ECEN 743: Reinforcement Learning

Deep Q-Learning

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Experience Replay and Target Network

Correlation and Forgetting in Q-Learning

- Q-learning with function approximation

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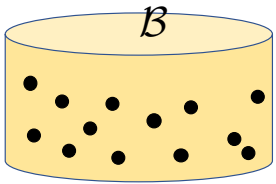
- Rapid forgetting: Information from not-so-frequent experiences (that would be useful later on) are not effectively used

Experience Relay

- Store samples of prior experience in replay buffer
- Sample from the replay buffer randomly for updating Q-value



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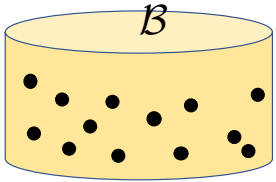

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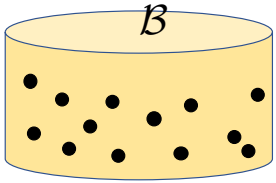

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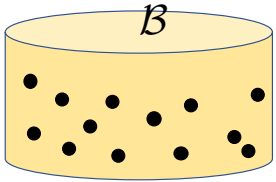

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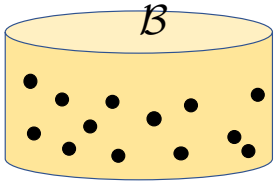

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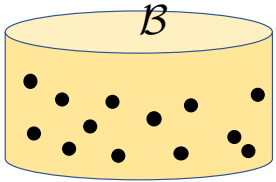

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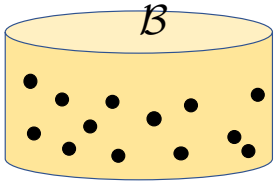

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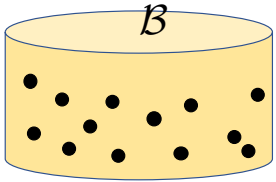

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- Experience replay breaks the temporal correlations by mixing data
- Experience replay uses (rare) experiences for more than just a single update

Target Network

- Q-learning gradient update:

$$w = w + \alpha \left(r_k + \gamma \max_b Q(w)(s_{k+1}, b) - Q(w)(s_k, a_k) \right) \nabla_w Q(w)(s_k, a_k)$$

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- Target network: A separate target network and keep it unchanged for multiple updates

$$w = w + \alpha \left(r_k + \gamma \max_b Q(w^-)(s_{k+1}, b) - Q(w)(s_k, a_k)\right) \nabla_w Q(w)(s_k, a_k)$$

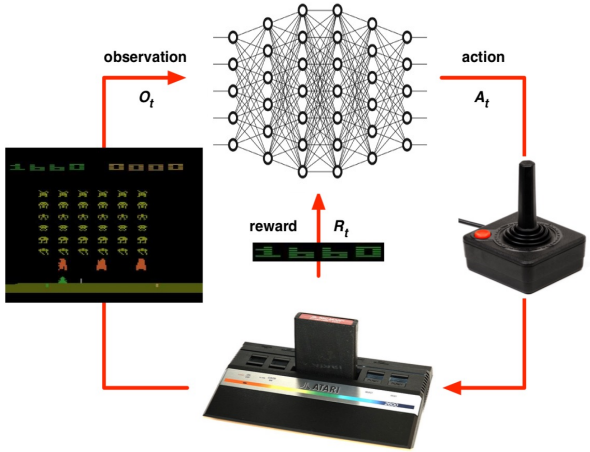
$w^- = w$, after every N steps

Deep Q Networks

Mnih et al, "Human-level control through deep reinforcement learning", *Nature*, 2015

Playing Atari Games using RL

- States: screen Images, 210 x 160 pixel, 128 color
- Actions: joystick positions, 18 different positions
- Reward: Game score



- Objective: Learn to win the game (a control policy that maximizes game score)

DQN for Atari

- Pre-processing
  - Each original frame is 210 x 160 pixel images with a 128-colour palette
  - Reduce it to to 84 x 84 images by pre-processing
  - Use the 4 most recent frame
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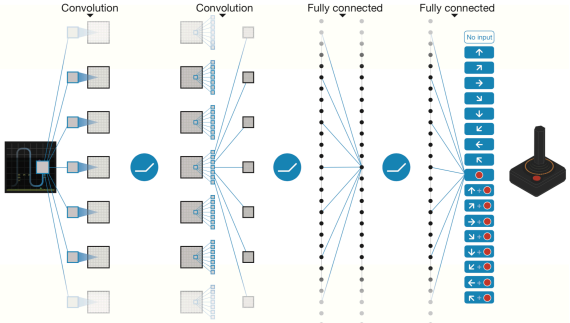

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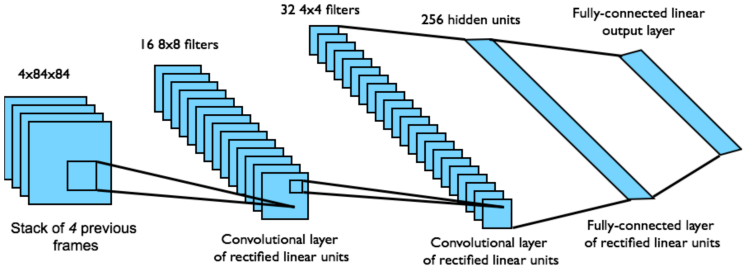
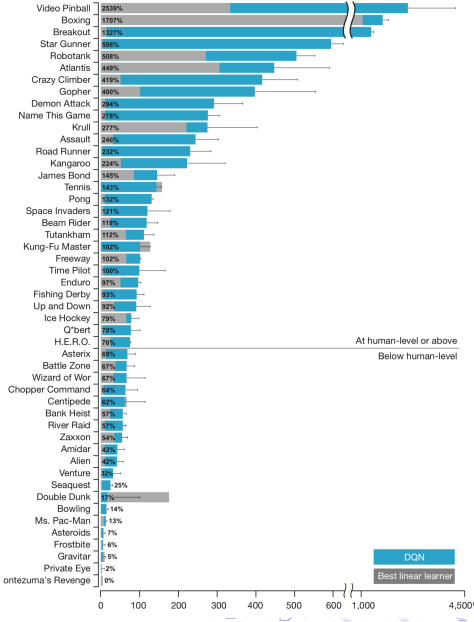


DQN Performance in Atari

- 100 x (DQN score - random play score)/(human score - random play score)
- Score for each game is averaged over 30 sessions on each game
- Professional human tester played using the same emulator
- DQN played better than the human player on 22 of the games
- DQN played better than the 75% of the human player performance on 29 of the games



DQN Training

- Training time: Training over 50 million frames. 38 days of game experience in total
- Replay memory: Recent 1 million frames
- Minibatch size: 32
- Target network update frequency: After every 10k parameter updates
- Action repeat: Repeat the same action for k (= 4) frames
- SGD: RMSProp, with learning rate 0.00025
- Exploration: Epsilon-greedy policy, with epsilon decreasing from 1.0 to 0.1 over first million frames and then fixed after
- Discount factor: 0.99

Effect of Replay and Target Network

Game	With replay, with target Q	With replay, without target Q	Without replay, with target Q	Without replay, without target Q
Breakout	316.8	240.7	10.2	3.2
Enduro	1006.3	831.4	141.9	29.1
River Raid	7446.6	4102.8	2867.7	1453.0
Seaquest	2894.4	822.6	1003.0	275.8
Space Invaders	1088.9	826.3	373.2	302.0

DQN Algorithm

Algorithm 1: deep Q-learning with experience replay.

Initialize replay memory D to capacity N

Initialize action-value function Q with random weights θ

Initialize target action-value function $\hat{Q}$ with weights $\theta^{-} = \theta$

For episode $= 1, M$ **do**

  Initialize sequence $s_1 = \{x_1\}$ and preprocessed sequence $\phi_1 = \phi(s_1)$

  For $t = 1$,T **do**

    With probability ε select a random action a_t

    otherwise select $a_t = \text{argmax}_a Q(\phi(s_t), a; \theta)$

    Execute action a_t in emulator and observe reward r_t and image x_{t+1}

    Set $s_{t+1} = s_t, a_t, x_{t+1}$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$

    Store transition $\left(\phi_t, a_t, r_t, \phi_{t+1}\right)$ in D

    Sample random minibatch of transitions $\left(\phi_j, a_j, r_j, \phi_{j+1}\right)$ from D

    Set $y_j = \begin{cases} r_j & \text{if episode terminates at step j+1} \\ r_j + \gamma \max_{a'} \hat{Q}\left(\phi_{j+1}, a'; \theta^{-}\right) & \text{otherwise} \end{cases}$

    Perform a gradient descent step on $\left(y_j - Q\left(\phi_j, a_j; \theta\right)\right)^2$ with respect to the network parameters θ

    Every C steps reset $\hat{Q} = Q$

  End For

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DQN Improvements

- Double DQN: [Van Hasselt et al., 2016][1], [Van Hasselt, 2010][2]
- Prioritized Experience Replay: [Schaul et al, 2016] [3]
- Rainbow DQN: [Hessel et al, 2018][4]

[1] H Van Hasselt, A Guez, D Silver, "Deep Reinforcement Learning with Double Q-Learning", *AAAI*, 2016
[2] Van Hasselt, "Double Q-Learning", *NeurIPS*, 2010
[3] Schaul et al, "Prioritized experience replay", *ICLR*, 2016
[4] Hessel et al, "Rainbow: Combining Improvements in Deep Reinforcement Learning", *AAAI*, 2018

DQN Improvements:
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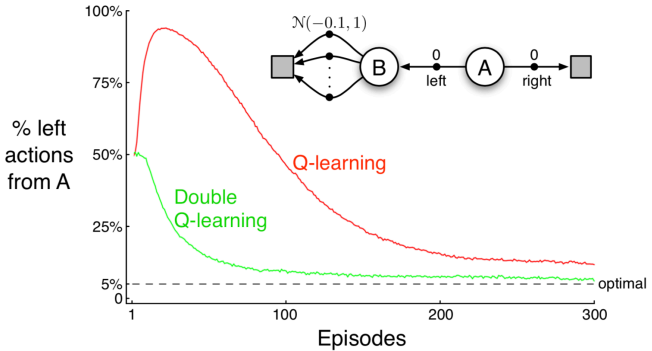
- Example 6.7, Sutton and Barto



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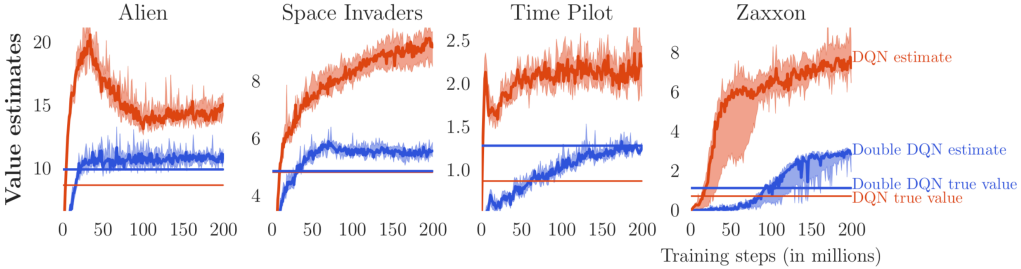
- Maximization bias is DQN, from [Van Hasselt et al., 2016]



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  3. Find $\hat{i}^* = \arg\max_i \hat{\mu}^i_A$ and get double estimator $\hat{\mu}_{\max} = \hat{\mu}^{\hat{i}^*}_B$
- We can show that the double estimator is an underestimator, i.e., $\mathbb{E}[\hat{\mu}_{\max}] \leq \max_i \mathbb{E}[Y^i]$

Double Estimator

Proof: Let $i^* = \arg\max_i \mathbb{E}[Y^i]$ be the true maximizer. Then,

$$\begin{aligned}
\mathbb{E}[\hat{\mu}_{\max}] &= \mathbb{E}[\hat{\mu}_B^{\hat{i}^*}] = \mathbb{P}(\hat{i}^* = i^*)\mathbb{E}[\hat{\mu}_B^{\hat{i}^*}|\hat{i}^* = i^*] + \mathbb{P}(\hat{i}^* \neq i^*)\mathbb{E}[\hat{\mu}_B^{\hat{i}^*}|\hat{i}^* \neq i^*] \\
&= \mathbb{P}(\hat{i}^* = i^*) \max_i \mathbb{E}[Y^i] + \mathbb{P}(\hat{i}^* \neq i^*)\mathbb{E}[\hat{\mu}_B^{\hat{i}^*}|\hat{i}^* \neq i^*] \\
&\leq \mathbb{P}(\hat{i}^* = i^*) \max_i \mathbb{E}[Y^i] + \mathbb{P}(\hat{i}^* \neq i^*) \max_i \mathbb{E}[Y^i] = \max_i \mathbb{E}[Y^i]
\end{aligned}$$

□

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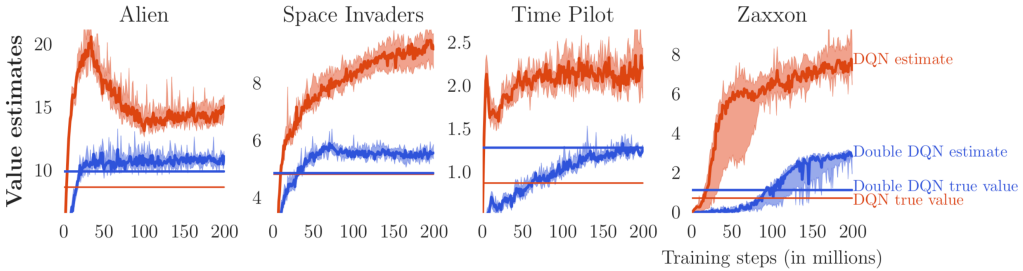
Double DQN Performance



Images from [Van Hasselt et al., 2016]

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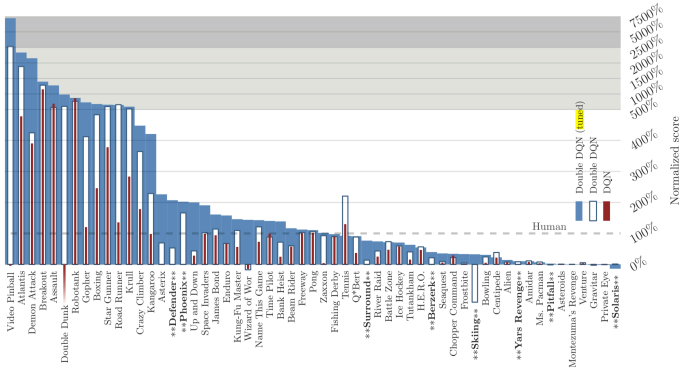

	no ops		human starts		
	DQN	DDQN	DQN	DDQN	DDQN (tuned)
Median	93%	**115**%	47%	88%	**117**%
Mean	241%	**330**%	122%	273%	**475**%

DQN Improvements
Prioritized Experience Replay

- Prioritized Experience Replay: [Schaul et al, 2016] [9]

[9] Schaul et al, "Prioritized experience replay", *ICLR*, 2016

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- Uniform sampling will not be able to make use of such informative samples effectively
- Can we prioritize the samples to accelerate the learning progress?

Prioritized Experience Replay

- Intuition: Prioritize a sample based on how much can learn from a transition (expected learning progress)

Image from [Schaul et al, 2016]

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- Proxy for this is TD error: For $e_i = (s_i, a_i, r_i, s_{i+1})$, TD error is defined as $\delta_i = r_i + \gamma \max_b Q(w)(s_{i+1}, b) - Q(w)(s_i, a_i)$

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- Prioritized experience replay: performance improvement

	DQN		Double DQN (tuned)		
	baseline	rank-based	baseline	rank-based	proportional
Median	48%	106%	111%	113%	128%
Mean	122%	355%	418%	454%	551%
> baseline	–	41	–	38	42
> human	15	25	30	33	33
# games	49	49	57	57	57

Image from [Schaul et al, 2016]

DQN Improvements:
Rainbow DQN

- Rainbow DQN: [Hessel et al., 2018][11]

[11] Hessel et al, "Rainbow: Combining Improvements in Deep Reinforcement Learning", *AAAI*, 2018

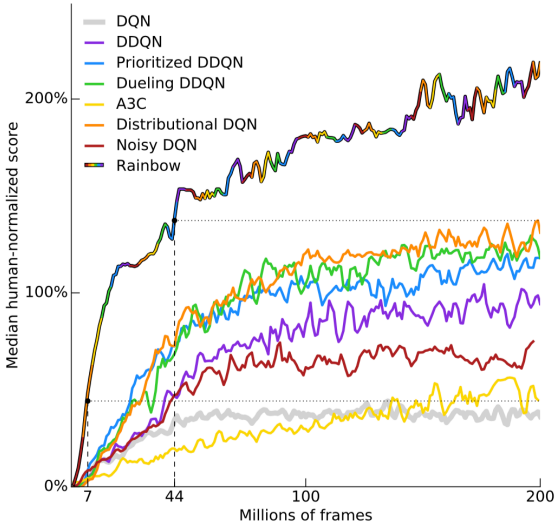
Rainbow DQN

Abstract

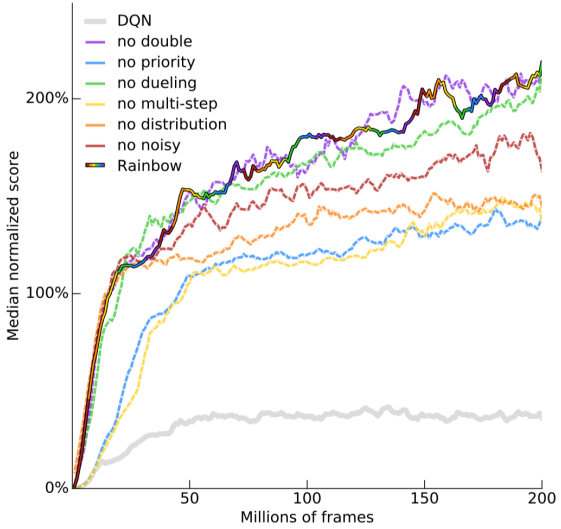
The deep reinforcement learning community has made several independent improvements to the DQN algorithm. However, it is unclear which of these extensions are complementary and can be fruitfully combined. This paper examines six extensions to the DQN algorithm and empirically studies their combination. Our experiments show that the combination provides state-of-the-art performance on the Atari 2600 benchmark, both in terms of data efficiency and final performance. We also provide results from a detailed ablation study that shows the contribution of each component to overall performance.

Image from [Hessel et al., 2018]

Rainbow DQN: Performance



Rainbow DQN: Ablation Studies



Agent57

- Agent57: Outperforming the human Atari benchmark [Link]
  - "We have developed Agent57, the first deep reinforcement learning agent to obtain a score that is above the human baseline on all 57 Atari 2600 games"