ECEN 743: Reinforcement Learning

Trust Region Methods

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References

- [AJKS, Section 3]

Provable Guarantees for Policy Optimization Algorithms

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- We know that classical policy iteration algorithm gives monotone improvement guarantee
- However, due to the unknown MDP and large state space, greedy improvement of $Q_\pi(s, a)$ for each state-action pair is infeasible
- How do we ensure provable improvement guarantees for policy optimization algorithms?

Performance Difference Lemma

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Lemma (Performance Difference Lemma)

Let $V_{\pi,\mu} = \mathbb{E}_{s\sim\mu}[V_\pi(s)]$. *For any two policies,* π *and* $\bar{\pi}$,

$$V_{\pi,\mu} - V_{\bar{\pi},\mu} = \frac{1}{(1-\gamma)}\mathbb{E}_{s\sim\rho_{\pi,\mu}(\cdot)}\mathbb{E}_{a\sim\pi(s,\cdot)}\left[A_{\bar{\pi}}(s,a)\right]$$

Performance Difference Lemma: Proof

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$$V_\pi(s) - V_{\bar{\pi}}(s) = \mathbb{E}_{\tau \sim P_{\text{traj},\pi}}\left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)\right] - V_{\bar{\pi}}(s)$$

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&= \mathbb{E}_{\tau \sim P_{\text{traj},\pi}} \left[\sum_{t=0}^{\infty} \gamma^t \left(r(s_t, a_t) + \gamma V_{\bar{\pi}}(s_{t+1}) - V_{\bar{\pi}}(s_t) \right) \right]
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Now, averaging over the initial state s with a given initial state distribution μ, we get

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V_{\pi,\mu} - V_{\bar{\pi},\mu} &= \frac{1}{(1-\gamma)} \mathbb{E}_{(s', a') \sim \rho_{\pi, \mu}} [A_{\bar{\pi}}(s', a')] \\
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Why PDL is Useful?

- PDL: $V_{\pi,\mu} - V_{\bar{\pi},\mu} = \frac{1}{(1-\gamma)}\mathbb{E}_{s\sim\rho_{\pi,\mu}}\mathbb{E}_{a\sim\pi(s,\cdot)}\left[A_{\bar{\pi}}(s,a)\right]$

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- Let π_k be the current policy. Then, for any policy π,

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  - However, we don't know $\rho_{\pi,\mu}$ and evaluating that expectation requires sampling according to π

Trust Region Methods

- "Ideal" policy update step:

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- Will this "practical" policy update step give an improvement?

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  2. $\pi_{k+1}(s,\cdot) = (1-\alpha)\pi_k(s,\cdot) + \alpha \pi'(s,\cdot)$ for all $s \in \mathcal{S}$

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$$\pi_{k+1} = \underset{\pi \in \texttt{trust-region}(\pi_k)}{\arg\max} \; \mathbb{E}_{s \sim \rho_{\pi_k, \mu}} \mathbb{E}_{a \sim \pi(s, \cdot)} \left[A_{\pi_k}(s, a) \right]$$

- How doe we ensure a trust region?
- Conservative Policy Iteration (CPI) algorithm:
  1. $\pi' = \arg\max_{\pi \in \Pi} \; \mathbb{E}_{s \sim \rho_{\pi_k, \mu}} \mathbb{E}_{a \sim \pi(s, \cdot)} \left[A_{\pi_k}(s, a) \right]$
  2. $\pi_{k+1}(s, \cdot) = (1 - \alpha) \pi_k(s, \cdot) + \alpha \pi'(s, \cdot)$ for all $s \in \mathcal{S}$
- In CPI, we get $\left\| \pi_{k+1}(s, \cdot) - \pi_k(s, \cdot) \right\|_1 \leq 2\alpha$

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Theorem (Monotone improvement)

Let $\bar{A}_k = \mathbb{E}_{s \sim \rho_{\pi_k,\mu}} \mathbb{E}_{a \sim \pi'(s,\cdot)}[A_{\pi_k}(s,a)]$. *Then,* $V_{\pi_{k+1},\mu} - V_{\pi_k,\mu} \geq \frac{\alpha}{(1-\gamma)}\left(\bar{A}_k - \frac{2\alpha\gamma}{(1-\gamma)^2}\right)$.

Set $\alpha = \frac{\bar{A}_k(1-\gamma)^2}{4\gamma}$. *Then,* $V_{\pi_{k+1},\mu} - V_{\pi_k,\mu} \geq \frac{\bar{A}_k^2(1-\gamma)}{8\gamma}$.

Auxiliary results

- We will state the following supporting result without proof (For proof, see [AJKS], Chapter 12)

Lemma

Suppose we have $\|\pi(s,\cdot) - \pi_k(s,\cdot)\|_1 \leq 2\alpha$ *for all* s. *Then, we have*

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- We will also use the following simple result

Lemma

Let z *be a random variable and* p, q *be two distributions. Then,*

$$|\mathbb{E}_{z\sim p}[f(z)] - \mathbb{E}_{z\sim q}[f(z)]| \leq \|p-q\|_1 \; \max_z |f(z)|$$

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Proof: From PDL,

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&= \alpha \ \mathbb{E}_{s\sim\rho_{\pi_{k+1},\mu}} \mathbb{E}_{a\sim\pi'(s,\cdot)}[A_{\pi_k}(s,a)]
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&\geq \alpha\left(\bar{A}_k - \frac{2\alpha\gamma}{(1-\gamma)^2}\right)
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Select α to get the maximum improvement. This will complete the proof.

$\square$

TRPO and PPO

References

- Schulman, et al. "Trust region policy optimization", *International Conference on Machine Learning (ICML)*, 2015.
- Schulman, et al. "Proximal policy optimization algorithms", 2017.
- [AJKS], Section 3

Some Distance Metric

- Total variation distance between two distributions p, q:

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- TRPO performs parameter update by solving the optimization problem

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- This reduces to NPG

$$\theta_{k+1} \leftarrow \theta_k + \sqrt{\frac{\alpha}{(\nabla V_{\pi_{\theta_k}})^\top F(\theta_k)^{-1} (\nabla V_{\pi_{\theta_k}})}} \, F(\theta_k)^{-1} (\nabla V_{\pi_{\theta_k}})$$

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- One obvious approach is: generate n i.i.d. samples $(z_i)_{i=1}^n$ according to p. Then, $\mathbb{E}_{z\sim p}[f(z)] \approx \frac{1}{n}\sum_{i=1}^n f(z_i)$

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- Suppose we can only generate i.i.d. samples according q. How do we get the estimate for $\mathbb{E}_{z \sim p}[f(z)]$?

Importance Sampling

- We have

$$\mathbb{E}_{z\sim p}[f(z)] = \mathbb{E}_{z\sim q}[\frac{p(z)}{q(z)}f(z)] \approx \frac{1}{n}\sum_{i=1}^{n}\frac{p(z_i)}{q(z_i)}f(z_i), \quad \text{where}, \quad z_i \sim q$$

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$$\begin{aligned}\text{Var}\left(\frac{p(z)}{q(z)}f(z)\right) &= \mathbb{E}_{z\sim q}\left[\left(\frac{p(z)}{q(z)}f(z)\right)^2\right] - \left(\mathbb{E}_{z\sim q}\left[\frac{p(z)}{q(z)}f(z)\right]\right)^2 \\ &= \mathbb{E}_{z\sim p}\left[\frac{p(z)}{q(z)}f^2(z)\right] - \left(\mathbb{E}_{z\sim p}[f(z)]\right)^2\end{aligned}$$

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- Importance sampling gives an unbiased estimate
- What is the variance of the importance sampling based estimate?

$$\begin{aligned} \text{Var}\left(\frac{p(z)}{q(z)} f(z)\right) &= \mathbb{E}_{z\sim q}\left[\left(\frac{p(z)}{q(z)} f(z)\right)^2\right] - \left(\mathbb{E}_{z\sim q}\left[\frac{p(z)}{q(z)} f(z)\right]\right)^2 \\ &= \mathbb{E}_{z\sim p}\left[\frac{p(z)}{q(z)} f^2(z)\right] - (\mathbb{E}_{z\sim p}[f(z)])^2 \end{aligned}$$

- Importance sampling weight, $p(z)/q(z)$, can be very large for some z. Then the variance can be very large.

Importance Sampling in RL

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- The optimization problem then can then be solved using a direct stochastic gradient ascent approach

Proximal Policy Optimization (PPO)

- PPO solves the following optimization problem

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- The constraints are enforced by a *clipping trick*

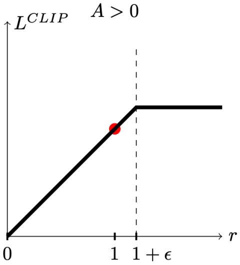
Proximal Policy Optimization (PPO)

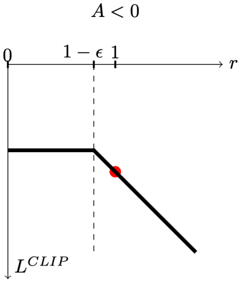
- For ensuring that $\pi_{\theta_k}(s,a)$ and $\pi_\theta(s,a)$ are not very different, PPO modifies the objective function as follows

$$L(\theta) = \mathbb{E}_{x\sim\rho_{\pi_{\theta_k}},\mu}\mathbb{E}_{a\sim\pi_{\theta_k}(s,\cdot)}\left[\min\left\{\frac{\pi_\theta(s,a)}{\pi_{\theta_k}(s,a)}A_{\pi_{\theta_k}}(s,a), \text{clip}\left(\frac{\pi_\theta(s,a)}{\pi_{\theta_k}(x,a)}; 1-\epsilon, 1+\epsilon\right)A_{\pi_{\theta_k}}(s,a)\right\}\right],$$

where

$$\text{clip}(z; 1-\epsilon, 1+\epsilon) = \begin{cases} 1-\epsilon & z \leq 1-\epsilon \\ 1+\epsilon & z \geq 1+\epsilon \\ z & \text{otherwise} \end{cases}$$





Proximal Policy Optimization (PPO)

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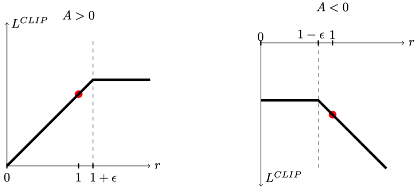

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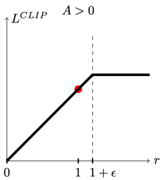

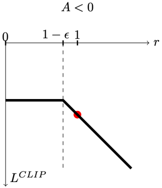

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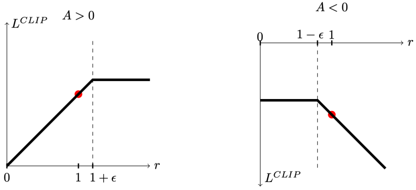

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- The *minimum* ensures that the objective function $L(\theta)$ is a lower bound of the original objective
- PPO optimizes the objective function using mini-batch stochastic gradient ascent (instead of the Taylor series expansion approach of NPG or TRPO)