Assignment - 2

Roll No. 202CD005

* Numerical Methods *

GOODLUCK Page No. 1
Date 10/05/21

Q.1 For the data (0,7), (1,11), (3,28), value of $L_0(2) + L_2(2)$ is (where $L_i(x) \Rightarrow$ lagrange's multipliers)

→ Sol^n: Given $x=2$,

$$L_0(2) = \frac{(x-1)(x-3)}{(0-1)(0-3)} = \frac{(2-1)(2-3)}{3}$$

$$\therefore L_0(2) = \frac{-1}{3}$$

$$L_2(2) = \frac{(x-0)(x-1)}{(3-0)(3-1)} = \frac{2\times 1}{3\times 2}$$

$$\therefore L_2(2) = \frac{1}{3}$$

$$\therefore L_0(2) + L_2(2) = \frac{-1}{3} + \frac{1}{3} = 0$$

$$\therefore \boxed{L_0(2) + L_2(2) = 0} \Leftarrow \text{Ans}$$

Q.2 Let $p(x)$ be the interpolating polynomial on the data (0,0), (0.5, y), (1,3) & (2,2). The coefficient of x^3 in $p(x)$ is 6. Then y must be?

→ solution:- Using Lagrange's method we have,

$$p(x) = L_0(x) \times y_0 + L_1(x) \times y_1 + L_2(x) \times y_2 + L_3(x) \times y_3$$

$$= L_0(x) \times 0 + \frac{(x-0)(x-1)(x-2)}{(0.5-0)(0.5-1)(0.5-2)} \times y$$

$$+ \frac{(x-0)(x-0.5)(x-2)}{(1-0)(1-0.5)(1-2)} \times 3$$

$$+ \frac{(x-0)(x-0.5)(x-1)}{(2-0)(2-0.5)(2-1)} \times 2$$

$$= 0 + \left(\frac{8}{3}x^3 - 8x^2 + \frac{16}{3}x\right)y$$

$$+ (-2x^3) + 5x^2 - 2x$$

$$+ \frac{1}{3}x^3 - \frac{1}{2}x^2 + \frac{1}{6}x$$

simplifying we, get,

$$p(x) = \left(-\frac{16}{3} + \frac{8}{3}y\right)x^3 + (14 - 8y)x^2$$

$$+ \left(-\frac{17}{3} + \frac{16}{3}\right)x.$$

But we have given that, coefficient of x^3 is 6.

∴ comparing two values we get.

$$6 = -\frac{16}{3} + \frac{8}{3}y$$

$$18 = -16 + 8y$$

$$\frac{34}{8} = y$$

$$\therefore \boxed{y = \frac{17}{4}} \leftarrow \underline{\text{Ans}}$$

Q.3 Let Q_1 & Q_2 are approximation of $\int_{-1}^{1} e^{x^3} dx$ using Trapezoidal rule with step sizes $h=0.1$ & $h=0.2$ resp. The absolute difference betn these 2-approximations Q_1 & Q_2 is?

→ ① finding, Q_1 :

$h = 0.1$, $a = -1$, $b = 1$ & $n = \frac{b-a}{h}$

$$\therefore n = \frac{1-(-1)}{0.1} = 20$$

$$\therefore \boxed{n = 20}$$

∴ The formula for trapezoidal method becomes,

$$I_1 = Q_1 = \frac{h}{2}\left[(y_0+y_{20}) + 2\left(\sum_{i=1}^{19} y_i\right)\right]$$

finding y_i's.

$x_0=-1, y_0 = f(-1) = 0.36787944$
$x_1=-0.9, y_1 = f(-0.9) = 0.48239114$

$x_2 = -0.8$	$y_2 = 0.59929578$
$x_3 = -0.7$	$y_3 = 0.709638211$
$x_4 = -0.6$	$y_4 = 0.80573530$
$x_5 = -0.5$	$y_5 = 0.88249690$
$x_6 = -0.4$	$y_6 = 0.93800499$
$x_7 = -0.3$	$y_7 = 0.97336124$
$x_8 = -0.2$	$y_8 = 0.992031914$
$x_9 = -0.1$	$y_9 = 0.9990004998$
$x_{10} = 0.0$	$y_{10} = 1.00$
$x_{11} = 0.1$	$y_{11} = 1.0010005$
$x_{12} = 0.2$	$y_{12} = 1.00803208$
$x_{13} = 0.3$	$y_{13} = 1.0273678$
$x_{14} = 0.4$	$y_{14} = 1.066092398$
$x_{15} = 0.5$	$y_{15} = 1.13314845$
$x_{16} = 0.6$	$y_{16} = 1.241102379$
$x_{17} = 0.7$	$y_{17} = 1.40916876$
$x_{18} = 0.8$	$y_{18} = 1.66862511$
$x_{19} = 0.9$	$y_{19} = 2.073006564$
$x_{20} = 1.0$	$y_{20} = 2.71828182$

$\therefore$ Q_1 will be,

$$\boxed{Q_1 = 2.155258069}$$

② Finding Q_2,

$h = 0.2$, $a = -1$, $b = 1$, $n = \frac{b-a}{h}$

$$n = \frac{1-(-1)}{0.2} = 10$$

$$\therefore I_2 = Q_2 = \frac{h}{2}\left[(y_0 + y_{10}) + 2 \times \sum_{i=1}^{9} y_i\right]$$

finding ~~y_i's~~ y_i's

$x_0 = -1$ $\quad y_0 = f(x_0) = 0.367879441 17$

$x_1 = -0.8$ $\quad y_1 = 0.5992957878$

$x_2 = -0.6$ $\quad y_2 = 0.8057353018 73$

$x_3 = -0.4$ $\quad y_3 = 0.938004899 95307$

$x_4 = -0.2$ $\quad y_4 = 0.9920319 1484$

$x_5 = 0.0$ $\quad y_5 = 1$

$x_6 = 0.2$ $\quad y_6 = 1.0080320855$

$x_7 = 0.4$ $\quad y_7 = 1.066092398 76$

$x_8 = 0.6$ $\quad y_8 = 1.2411023790006$

$x_9 = 0.8$ $\quad y_9 = 1.66862511013 9667$

$x_{10} = 1.0$ $\quad y_{10} = 2.718281828 4$

$\therefore$ Q_2 will be,

$$\boxed{Q_2 = 2.17240012}$$

$$|Q_2 - Q_1| = |2.17240012 - 2.155258069|$$

$$\therefore \boxed{|Q_2 - Q_1| = 0.017142053} \Leftarrow \underline{\underline{\text{Ans}}}$$

Q.4 Let 502.5 be the approximate value for $\int_1^{17} f(x)\,dx$ when simpson's $1/3^{rd}$ rule is used with 2-subinterval division.

If simpson's $\frac{1}{3}^{rd}$ rule with four subinterval division is used for $[1, 17]$, the approximate value of above integral is:

$\rightarrow$ let $I_1 = 502.5$ corresponding to $n_1 = 2$.

$a = 1$ & $b = 17$ $\Rightarrow$ $h_1 = \frac{17-1}{2} = 8 \Rightarrow x_0 = 1, x_1 = 9,$
$x_2 = 17$

$\therefore$ By simpson's $\frac{1}{3}^{rd}$ formula we have.

$$I_1 = 502.5 = \frac{h_1}{3}\left[y_0 + y_2 + 4 \times y_1\right]$$

Roll No. 202CD005

$$502.5 = \frac{8}{3}\left[f(1) + f(17) + 4 \times f(9)\right] \quad -①$$

Illy for $n_2 = 4$, we have, $h_2 = \frac{16}{4} = 4$

x_i's $\Rightarrow x_0 = 1,\ x_1 = 5,\ x_2 = 9,\ x_3 = 13,\ x_4 = 17.$

$$\therefore I_2 = \frac{h_2}{3}\Big[f(x_0) + f(x_4) + 2 \times [f(x_1)] + 4[f(x_1) + f(x_3)]\Big]$$

$$I_2 = \frac{4}{3}\left[f(1) + f(17) + 2 \times f(9) + 4f(5) + 4f(13)\right] \quad -②$$

subtracting eq^n ① from ② we get.

$$I_2 - 502.5 = \frac{4}{3}\Big[f(1) + f(17) + 2f(9) + 4f(5) + 4f(13) - 2f(1) - 2f(17) - 8f(9)\Big]$$

$$I_2 - 502.5 = \frac{4}{3}\Big[2f(9) + 4f(5) + 4f(13) \underbrace{- f(1) - f(17) - 4f(9)}_{\text{clubbing together}} - 4f(9)\Big]$$

$$I_2 = 502.5 - \frac{4}{3}\left[f(1) + f(17) + 4f(9)\right]$$

$$+ \frac{4}{3}\left[2f(9) + 4f(5) + 4f(13) - 4f(9)\right]$$

Roll No. 202CD005

As, $502.5 = \frac{8}{3}[f(1) + f(17) + 4f(9)]$

$\therefore \frac{502.5}{2} = \frac{4}{3}[f(1) + f(17) + 4f(9)]$

$\therefore$

$I_2 = 502.5 - \frac{502.5}{2} + \frac{8}{3}\left[2f(5) - f(9) + 2f(13)\right]$

$$\therefore \boxed{I_2 = \frac{502.5}{2} + \frac{8}{3}\left[2f(5) - f(9) + 2f(13)\right]}$$

$\Leftarrow$ Ans

Q.5 The coefficient b of the eq. of quadratic polynomial of form $y = ax^2 + b$ that best represents data (-1, 3.1), (0, 0.9), (1, 2.9) (1.5, 2) using least-square approach is?

→ Here, $y' = ax^2 + b$,

Least square error is

$$E = \sum_{i=0}^{n} (y_i - y_i')^2$$

$$\therefore E = \sum (y - (ax^2 + b))^2$$

Roll No. 202 CD005

finding normal eq^{ns},

diff^n w.r.t. a, we get.

$$\frac{dE}{da} = 2\Sigma(y-(ax^2+b))(x^2)$$

$$\therefore \quad 0 = \Sigma x^2 y - \Sigma x^2(ax^2+b)$$

$$\therefore \quad \boxed{\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^2} \quad - (1)$$

Illy, diff^n w.r.t. b, we get,

$$\frac{dE}{db} = 2\Sigma(y-(ax^2+b))$$

$$0 = \Sigma(y-(ax^2+b))$$

$$\therefore \quad \boxed{\Sigma y = a\Sigma x^2 + nb} \quad - (2)$$

These eqns can be written in matrix form as.

$$\begin{bmatrix} n & \Sigma x^2 \\ \Sigma x^2 & \Sigma x^4 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} \Sigma y \\ \Sigma x^2 y \end{bmatrix}$$

Roll No. 202CD005

GOODLUCK Page No. 10
Date 10/05/21

	x	y	x^2	x^4	x^2y
	-1	3.1	1	1	3.9
	0	0.9	0	0	0
	1	2.9	1	1	2.9
	1.5	2	2.25	5.0625	4.5
$\Sigma \Rightarrow$	1.5	8.9	4.25	7.0625	10.5
	$= \Sigma x$	$= \Sigma y$	$= \Sigma x^2$	$= \Sigma x^4$	$= \Sigma x^2 y$.

$$\therefore \begin{bmatrix} 4 & 4.25 \\ 4.25 & 7.0625 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 8.9 \\ 10.5 \end{bmatrix}$$

Converting in upper triangular form.

Using $R_2 - \frac{4.25}{4} R_1$

$$\begin{bmatrix} 4 & 4.25 \\ 0 & 2.5468 \end{bmatrix} \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 8.9 \\ 1.8437 \end{bmatrix}$$

Using back substituation.

$\therefore \quad 2.5468a = 1.8437$ — from R_2.

$\therefore \; a = 0.4$[illegible] $\Rightarrow$ $\boxed{a = 0.410}$

& from R_1,

$4b + 4.25a = 8.9$

$\therefore \; \boxed{b = 1.789375}$ $\Leftarrow$ Ans