Knowledge-based approaches for the creative synthesis of mechanisms

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Two methods for mechanism synthesis using AI techniques are described, one consisting of forward design reasoning and one of backward design reasoning. Results are given for various examples. Assessment of the effects of errors (joint clearances and link length inaccuracies) is also discussed. Calculations on a Schmidt coupling and a Freudenstein parallel-jaw, straight-line plier-wrench mechanism are given to illustrate the effectiveness of the expert system.

computer-aided design, knowledge-based methods, mechanism design

Since the beginning of civilization, man has required techniques to move or lift heavy objects or, in a more rigorous sense, has required the ability to *control the motion* of various types of objects, such that useful work could be accomplished. Usher[1] says that the Greek term 'Mechanics' was taken to mean the 'lifting of heavy weights'. In moving beyond this rather simplistic definition of mechanics, the term mechanism, as described by Franz Reuleaux[2] (generally considered to be the founding father of modern mechanism theory), can be described as 'a combination of resistant bodies so arranged that by their means the mechanical forces of nature can be compelled to do work accompanied by certain determinate motions'.

The individual bodies that comprise a mechanism are called *links*, while the connections between the links are called *joints*. When combined together, they comprise what are called *kinematic pairs*, of which various types exist (Figure 1). When a series of links and joints are interconnected to form a chain, they are referred to as a *kinematic chain*. If one of the links of this kinematic chain is held fixed, it is called a *mechanism*. Mechanisms containing only revolute and/or prismatic joints are called *linkages*. The number of independent loops, denoted as L_{ind}, of a mechanism is indicative of the degree of complexity of the mechanism (Figure 2), and can be calculated as follows:

$$L_{ind} = 1 + j - l \tag{1}$$

where j is the number of joints and l is the number of links.

In describing the motion of mechanisms, the concept of degree of freedom is required. The degree of freedom

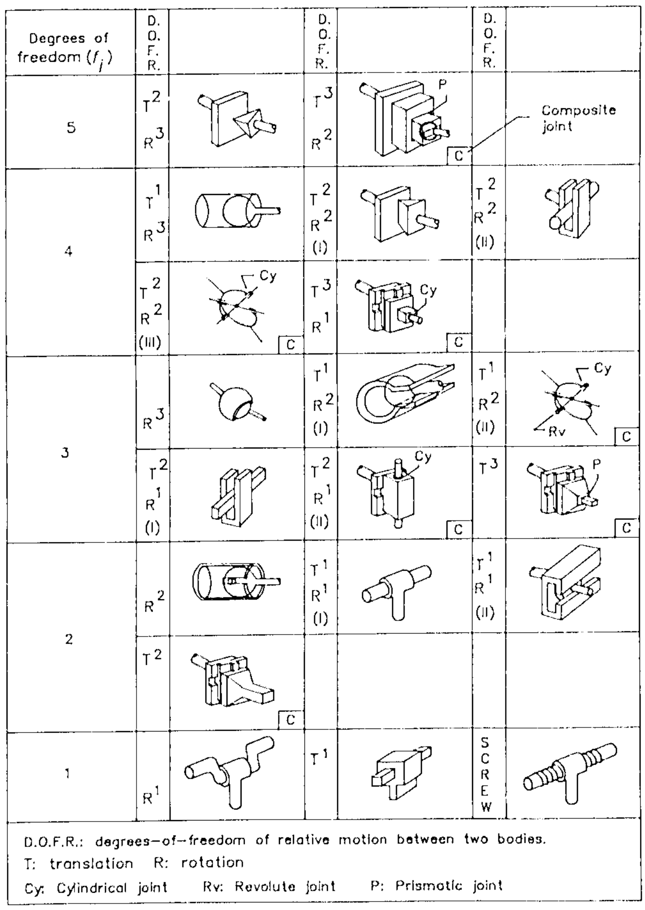

Figure 1. Types of kinematic pairs

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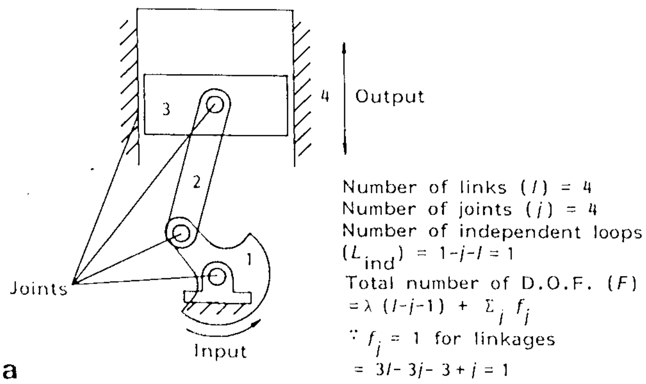


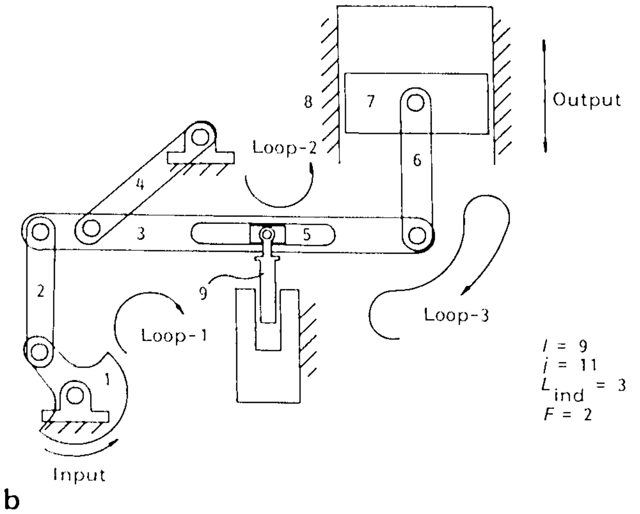


Figure 2. Degree-of-freedom identification: (a) traditional engine mechanism, (b) variable-stroke engine mechanism

F of a mechanism constitutes the minimum number of coordinates necessary to completely describe its configuration, and is determined by the number of inputs required to drive a mechanism, as well as the number of required outputs, specific application requirements, the degree of complexity of the mechanism and whether or not the mechanism is required to be adjustable. The general degree-of-freedom equation (Figure 2) may be expressed as[3]

$$F = \lambda(l - j - 1) + \sum_{i=1}^{j} f_i \tag{2}$$

where λ is the mobility of the space in which the mechanism operates ($\lambda = 3$ for a general plane mechanism, $\lambda = 6$ for spatial mechanisms), and f_i is the degree of freedom of relative motion permitted by the ith joint. A mechanism with zero degrees of freedom degenerates into a *structure*.

To place the topic of mechanism synthesis in proper perspective among other engineering research disciplines, it is important to note that mechanism design represents an extraordinarily pervasive engineering technology. Nearly every mechanical device that is conceived, designed and fabricated typically contains a mechanism in one form or another. The list of possibilities is extensive, and includes automobiles, aircraft, spacecraft, watercraft, home appliances, heating systems, childrens toys, baby carriages, shopping carts, computers, typewriters, desks, lamps, and movable windows. Figure 3 depicts several examples of mechanisms, and the manner in which the motion characteristics of each is commonly specified, according to path generation, rigid body guidance, or function generation. To clarify the manner in which motion characteristics can be used to synthesize a particular mechanism, consider that a double-stroke pump mechanism can be designed based on the fact that the path through which a point on the coupler link (i.e. point P) moves, traces out a curve, called the coupler curve, that has a double peak. The specific shape of the coupler curve depends on the ratio of the lengths of the links that comprise the mechanism. The process whereby the ratio of the link lengths are varied to achieve a desired coupler curve is typically referred to as *dimensional synthesis*.

MAPPING OF STRUCTURE-TO-FUNCTION

Having established the importance of mechanisms in everyday life, a critical problem facing mechanism

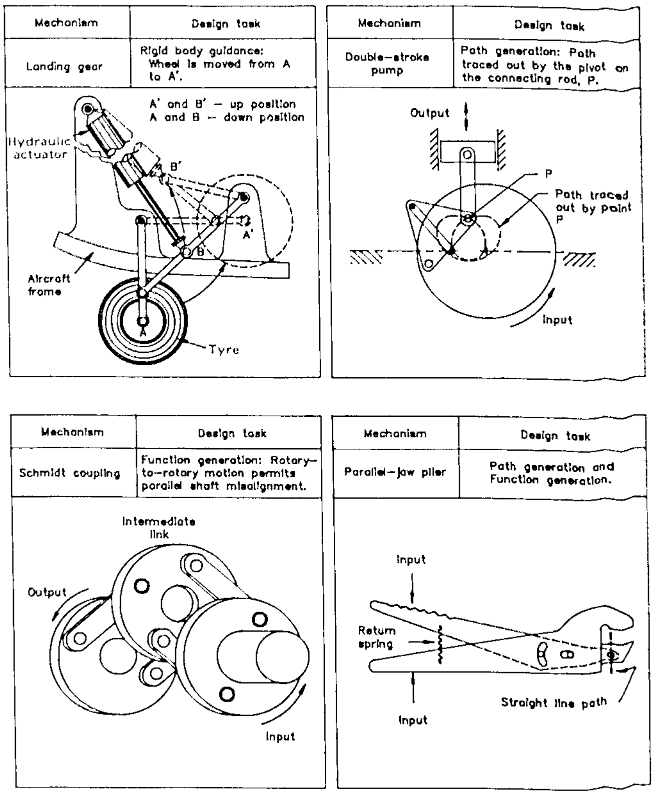


Figure 3. Examples of several different mechanisms

researchers in the opinion of the authors is the search to establish a consistent, extensible mapping between the function and structure of mechanisms. Functionality represents that property of a mechanism which defines the task(s) for which it is to be utilized. In contrast, the structure of a mechanism defines the structural elements that comprise the mechanism; that is, the type and number of links and joints, and the manner in which these elements are interconnected to form the mechanism. Much of the pioneering research undertaken on the subject of separating function and structure, as a means of systematically designing mechanisms, can be attributed to F. Freudenstein[4,5]. The following motion planning problem exemplifies the manner in which mechanism design problems are formulated, and should be contemplated as an aid in developing an understanding of the concept of structure and function in mechanism synthesis. This concept is subtle and elusive, but very important.

A mechanical engineer is faced with the task of creating a mechanism to pump a gas in an efficient manner. In designing a pump for gases, the engineer is provided with certain details that describe the volume flow rate, temperature and pressure of the gas to be pumped. In addition, the gas must be brought to a relatively high level of pressure as it moves through the pump. The requirement on efficiency and the need for a significant elevation in pressure level might suggest that a two-stage pump be utilized for this application.

The resulting question to be answered in this problem statement is, how might the engineer decide on the structure of a mechanism which has the potential to fulfil the required (i.e. prespecified) functionality? This problem clearly exemplifies the manner in which a typical mechanism synthesis problem might be stated, and while seemingly simple, it is not a trivial, nor even a straightforward, mechanism synthesis problem. In short, the solution is not at all apparent, except possibly to the experienced mechanism design engineer. So how then does one automate the process of designing a mechanism? This will be discussed in the following.

TRADITIONAL APPROACHES TO MECHANISM SYNTHESIS

Applying traditional approaches available for mechanism synthesis, the typical mechanical engineer in attempting to solve the problem described above would proceed to call upon prior experience. If this person is at all like most mechanical engineers, the prior experience probably contains a limited amount of knowledge about mechanisms. However, possessing sufficient knowhow and some resourcefulness, the engineer could expand on limited talents by obtaining assistance from various references that might include the famous Hrones–Nelson Atlas[6], or other handbooks of mechanism design containing compilations of previously thought-out mechanisms, capable of producing specific programmed motions for various applications. These include the *Handbook of Ingenious Mechanisms* by Jones[7], a rather comprehensive series entitled *Mechanisms in Modern Engineering Design* by Artobolevsky[8], and an atlas called *Mechanisms, Linkages, and Mechanical Controls* by Chironis[9]. Each of these compendia lists various mechanisms in accordance with practical, functional design requirements.

Following a traditional design methodology, the mechanical designer would attempt to uncover various pump or compressor mechanisms in each of the compendia. If there were none to be found, the designer might attempt to locate some type of reciprocating engine mechanism and modify it to meet the prespecified design requirements. As an alternative example problem in which a traditional design approach may succeed, consider a situation requiring a mechanism capable of producing intermittent motion, in order to move bottles in a controlled manner along an assembly line under a capping machine. A Geneva mechanism could be selected from a number of alternatives listed in Chapter 2 of the compendia entitled *Mechanisms, Linkages, and Mechanical Controls* by Chironis[9], under the heading of intermittent motion. Figure 4 depicts such a mechanism, and the manner in which bottles might be conveyed through the capping process. The precise dimensions of the mechanism would still have to be selected manually, following a trial and error process, and computer-based kinematic simulation would not be directly possible using this manual design approach.

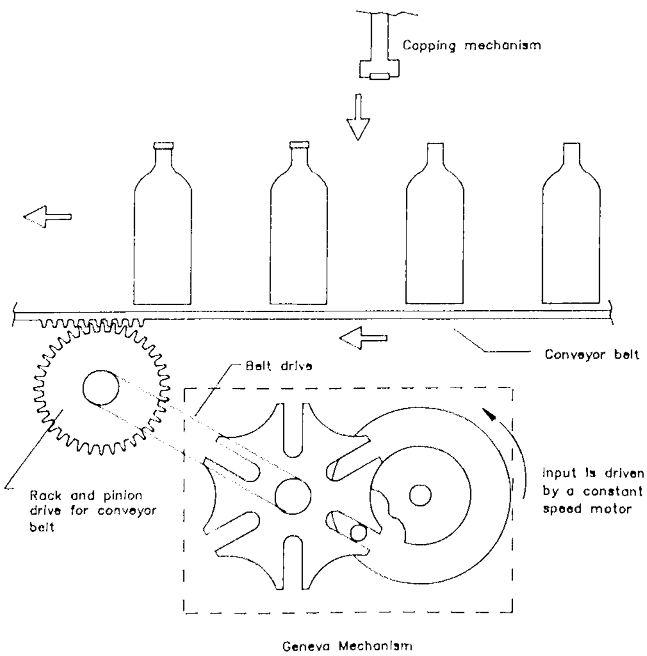


Figure 4. Geneva mechanism used to cap bottles: the mechanism provides required intermittent motion of the conveyor when placing caps on bottles

AUTOMATING THE MAPPING OF STRUCTURE-TO-FUNCTION IN MECHANISM SYNTHESIS

Having expounded the concepts and stressed the importance of structure and function, as it relates to mechanism synthesis, it is now necessary to develop an approach for representing these concepts in a form suitable for computer representation and manipulation in an attempt to automate the process of mechanism synthesis. The underlying mathematical theory applicable to representing structure in general, and in particular that of mechanisms, is graph theory. Graph theory provides a compact and complete mathematical medium for representing the structure of mechanisms. Its ability to retain function information, on the other hand, requires additional work to expand its present limited applicability.

Figure 5 depicts the physical representation of the structure of two different variable-stroke engine mechanisms, and the manner in which graph theory can be used to represent the structure of the mechanisms in a convenient and readily understood form[3]. The graphs store information about the connectivity (i.e. which link is connected to which joint) of the mechanisms. The connectivity matrix enables the contents of a graph to be stored in a compact form. The Lisp programming language can be effectively utilized to represent the contents of the connectivity matrix of a mechanism in a list-type of data structure, which can be rapidly manipulated (enumerated and searched through) in Lisp[10]. Finally, determination of the 'equivalence' or *isomorphism* of two graphs must be considered, since when undertaking a search through a large number of graphs it is critically important to avoid unnecessary, repetitive evaluations of the same graph. The concept of isomorphism is used in this context to mean that it is possible to find a one-to-one correspondence between the vertices of two graphs such that a pair of vertices are adjacent in one graph if and only if the corresponding pair of vertices are adjacent in the other graph[11].

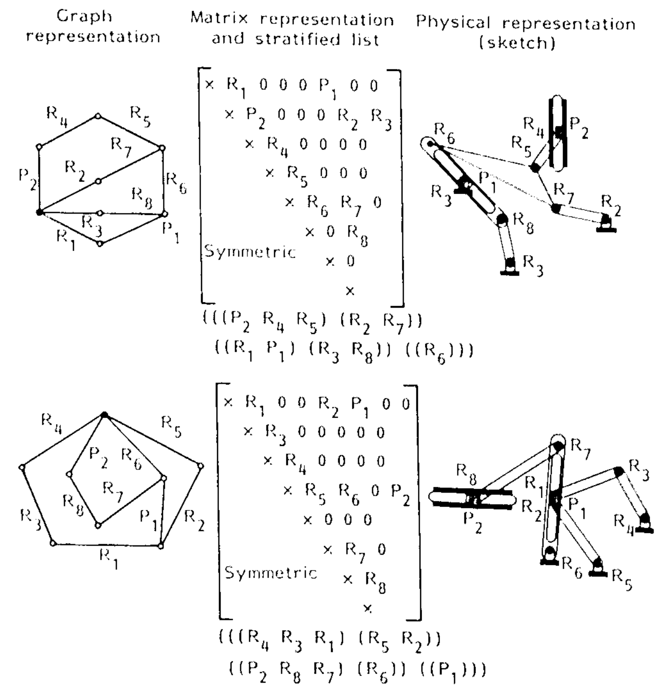


Figure 5. Graph, matrix and physical (sketch) representations of a variable-stroke engine mechanism. Graph edges represent joints and graph vertices represent links: R, revolute joint; P, prismatic joint: o, moving link; •, fixed link (ground link)

KNOWLEDGE-BASED MECHANISM SYNTHESIS

Artificial intelligence (AI) programming techniques are necessary if one expects to obtain solutions to problems which demonstrate inherent numerical intractability. Such problems are referred to as NP-complete (NPC), or nondeterministic polynomial time complete problems[12]. The CPU time required to solve an NP-complete problem, based on known algorithms, grows exponentially wth the 'size' of the problem. There exist no polynomial time transformations for NPC problems, nor are there any polynomial time algorithms capable of solving any NP-complete problems, therefore these problems are considered to be 'open' or unsolved problems, in the sense that enormous and impractical search times would be required to search through the entire space of potential solutions.

The potential to solve these NP-complete problems depends on the availability of certain heuristics (i.e. rules of thumb, or empirical knowledge that can be used to help guide a search through a large number of alternative, potential solutions). At the present time, creative mechanism design (i.e. synthesis) is inherently intractable from the standpoint of NP-completeness, since there exists no one-to-one mapping between structure and function. As a result, design heuristics are required if solvability is expected (Figure 6).

In developing knowledge-based approaches to mechanism synthesis, it was decided that two alternative solution strategies should be followed. The first is based on *forward design reasoning*, and the second is based on *backward design reasoning*. Both approaches rely on searching through a tree or 'space' of potential solution nodes or 'states'. In simple terms, forward reasoning begins with basic information which defines a problem, and attempts to proceed toward a solution state, through various intermediate states along the way (Figure 7(a)). Backward reasoning, in contrast begins with prespecified solution states and proceeds backward, through intermediate states, toward goal states (Figure 7(b)). The application of these basic reasoning strategies to mechanism design will be discussed in the following.

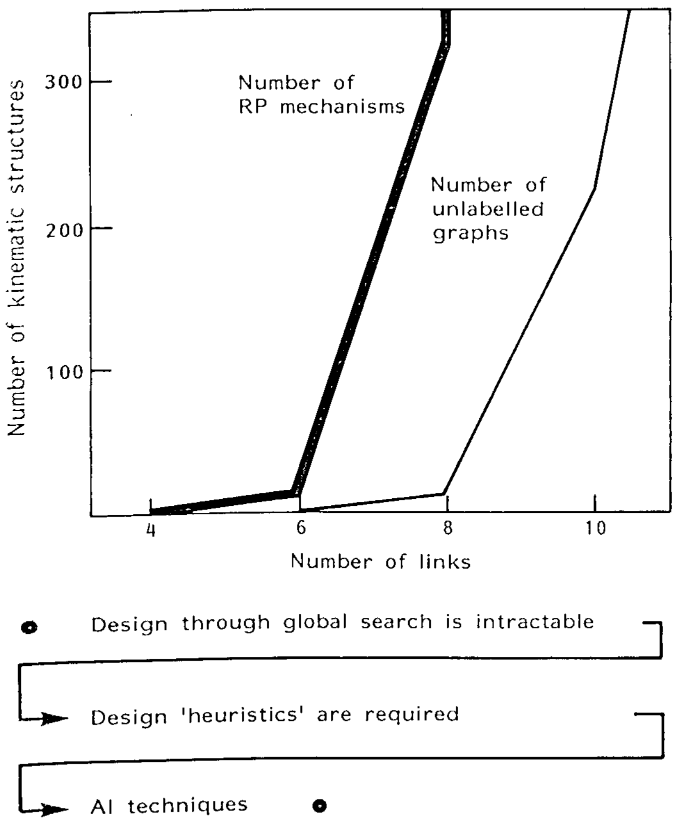


Figure 6. Creative mechanism design is an NP-complete problem. The number of mechanisms (kinematic structures) containing only revolute (R) and prismatic (P) pairs, and the number of their corresponding graphs grow exponentially with the number of links. (RP mechanisms are those containing only revolute and prismatic joints)

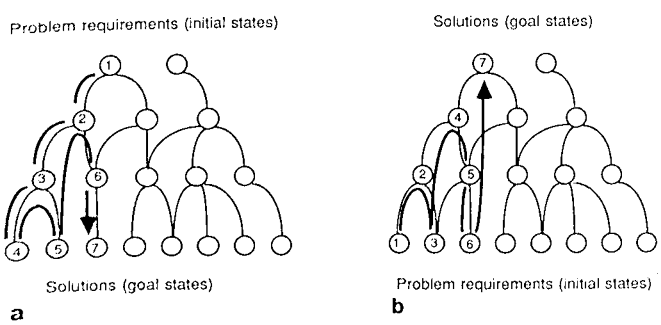


Figure 7. Schematic representation of forward and backward reasoning. The numbered circles represent initial, intermediate or goal states in the synthesis (search) procedure

Since graph theory can be effectively used to represent the structure of mechanisms in a compact manner, heuristic search techniques have been applied to enumerate graphs corresponding to mechanisms, so that unreasonable search times can be avoided. Using the forward reasoning strategy, called Mechanism Expert {Mecxpert}, basic information that defines the problem is provided to the Mecxpert system, and includes as a partial list[11]:

- number of inputs (i.e. how many inputs does the mechanism require)
- number of outputs (i.e. how many outputs does the mechanism require)
- input(s) which must be grounded
- input(s) which must be sliders (i.e. prismatic joints)
- whether there will be a control or guidance function within the mechanism
- maximum number of links
- minimum number of independent loops

Along with this information, the following rules (predefined knowledge) are also used to pin down (i.e. limit) more completely the search through the space of graphs corresponding to mechanisms that may satisfy the prespecified functional requirements, and any known structural requirements that characterize the problem:

Rule 1: If the mechanism is a path generator, then the output link must be a floating link.
Rule 2: If the mechanism is a function generator, then the output link must be in contact with the ground.
Rule 3: If there are more than two slider (prismatic) joints in any single loop, then the topology of the mechanism is invalid.
Rule 4: If there is a need for a guidance or control loop in the mechanism, then the output link should not belong to the loop that contains the input. This implies that the minimum number of independent loops must be greater than or equal to 2 (i.e. $L_{ind} \geq 2$).
Rule 5: If a double-peaked coupler curve is required, it can be generated using a crank-rocker four bar mechanism.

⋮

Rule *N*: The total number of independent loops cannot be less than {the required number of links which are adjacent with the ground link} − 1.

A typical design session between the mechanical engineer and the Mecxpert knowledge-based mechanism design system, for the synthesis of the pump mechanism, would apply the knowledge specified above, and would combine equations (1) and (2) producing the following equation

$$\sum_{i=1}^{J} f_i = F + \lambda L_{ind} \qquad (3)$$

Based on equation (3) with $L_{ind} = 3$, $F = 1$ and $\lambda = 3$ (for a general plane mechanism), the sum of the degrees of freedom for all the joints would be 10. Notice that the minimum number of independent loops is 3 in this case, since two separate output loops are required, one for each piston, in addition to an input loop (four bar mechanism which generates the double-peaked coupler curve). Since high load-carrying capability would be a specified design requirement for a pump, only revolute (R) and prismatic (P) joints, each having one degree of freedom ($f_i = 1$) would be included in the design, since these joints provide large area contact and can therefore transmit high loads. Based on this result, equation (1) can be used to determine that the number of links equals 8 (i.e. $l = 8$). With known values for j and l, appropriate graphs can be enumerated (labelling the graphs in as many nonisomorphic ways as possible) and different joint types can be assigned to the edges of the graphs in a way that ensures the satisfaction of equation (3). Samplings of both potentially plausible and nonplausible results generated by this process, using Mecxpert, are depicted, respectively, in Figures 8 and 9.

If this new design is carefully reviewed, it can be seen that the first stage piston performs two compression strokes for each compression stroke of the secondary stage piston. This feature would allow for a pump design

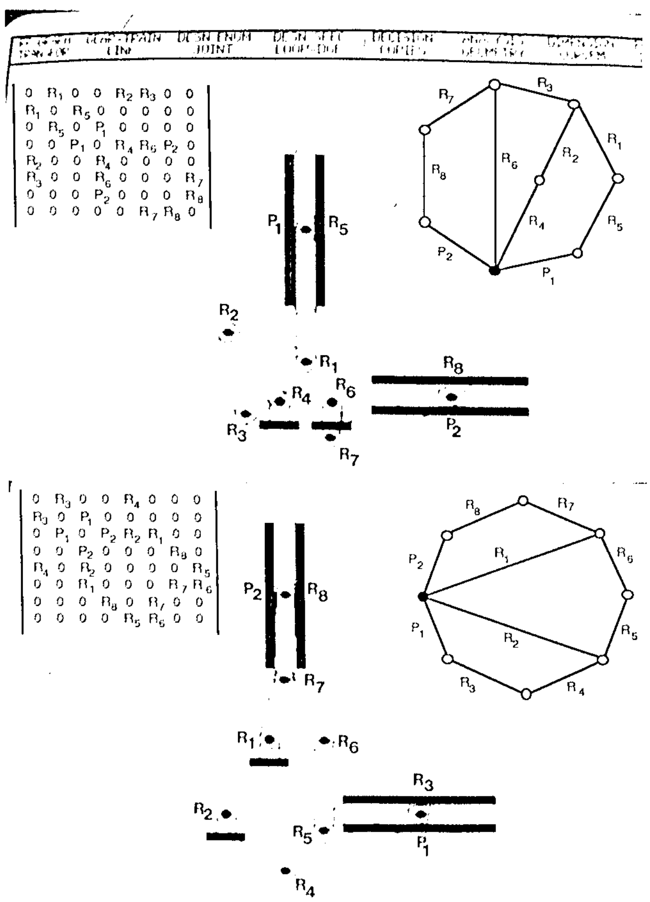


Figure 8. Two potentially plausible mechanisms, generated by Mecxpert, that are capable of generating a double-peaked coupler curve for use in a two-stage pump

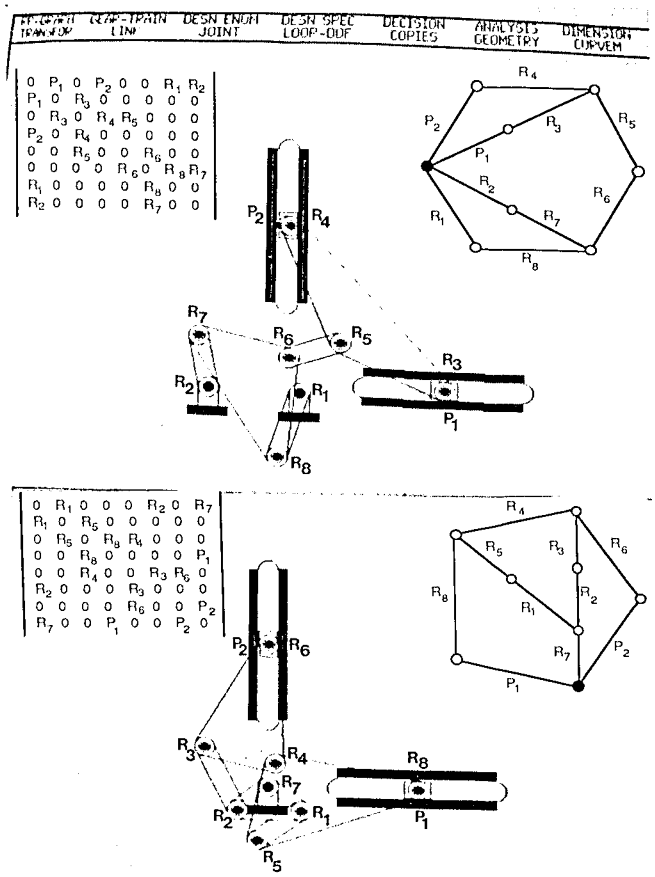


Figure 9. Two nonplausible mechanisms generated by Mecxpert that are not capable of generating a double-peaked coupler curve

with a second stage piston that is larger than that of the first stage (in most two-stage pumps, the second stage piston is smaller than that of the first stage). This design allows the volume flow rate of the pump to be increased over that possible with a smaller secondary piston. While kinematically correct structures have been enumerated during this stage of the design process (i.e. the conceptual stage), exact details concerning timing of the valves with the pistons would have to be studied during the detailed design stage, in order to maximize the thermodynamic efficiency of the pump[14]. In addition, a dynamic analysis would be required to ensure proper balancing of the mechanism.

The primary advantage of forward design reasoning is its ability to enumerate, in a systematic and unbiased manner, the structures for all mechanisms with the potential to satisfy the design requirements. The disadvantages of this approach are that it is not guaranteed to produce a solution, and it may require somewhat lengthy design times. However, this is the price which has to be paid for a thorough search through the space of potential design solutions.

The backward reasoning strategy is called Pattern Matching Synthesis {PMS}[15]. This approach begins with mechanism functional design requirements in the form of prespecified coupler curve patterns (Figure 10). These coupler curve patterns represent the goal states in the design search space. One of the ways in which design engineers typically select mechanisms is in accordance with a prespecified path to be traced out by a point on the mechanism (Figure 3). These coupler curves can be mapped in a systematic, one-to-one manner to the structures that generate the patterns. This process has been automated by adopting a machine learning approach based on neural network computing[15]. Using this approach, a mechanism design problem can be solved provided the desired goal state (i.e. coupler curve pattern) has been prespecified in the systems knowledge base of coupler curves. In addition, modifications can be made to the dimensions of the mechanism(s) which have been prestored and which are used to produce specific classes of coupler curve patterns (i.e. this constitutes the redesign process) in an attempt to 'tune' the mechanism to match a required coupler curve pattern more closely.

The primary advantage of backward design reasoning is its ability to guarantee a solution to a mechanism design problem provided the desired solution state has been previously introduced into the system. The development of a large knowledge base of curve patterns would satisfy this requirement. The authors' research is currently directed towards achieving this goal. In addition, the process of generating a correct solution, based on the PMS approach, tends to avoid lengthy design times.

Mechanical engineers faced with the previously described mechanism design problem (i.e. pump design problem) now have an innovative, computer-based tool at their disposal for the systematic representation, enumeration, kinematic analysis and graphical animation[16,17] of the structures of mechanisms. In attempting to apply the backward reasoning approach to synthesize a double-stroke, two-stage pump mechanism, a mechanical engineer would peruse the knowledge base of possible coupler curve patterns and

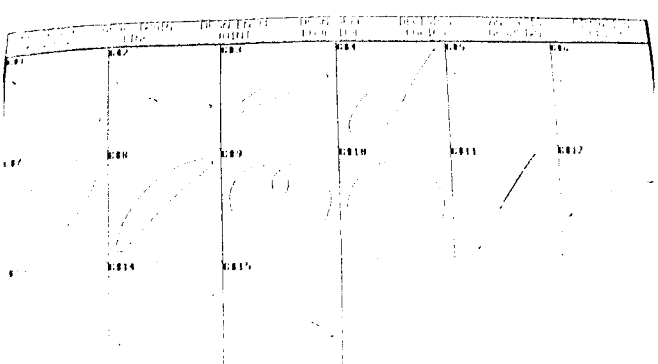


Figure 10. Classification of 356 coupler curves into 15 groups

would notice that pattern 9 has the potential to provide a double-stroke, reciprocating-type of motion (Figure 10). Upon further investigation of the structures of the mechanisms producing this pattern, the engineer could assess the effects of varying the dimensions of the links to achieve some desirable intake–exhaust characteristics of the thermodynamic cycle provided by the pump (Figures 11 and 12). The final design of the first stage might appear as in Figure 13.

REDESIGN: EVALUATING THE KINEMATIC EFFECTS OF JOINT CLEARANCES AND LINK LENGTH TOLERANCES ON KINEMATIC PERFORMANCE

One domain in which there is a critical need for improved computational design tools is in determining (assessing) the effects of errors (e.g. joint clearances

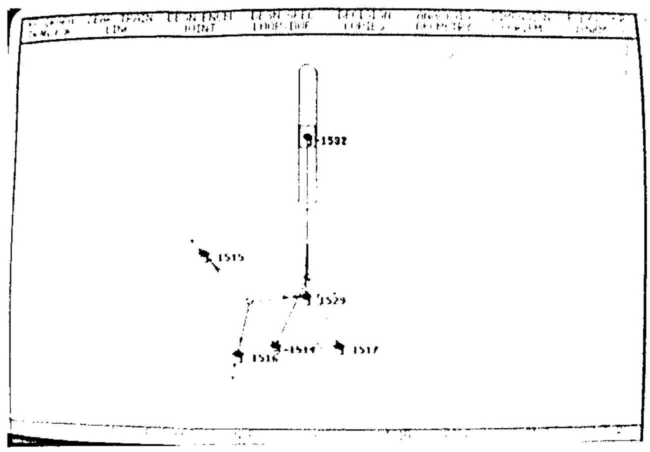


Figure 11. First stage of the two-stage pump mechanism with markers (+) indicating the repositioning of three of the joints

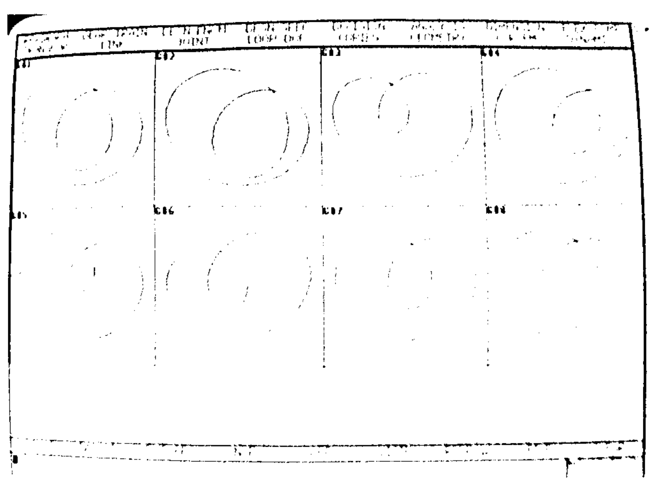


Figure 12. Categorization of coupler curves for the first stage of the two-stage pump mechanism

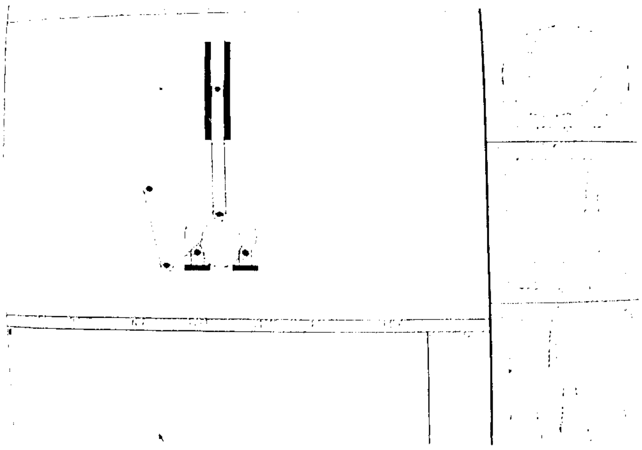


Figure 13. First stage of the two-stage pump mechanism that provides double peaks for each input cycle, generated and analysed by the Mecxpert system

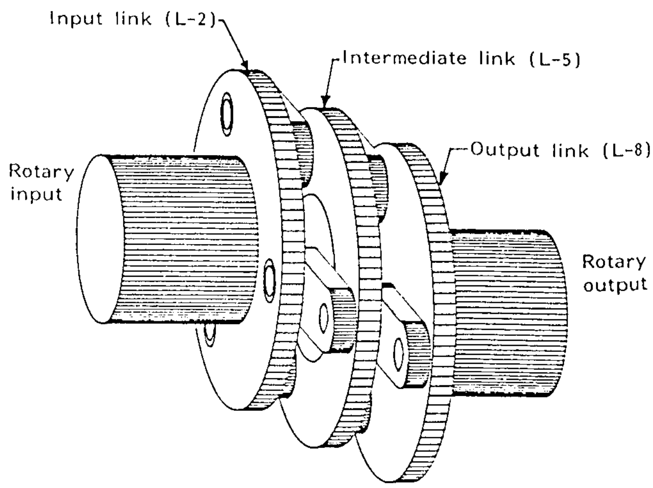


Figure 14. Schmidt coupling (isometric view)

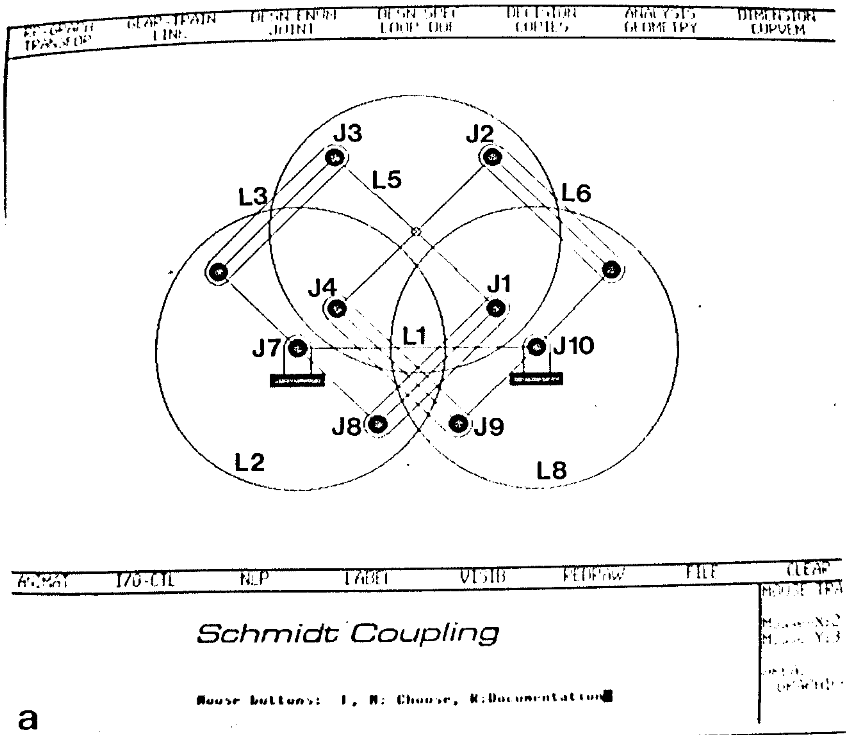
and link length inaccuracies) on the performance of various mechanism designs. As far as the authors are aware, there are no currently available software tools for mechanism design that include such a capability.

Manufacturing inaccuracies can produce errors in the desired kinematic response of mechanisms. Assessing the effects of such errors on the performance of mechanisms provides a systematic approach for *quantitatively* evaluating the susceptibility of a mechanism design to manufacturing inaccuracies, i.e. its *inherent resistance* to manufacturing inaccuracies. Clearly, significant insight about the kinematic 'quality' of a design can be gained through this type of analysis. The approach to the solution of this type of problem considers link length tolerances as well as joint clearances, both resulting from manufacturing (machining) errors, and relies on the principle of virtual work[18]. The approach is sufficiently general, such that it can be applied to both planar and spatial mechanisms subject to specified manufacturing operations (e.g. milling, boring, slotting).

To demonstrate the practical utility of this type of analysis to representative, real-world mechanism design problems, Figures 14, 15(a) and 16(a) show, respectively, a Schmidt coupling (isometric and front views), and Freudenstein's parallel-jaw, straight-line plier mechanism[19]. The Schmidt coupling permits a shaft to transmit a torque to another shaft that is parallel to itself, while allowing transverse motion between the shafts. This prevents transverse vibrations being transferred from the input shaft attached to link L2, through the intermediate link (L5) of the coupling, to the output shaft attached to link L8. For this example, the radius of each disc was chosen to be 6 cm. The mechanical error and the error sensitivity produced at a specified point on the mechanism (in this case on link 8 – the output link) due to an assumed 0.03 mm link length tolerance (due to a grinding operation) and a 0.01 mm joint clearance (due to a diamond boring operation) was calculated. Figure 15(b) depicts the root-mean-square error sensitivity (i.e. error sensitivity averaged over one kinematic cycle) at link 8 due to joint clearances (left) and link length tolerances (right). Figure 15(c) depicts the total mechanical error (top), the error sensitivity as a function of joint clearances (middle), and the error sensitivity as a function of link length tolerances (bottom), respectively on link 8. A similar kinematic error analysis was performed for the parallel-jaw plier mechanism. This plier-wrench was designed to permit its use in a confined work space. The value assumed for the link length tolerances were 0.06 mm (due to a milling operation) and 0.06 mm for the joint clearances (due to a boring operation). Figure 16(b) depicts the root-mean-square error sensitivity (i.e. error sensitivity averaged over one kinematic cycle) at joint-5 due to joint clearances (left) and link length tolerances (right). Figure 16(c) depicts the x-component of the total translational mechanical error (top), the x-component of the translational error sensitivity due to clearances at joints one, two, three and four (middle), and the x-component of the translational error sensitivity due to link length tolerances of links two, three and four (bottom), respectively at joint-5 (attached to the bottom jaw). It is interesting, informative, and quite important, from a design and manufacturability standpoint, to note that the plier-wrench mechanism is inherently more tolerant to joint clearance and link length tolerance than is the Schmidt coupling, since its error and error sensitivity curves demonstrate comparatively smaller values.

CONCLUSIONS

Mechanisms, and the underlying engineering technology required to synthesize them, represent an extraordinarily pervasive subdiscipline within the field of mechanical



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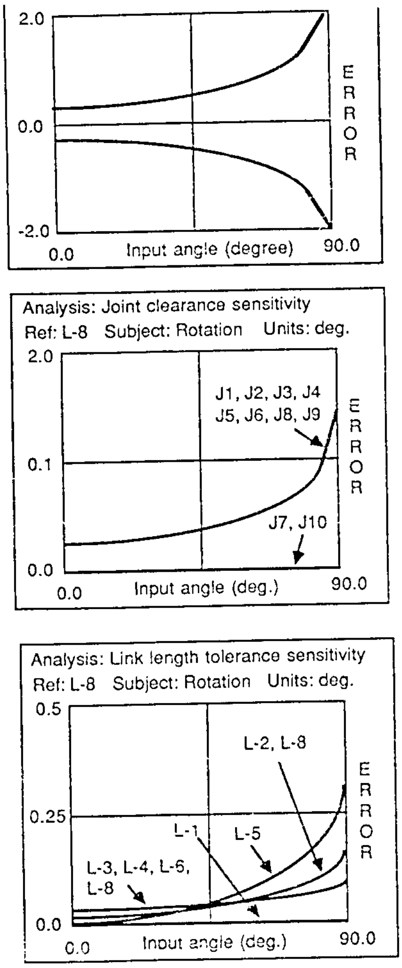

Joint clearance analysis		
Joint Name	Clearance	RMS Sensitivity at L-8
J1	0.01 mm	0.08 deg/.01mm
J2	0.01 mm	0.08 deg/.01mm
J3	0.01 mm	0.08 deg/.01mm
J4	0.01 mm	0.08 deg/.01mm
J5	0.01 mm	0.08 deg/.01mm
J6	0.01 mm	0.08 deg/.01mm
J7	0.01 mm	0.0 deg/.01mm
J8	0.01 mm	0.08 deg/.01mm
J9	0.01 mm	0.08 deg/.01mm
J10	0.01 mm	0.0 deg/.01mm

Link length tolerance analysis		
Link Name	Clearance	RMS Sensitivity at L-8
L-1	0.02 mm	0.0 deg/0.01mm
L-2	0.02 mm	0.07 deg/0.01mm
L-3	0.02 mm	0.08 deg/0.01mm
L-4	0.02 mm	0.08 deg/0.01mm
L-5	0.02 mm	0.14 deg/0.01mm
L-6	0.02 mm	0.08 deg/0.01mm
L-7	0.02 mm	0.08 deg/0.01mm
L-8	0.02 mm	0.07 deg/0.01mm

b

c

Figure 15. (a) Schematic representation of a Schmidt coupling; (b) joint and link length clearance analyses; (c) top, maximum error produced by joint clearances (0.01 mm) and link length tolerances (0.02 mm); middle, error sensitivity due to joint clearance (degrees per 0.01 mm); bottom, error sensitivity due to link length tolerance (degrees per 0.01 mm)

engineering. Knowledge-based approaches to mechanism synthesis have the potential to advance existing design methodologies, such that more creative designs can be achieved in a systematic manner. While forward synthesis strategies offer generality, they can be time-consuming. Backward synthesis strategies, in contrast, rely on the scope of their knowledge base, but can produce solutions in an expeditious manner.

The authors believe that industrial mechanism designers should seek to use these new, knowledge-based computer-aided design tools if they expect to create designs in a timely manner, which are truly innovative and able to satisfy complex functional requirements.

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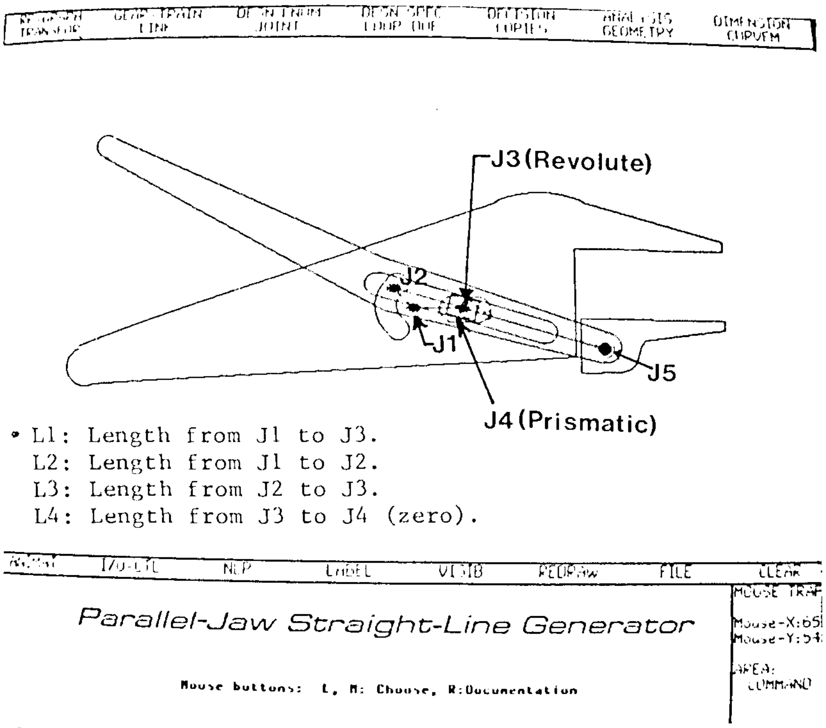
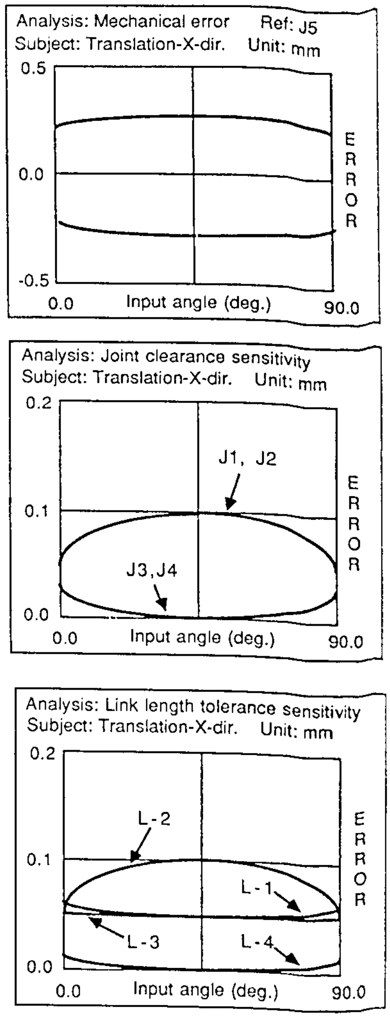


a

Joint clearance analysis		
Joint Name	Clearance	RMS Sensitivity at J5
J1	0.06 mm	0.083 mm/0.1mm
J2	0.06 mm	0.083 mm/0.1mm
J3	0.06 mm	0.02 mm/0.1mm
J4	0.06 mm	0.02 mm/0.1mm

Link length tolerance analysis		
Link Name	Clearance	RMS Sensitivity at J5
L-1	0.06 mm	0.054 mm/0.1mm
L-2	0.06 mm	0.083 mm/0.1mm
L-3	0.06 mm	0.049 mm/0.1mm
L-4	0.06 mm	0.01 mm/0.1mm

b



c

Figure 16. (a) Schematic representation of Freudenstein's parallel-jaw plier: (b) root mean square error sensitivity (error at a specified point due to each increment of joint clearance and link length tolerance) for the case when the output is located at J5; (c) top, maximum error produced by joint clearances (0.06 mm) and link length tolerances (0.06 mm); middle, error sensitivity due to joint clearance (0.1 mm/0.1 mm); bottom, error sensitivity due to link length tolerance (0.1 mm/0.1 mm)

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