Symbol Detection in OTFS modulation

A thesis report submitted for BTP phase II

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Abstract

OTFS modulates the signal in delay doppler domain , unlike other modulation schemes which modulate in time frequency domain . The basis transmitter and receiver pulses along with their windowing functions used for sampling the signals in 2-d lattices can be designed in such a way that it leads to an almost constant fade with negligible symbol and doppler interference.Methods have been developed for channel estimation and symbol detection of OTFS based on MCMC sampling . Although those algorithms perform well in perfect channel estimation their performance decreases by a huge margin at imperfect channel estimation at higher SNR . A particular method developed for improving their performance at imperfect channel state information (CSI) is introducing the temperature parameter which comes with the tradeoff of performance at lower SNR in traditional Gibbs sampling based algorithms. Here we propose and simulate a Gibbs sampling based algorithm which gives same performance in perfect CSI , is able to maintain its performance at higher as well as lower SNR in imperfect CSI when incorporated with the temperature parameter .

1 Contents

2 Introduction

Wireless services is an integral part of our life , and a variety of new services, such as smart homes,high quality movie streaming and other higher data requirements devices are emerging. They will require higher data rate than the existing LTE. That's why 5G network is in utmost demand.

A important factor of these 5G systems will be millimeter-length bands which provide a much larger bandwidth.
But real problem arises in frequency-dispersive channels where due to movement of user Doppler shift is introduced and as the carrier frequency increase these fading characteristics becomes more evident.

We used a emerging modulation technique named Orthogonal Time Frequency Space(OTFS) which operates in delay-Doppler domain as impulse response is localized in this domain rather than in time-frequency domain and our analysis can be simplified.

Further Symbol detection techniques can be applied to otfs waveform when noise causes distortion in signal.

3 Theory

Mobile communication channels suffers from both time dispersion due to multipath channel effects and also suffers from frequency dispersion caused due to Doppler shifts which causes Inter-channel Interference.
Orthogonal time frequency space(OTFS) offers us a way to combat this problem by signalling in the delay-Doppler domain. Due to Heisenberg's Uncertainity principle , a signal can't be localised in both time and frequency but OTFS waveform can be localised in delay-Doppler plane.
We have used OTFS modulation and demodulation techniques to transmit our signal but still this signal suffers from random noise additions. So for OTFS symbol detection we used modified Gibbs sampling based OTFS detection.

Through a transform called 2D inverse sympletic transform and windowing , information symbols x[k,l] residing in the delay-Doppler domain are mapped to the time-frequency domain symbols X[p,q]. This is our pre-processing step. Further Heisenberg transform transforms these symbol into the signal which will be transmitted x(t).Channel impulse response
h(τ, ν)
transforms it into received signal y(t) .
Wigner transform transforms the signal back to its time-frequency domain symbols.Then OTFS reverse transform is used which consists of receive windowing and SFFT to get information symbols received.

Now due to noise additions our received signal is Y = Hx + V where V is noise . For OTFS signal detection, Monte carlo Markov chain based sampling methods called Gibbs Sampling can be used.

$\hat{\mathrm{x}}_{ML} = argmin\|y - Hx\|^2$

In Gibbs sampling random initial vector is chosen. In each iteration all the coordinates are updated and the update in
t+1$^{th} iteration is obtained from the sampling distributions$:

$\mathrm{x}_i^{t+1} = p(x_1^t | x_2^t, \cdots, y, H)$

The vector obtained is passed to the next iteration for the next set of coordinate updates. After certain number of iterations the distributions converge to a certain distributiion to simulate for drawing samples. Finally the detected symbol vector is used to calculate the least ML cost in all coordinates. We can also use a temperature parameter to reduce the number of iterations.

4 Method

4.1 Chapter1: Final relation b/w Channel matrix and Transmitted symbol:

The relation between received signal y[k,l] and the channel parameters can be given by

$$\text{y[k,l]} = \frac{\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\sum_{i=0}^{p-1}\frac{(1-e^{2j\pi M\triangle f(\frac{l-m}{M\triangle f}-\tau_i)})(1-e^{2j\pi NT(\frac{k-n}{NT}-v_i)})}{(1-e^{2j\pi\triangle f(\frac{l-m}{\triangle f}-\tau_i)})(1-e^{2j\pi T(\frac{k-n}{NT}-v_i)})}\times x[n,m]\times h_i}{M\times N} + \text{v[k,l]}$$

Where $\text{h}_i = a_i e^{-j*2*\pi*\tau_i*\nu_i}$
$\text{l} \in 0,1,2..M-1, k \in 0,1,2..N-1$
v[k,l] is AWGN.

Since τ_i
is an integral multiple of T and T is an integral multiple of

$$\frac{1}{M \times \triangle f}$$

the above equation can be reduced to

$\text{y[k,l]}=\sum_{n=0}^{N-1}\sum_{i=0}^{p-1} x[n,f(l,i)] \times H(k,n,i) + v[k,l]$
Where H(k,n,i)= $\text{h}_i \times \frac{1-e^{-j2v_i\pi(\frac{k-n}{NT})NT}}{1-e^{-j2v_i\pi(\frac{k-n}{NT})T}} \times \frac{1}{N}$

and f(l,i)is given by

M-—l-s $\times i|modM \quad if \quad l-s\times i \leq 0$
—l-s $\times i|modM \quad if \quad l-s\times i > 0$

Where T=

$$\frac{s}{M \times \triangle f}$$

i.e it's an integral multiple of

$$\frac{1}{M \times \triangle f}$$

for satisfying the bi orthogonality condition.In our channel setting we have taken T=

$$\frac{1}{\triangle f}$$

to minimize the dimensionality of previous equation as we will be dealing with imperfect CSI we will be iterating the algorithms multiple times for evaluating the performance,the new equation becomes

y[k,l]=$\sum_{n=0}^{N-1}\sum_{i=0}^{p-1} x[n,l] \times H(k,n,i) + v[k,l]$

We will simulate our algorithms by keeping l constant and n $\in 0,1,2..N-1, k \in 0,1,2..N-1$.
So our final matrix equation of dimensionality N giving the relation between the transmitted signal and received signal is

Y=HX+V where
Y,X,V are NX1 vectors where the kth element is given by
Y[k]=y[k,l]
X[k]=x[k,l]
and the elements of the matrix H are given by

H[k,n]=$\sum_{i=0}^{p-1} H(k,n,i)$
where k,n $\in 0,1,2...N-1$

4.2 Chapter2: Algorithms

Here we propose a modified variant of the traditional Gibbs sampling method which always gives better results than the traditional Gibbs sampling method and compare it with our Gibbs sampling based algorithm. The presented two algorithms work for both perfect and imperfect CSI(channel state Information) condition . We explain the reasons for bringing the changes in the traditional Gibbs sampling algorithm to make it work for imperfect CSI.

To detect x̂ for given y and H.
Let the length of the vector to be detected ,x be L. Let C be the set of possible symbols.

C= z1, z2...$z_k (In our case they are 4qam symbols)$
Let y-Hx=d(x)
$e^{-\|y-Hx\|/\sigma^2} = P(x)$
No of iterations = n_iter
Maximum no of steps = maxsteps
Both are pre defined variables depending on the dimension of the vector to be detected
Initialize:
1: $\hat{x}=H^{\dagger}y$
where

$$H^{\dagger}$$

= pseudoinverse of H (it would have been maximum likelihood solution as
$\|y - Hx\|$
will be minimum but necessarily $H^{\dagger}y \notin C^L$,
all the L components of the vector are not necessarily symbols)

2: min=infinity

Pseudocode1(our algorithm)

```
for i=1 to n_iter
    x̂ = H†y
        for j=1 to maxsteps
            for m=1 to L (loop for obtaining the required PMF)
                find k ∈ 1,2..K s.t  d(x̂)  for  x_m = z_k is minimum
                A[m]=z_k
                (array used for storing the symbol which minimizes d )
                p[m]=P(x̂) at x_m = z_k
                if x_m == A[m]
                    p[m]=0(no need to change the
                    mth symbol of the vector if it's already equal to
the best possible symbol)
            Sample t ∈ 1,2,..L  with  PMF p[t]/∑_{t=1}^{L} p[t]
            x_t = A[t]
            if x̂ ∈ C^L  and  d(x̂) < min
                x_detected = x̂
```

min=d($\hat{x}$)

Return $x_{detected}$

Pseudocode 2(modified variant based on Gibbs sampling)

```
for i =1 to n_iter
    for j=1 to L
        for k=1 to K
            p[k]=P(x̂) at x_j = z_k
        if all p[k] 's are computationally existing values
            Sample t ∈ 1,2,..K  with  PMF p[t]/∑_{t=1}^{K} p[t]
        else
            (choose best symbol which gives minimum distance )
            find t ∈ 1,2,3...K s.t d(x)  for  x_j = z_t  is minimum
        x_j = z_t
        if x̂ ∈ C^L  and  d(x̂) < min
            min= d( x̂)
            x_detected = x̂
Return x_detected
```

As we can see we have brought 3 major changes in algorithm 2 which is the modified algorithm.

1: We have started the vector from $H^{\dagger}y$,

it has been observed that starting the iteration from MMSE or zero forcing solution gives better results , as we had started the algorithm 1 (our algorithm) from $H^{\dagger}y$,

we started algorithm 2 also from the same starting point.

2: We are finding the best vector by comparing d(x) after every change in symbol rather than returning the vector after all the iterations , it gives better results by a huge margin for lower SNR.

3: We observe the term P(x) in case of imperfect CSI $e^{-\|y-(H+\triangle H)x\|/\sigma^2}$

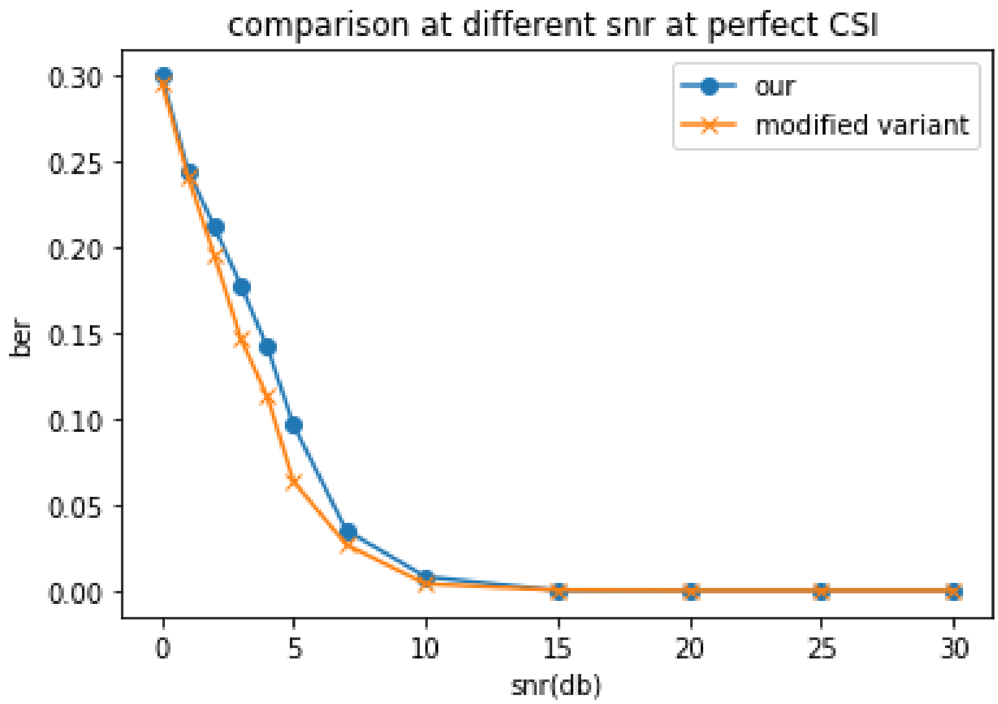
where $H_{imperfect} = H + \triangle H$
is the imperfect channel matrix observed at the receiver. While simulating the algorithm for imperfect CSI we produce H at the receiver by adding noise to the actual channel matrix , and rarely it happens that so much noise is added that the term $\|y - (H + \triangle H)x\|$
becomes significantly large compared to σ^2
such that it becomes difficult to compute the PMF. So rather than computing the PMF and sampling accordingly , we assign the symbol which produces minimum d(x) at that instant.

5 Results:

We run both the algorithms using 4QAM symbol set for the following channel condition :
No of delay tap components p=5
N=10,
T=2.1 μs
f=47.6 kHz

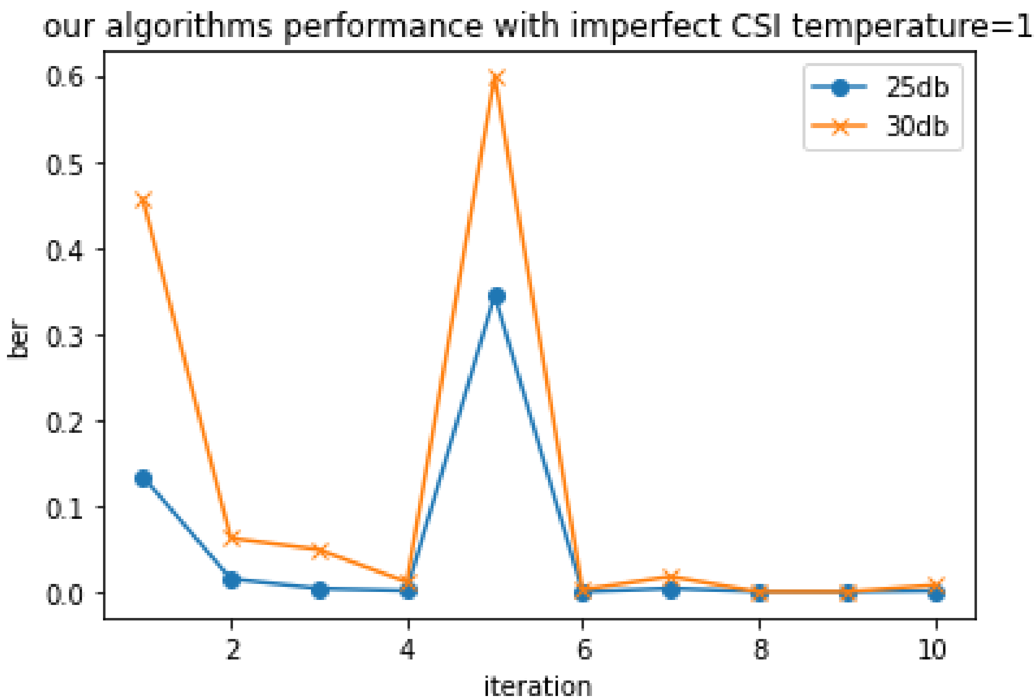
Path index(i)	1	2	3	4	5
a_i	1	0.7	0.5	0.4	0.1
υ_i	0/T	0.84/T	0.63/T	1.05/T	1.89/T
τi	$0 \times T$	$1 \times T$	$2 \times T$	$3 \times T$	$4 \times T$

5.1 Graph1



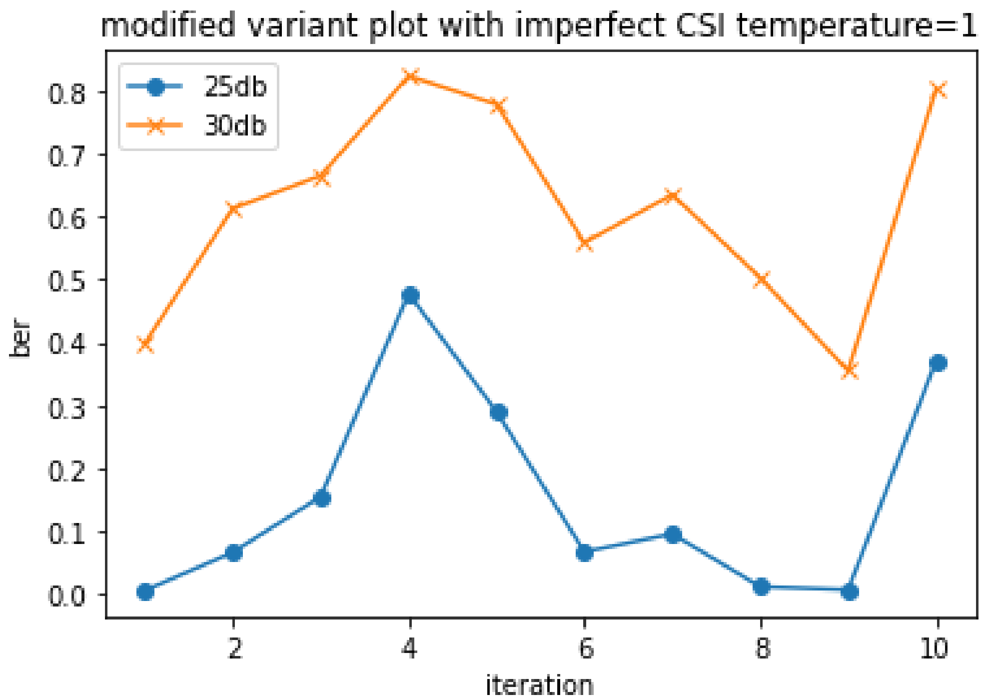
Perfect CSI:
We take the following values for running our algorithm and the modified variant:
L=10
n_iter=5(for our algorithm with L=10. 5 iterations are enough to give good results)
maxsteps=20
n_iter=10(for modified variant,as it shows nearly no change after that)
We can see from the above graph that our algorithm performs nearly as good as the modified variant in case of perfect CSI.

5.2 Graph2



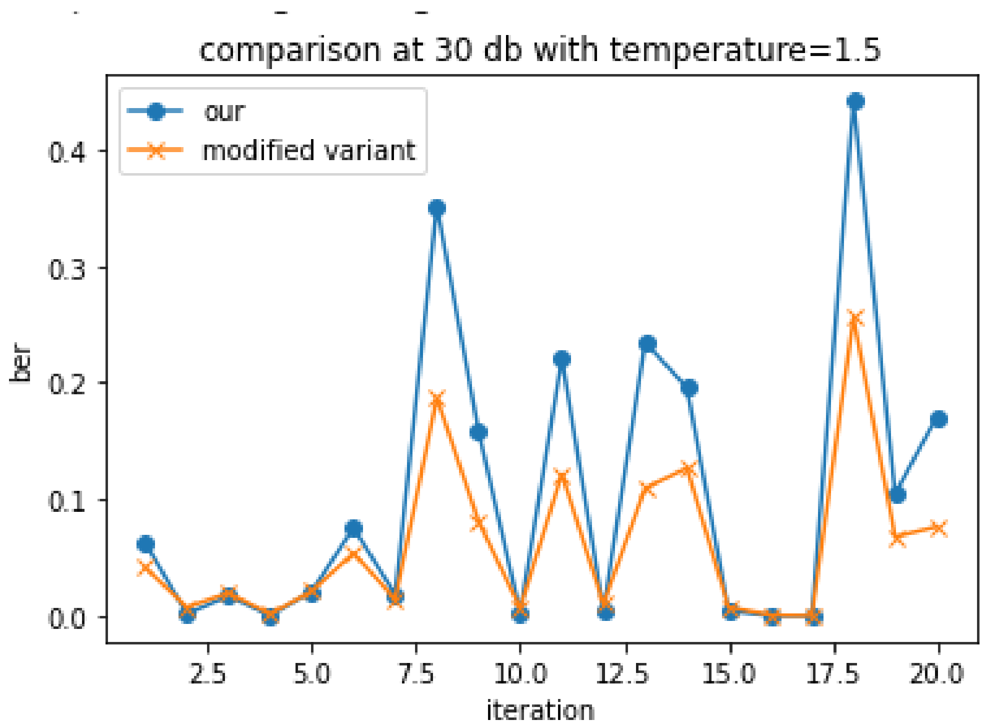
Imperfect CSI:
We add complex Gaussian noise of variance 0.01 to H and iterate both the algorithms 10 times , we observe that the accuracy starts somewhat diverging at higher SNR 25,30 db , as y-(H+$\triangle H)x$
becomes much larger compared to the term σ^2
and P(x) changes significantly. We plot the BER with respect to iteration for 25 and 30 db first for our algorithm and then the modified variant. We also observe that the modified variant gives worse performance compared to our algorithm at 30 db.

5.3 Graph3



modified variant plot with imperfect CSI temperature=1
25db
30db
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
ber
2
4
6
8
10
iteration

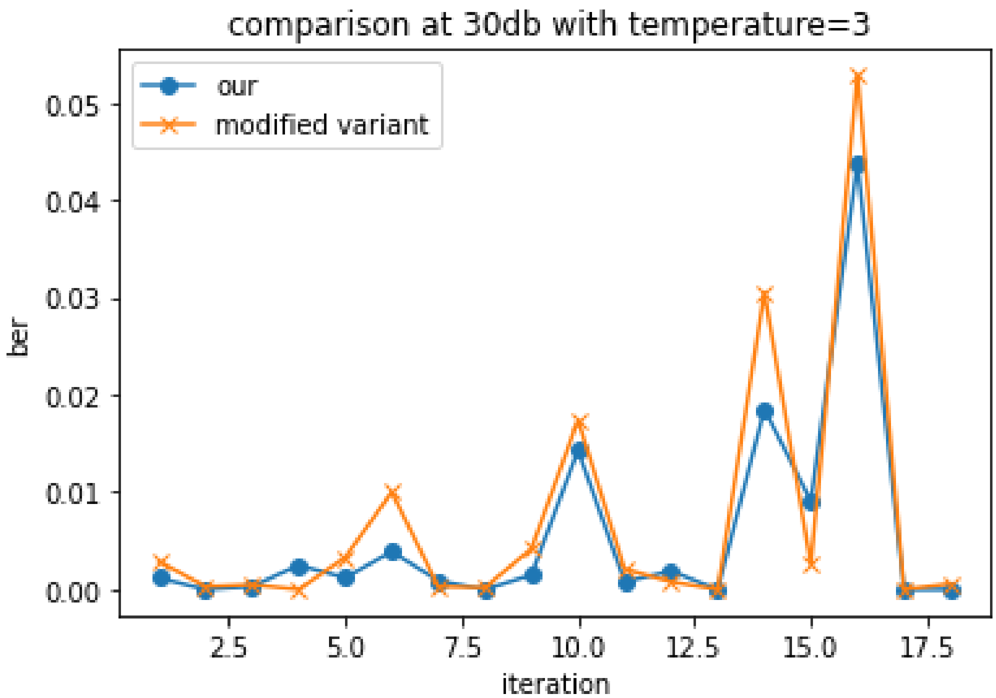
5.4 Graph4



Incorporating the temperature parameter $\alpha :-$
To improve the performance at higher SNR we include the temperature parameter , but it comes with the decrease in performance in lower SNR in the traditional Gibbs sampling algorithm as well as in its modified variant, but in our algorithm it shows little or no change in BER of lower SNR as well.
We first plot the BER for 30 db for our algorithm and the modified variant for
$\alpha = 1.5$.
The values of n_iter , maxsteps , remain the same for both the algorithms .
While iterating 10 times we keep the H values obtained in each iteration and use that value of H where it had given worse performance to finetune

the values of n_iter,α
and maxsteps.
We observe that the worst performance is given in iteration 18 , accuracies for our algorithm and modified variant are 0.5597 and 0.7439 respectively.

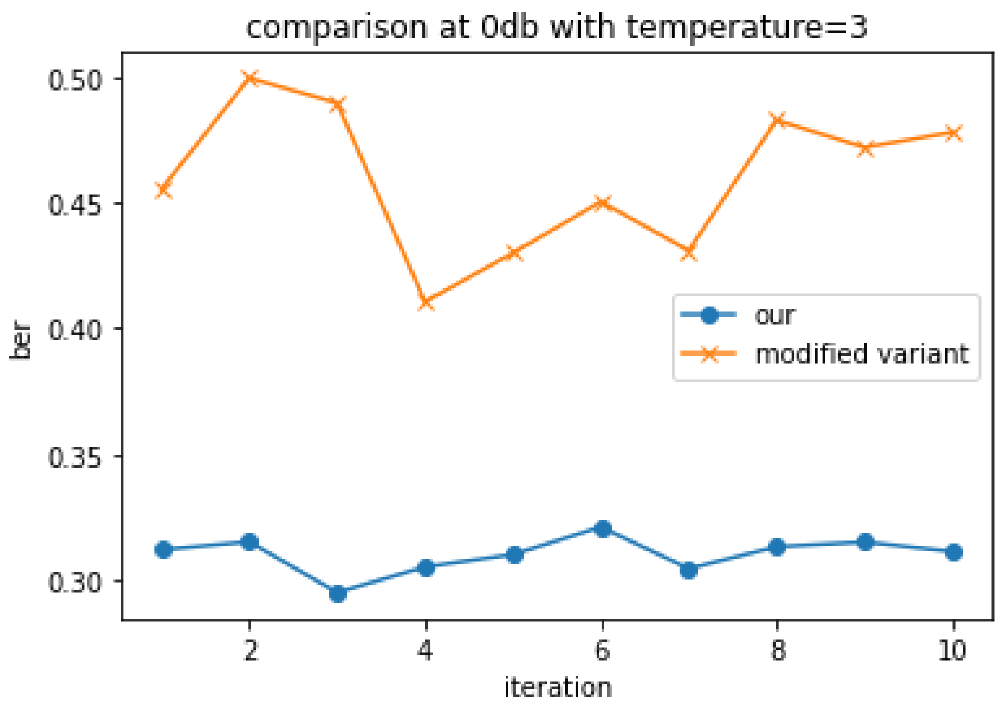
5.5 Graph5



We increase the values as follows
$\alpha = 3$
n_iter = 5(for our algorithm)
maxsteps = 25
n_iter = 15(for modified variant)
We observe that the new accuracies obtained after changing the values as

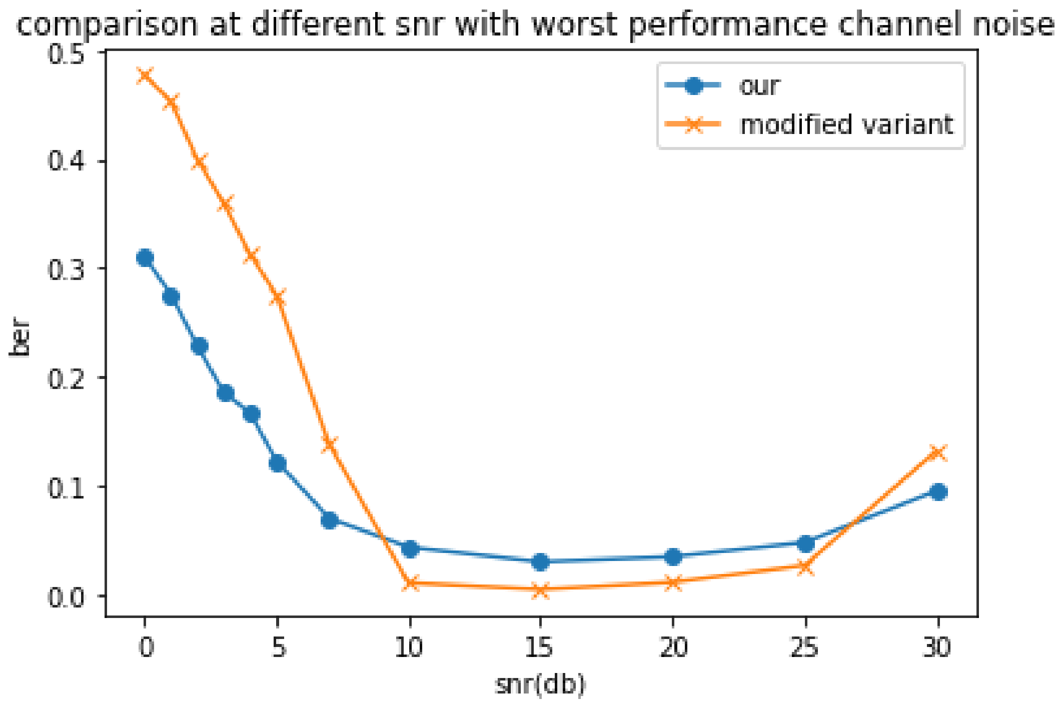
stated above are 0.9047 and 0.8805 respectively which is a significant increase from the previous accuracies.
We plot the results of our algorithm and the modified variant at 30 db .We can observe that there is significant increase in accuracies at higher SNR by incorporating the temperature parameter.

5.6 Graph6



But increasing the value led to significant decrease in accuracy at lower SNR for the modified variant of Gibbs sampling algorithm, we plot the accuracies at 0db for our algorithm and the modified variant after 10 iterations and we can observe that the modified variant algorithm gives far worse performance than our algorithm , whereas the accuracies in our algorithm are nearly the same as in perfect CSI.

5.7 Graph7



Finally we plot the performance of our algorithm with imperfect CSI , we keep the channel values the same where the algorithm had performed the worse among the 20 iterations i.e where it had attained an accuracy of 0.5597 at $\alpha = 1.5$.

6 Conclusions

Although the modified variant of Gibbs sampling algorithm gives good results in perfect CSI.
the results start diverging and decreasing at higher SNR , this problem can be solved by incorporating the temperature parameter , which decreases the performance at lower SNR by a huge margin , we have tried by increasing the n_iter variable in modified variant upto as large as 200, but it still gives lower accuracy than our algorithm at lower SNR, whereas the algorithm we proposed has been able to successfully give nearly the same results at lower SNR even after introducing the temperature parameter, so the algorithm we proposed can be used as a unified algorithm which is able to perform in imperfect CSI in high SNR , with almost no tradeoff of accuracy in lower SNR.
Disadvantages of the algorithm include the need to tune the variables n_iter, maxsteps where it gives sufficiently good results ,and higher computational complexity.

7 Bibliography

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