Q2.1 PCA of Yale Face Database

Importing Libraries

In [1]:
```python
import numpy as np
from matplotlib.image import imread
import matplotlib.pyplot as plt
import scipy.io
import copy

plt.rcParams['figure.figsize'] = [8,4]
```

Importing Yale Faces Database from .mat file

scipy.io.loadmat() function imports .mat file as dictonary

In [2]:
```python
data = scipy.io.loadmat('./Yale_64x64.mat')
print(type(data))
```

```
<class 'dict'>
```

Dictonary to Numpy Array

In [3]:
```python
A = np.array(data['fea']).T
```

In [4]:
```python
print(A.shape)
```

```
(4096, 165)
```

Sample Image/Face from database

In [5]:
```python
img = plt.imshow(A[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

Centering Matrix in terms of Columns and Rows

1. Subtracting Column Mean

In [6]:
```python
Amean = A.mean(axis=1,keepdims=True)
Am = A - Amean
```

In [7]:
```python
img = plt.imshow(Am[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

2. Subtracting Row Mean

In [8]:
```python
Amean = Amean.mean(axis=0,keepdims=True)
Am = Am - Amean
```

In [9]:
```python
img = plt.imshow(Am[:,1].reshape(64,64).transpose())
img.set_cmap('gray')
plt.axis('off')
plt.show()
```

Calculating SVD

In [10]:
```python
U,D,Vt = np.linalg.svd(Am)          # Complete SVD i.e. calculation corresponding to ze

# U,D,Vt = np.linalg.svd(Am, full_matrices=False)    # Economy SVD i.e. Calculations c

D = np.diag(D)
```

In [11]:
```python
print(U.shape,D.shape,Vt.shape)
```

```
(4096, 4096) (165, 165) (165, 165)
```

Finding number of eigen values with least significance

In [12]:
```python
i = 0
n = len(D)
while(i<n):
    if abs(D[i][i]) <10:
        break
    i += 1
eig_vals_with_least_significance = n - i
```

In [13]:
```python
print(eig_vals_with_least_significance)
```

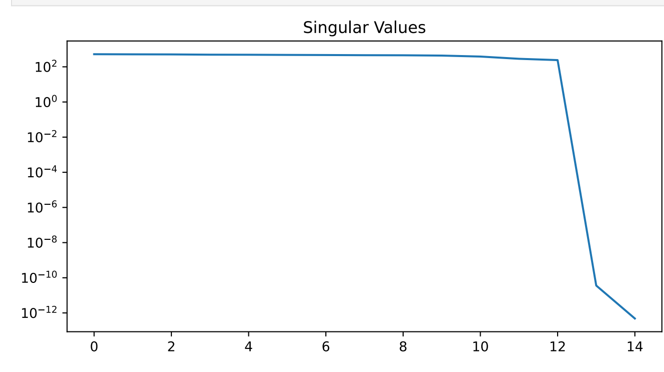
```
2
```

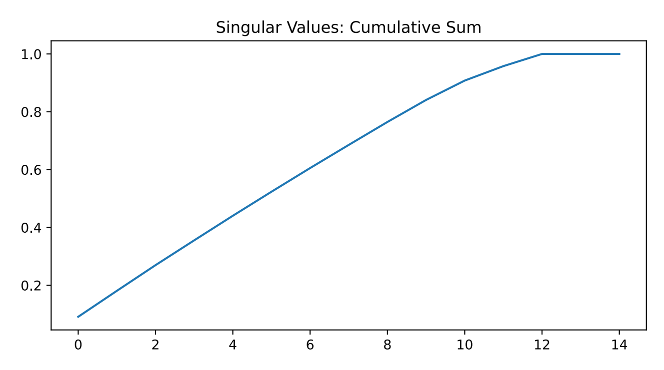
Visualizing singular values by plotting graph

1. Singular values vs Count

2. (Cumulative sum/Total sum) vs Count

In [14]:
```python
d = D[150:,150:]

plt.figure(1)
plt.semilogy(np.diag(d))
plt.title('Singular Values')
plt.show()

plt.figure(2)
plt.plot(np.cumsum(np.diag(d))/np.sum(np.diag(d)))
plt.title('Singular Values: Cumulative Sum')
plt.show()
```

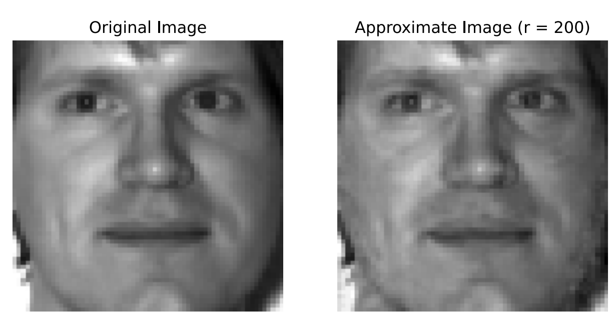


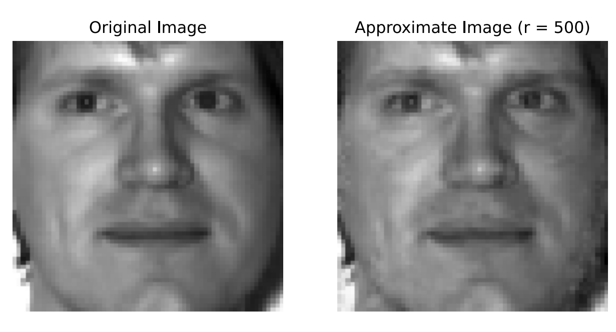


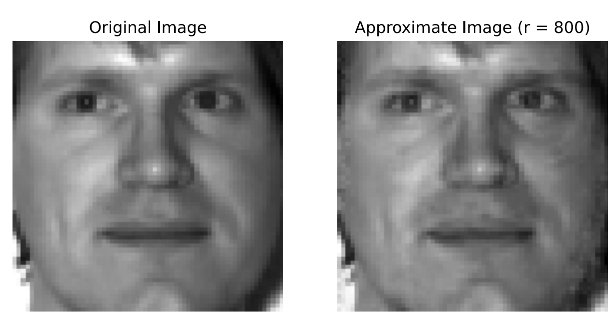
In Sample Projection and Prediction

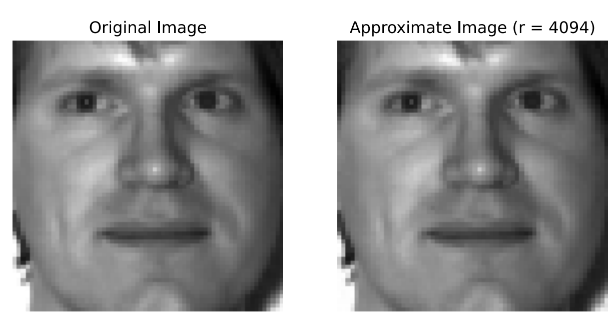
In [15]:
```python
sample_size = 150

def InSampleProjectionAndReconstruction(image_number):

    j = 0

    for r in (50, 100, 200, 500, 800, 2000, 4096, 4096-eig_vals_with_least_significanc

        # Construct approximate image
        u = U[:,:r]

        # Projection
        A_train_model = np.matmul(u.T,A[:,:sample_size])

        # Reconstruction
        A_train_pred = np.matmul(u,A_train_model)
        Fimg = A_train_pred

        plt.figure(j+1)
        j += 1

        plot1 = plt.subplot(121)
        img =  plt.imshow(A[:,image_number].reshape(64,64).transpose())
        img.set_cmap('gray')
        plt.title(f'Original Image')
        plt.axis('off')

        plot2 = plt.subplot(122)
        img2 = plt.imshow(Fimg[:,image_number].reshape(64,64).transpose())
        img2.set_cmap('gray')
        plt.axis('off')
        plt.title(f'Approximate Image (r = {r})')
        plt.show()
```

In [16]:
```python
InSampleProjectionAndReconstruction(0)
```









Out off Sample Projection and Prediction

In [17]:
```python
def outOffSampleProjectionAndReconstruction(image_number):
    if(image_number>=sample_size):
        j = 0
        for r in (50, 100, 200, 500, 800, 2000,4096, 4096-eig_vals_with_least_signific

            # Construct approximate image
            u = U[:,:r]

            # Projection
            A_test_model = np.matmul(u.T,A[:,image_number])

            # Reconstruction
            A_test_pred = np.matmul(u,A_test_model)
            Fimg = A_test_pred

            plt.figure(j+1)
            j += 1

            plt.subplot(121)
            img =  plt.imshow(A[:,image_number].reshape(64,64).transpose() )

            img.set_cmap('gray')
            plt.axis('off')
            plt.title(f'Original Image (r = {r})')

            plt.subplot(122)
            img2 = plt.imshow(Fimg.reshape(64,64).transpose())
            img2.set_cmap('gray')
            plt.axis('off')
            plt.title(f'Approximate Image (r = {r})')
            plt.show()

    else:
            print("Object Belongs to Sample")
```

In [18]:
```python
outOffSampleProjectionAndReconstruction(155)
```