degree sequence in non-increasing order we get,

$\pi_1 = \{3,3,3,1,1,1,1,1,1,1,1,1,0\}$

Iteration 2:

$\pi_1' = \{2,2,0,1,1,1,1,1,1,1,1,0\}$

Sorting degree sequence in non-increasing order we get,

$\pi_2 = \{2,2,1,1,1,1,1,1,1,1,0,0\}$

Iteration 3:

$\pi_2' = \{1,0,1,1,1,1,1,1,0,0\}$

Sorting degree sequence in non-increasing order we get,

$\pi_3 = \{1,1,1,1,1,1,1,1,0,0,0\}$

Iteration 4:

$\pi_3' = \{0,1,1,1,1,1,1,0,0,0\}$

Sorting degree sequence in non-increasing order we get,

$\pi_4 = \{1,1,1,1,1,1,0,0,0,0\}$

Iteration 5:

$\pi_4' = \{0,1,1,1,1,0,0,0,0\}$

Sorting degree sequence in non-increasing order we get,

$\pi_5 = \{1,1,1,1,0,0,0,0,0\}$

Iteration 6:

$\pi_5' = \{0,1,1,0,0,0,0,0\}$

Sorting degree sequence in non-increasing order we get,

$\pi_6 = \{1,1,0,0,0,0,0,0\}$

Iteration 7:

$\pi_6' = \{0,0,0,0,0,0,0\}$

As the degree sequence reduced to zeros, thus the it is possible to draw a graph with given vertex degree sequence (i.e. given degree sequence is **graphic**).

Drawbacks of Havel-Hakimi Algorithm:

1. Time complexity of this algorithm is $O(n^2)$. Hence it's fairly slow.
2. Only tells if graph is possible or not, does not tell anything about graph representation.
3. Does not tell anything about connectivity of graph.
4. Does not tell anything about presence or absense of cycle in graph.