ECEN 628 ASSIGNMENT 3
AAKASH DESHMANE 133008022

1)

$$G(s) = \frac{s+1}{s^2(s+p)}$$
$$F(s) = \frac{s-1}{s(s+3)(s^2-2s+1.25)}$$

a) To find whether Vertex Lemma is applicable or not, we must first find the difference polynomial and check if it can be written in the standard form or not.

$$\delta_0 = s^t(as+b)P(s)$$

In our given system we get the following difference polynomial. (MATLAB code attached at the end)

$$\delta_0 = -4s^6 - 4s^5 + 19s^4 - 15s^3$$

In this case, we cannot write the polynomial in the standard form. Hence, Vertex Lemma is not sufficient to guarantee robust stability. It is necessary to check robustness using Kharitnov's polynomials,

ROBUSTNESS FROM KHARITNOV'S POLYNOMIAL'S

Closed loop characteristic polynomial is:

$$s^7 + (p+1)s^6 + \left(p - \frac{19}{4}\right)s^5 + \left(\frac{15}{4} - \frac{19*p}{4}\right)s^4 + \frac{15ps^3}{4} + s^2 - 1$$

(MATLAB code attached below)

P ranges in [1,5].
Intermediate polynomials:

$$K^{even}max = 6s^6 - 20s^4 + s^2 - 1$$
$$K^{even}min = 2s^6 - s^4 + s^2 - 1$$
$$K^{odd}max = s^7 + 0.25s^5 + 18.75s^3$$
$$K^{odd}min = s^7 - 3.75s^5 + 3.75s^3$$

Kharitnov's polynomials are:

$$K_1 = s^7 + 2s^6 - 3.75s^5 - s^4 + 3.75s^3 + s^2 - 1$$
$$K_2 = s^7 + 2s^6 + 0.25s^5 - s^4 + 18.75s^3 + s^2 - 1$$
$$K_3 = s^7 + 6s^6 + 0.25s^5 - 20s^4 + 18.75s^3 + s^2 - 1$$
$$K_4 = s^7 + 6s^6 - 3.75s^5 - 20s^4 + 3.75s^3 + s^2 - 1$$

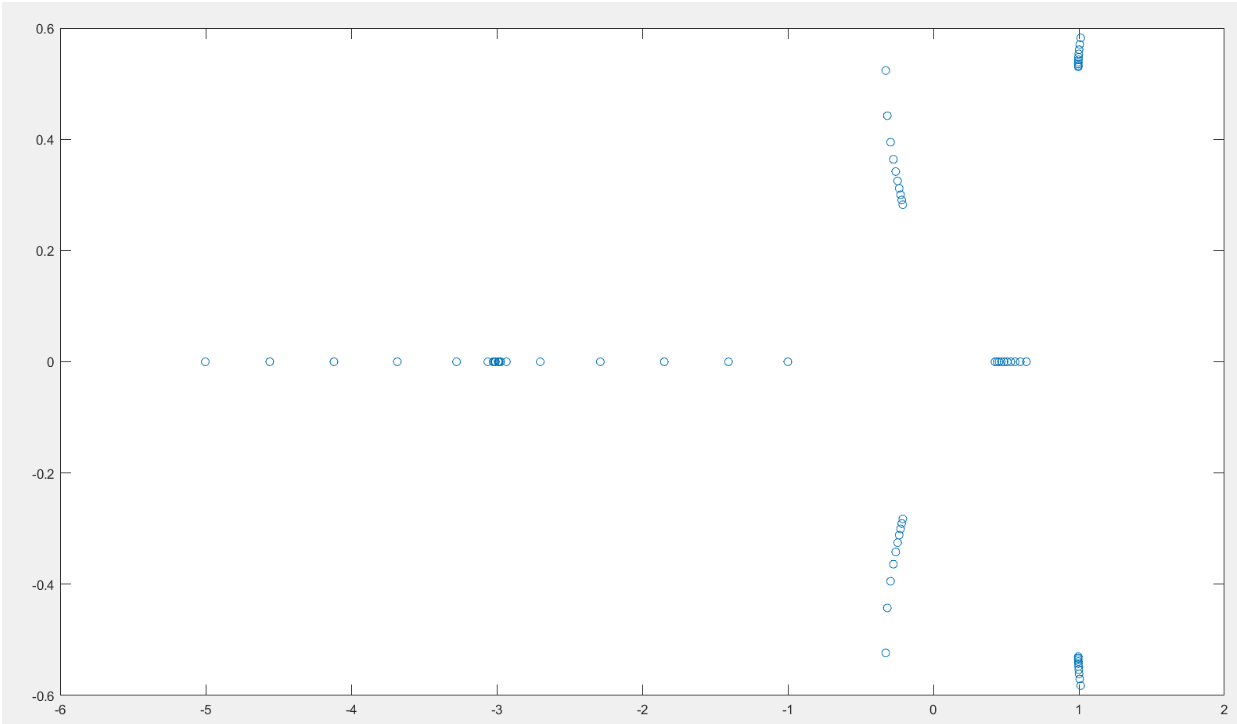
Testing the Hurwitz of these polynomials:
For K1:

1	-3.75	3.75	0
2	-1	1	-1
-3.25	3.25	0.5	0
1	1.3076923076923077	-1	0
7.5	-2.75	0	0
1.6743589743589744	-1	0	0
1.7293261868300152	0	0	0
-1	0	0	0

Table 1. Routh Hurwitz Table of the family of polynomials

As we can see, there is a sign change, hence the system is unstable under parametric perturbations. If one of the polynomials is unstable, there is no point in checking the Hurwitz criteria of the other 3 polynomials.
We can conclude that the system is unstable.

b) If we plot the root locus of the family of polynomials with varying p, we get the following graph. From the graph it is evident that the family of polynomials is unstable as there are roots seen in the right half of the complex plane as p traverses in the interval [1,5].
Hence, this verifies our results we got from Kharitnov's polynomials that our system is robustly UNSTABLE.



Plot 1. Root locus plot of the closed loop polynomial

2)

$$n(s,a) = s^2 + (3-a)s + 1$$
$$d(s,a) = s^3 + (4+a)s^2 + 6s + 4 + a$$

Closed loop characteristic polynomial is :

$$\delta = d(s,a) + k * n(s,a)$$
$$= s^3 + (a + k + 4)s^2 + (3k - ak + 6)s + a + k + 4$$

The stability conditions for the family of polynomials can be found out by the Routh Hurwitz criterion easily.

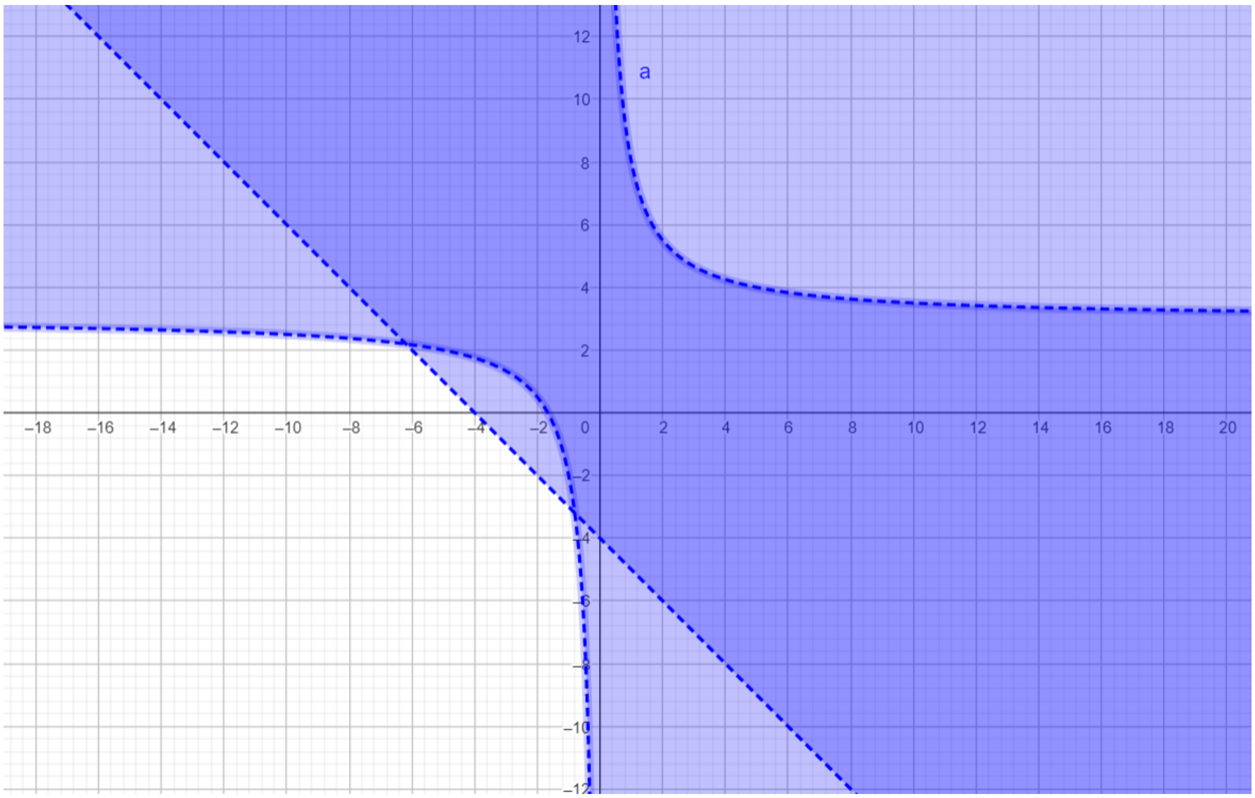
Applying RH:

S3	1	3k – ak + 6
S2	a + k + 4	a + k + 4
S1	3k – ak + 5	0
S0	a + k + 4	0

From the RH table, we get our inequalities,

$$(3 - a)k + 5 > 0$$
$$a + k + 4 > 0$$

Graphing these conditions, we get:



Plot 2. Stable and Unstable regions of the family of polynomials.

3)

$$n(s, a) = s + a$$
$$d(s, a) = s^3 + 2as^2 + as - 1$$

Closed loop characteristic polynomial is :

$$\delta = d(s, a) + k * n(s, a)$$
$$= s^3 + 2as^2 + (a + k)s + ak - 1$$

a ranges [2,3].

Intermediate polynomials:

$$K^{even}max = 6s^2 + 3k - 1$$
$$K^{even}min = 4s^2 + 2k - 1$$
$$K^{odd}max = s^3 + (3 + k)s$$
$$K^{odd}min = s^3 + (2 + k)s$$

Kharitnov's polynomials are:

$$K_1 = s^3 + 4s^2 + (2 + k)s + 2k - 1$$
$$K_2 = s^3 + 4s^2 + (3 + k)s + 2k - 1$$
$$K_3 = s^3 + 6s^2 + (3 + k)s + 3k - 1$$
$$K_4 = s^3 + 6s^2 + (2 + k)s + 3k - 1$$

Constructing the RH table for all the Kharitnov polynomials:
K1:

S3	1	2+k
S2	4	2k - 1
S1	9 + 2k	0
S0	2k - 1	0

This gives us the following inequality:

$$k > -\frac{9}{2}$$
$$k > \frac{1}{2}$$

K2:

S3	1	3+k
S2	4	2k - 1
S1	13 + 2k	0
S0	2k - 1	0

This gives us the following inequality:

$$k > -\frac{13}{2}$$
$$k > \frac{1}{2}$$

K3:

S3	1	3+k
S2	6	3k - 1
S1	19 + 3k	0
S0	3k - 1	0

This gives us the following inequality:

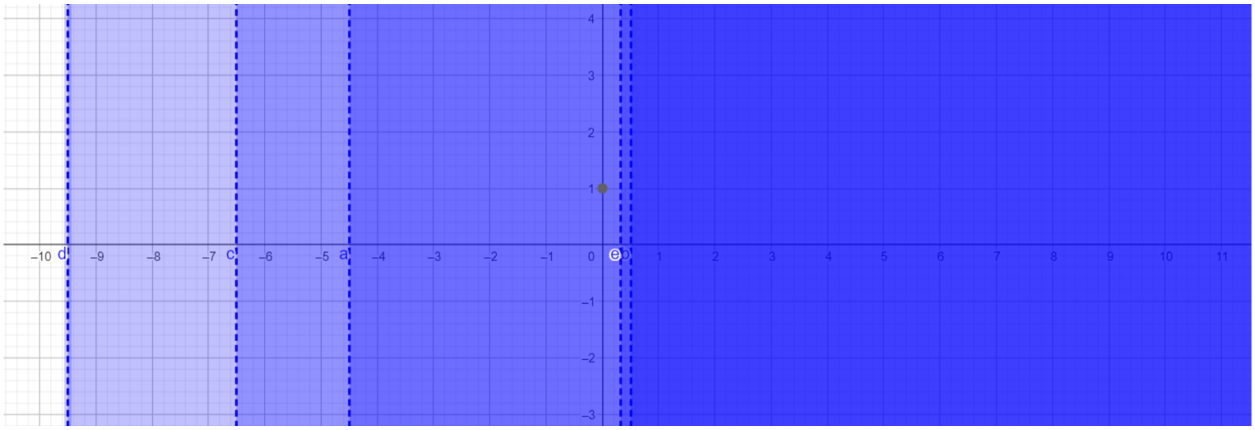
$$k > -\frac{19}{3}$$
$$k > \frac{1}{3}$$

K4:

S3	1	3+k
S2	6	3k - 1
S1	19 + 3k	0
S0	3k - 1	0

This gives us the following inequality:

$$k > -\frac{13}{3}$$
$$k < \frac{1}{3}$$



Plot 3. Region of stability for the given family of polynomials.

From all the inequalities, we get that the stable region for k lies between:

$$k > -\frac{9}{2}$$
$$k < \frac{1}{3}$$

Hence, we have our robustly stabilizing controller for the family of polynomials to be K ϵ (-4.5, 0.33)

10.9) Part 1 is solved in MATLAB by using Edge Theorem and finding the root space of the given family, code attached below.

Assuming all nominal parameters to be zero and all weights to be 1, we get:
Stability margin via L2 norm:

Characteristic eq:

$$p_1 s^3 + \left(\frac{17p_1}{2} - \frac{13p_2}{2}\right)s^2 + \left(\frac{25p_2}{2} - \frac{19p_1}{2}\right)s + p2 = 0$$

Substituting s = jw and Rearranging in matrix form:

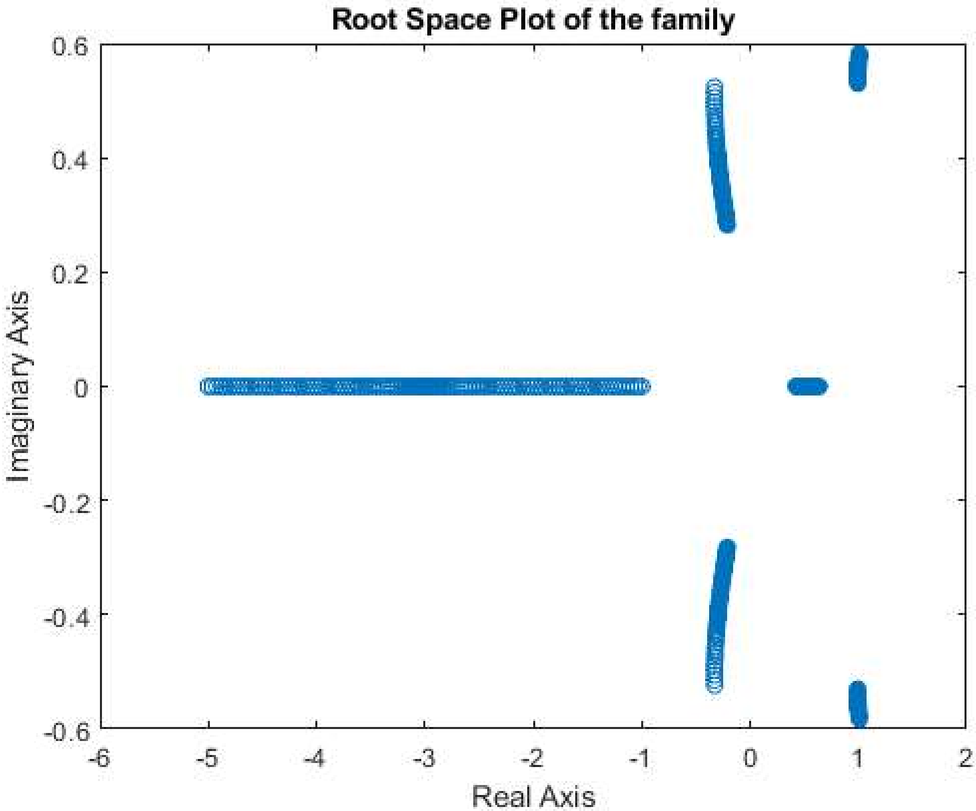
$$\begin{matrix} 0 & 0 \\ 0 & 0 \end{matrix} \begin{matrix} p_1 \\ p_2 \end{matrix} = \begin{matrix} 0 \\ 0 \end{matrix}$$

Stability margin is 0 for the range of all p1 and p2 values. This is evident from the plot from the Matlab code as well as there is one root at (0,0) in the complex plane. This means that the system has MARGINAL robust stability with no margins.
If the system is marginally robustly stable, then we do not need to find the l_infinity norm of the system.

ECEN 628 ASSIGNMENT 3

AAKASH DESHMANE 133008022 Q.10.1

```
%___________________________________________________________________________
clc
clear
syms s p
p2 = s + 1;
p1 = s^3;
p0 = s^2;
q2 = s - 1;
q1 = s*(s + 3)*(s^2 - 2*s + 1.25);
d1 = q2*p2 + q1*(p1 + p0);
d2 = q2*p2 + q1*(p1 + 5*p0);
d0 = d1 - d2;
%d = q2*p2 + q1*(p1 + p*p0)
n = length(linspace(1,5,10));
sol = [];
f = 1;
% p = 2;
delta = q2*p2 + q1*(p1 + p*p0);
for lambda = linspace(1,5,100)
    d = subs(delta,p,lambda);
    solution = vpasolve(d,s);
    r = double(real(solution));
    i = double(imag(solution));
    sol = [sol;
           r i];
    % f = f + 10;
end
plot(sol(:,1),sol(:,2),'o')
xlabel('Real Axis')
ylabel('Imaginary Axis')
title('Root Space Plot of the family')
```



Root Space Plot of the family
Imaginary Axis
Real Axis
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-6
-5
-4
-3
-2
-1
0
1
2

ECEN 628 ASSIGNMENT 3

AAKASH DESHMANE 133008022 Q.10.9

```
%

clear
clc
syms s p1 p2
format long
tstart = cputime;
% Closed loop characteristic polynomial

eqn = p1*s*(s + 9.5)*(s - 1) - p2*(6.5*s + 0.5)*(s - 2);
sol = solve(eqn,s,'MaxDegree',3);

n = 20;
p = zeros(3*n^4,2);
count = 1;

% Iterating through all family of polynomials

for p_1 = linspace(1,1.1,n)
    for p_2 = linspace(1.2,1.25,n)

        % Solving for the particular polynomial
        s1 = double(subs(sol,{p1,p2},{p_1,p_2}));

        % Calculating 1st root
        x = s1(1);
        r = real(x);
        i = imag(x);
        p(count,:) = [r i];
        count = count + 1;

        % Calculating 2nd root
        x = s1(2);
        r = real(x);
        i = imag(x);
        p(count,:) = [r i];
        count = count + 1;

        % Calculating 3rd root
        x = s1(3);
        r = real(x);
        i = imag(x);
        p(count,:) = [r i];
        count = count + 1;

    end
end

% Stability Condition
if max(p)>0
```

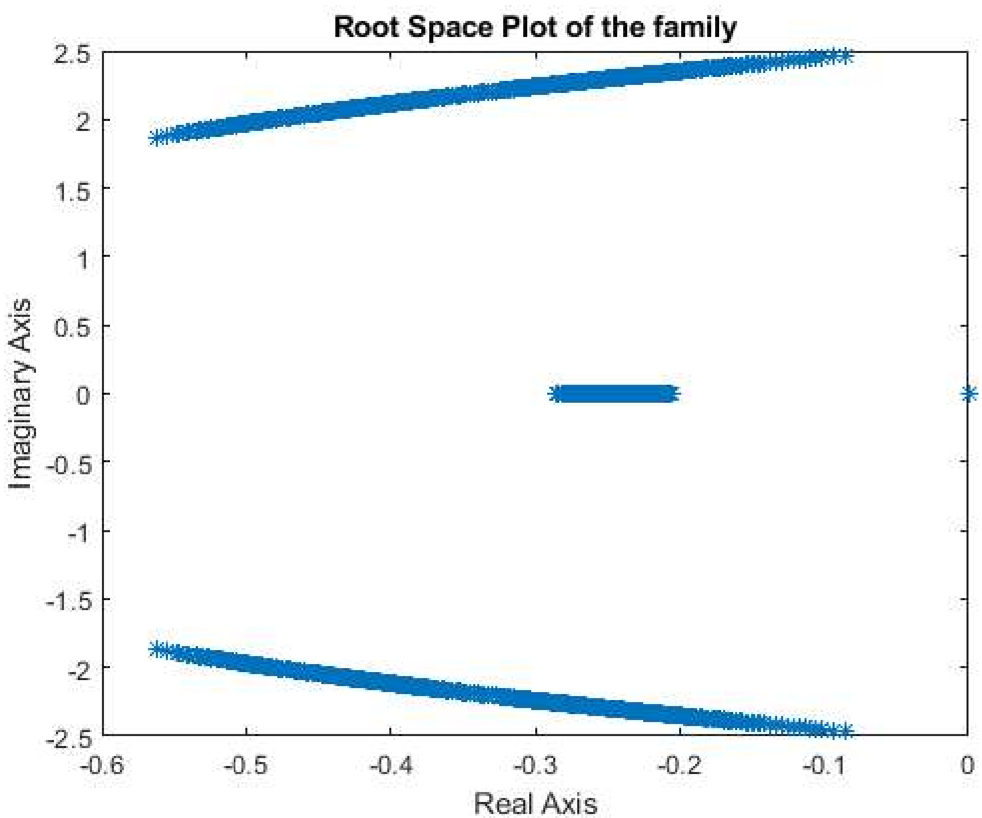
```
    disp('Roots are present in right half of complex plane. Hence, System is NOT ROBUSTLY STABLE!')
else
    disp('System is R0BUSTLY STABLE!')
end

% Plotting
plot(p(:,1),p(:,2),'*')
xlabel('Real Axis')
ylabel('Imaginary Axis')
title('Root Space Plot of the family')
tend = cputime - tstart
```

```
System is R0BUSTLY STABLE!

tend =

   1.937500000000000
```



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ECEN 628 ASSIGNMENT 3

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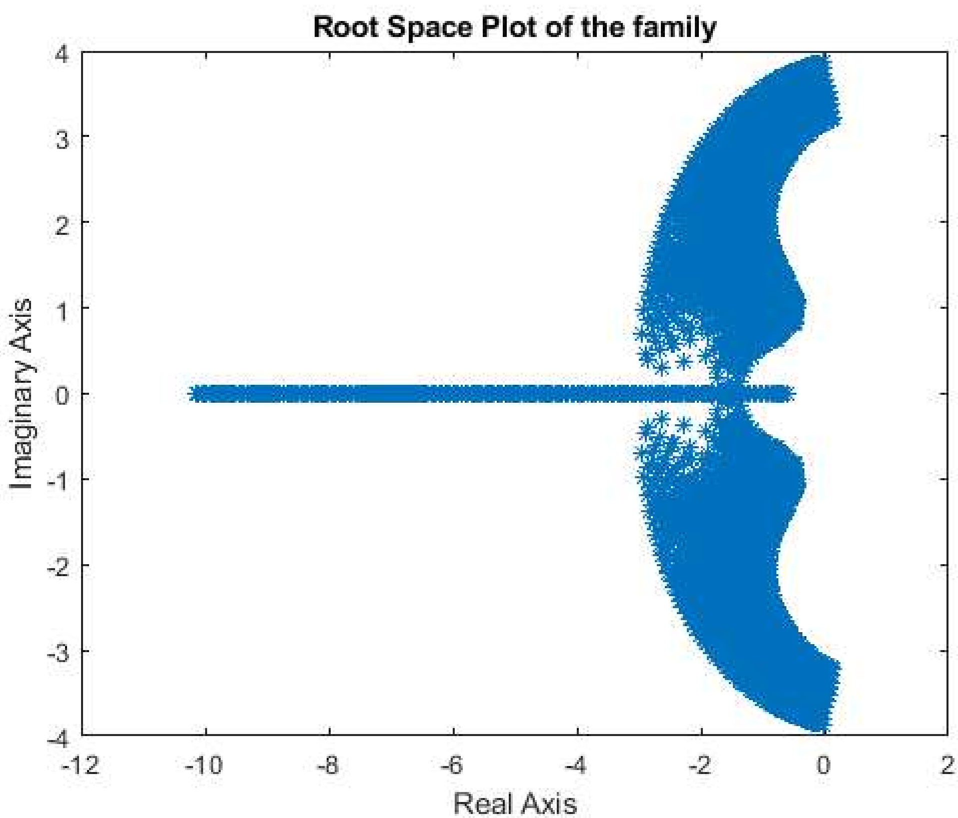
```
%_______________________________________________________________________

clear
clc
syms s a1 b1 c1 d1
format long
tstart = cputime;
% Closed loop characteristic polynomial

eqn = s^3 + (a1 + 3*b1)*s^2 + c1*s + d1;
sol = solve(eqn,s,'MaxDegree',3);

n = 10;
p = zeros(3*n^4,2);
count = 1;

% Iterating through all family of polynomials

for a = linspace(1,2,n)
    for b = linspace(0,3,n)
        for c = linspace(10,15,n)
            for d = linspace(9,14,n)

                % Solving for the particular polynomial
                s1 = double(subs(sol,{a1,b1,c1,d1},{a,b,c,d}));

                % Calculating 1st root
                x = s1(1);
                r = real(x);
                i = imag(x);
                p(count,:) = [r i];
                count = count + 1;

                % Calculating 2nd root
                x = s1(2);
                r = real(x);
                i = imag(x);
                p(count,:) = [r i];
                count = count + 1;

                % Calculating 3rd root
                x = s1(3);
                r = real(x);
                i = imag(x);
                p(count,:) = [r i];
                count = count + 1;

            end
        end
    end
end
```

```
% Stability Condition
if max(p)>0

    disp('Roots are present in right half of complex plane. Hence, System is NOT ROBUSTLY STABLE!')
else
    disp('System is R0BUSTLY STABLE!')
end

% Plotting
plot(p(:,1),p(:,2),'*')
xlabel('Real Axis')
ylabel('Imaginary Axis')
title('Root Space Plot of the family')
tend = cputime - tstart;
```

```
Roots are present in right half of complex plane. Hence, System is NOT ROBUSTLY STABLE!
```



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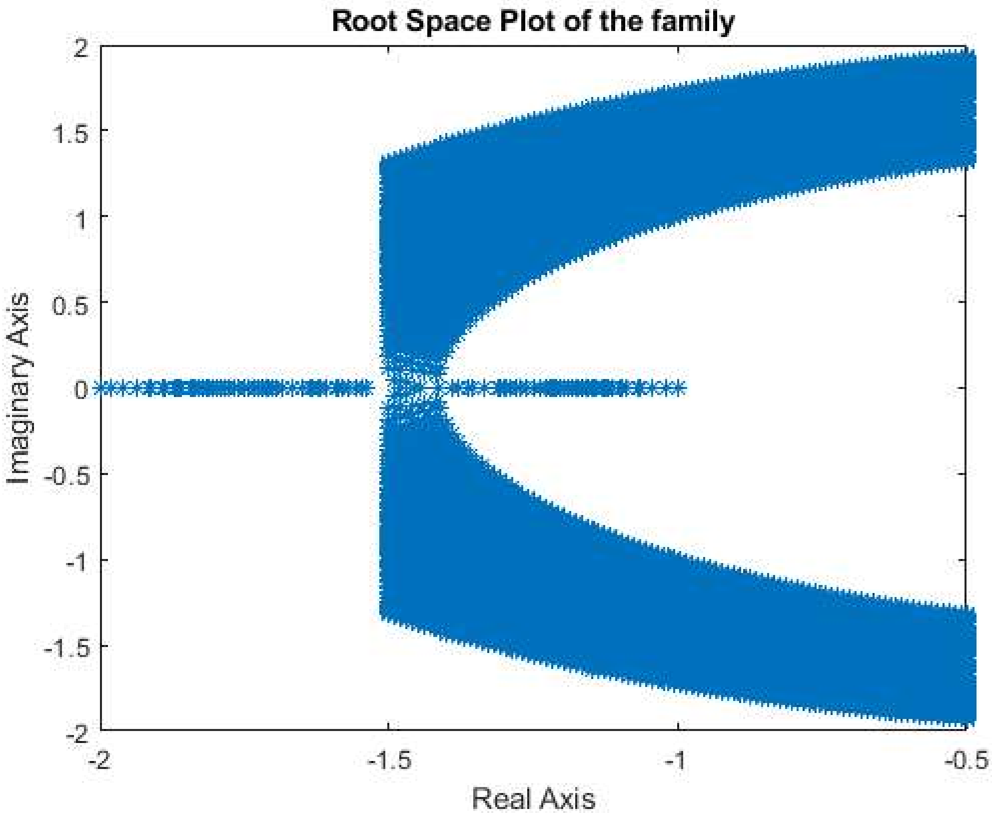
ECEN 628 ASSIGNMENT 3

AAKASH DESHMANE 133008022 Q.10.21

```
%_________________________________________________________________________

clear
clc
syms s a b
tstart = cputime;

% Closed loop characteristic polynomial
eqn = s*(s - 1) + a*s + b;
sol = solve(eqn,s);

count = 1;
n = 100;
p = zeros(2*n^2,2);

% Iterating through all family of polynomials
for kp = linspace(2,4,n)
    for ki = linspace(2,4,n)

        % Solving for the particular polynomial
        s1 = double(subs(sol,{a,b},{kp,ki}));

        % Calculating 1st root
        x = s1(1);
        r = real(x);
        i = imag(x);
        p(count,:) = [r i];
        count = count + 1;

        % Calculating 2nd root
        x = s1(2);
        r = real(x);
        i = imag(x);
        p(count,:) = [r i];
        count = count + 1;

    end
end

% Stability Condition
if max(p)>0

    disp('Roots are present in right half of complex plane. Hence, System is NOT ROBUSTLY STABLE!')
else
    disp('System is ROBUSTLY STABLE!')
end

% Plotting
plot(p(:,1),p(:,2),'*')
xlabel('Real Axis')
ylabel('Imaginary Axis')
```

```
title('Root Space Plot of the family')
tend = cputime - tstart;
```

```
System is R0BUSTLY STABLE!
```



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