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Subject: Selected Topics in Graph Theory

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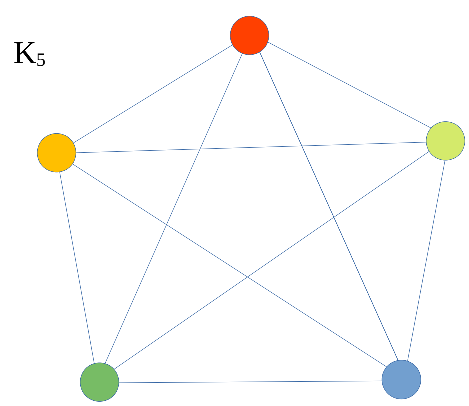
Assignment Number: 3

Q1. Give an example of a graph G for which $\chi(G) = 1 + \Delta(G)$.

Ans:

Any complete graph with n vertices follows above property. It comes from the Brooke's theorem.

We will consider a complete graph G with n = 5 vertices, then there will be n(n-1)/2 = 10 edges.



Here Maximum degreee of a vertex is 4 (as graph is complete it is also regular meaning all vertices has same degree) and chromatic number of graph is 5.

Q2. Prove or disprove: Orthogonal equivalance among n x n matrices is equivalance relation.

Ans:

Consider a graph G and let G' be its complement. Then these two graphs (G and G') being isomorphic means in particular that we have a bijctive vertex map f : V(G) $\rightarrow$ V(G') so there are equally many edges between u and v in G as there are between $f_1(u)$ *and* $f_1(u')$ in G'. If now V(G) = $\{u_1,...,u_n\}$ and V(G') = $\{u_1',...,u_n'\}$ then there must be a permutation σ such that $f_1(u_i) = u'_{\sigma(i)}$ for each i belongs to {1,...,n}. Then graphs G and G' are isomorphic. This also means that adjacency matrices A(G) and A(G') are orthogonally equivalent with respect to any labelling of their vertices. Hence, orthogonal equivalance among n x n matrices is equivalence relation

Q3. Let G be a simple graph with vertex labeling V(G) =$\{u_1,u_2,...,u_n\}$. Let k be a natural number greater than zero. Prove that the entry a_{ij}k in A^k(G) is the numberof distinct walks from u_i to u_j of length k in G.

Ans:

Let G be a graph with adjacency matrix A, and vertices $u_1, . . . , u_n$. We proceed by induction on k to obtain the result.

Base Case:

Let k = 1. A^1 = A. aij= the number of edges from u_i to u_j = the number of walks of length 1 from u_i to u_j.

Inductive Step:

Assume true for k. Let bij be the ijth entry of A^k, and let a_{ij} be the ijth entry of A. By the inductive hypothesis b_{ij} is the number of walks of lengthk from u_i to u_j.

Consider the ijth entry of $A^{k+1} = A*A^k = a_{i1}b_{1j} + a_{i2}b_{2j} +. . .+ a_{in}b_{2n} = \sum a_{im}b_{mj}$ (where m goes from 1 to n)

Consider $a_{i1}b_{1j}$ = number of walks of length k from u_1 to u_j times the number of walks of length 1 from u_i to u_1

= the number of walks of length k+ 1 from u_i to u_j, where u_1 is the second vertex. This argument holds for each m, i.e. $a_{it}b_{tj}$

= number of walks from u_i to v_j in which u_m is the second vertex

So the sum is the number of all possible walks from u_i to u_j. Hence proved.

Q4. Write an algorithm to construct a graph G for which κ(G) = a, λ(G) = b and δ(G) = c, where a, b and c are integers such that 0 < a ≤ b ≤c.

Ans:

Algorithm:

Given integers k,ℓ,d with $1\le k\le \ell\le d$, here's how you can construct a graph G with $\kappa(G)=a$, $\lambda(G)=$b, and $\delta(G)=c$.
Take five disjoint sets V_1, V_2, V_3, V_4, V_5 with $|V_1| = 1$, $|V_2| = c$, $|V_3| = b$, $|V_4| = a$, $|V_5| = c$, and take a surjection $f:V_3 \to V_4$.

The vertex set of G is $V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$.

For the edge set of G take all edges xy where $\{x,y\} \subseteq V_1 \cup V_2$ or $\{x,y\} \subseteq V_2 \cup V_3$ or $\{x,y\} \subseteq V_4 \cup V_5$, and all edges xy where $x \in V_3$ and $y=f(x) \in V_4$.

We get a graph G with above mentioned constraints, such that G can be disconnected by removing either the a-vertices in V_4 or the b-edges between V_3 and V_4, and that the vertex in V_1 has degree c.