ECEN 743: Reinforcement Learning

TD Learning and Q-learning

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References

- [SB, Chapter 5-6]
- "Neuro-Dynamic Programming", D. Bertsekas and J. Tsitsiklis, Chapter 5

TD Learning: Model-Free Policy Evaluation

MDP Questions

- How do we find the value of a policy π?
  - Policy evaluation iteration
- How do we find the optimal value function V^*?
  - Value iteration
- How do we find the optimal policy?
  - Value iteration, policy iteration

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- Need to learn from the observed sequence of states, actions, and rewards

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- Assumption: The agent can interact with the environment (real-world environment or a simulator) to generate data

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- Generate trajectories $\tau^k, 1 \leq k \leq K$, according to policy π with $s_0^k = s$

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- Compute the return of the trajectory τ^k as

$$G^k(s) = r_0^k(s_0, a_0) + \gamma r_1^k(s_1, a_1) + \gamma^2 r_2^k(s_2, a_2) + \ldots$$

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- Problems with naive MC policy evaluation:
  - We need to generate K trajectories for each and every state!

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- Compute the return of the trajectory τ^k at timestep t as as

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- We only need a (very long) *single trajectory* actually!

Incremental MC Updates

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$$= \frac{1}{m+1} y_{m+1} + \frac{m}{m+1}\mu_m = \mu_m + \frac{1}{m+1}(y_{m+1} - \mu_m)$$

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$$\begin{aligned}\mu_{m+1} &= \frac{1}{m+1} \sum_{i=1}^{m+1} y_i = \frac{1}{m+1} y_{m+1} + \frac{1}{m+1} \sum_{i=1}^{m} y_i \\ &= \frac{1}{m+1} y_{m+1} + \frac{m}{m+1} \mu_m = \mu_m + \frac{1}{m+1}(y_{m+1} - \mu_m)\end{aligned}$$

- Incremental MC: $V_{t+1}(s_t) = V_t(s_t) + \frac{1}{n_t(s_t)}(G_t - V_t(s_t))$ and $V_{t+1}(s) = V_t(s)$ for all $s \neq s_t$

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- In most practical problems, we use a constant step size

$$V_{t+1}(s_t) = V_t(s_t) + \alpha(G_t - V_t(s_t))$$

Monte-Carlo Policy Evaluation

- MC methods learn directly from episodes of experience
- MC is model-free

- MC learns from complete episodes: no bootstrapping
- MC does not really exploit the properties of the underlying MDP
- MC estimate has generally high variance

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- TD Target: $r_t + \gamma V(s_{t+1})$
- TD Error: $r_t + \gamma V(s_{t+1}) - V(s_t)$

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- TD will (provably) converge to V_π under suitable assumptions
- Proof requires stochastic approximation theory. We will discuss this later in the course

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Let $H : \mathbb{R}^n \to \mathbb{R}^n$ be a contraction w.r.t. $\|\cdot\|$ with contraction factor γ. Then, for any $\alpha \in (0, 1)$, the function $F : \mathbb{R}^n \to \mathbb{R}^n$, defined as $F = (1 - \alpha I) + \alpha H$ is also a contraction.

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- Consider the operator $F_\pi = (1 - \alpha)I + \alpha T_\pi$, which is a contraction by the above result

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- Then, the iteration $V_{k+1} = (1-\alpha)V_k + \alpha T_\pi V_k$ will converge to V_π
- Rewrite the iteration as

$$V_{k+1}(s) = V_k(s) + \alpha(T_\pi V_k(s) - V_k(s)) = V_k(s) + \alpha(\mathbb{E}_{a\sim\pi(s,\cdot),s'\sim P(\cdot|s,a)}[r(s,a) + \gamma V_k(s')] - V_k(s))$$

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$$V_{k+1}(s) = V_k(s) + \alpha(T_\pi V_k(s) - V_k(s)) = V_k(s) + \alpha(\mathbb{E}_{a\sim\pi(s,\cdot),s'\sim P(\cdot|s,a)}[r(s,a) + \gamma V_k(s')] - V_k(s))$$

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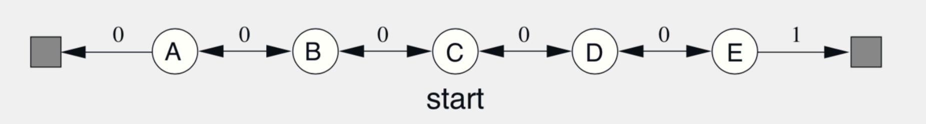
MC vs TD: Random Walk Example (from [SB])



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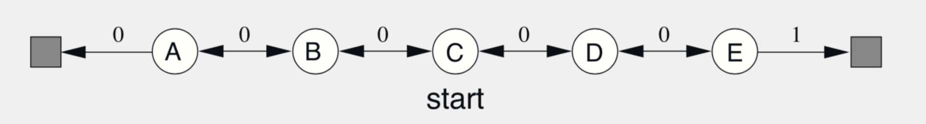

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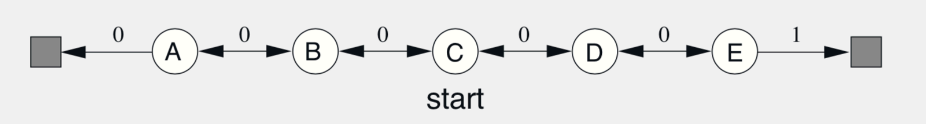

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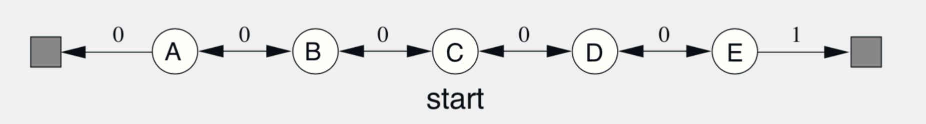

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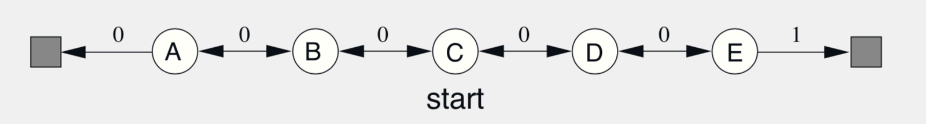

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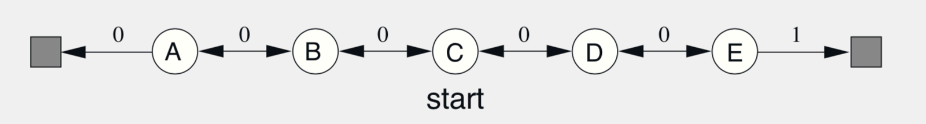

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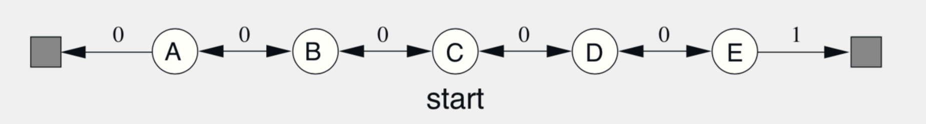

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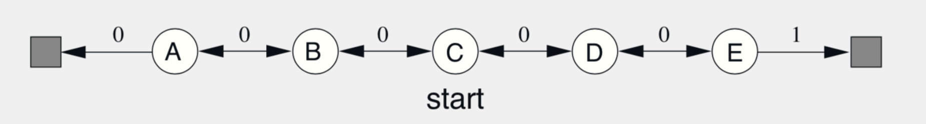

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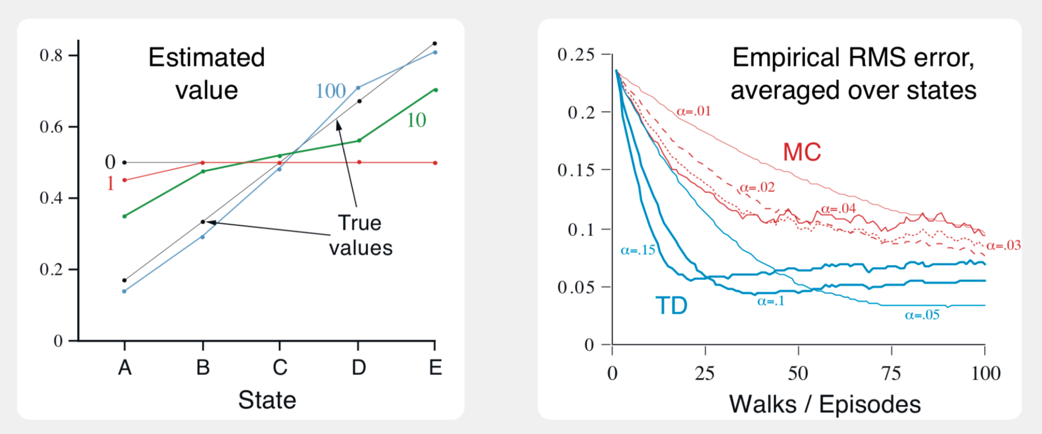
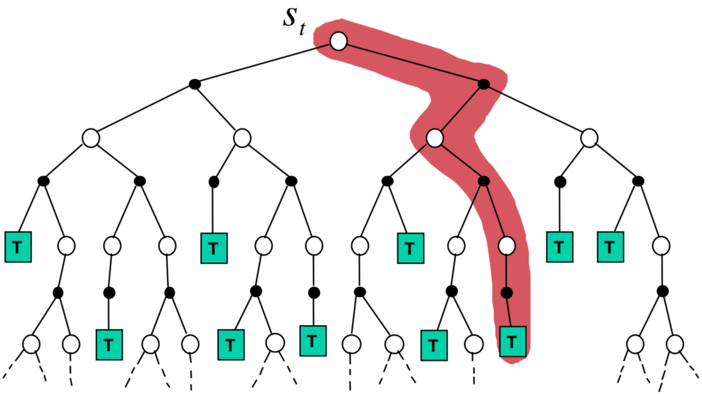
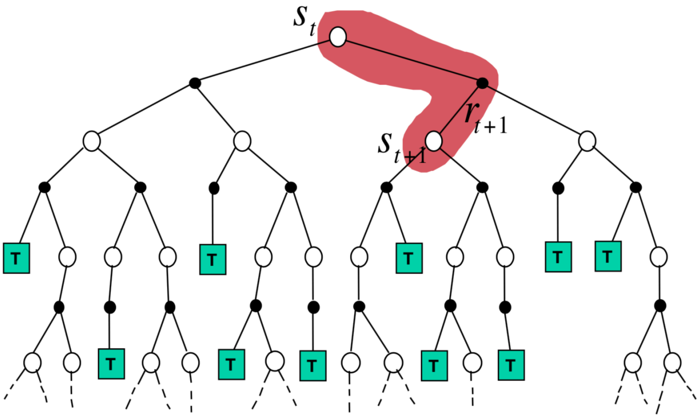


MC Backup



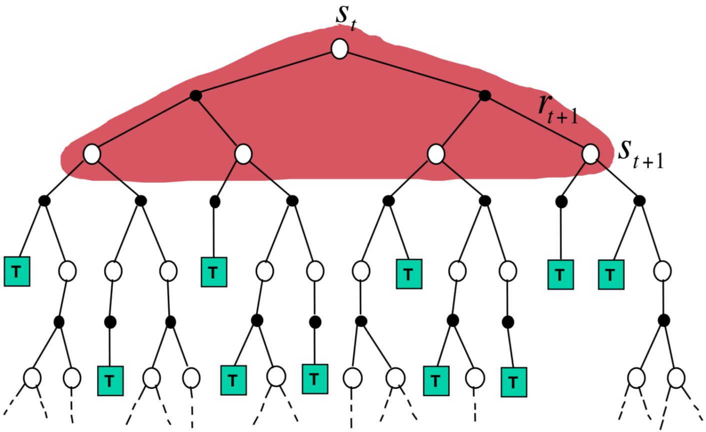
$$V_{t+1}(s_t) = V_t(s_t) + \frac{1}{n(s_t)}\left(G_t - V_t(s_t)\right)$$

TD Backup



$$V_{t+1}(s_t) = V_t(s_t) + \alpha(r_t + \gamma V_t(s_{t+1}) - V_t(s_t))$$

DP Backup



$$V_{t+1}(s_t) = r_t + \gamma \mathbb{E}[V_t(s_{t+1})]$$

Q-Learning: Model-free Learning of the Optimal Q-function

Reinforcement Learning Questions

- How do we learn the value of a policy π?
- How do we learn the optimal action-value function Q^*?
- How do we learn the optimal policy π^*?

... without the knowledge of the model P

- Need to learn from the observed sequence of states, actions, and rewards

Optimal Q-value Function

- Q-value function of a policy π

$$Q_\pi(s,a) = \mathbb{E}[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) | s_0 = s, a_0 = a]$$

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$$\pi^*(s) = \arg\max_{a} \; Q^*(s,a)$$

- Can we learn Q^* only using the trajectory data?

Intuitive Idea

- Recall the Bellman operator for Q-value function

$$TQ(s,a) = r(s,a) + \gamma \mathbb{E}_{s' \sim P(\cdot|s,a)}[\max_b Q(s',b)]$$

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- T is a contraction. Then, $F = (1 - \alpha I) + \alpha T$ is also a contraction. So, the iteration

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  - In general, ϵ-greedy is not the best exploration strategy.
  - However, we use this in deep RL most of the time.
  - There are many clever exploration strategies. This is an active area of research.

Q-Learning Algorithm

Algorithm Q-Learning Algorithm

Initialization: Initial Q-value, Q_0, a behavior policy π_b, $t = 0$, initial state s_0
for each $t = 0, 1, 2, \ldots$ **do**
    Observe the current state s_t
    Take action a_t according to the behavior policy π_b, $a_t \sim \pi_b(s_t, \cdot)$
    Observe the reward r_t and the next state s_{t+1}
    Update Q:

$$Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha_t \left(r_t + \gamma \max_b Q_t(s_{t+1}, b) - Q_t(s_t, a_t)\right)$$

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7: **end for**

Theorem (Convergence of Q-learning)

If: (i) all state-action pairs are visited infinitely often, and
(ii) step size satisfies Robbins-Munro condition, $\sum_t \alpha_t = \infty, \sum_t \alpha_t^2 < \infty$,
then Q-Learning will converge, i.e., $Q_t \to Q^*$ *almost surely.*

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Algorithm Q-Learning Algorithm

Initialization: Initial Q-value, Q_0, initial state s_0
for each $t = 0, 1, 2, \ldots$ **do**
    Observe the current state s_t
    Take action $a_t \sim \epsilon$-greedy$(Q_t)(s_t, \cdot)$
    Observe the reward r_t and the next state s_{t+1}
    Update Q:

$$Q_{t+1}(s_t, a_t) = Q_t(s_t, a_t) + \alpha_t \left(r_t + \gamma \max_b Q_t(s_{t+1}, b) - Q_t(s_t, a_t)\right)$$

end for