ECEN 743: Reinforcement Learning

Inverse Reinforcement Learning

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References

- "Imitation Learning Tutorial",Yisong Yue, Hoang M. Le, ICML 2018. [Link]
- [AJKS, Chapter 15]

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- Designing rewards which can induce the desired behavior of the system is incredibly challenging

RL without Reward

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- IRL objective: Estimate r and use that to recover the expert policy

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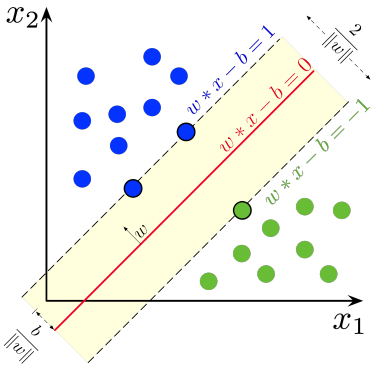
Maximum Margin Learning in SVM

- Support Vector Machine (SVM): Find a hyperplane that separates two classes
- There are infinitely many solutions
- How do we select one?



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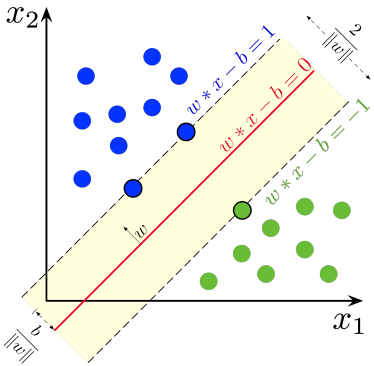


- Maximum margin principle

$$\max_{w} \frac{1}{||w||}$$

s.t. w separates the data set

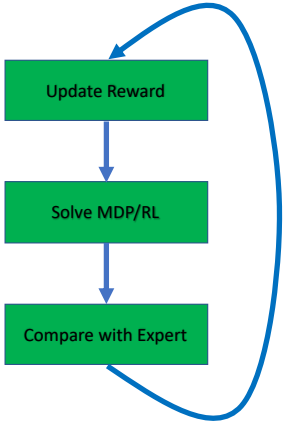
Maximum Margin IRL

Algorithm 1 Maximum Margin IRL

1: Initialization: Set $i = 0$. Select initial policy π_0. Estimate $\rho^{\phi}_{\pi_0}$ by generating data according to π_0 using the simulator
2: **for** $i = 1, 2, \ldots$ **do**
3: Solve for w_i, m_i:

$$\max_{m,w} \; m$$
$$\text{s.t.} \;\; w^\top \rho^{\phi}_{\pi^*} \geq w^\top \rho^{\phi}_{\pi_j} + m, \;\; j = 0, 1, \ldots, i-1$$
$$||w||_2 \leq 1$$

4: **if** $m_i \leq \epsilon$ **then**
5: Terminate the loop
6: **end if**
7: Learn the optimal policy π^i for the MDP using rewards $r(s,a) = w_i^\top \phi(s,a)$
8: Estimate $\rho^{\phi}_{\pi_i}$
9: **end for**



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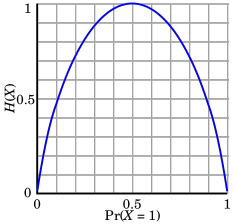

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  - This is maximum entropy RL where because the objective is

$$\max_{\pi} \left(\mathbb{E}_{(s,a)\sim\rho_\pi}[w^\top \phi(s,a) + H(\pi(s,\cdot))]\right)$$

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- The inner optimization problem (find the best policy given w)

$$\max_{\pi} \left(\mathbb{E}_{(s,a)\sim\rho_\pi}[w^\top \phi(s,a)] - \mathbb{E}_{(s,a)\sim\rho_\pi}[\log \pi(s,a)]\right)$$

  - Find the policy that maximizes the value function with an additional entropy regularization cost
  - This is maximum entropy RL where because the objective is

$$\max_{\pi} \left(\mathbb{E}_{(s,a)\sim\rho_\pi}[w^\top \phi(s,a) + H(\pi(s,\cdot))]\right)$$

- Outer optimization problem (update w)

$$w \leftarrow w + \alpha \left(\mathbb{E}_{(s,a)\sim\rho_\pi}[\phi(s,a)] - \mathbb{E}_{(s,a)\sim\rho_{\pi^*}}[\phi(s,a)]\right)$$

A Relaxed Problem

- Maximum Entropy IRL:

$$\max_{\pi} \; H(\pi) \;\; \text{s.t.} \;\; \mathbb{E}_{(s,a)\sim\rho_{\pi}(\cdot)}[\phi(s,a)] = \mathbb{E}_{(s,a)\sim\rho_{\pi^*}(\cdot)}[\phi(s,a)]$$

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Discriminator as Pseudo-Reward Function

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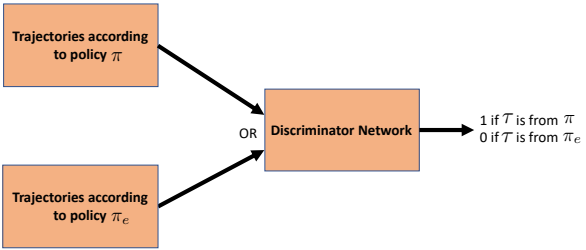

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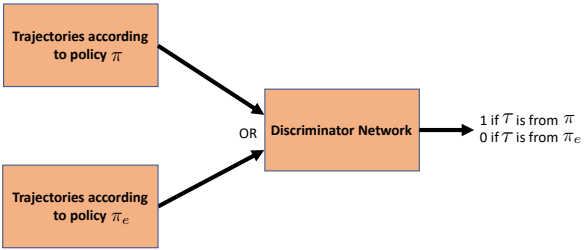

- This can be written as

$$d(\rho_\pi, \rho_{\pi_e}) = \max_D \quad \mathbb{E}_{(s,a)\sim\rho_\pi}[\log D(s,a)] + \mathbb{E}_{(s,a)\sim\rho_{\pi_e}}[\log(1 - D(s,a))]$$

GAIL

- Generative Adversarial Imitation Learning (GAIL)[3]

$$\min_{\theta} \max_{w} \quad \mathbb{E}_{(s,a)\sim\rho_{\pi_\theta}}[\log D_w(s,a)] + \mathbb{E}_{(s,a)\sim\rho_{\pi_e}}[\log(1 - D_w(s,a))] - \lambda H(\pi_\theta)$$

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- For a given discriminator parameter w, the policy optimization problem is

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- Update θ (policy):

$$\theta \leftarrow \theta - \beta_k \; \nabla_\theta \Big(\mathbb{E}_{(s,a)\sim\rho_{\pi_\theta}}[\log D_w(s,a)] - \lambda H(\pi_\theta)\Big)$$

[3] Ho and Ermon, "Generative adversarial imitation learning", NeurIPS, 2016.

Generative Adversarial Networks

References for GAN

- "Introduction to GANs", CVPR 2018 Tutorial on GANs by Ian Goodfellow [Link]
- Goodfellow et al. “Generative adversarial networks”, NeurIPS, 2014

- Figures from the above two references are used in this lecture

Generative Models

- We have a set of data $\{x_i\}_{i=1}^n$

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- Can we estimate a model p_{model} and generate samples $x_i' \sim p_{\text{model}}(\cdot)$, according to this model?

Generative Models

- Generative models: Given training data, generate new samples from same distribution

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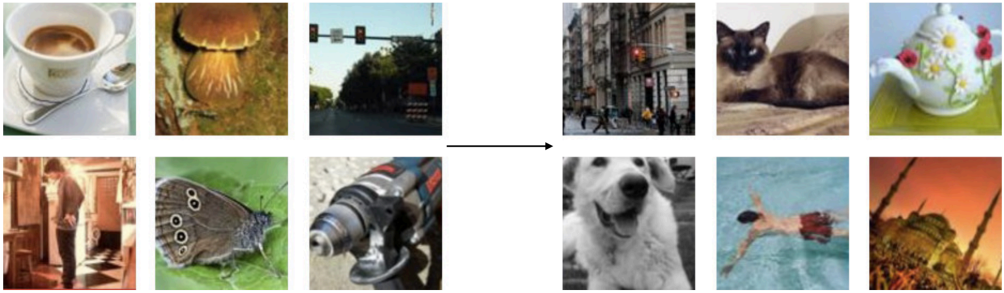
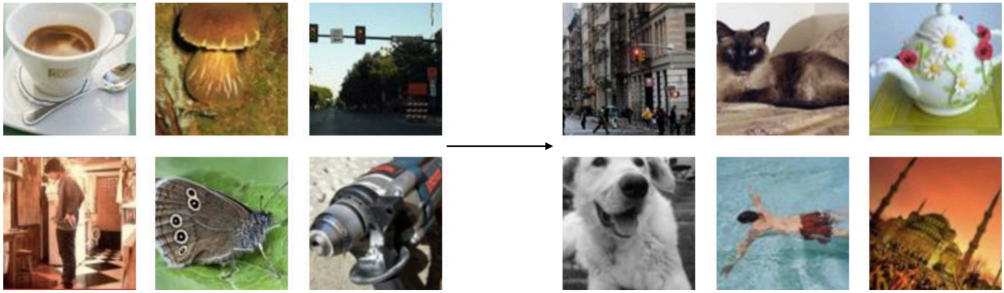


Training examples

Model samples

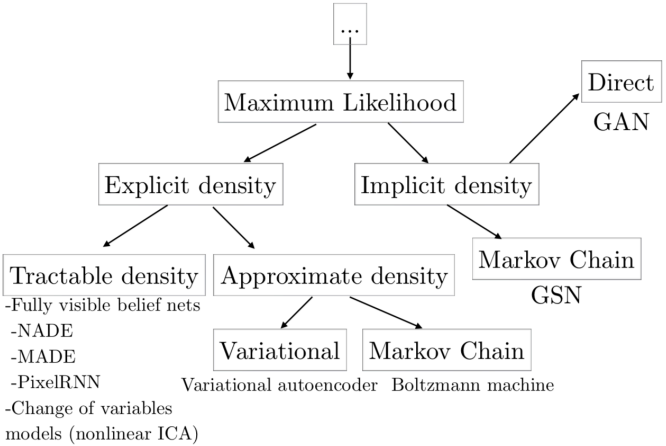
Generative Models

- Generative models: Given training data, generate new samples from same distribution



- Given the training data, say, samples from ImageNet, can we generate new images that are not in the original training data?

Generative Models



Generative Adversarial Network (GAN)

- GAN: A neural network that takes a random noise sample as the input and output a realistic image which was not in the training data!

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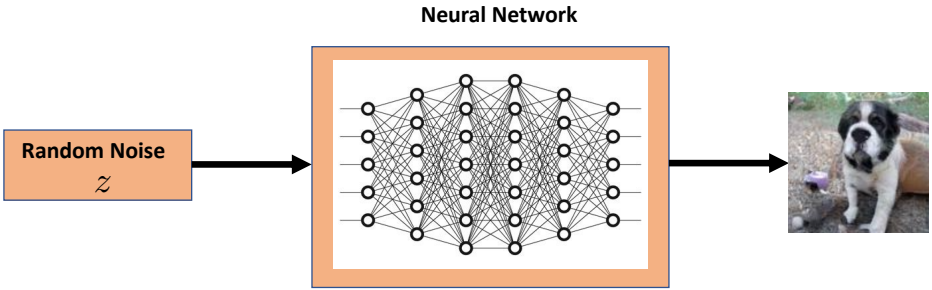
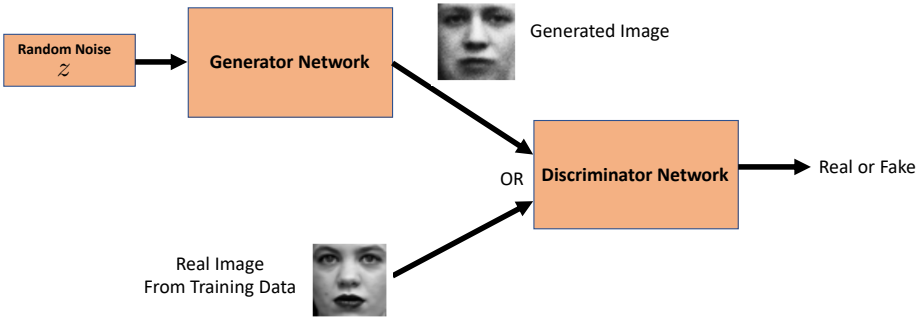


Generator and Discriminator



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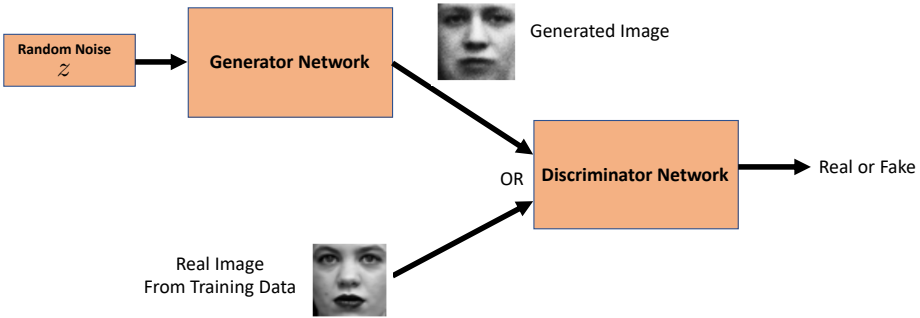

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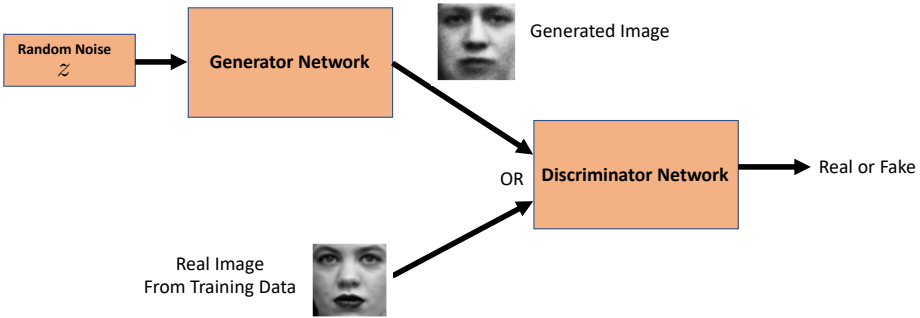

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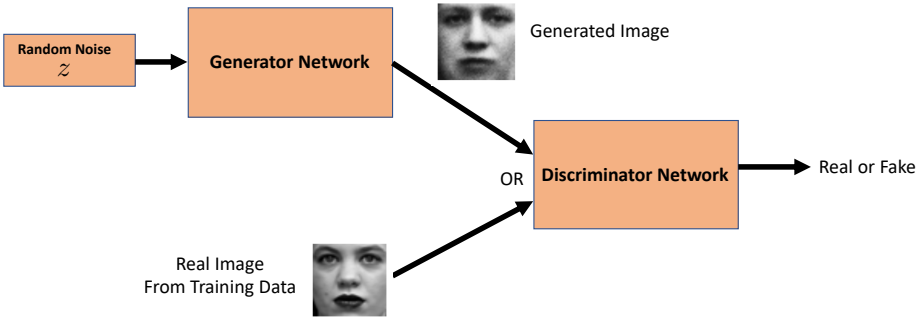

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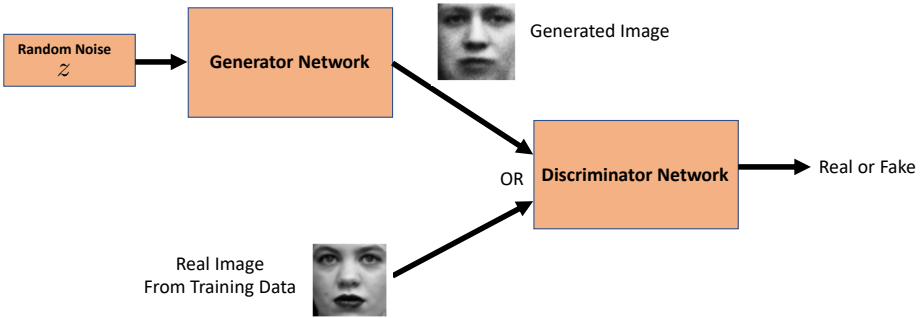

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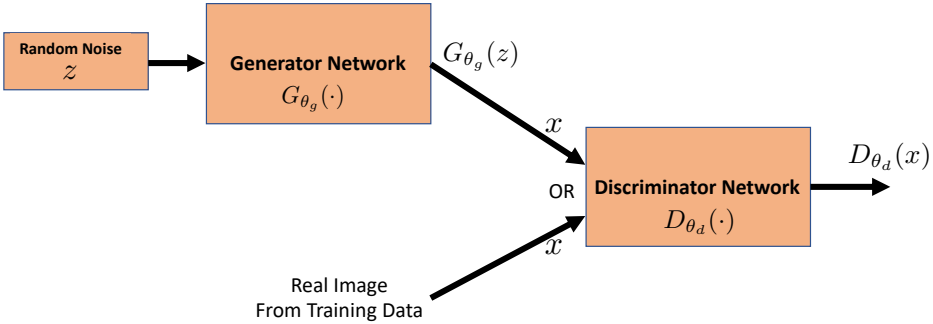
  - "analogous to the police, trying to detect the counterfeit currency"

GAN Objective



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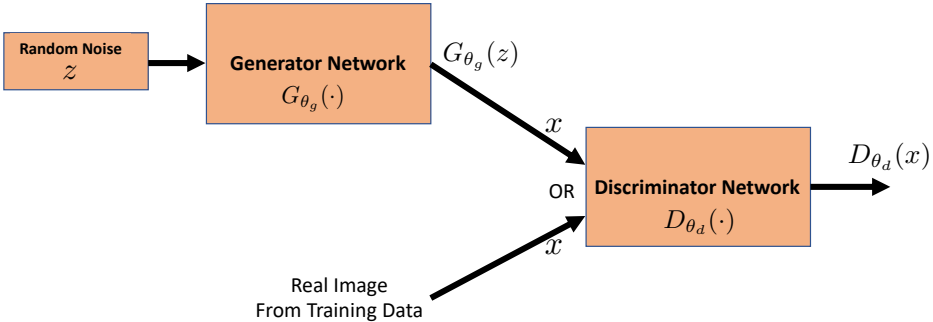

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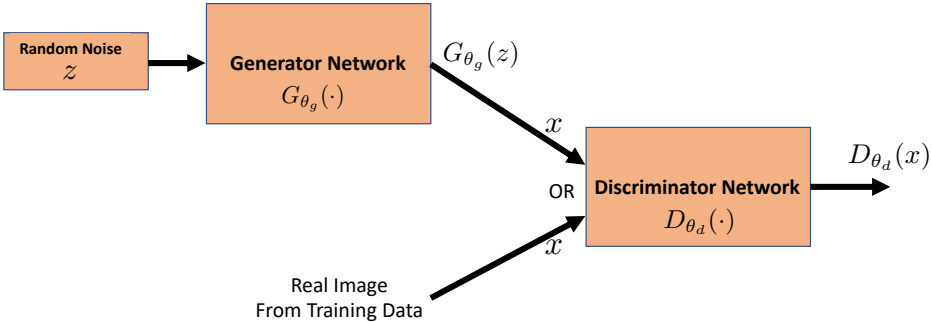

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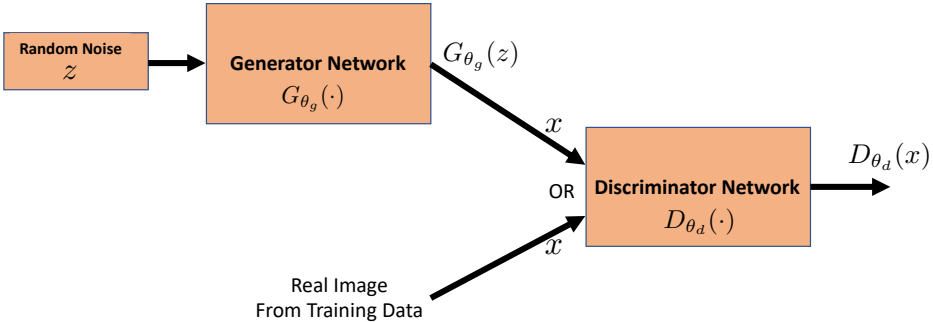

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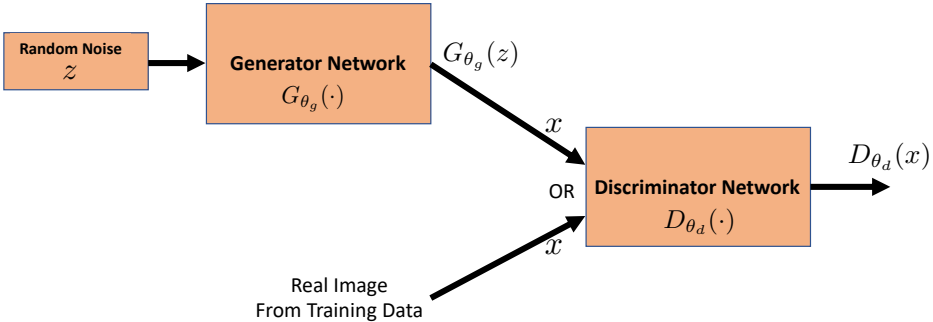

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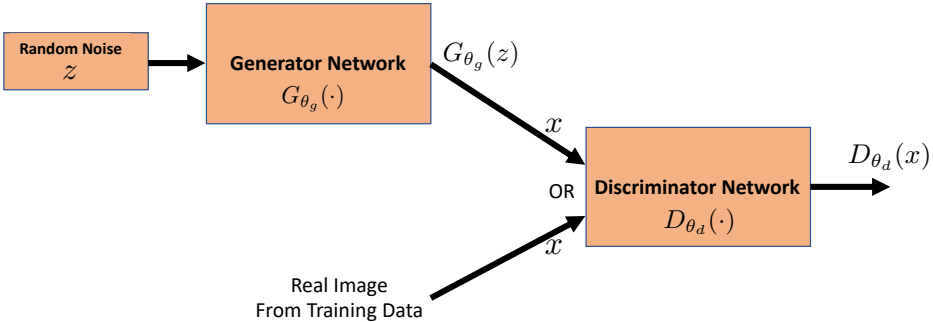

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  - On the other hand, we want $D_{\theta_d}(G_{\theta_g}(z))$ to be close to zero.
- Our goal is to train generator and discriminator such that after training we will have $p_g \approx p_{\text{data}}$

Game and Value Function

- Discriminator objective

$$\max_{D} \quad \mathbb{E}_{x \sim p_{\textbf{data}}(\cdot)}[\log D(x)] + \mathbb{E}_{z \sim p_z(\cdot)}[\log(1 - D(G(z)))]$$

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Training GAN

- Gradient ascent on discriminator

$$\max_{\theta_d} \mathbb{E}_{x \sim p_{\textbf{data}}(\cdot)}[\log D_{\theta_d}(x)] + \mathbb{E}_{z \sim p_z(\cdot)}[\log(1 - D_{\theta_d}(G_{\theta_g}(z)))]$$

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- Gradient ascent on discriminator

$$\max_{\theta_d} \mathbb{E}_{x \sim p_{\mathbf{data}}(\cdot)}[\log D_{\theta_d}(x)] + \mathbb{E}_{z \sim p_z(\cdot)}[\log(1 - D_{\theta_d}(G_{\theta_g}(z)))]$$

- Gradient descent on generator

$$\min_{\theta_g} \mathbb{E}_{z \sim p_z(\cdot)}[\log(1 - D_{\theta_d}(G_{\theta_g}(z)))]$$

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- Gradient ascent on generator with modified objective

$$\max_{\theta_g} \mathbb{E}_{z \sim p_z(\cdot)}[\log D_{\theta_d}(G_{\theta_g}(z))]$$

GAN Training

Algorithm GAN Training

for number of training iterations **do**
  for k steps **do**
    Sample minibatch of m noise samples $\{z^{(1)}, \ldots, z^{(m)}\}$ according p_z.
    Sample minibatch of m data samples $\{x^{(1)}, \ldots, x^{(m)}\}$ according p_{data}.
    Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D(x^{(i)}) + \log \left(1 - D(G(z^{(i)})) \right) \right].$$

  end for
  Sample minibatch of m noise samples $\{z^{(1)}, \ldots, z^{(m)}\}$ according p_z
  Update the generator by ascending its stochastic gradient:

$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log D(G(z^{(i)}))$$

end for

GAN Training

Proposition

For a fixed G, the optimal discriminator D_G^* is given by $D_G^* = \frac{p_{\textbf{data}}(x)}{p_{\textbf{data}}(x)+p_g(x)}$

GAN Training

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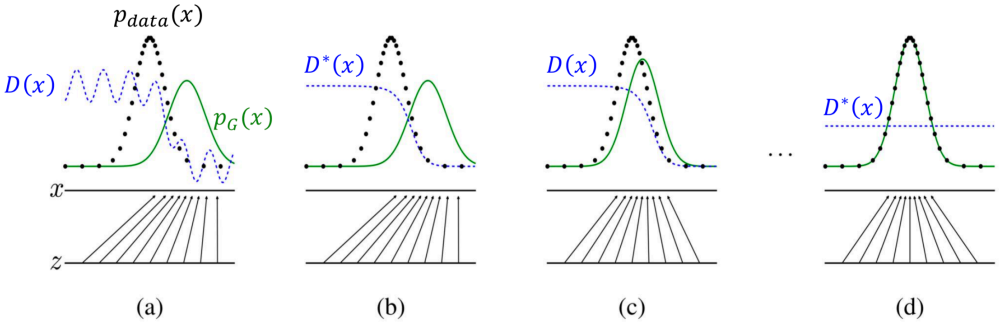
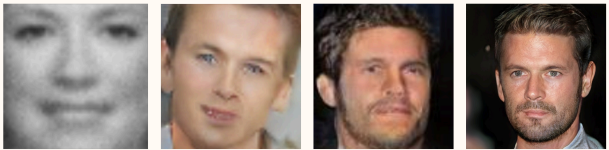
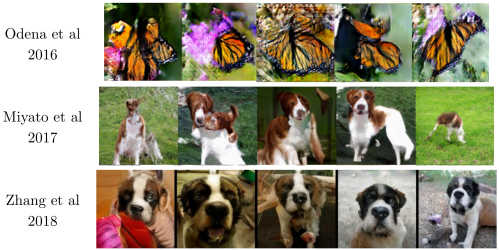


GAN Examples





Images are taken from CVPR 2018 Tutorial slides by Ian Goodfellow

GAN Examples

- GAN vector algebra

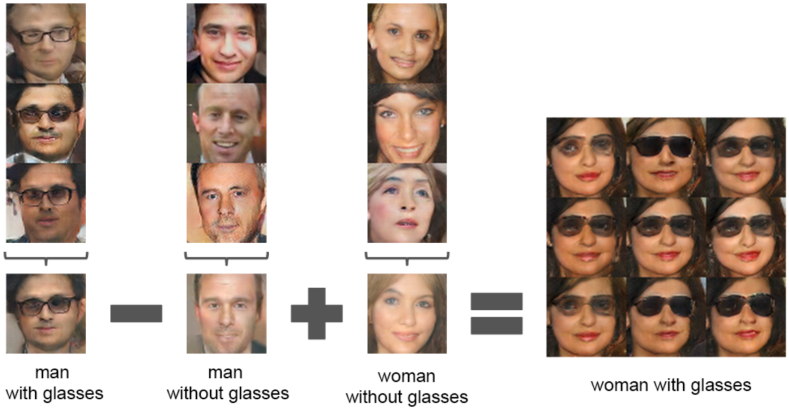


Image from: Radford and Chintala, "Unsupervised representation learning with deep convolutional generative adversarial networks", 2016

GAN Examples

- Image-to-Image Translation

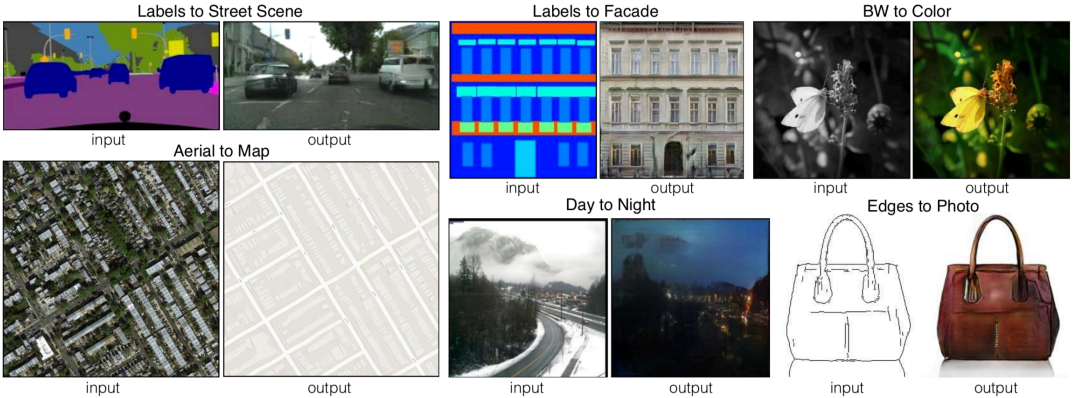


Image from: Isola et al, "Image-to-Image Translation with Conditional Adversarial Networks", 2017

GAN Examples

- Text-to-Image Synthesis

this small bird has a pink breast and crown, and black primaries and secondaries.

this magnificent fellow is almost all black with a red crest, and white cheek patch.

the flower has petals that are bright pinkish purple with white stigma

this white and yellow flower have thin white petals and a round yellow stamen

Image from: Reed et al, "Generative Adversarial Text to Image Synthesis", 2016

GAN Examples

Image from: Brock et al, "Large Scale GAN Training for High Fidelity Natural Image Synthesis", 2019