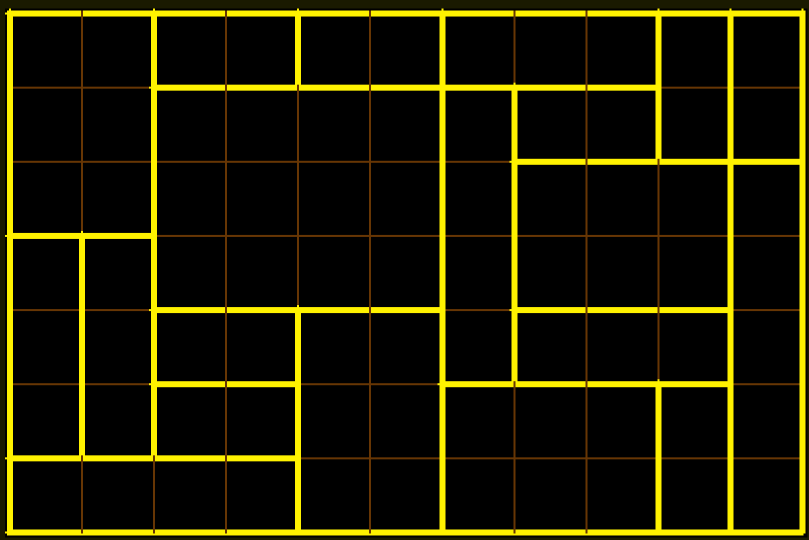
Rectangles, parts 1-2-3

We want to investigate the number of ways to fill an $m \times n$ rectangle with non-square rectangles. Here is an example of a correct filling for an 7×11 rectangle:



Let $f(m, n)$ be the number of ways to fill an $m \times n$ rectangle with non-square rectangles. As an example (you should try to find these values using paper and pencil to warm up):

$$f(1,2) = 1$$
$$f(2,2) = 2$$
$$f(2,3) = 5$$
$$f(3,3) = 18$$

Note: for each of these 3 questions, the format of the flag is `FLAG-XYZ...`, where `XYZ...` is the answer.

Question 1 - **Compute** $f(5,5)$

Question 2 - **Compute** $f(2,20203)$ **mod** 999999937

Question 3 - **Compute** $f(7,11)$ Note: for each of these 3 questions, the format of the flag is `FLAG-XYZ...`, where `XYZ...` is the answer.

Twadolicious matrices part 1

Let n-twadolicious matrix be defined as a $n \times n$ square matrix such that every row and column of the matrix contains exactly two ones and $n-2$ zeroes. Here are 3 examples of 5-twadolicious matrices:

1 0 0 1 0	0 1 1 0 0	0 1 0 0 1
0 0 1 0 1	0 0 1 1 0	1 0 1 0 0
1 1 0 0 0	1 1 0 0 0	0 0 0 1 1
0 0 1 1 0	0 0 0 1 1	0 1 0 1 0
0 1 0 0 1	1 0 0 0 1	1 0 1 0 0

Let us denote $tot(n)$ the number of possible n-twadolicious matrices. Examples: $tot(2) = 1$, $tot(3) = 6$, $tot(5) = 2040$.

Compute $tot(25)$.

Twadolicious matrices part 2

Two n-twadolicious matrices will be called twado-similar if it is possible to transform one into the other by swapping lines or swapping columns any number of times. Two n-twadolicious matrices that are not twado-similar will be called strongly distinct.

In the example of the previous question,

1 0 0 1 0	0 1 1 0 0	0 1 0 0 1
0 0 1 0 1	0 0 1 1 0	1 0 1 0 0
1 1 0 0 0	1 1 0 0 0	0 0 0 1 1
0 0 1 1 0	0 0 0 1 1	0 1 0 1 0
0 1 0 0 1	1 0 0 0 1	1 0 1 0 0

The first two matrices are twado-similar, since you can transform the first into the second by doing these three operations:

1. Swap rows 1 et 5
2. Swap columns 4 et 5
3. Swap columns 3 et 4

However, swapping lines and columns in any of these two matrices will never lead to the third one, which is therefore strongly distinct to the two first matrices.

Let $ceq(n)$ be the size of the biggest set of pairwise strongly distinct n-twadolicious matrices.

The following table contains values of $ceq(n)$ for some values of n.

n	ceq(n)	tot(n)
2	1	1
3	1	6
5	2	2040
7	4	3110940
11	14	158815387962000
18	88	HUGE

Compute $ceq(666)$**.**

Twadolicious matrices part 3

Now, let us investigate the size of the equivalence class of some n-twadolicious matrix (i.e, the number of n-twadolicious matrices that are twado-similar to some n-twadolicious matrix M).

Looking at the same example,

1 0 0 1 0	0 1 1 0 0	0 1 0 0 1
0 0 1 0 1	0 0 1 1 0	1 0 1 0 0
1 1 0 0 0	1 1 0 0 0	0 0 0 1 1
0 0 1 1 0	0 0 0 1 1	0 1 0 1 0
0 1 0 0 1	1 0 0 0 1	1 0 1 0 0

The first and second matrices are in the same equivalence class, which contains 1440 matrices. The third one is in the only other existing equivalence class for 5-twadolicious matrices, which contains 600 matrices. This sums up to $tot(5) = 1440 + 600 = 2040$ 5-twadolicious matrices, which matches the result of the table in part 2.

The file twadomatrix.txt contains a 2020-twadolicious matrix. Let x be the number of matrices in its equivalence class. As x is very large, do not compute x directly. Instead, **compute the value of** x **mod** 999999937.

Note: 999999937 is a prime number.

Note: in the purpose of saving space, twadomatrix.txt indicates, for each of its rows, which two columns contain the "1"'s.