ECEN 743: Reinforcement Learning

Markov Decision Process (MDP): Introduction

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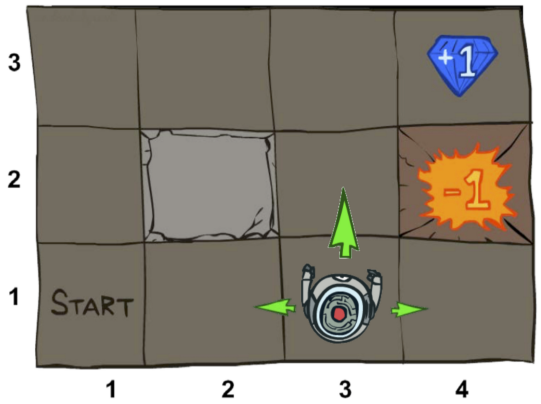
References

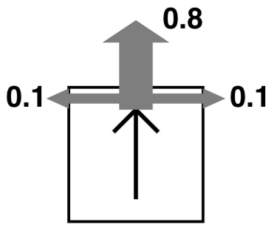
- [SB, Chapter 3-4]
- [DB, Chapter 2]
- [AJKS, Chapter 1]
- Puterman, *Markov Decision Process*, Chapter 1-3, 5-6

- Acknowledgment: Some figures for this lecture are taken from UC Berkeley CS188 course, with permission.

Grid World Example

- Agent lives in a grid world
- This world is stochastic





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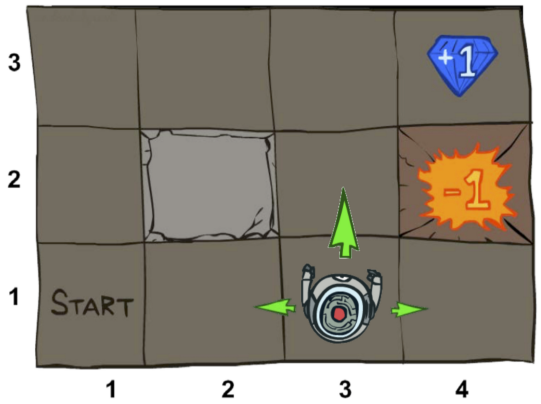
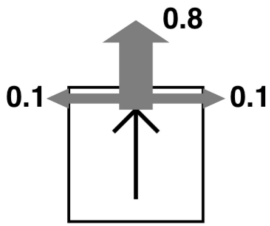
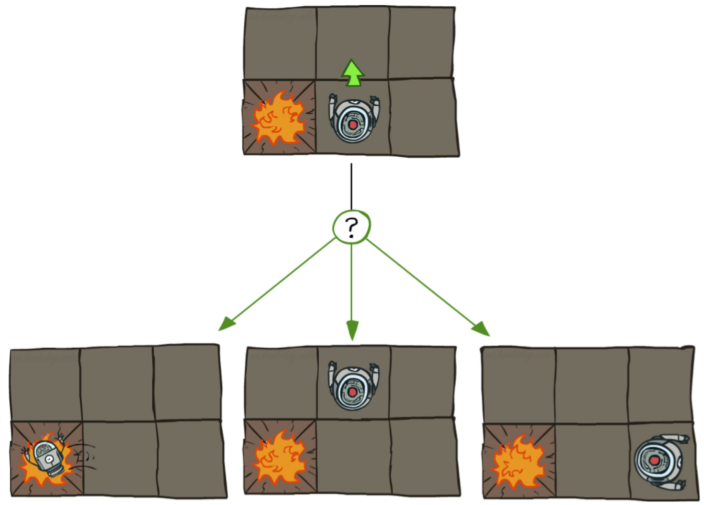




- Deterministic vs stochastic world:



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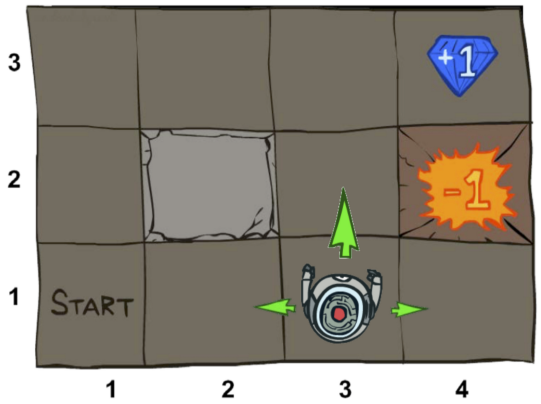

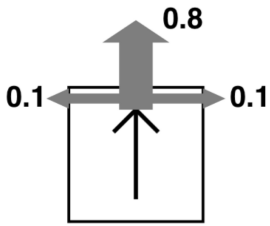

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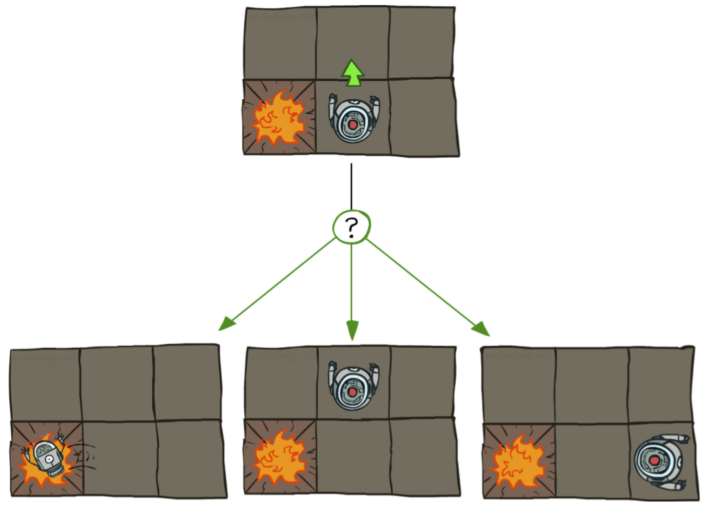

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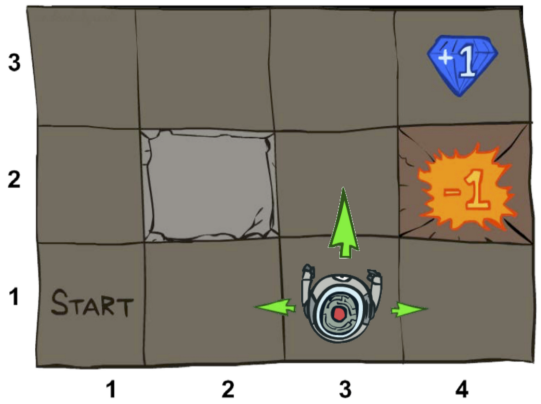

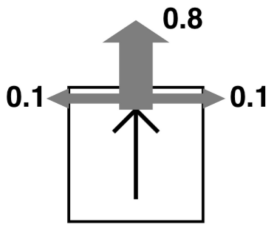

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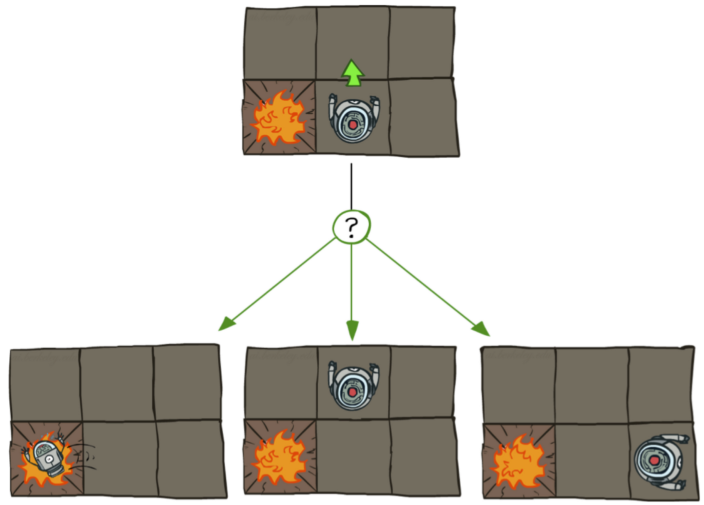

- Why do we need to model a stochastic world?
  - Inherent uncertainties in the system
  - Incomplete information about the system

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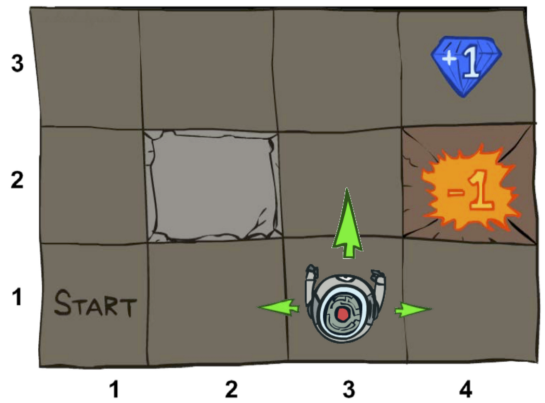
Markov Decision Process (MDP)

- A set of states $\mathcal{S}$ (state space)
- A set of action $\mathcal{A}$ (action space)
- Probability transition function P
  - $P(s'|s,a)$ represents the probability of moving from state s to state s' if action a is taken
  - Also called model of the system
- Reward function $r(s,a)$



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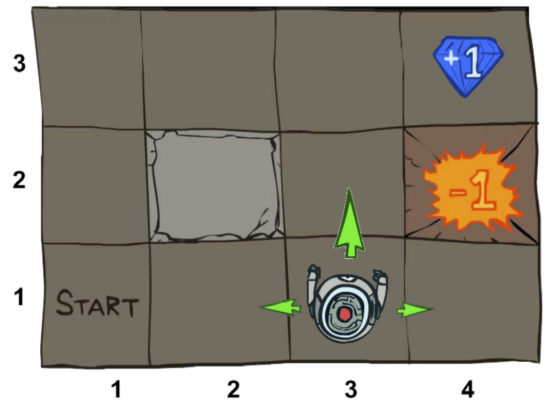

- MDP is an extremely useful mathematical formalism for modeling and solving many real-world control problems

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  - Tremendous reduction in memory/computation

MDP Examples

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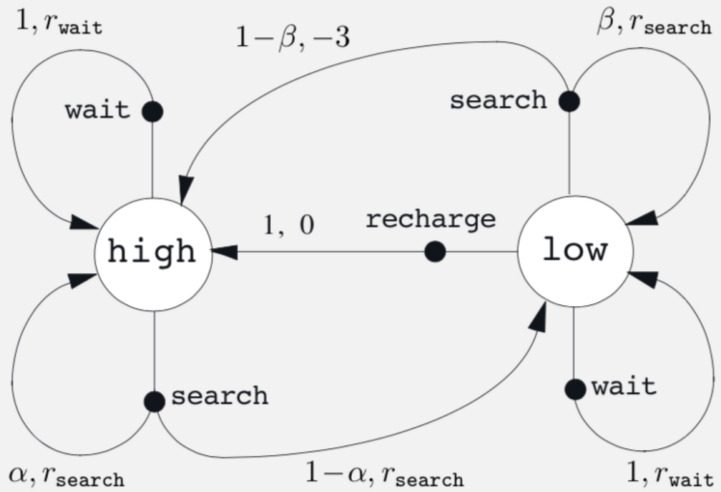
  - What are the states and actions of the system?
- Modeling depends on the goal and convenience!

MDP Modelling: Recycling Robot

- States (of charge): {high, low}
- Actions: {search, wait, recharge}
- Reward for searching, waiting, and penalty for running out of charge

s	a	s'	$p(s' \mid s, a)$	$r(s, a, s')$
high	search	high	α	$r_{\texttt{search}}$
high	search	low	$1-\alpha$	$r_{\texttt{search}}$
low	search	high	$1-\beta$	-3
low	search	low	β	$r_{\texttt{search}}$
high	wait	high	1	$r_{\texttt{wait}}$
high	wait	low	0	-
low	wait	high	0	-
low	wait	low	1	$r_{\texttt{wait}}$
low	recharge	high	1	0
low	recharge	low	0	-



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- Why discount?
  - Mathematically convenient to discount rewards

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  - Now or later?: {0.1, 0.1, 10} or {10, 0.1, 0.1}
- It is reasonable to maximize the sum of rewards
- It is also reasonable to prefer rewards now to rewards later
- One approach: value of rewards decay exponentially

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  - It is sometimes possible to use undiscounted cumulative rewards processes (i.e., $\gamma = 1$), if all sequences terminate

Value of a Policy

- Value of a policy evaluated at state s is the expected cumulative (discounted) rewards obtained by taking action according that policy, starting from s

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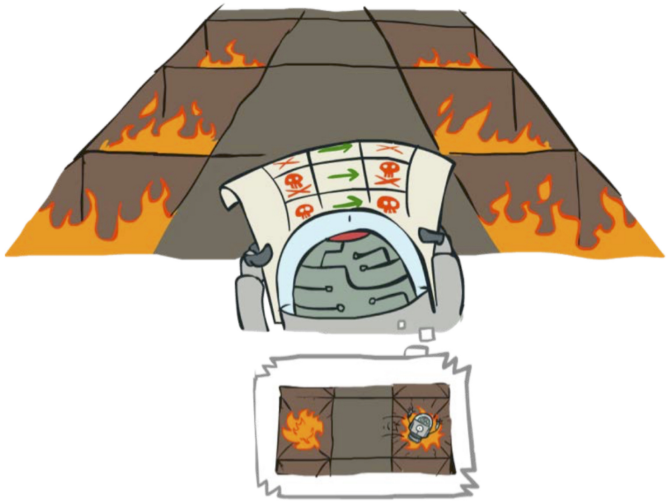

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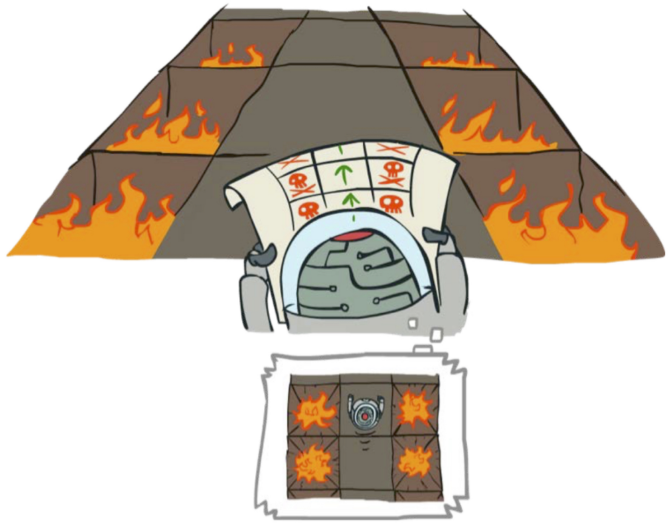

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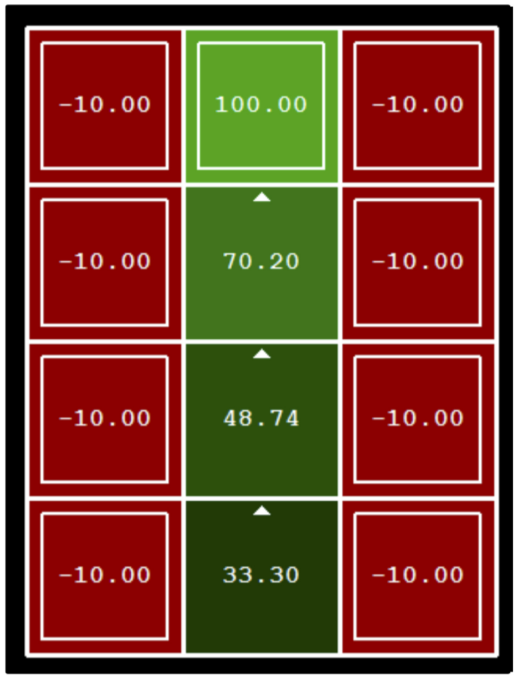

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- **Reading assignment**: Proof of the above theorem, see [AJKS, Chapter 1], [BDP, Chapter 1] [Puterman, Chapter 5]
- For the rest of the semester, we will focus only on stationary policies

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Review: Norm of a Vector

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