Optimal Synthesis of Mechanisms for Path Generation Using Fourier Descriptors and Global Search Methods

Irfan Ullah*
Assistant Professor.
Department of Mechanical Engineering,
NWFP University of Engineering & Technology,
Peshawar, Pakistan

Sridhar Kota
Associate Professor.
Department of Mechanical Engineering and Applied Mechanics,
The University of Michigan,
Ann Arbor, Michigan 48109

Generally, success in synthesis of mechanisms for path generation is limited to finding a reasonable local optima at best in spite of a very good initial guess. The most widely used Structural Error objective function is not effective in leading to practical solutions as it misrepresents the nature of the design problem by requiring the shape, size, orientation and position of the coupler curve to be optimized all at once. In this paper, we present an effective objective function based on Fourier descriptors that evaluates only the shape differences between two curves. This function is first minimized using a stochastic global search method derived from simulated annealing followed by Powell's method. The size, orientation and position of the desired curve are addressed in a later stage by determining analogous points on the desired and candidate curves. In spite of highly non-linear mechanisms design space, our method discovers near-global and practical solutions consistently without requiring any initial guess.

Introduction

In dimensional synthesis of path generating mechanisms, it is required to determine the dimensions of the links of, say, a four-bar mechanism such that a point on the coupler link of the mechanism traces out the desired trajectory. Optimal synthesis methods (Fox and Willmert, 1967; Angeles et al., 1988) are used for synthesizing mechanisms for path generation. Most, if not all, of these methods, require an initial mechanism design that generates a coupler curve which is reasonably similar to the desired trajectory. Typically a four-bar linkage is chosen, as the initial guess, either from mechanism catalogs or from previous experience. Next, the difference between this candidate (coupler) curve and the desired trajectory is expressed as a mathematical function of the dimensions of the mechanism. This objective function is then minimized by varying the mechanism dimensions. For this purpose, the mathematical theory of minimization of functions of several variables (Scales, 1985) is employed. The resulting design will perform the desired task with the minimum error.

Structural Error Function and Its Drawbacks

Optimal synthesis procedures commonly minimize the ''structural error'' objective function, depicted in Fig. 1. The position of the coupler point can be expressed as a function of the dimensions of the mechanism and the angle of the input link, θ_2:

$$P = P(r, \theta_2)$$

where r represents the nine parameters required to define a four-bar mechanism:

$$r = (r_0, r_1, r_2, r_3, r_4, r_7, \theta_0, \theta_1, \gamma)$$

The coupler curve is then compared to the desired curve point by point, as shown in Fig. 1, taking the mean squared distance between the two curves over a number of points as the structural error:

$$SE = \frac{1}{n} \sum_{i=1}^{n} |P_i Q_i|^2$$

Figure 2 shows a desired curve and a possible solution mechanism along with its coupler curve. It is clear that the structural error will be high in this case; yet, visual observation reveals that the shape of the solution curve is very similar to the desired shape. By translating, rotating and scaling the solution mechanism appropriately, without changing relative dimensions of various links, the solution curve can be made to coincide with the desired curve in shape, position, orientation and size. The difficulty however, is to find a mechanism that meets the shape specifications. Therefore, it is important to focus on shape specifications before tackling others.

Drawbacks of the Structural Error Function:

- It attempts to compare the shape, size, orientation, and location of a desired curve with a candidate curve all at once thereby simultaneously limiting the search space and making the search intractable.
- It introduces timing requirement as an artificial constraint to facilitate the solution of an ill-posed problem.

In this paper, an alternative objective function is presented that compares purely the shape of two plane, closed curves without being affected by the location, size, or orientation differences between the curves. The nine variables defining a four-bar mechanism can in fact be divided into two groups: those that determine the shape of the coupler curve (r_1/r_2, r_3/r_2, r_4/r_2, r_7/r_2, g) and those determining the other properties (r_2, r_0, θ_0, θ_1). The latter four are assigned fixed values during the optimal synthesis, thereby reducing the dimensions of the design space and simplifying the problem.

Fourier Descriptors

Fourier Descriptors were introduced in the early 60's as a set of numbers embodying the shape of a closed curve, and have

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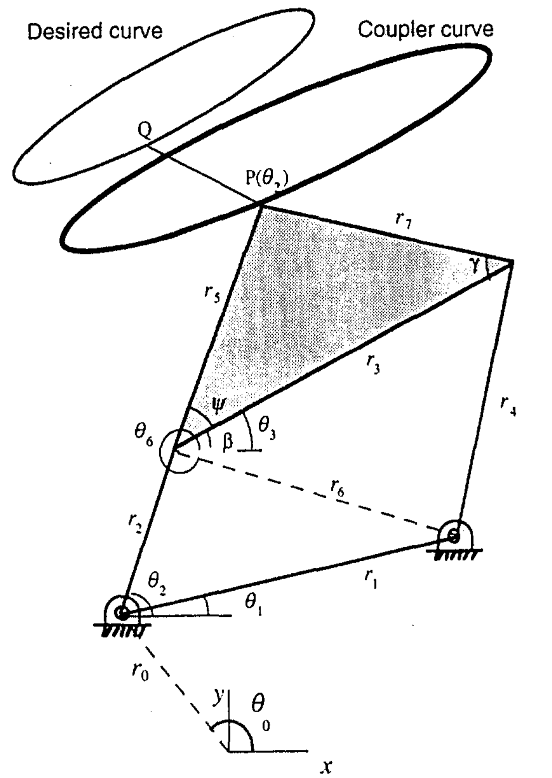

Fig. 1 Calculation of structural error for a four-bar mechanism

been used successfully in the field of pattern recognition (Zahn and Roskies, 1972). To define Fourier Descriptors, consider a simple, closed curve traced clockwise (Fig. 3). The tangent to the curve at any point makes an angle θ with the positive x-axis. When expressed as a function of the arc length 1, angle θ can be used to represent the shape of the curve. To remove the effect of orientation, define the cumulative angular function $f(l)$ as the net angular bend between the starting point and point l:

$$\phi(l) = \theta(l) - \theta(0)$$

Next, a variable change is introduced to make the function independent of the total length L:

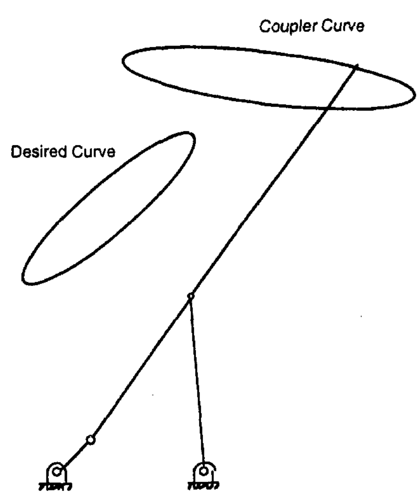

Fig. 2 A desired curve and a possible solution not recognized by the structural error function

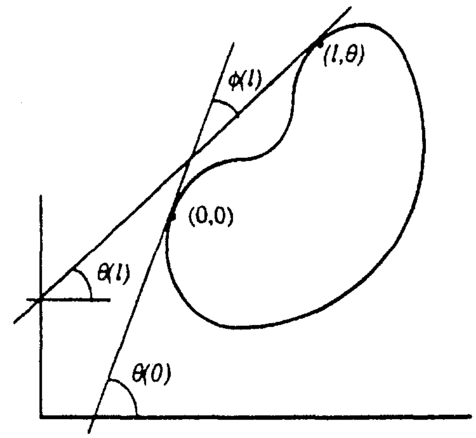

Fig. 3 Angle functions of a plane, closed curve

$$t = \frac{2\pi l}{L}$$

Since the curve is simple, clockwise oriented and closed, it will have a net angular bend of -2π, i.e. in terms of the new variable t, $f(0) = 0$ and $f(2\pi) = -2\pi$. A linearly increasing angle is added to f to obtain the cumulative angular deviant function f^* which describes pure shape and is invariant under translations, rotations and changes of perimeter L:

$$\phi^*(t) = \phi\left(\frac{Lt}{2\pi}\right) + t \quad t \in [0, 2\pi]$$

As the curve is traced continuously, the function f^* repeats (with a period of 2π) and can thus be expanded in a Fourier series:

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty} (a_k \cos kt + b_k \sin kt), \text{ or in polar form}$$

$$\phi^*(t) = \mu_0 + \sum_{k=1}^{\infty} A_k \cos (kt - \alpha_k)$$

where (A_k, α_k)—called the kth harmonic amplitude and phase—are polar coordinates of (a_k, b_k). Either set is termed as the Fourier descriptors of the curve.

For the specific case of curves represented by a set of points (i.e. polygons), the Fourier descriptors are given by Zahn and Roskies (1972) as:

$$\mu_0 = -\pi - \frac{1}{L} \sum_{i=1}^{n} l_i \Delta\phi_i$$

$$a_k = \frac{-1}{k\pi} \sum_{i=1}^{n} \Delta\phi_i \sin \frac{2\pi k l_i}{L}$$

$$b_k = \frac{1}{k\pi} \sum_{i=1}^{n} \Delta\phi_i \cos \frac{2\pi k l_i}{L}$$

where $\Delta\phi_i$ is the change in angular direction at vertex V_i and l_i is the length of the curve up to V_i (see Fig 4).

The cumulative angular deviant function f^*, and therefore the Fourier descriptors, are invariant under rotation, translation or scaling of the curve. Furthermore, the harmonic amplitudes A_k are also independent of the particular starting point used on the curve and the sense of the curve (clockwise or counter-clockwise). However, the phase angles α_k are affected by these latter. In comparing curve shapes for the purpose of mechanism synthesis, the starting point on a curve and the direction of traversing the curve are not relevant since these can be changed simply by changing the starting angle and direction of input

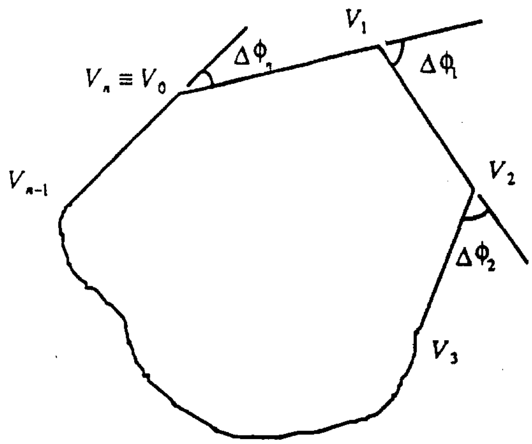

Fig. 4 Terminology for a polygonal curve

crank motion. It follows that phase angles cannot be used directly for curve comparison. To extract the shape information contained in phase angles while filtering out the effect of starting point, define the following simple functions:

$$F_{kj} = j^*\alpha_k - k^*\alpha_j$$

where $j^* = j/gcd(j, k)$ and $k^* = k/gcd(j, k)$ and gcd denotes the greatest common divisor. It can be shown (Zahn and Roskies, 1972) that F_{kj} are independent of the starting point on the curve.

Given two plane, closed curves C and C' with Fourier descriptors (A_k, α_k) and (A'_k, α'_k) respectively, we define the amplitude deviation Ampdev and the angle deviation Angdev respectively as

$$\text{Ampdev} = \sum_{k=1}^{M} |A'_k - A_k|$$

$$\text{Angdev} = \frac{1}{\max(w_{kj})} \sum_{k=2}^{M} |(F'_{k1} - F_{k1})w_{k1}| + |(F'_{32} - F_{32})w_{32}|$$

The weights w_{kj} are used because the phase angles of the harmonics with smaller amplitude are less significant than those of the harmonics with larger amplitudes. The weights are given by

$$w_{kj} = \min\,[\min\,(A'_k, A'_j),\ \min\,(A_k, A_j)]$$

The series for Angdev involves $M - 1$ terms containing F_{k1}, $k = 2, \ldots, M$, but not F_{11}, since, by definition, $F_{kk} = 0$ for all k. A separate term (containing F_{32}, chosen arbitrarily) is added in order to have M linear functions of M phase angles. In calculating Angdev, we ensure

$$-\pi < F'_{kj} - F_{kj} \leq \pi$$

by adding or subtracting 2π as necessary, because phase angles and their functions are known mod (2π) only.

Fourier Deviation Fdev is now defined as a combination of Ampdev and Angdev:

$$\text{Fdev} = m^*\text{Ampdev} + n^*\text{Angdev}$$

where m and n are parameters whose value can be changed to emphasize one or the other of the two component deviations.

Validation of Fourier Deviation

For the purpose of validation, let us consider the coupler curve C_d of a four-bar mechanism with the following dimensions as the "desired curve":

$$r = (0, 3.08, 1, 3.5, 3.5, 5.9, 0, 0, \pi)$$

By varying the coupler angle γ (see Fig. 1) from 0 to 2π in 20 steps, we obtain a set of curves C_i, $i = 0, \ldots, 19$. Figure 5 shows a plot of the magnitude deviation Ampdev, the angle deviation Angdev, and the Fourier Deviation Fdev (= Ampdev

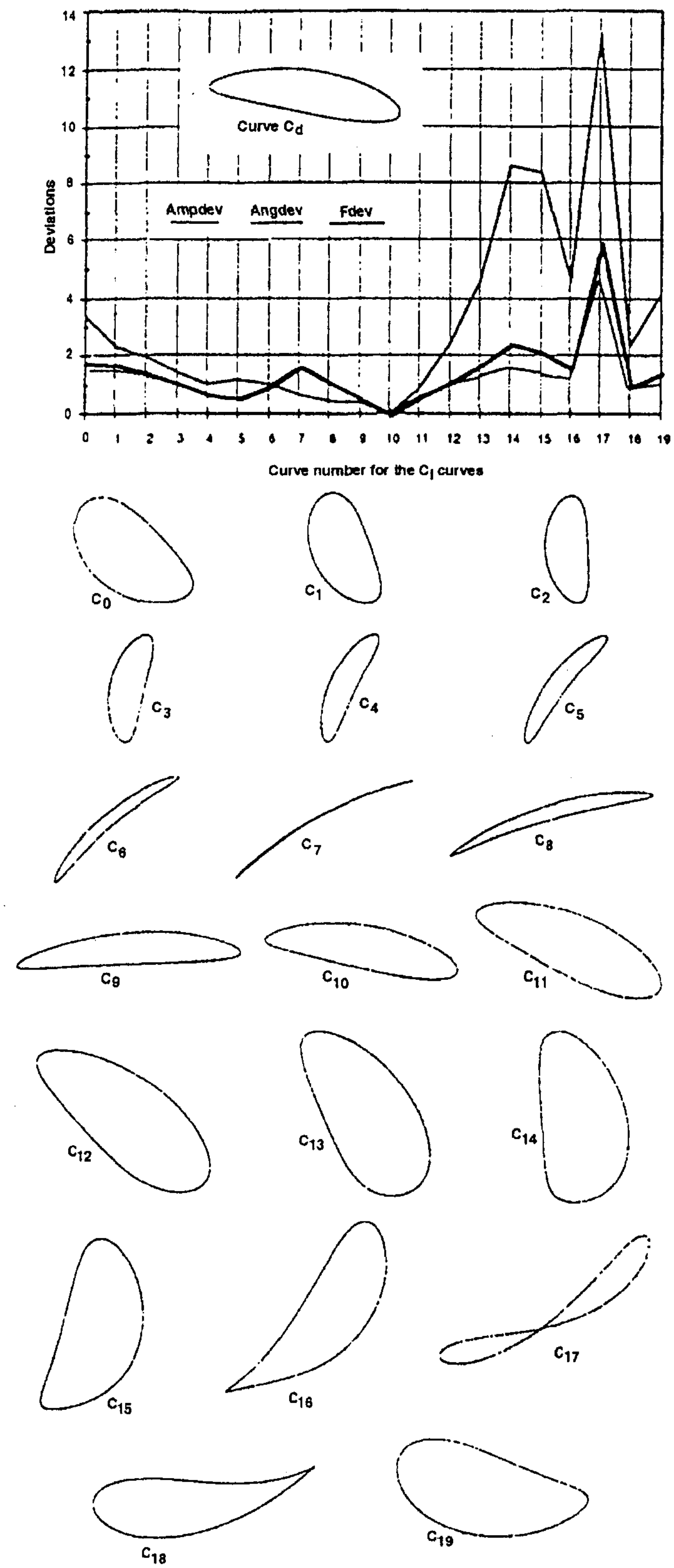

Fig. 5 An example for validation of Fourier deviation

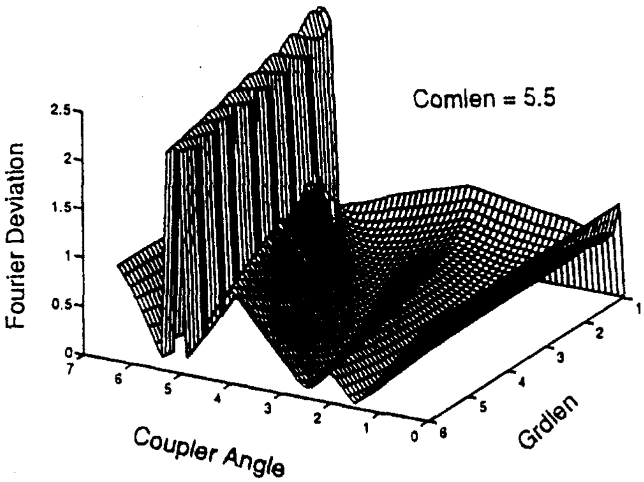

Fig. 6 Fourier deviation function as a function of the three parameters required to define a symmetrical coupler curve mechanism

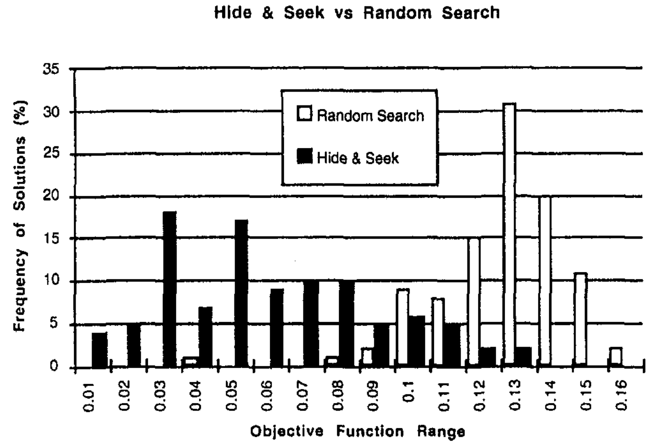

Fig. 8 Comparison of Hide-and-Seek with random search

+ 0.1 *Angdev) between the desired curve, C_d and the 20 generated candidates or coupler curves C_i ($i = 0, \ldots, 19$). The curve C_d and the twenty C_i ($i = 0, \ldots, 19$) curves are also shown in the figure for visual comparison. In calculating the deviations, the first 20 harmonics from the Fourier series were included.

Examination of the figure shows that Fourier deviation decreases as the shape of curves C_i ($i = 1, \ldots, 20$) becomes closer to the curve C_d and increases when the shape becomes progressively unlike curve C_d. Sharp peaks are produced whenever a curve with a double loop is compared with a single loop curve. Fourier deviation function values of 0.1 or less are considered very good matches and usually the shape differences between the two curves in question cannot be distinguished by naked eye. In this example the Fourier deviation value is zero for curve C_{10} as this curve and the C_d are one and the same. This example illustrates the effectiveness of Fourier deviation function in capturing the shape differences between any two curves.

Visualization of Fourier Deviation Function

Symmetrical coupler curves can be generated by four-bar mechanisms which satisfy the condition that the length of the coupler link AB, the length of the follower link BBo, and the distance BP are all equal. This length is termed as common length. By setting the crank to be of unit length, as a scaling factor, the total number of variables can be reduced to three. Therefore, for symmetrical four-bar mechanisms, the Fourier deviation is a function of only three parameters—common length, ground link length and the angle ABP and hence it is possible to visualize the complexity of the design space to some extent by drawing a series of two-dimensional plots. One such plot is shown in Fig. 6 for the example given above. Here the coupler angle and ground-link length are varied simultaneously for a fixed value of common length in each plot. Flat portions of the plots indicate infeasible region (non-Grashof four-bars) for which the objective function was not calculated. The plots indicate that the Fourier deviation function, although obviously non-linear and multimodal, is not overly complex, at least for this example. Corresponding structural error function would be much more complex and intractable for most search techniques.

Structural Error Versus Fourier Deviation Objective Function

To compare the effectiveness of structural error and Fourier deviation as objective functions used for synthesis of path-generating mechanisms, an example problem was solved in the following manner: A local optimization method was run 50 times on each objective function, starting from (the same) 50

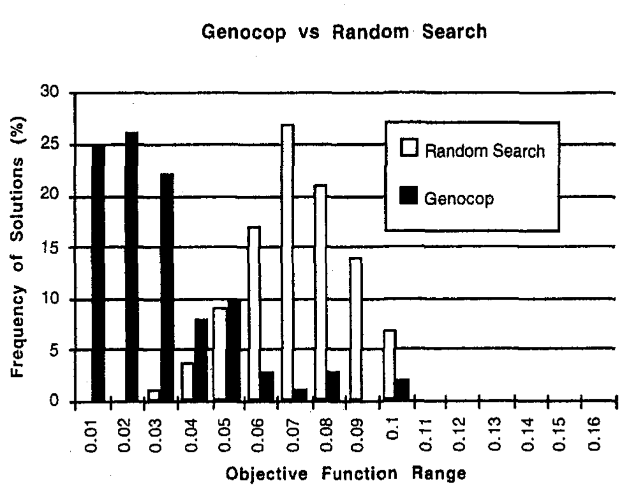

Fig. 7 Comparison of Genocop with random search

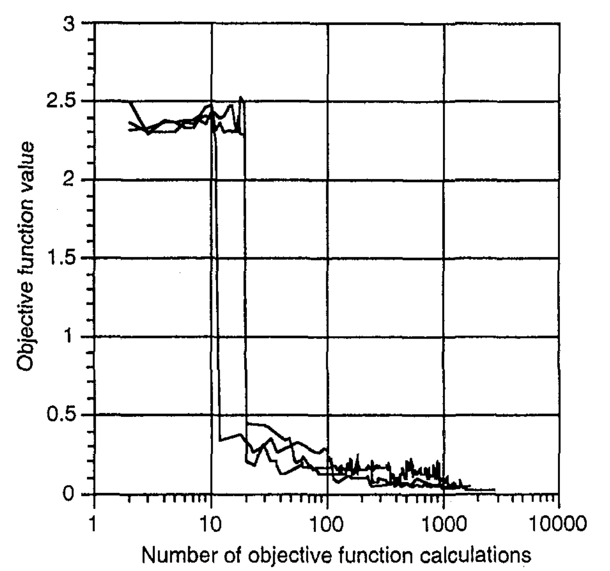

Fig. 9 Objective function value for three runs of Hide-and-Seek and Powell's method on first example problem

Table 1 Results of synthesis of example 1

r_1/r_2	r_3/r_2	r_4/r_2	r_7/r_2	γ	θ_1	x_0	y_0	r_2	Branch	Fdev
8.0269	4.6559	4.5483	3.2066	1.7040	0.6452	2.0083	8.9580	1.7982	1	0.5888
6.6110	5.2338	2.5536	6.7721	1.8448	4.9031	11.771	5.3811	1.4117	1	0.4488
3.3379	2.6428	2.0894	4.9062	3.1954	4.4217	7.5291	5.9013	1.0697	2	0.6617
4.5305	3.7512	1.9509	4.9574	2.2171	5.1523	8.9838	3.4854	1.3305	1	0.5248
8.2566	2.8269	6.6148	2.2536	4.5603	3.1208	2.2487	5.1483	1.8058	2	0.4790

randomly generated points in the design space. The random points were generated uniformly over a hypercube formed by prescribing a maximum link-length ratio of 6.0. For the optimization, Powell's method of conjugate directions was employed.

The desired curve used is the symmetric coupler curve described in example above. Using the Fourier deviation function, the optimization algorithm was able to locate the global minimum in 23 of the 50 runs. The average number of function evaluations in runs converging to the global minimum was 772. In contrast, in the case of structural error function, only four runs out of the 50 converged to the global minimum (fourier deviation function value of zero), with each run taking 2965 function evaluations on the average. These results indicate that the space of Fourier deviation function is relatively easier to search and thus more suitable for use in solving the synthesis problem.

Global Optimization

The design space of mechanisms contains a large number of local minima. A local optimization algorithm starting from a random point will converge to the nearest local minimum which may be an unsatisfactory design. This problem is generally averted by specifying a reasonably good design as the starting point, using a catalog of coupler curves or previous design experience. To avoid dependence on specialized knowledge and also to search the entire design space (rather than just the vicinity of the starting point), it is necessary to utilize a global search algorithm.

Global optimization is an active area of research, with a number of different methods proposed in the literature, each with its own limitations (Dixon and Szego, 1978). Except in the case of certain simple functions, convergence to the global minimum is not guaranteed by any algorithm. However, reasons exist for favoring the probabilistic methods over deterministic methods (Gomulka, 1978). Among the probabilistic or stochastic optimization methods, one can include the two-phase methods (Rinnooy Kan and Timmer, 1984), simulated annealing (Kirkpatrick et al., 1983) and the genetic algorithms (Holland, 1975). Although the performance of these algorithms is problem-dependent, the available evidence indicates that simulated annealing performs about as well as the best of two-phase methods (Dekker and Aarts, 1991). Hide and Seek, the global optimization algorithm used in this research, is an extension of the original simulated annealing to continuous spaces (Romeijn and Smith, 1992). Genetic algorithms, by their nature, require a very large number of function evaluations and do not seem to provide any special advantage over simulated annealing.

The strength of stochastic optimization lies in search of the entire design space to locate a region of low function values likely to contain the global minimum. In the later stages of the search, however, these methods become very inefficient. It is thus advisable to switch to an efficient local search method towards the end.

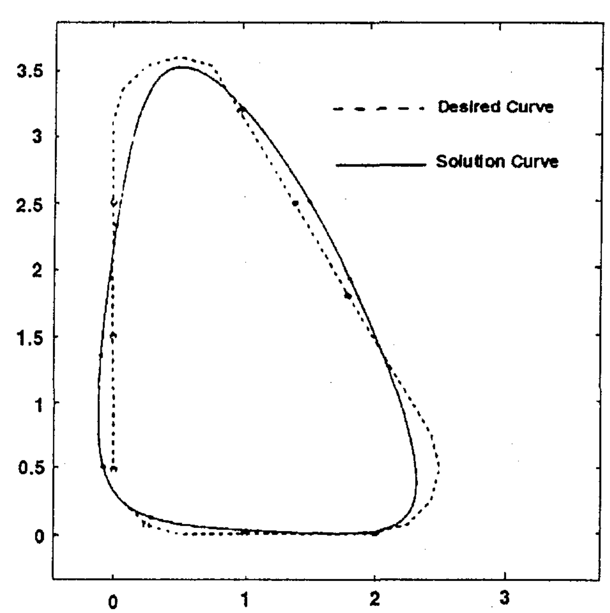

Fig. 10 Solution of the first synthesis example

Comparisons of Global Search Methods

We have made extensive quantitative comparisons of different search methods, including two different genetic algorithms, GAucsd (Shraudolph and Grefenstette, 1992) and Genocop (Michalewicz, 1992), and Hide & Seek, with pure random search to identify minimum fourier deviation function value on a given mechanism problem. Pure random search is the simplest of the stochastic methods of global optimizations and is therefore used as a base method for comparison. In this method, the function is evaluated at 50,000 randomly chosen points and the smallest of the objective function values are recorded. The random point is chosen from a uniform distribution over the hypercube formed by variable bounds. Points violating the Grashof constraints are treated in the same manner as by the particular genetic algorithm with which comparison is being made. GAuscd is a traditional genetic algorithm using binary representation and simple crossover and mutation operators. Genecop uses real number representation and special operators designed to handle linear constraints without requiring a penalty. Experiments for this case follow the same method as with GAucsd, except that in the Random Search, infeasible points are simply rejected, rather than imposing a penalty.

The example problem has a known global minimum of zero. In each run, a population of 100 individuals is evolved for a maximum of 50,000 function evaluations approximately. The lowest objective function value was noted for each run. These results were compared with 100 runs of the pure random search. The results indicate that while both genetic algorithms perform significantly better than Random Search, Genocop does better than GAucsd on this problem. The results are shown in Fig. 7.

Hide & Seek was used to solve the same problem as the genetic algorithms and compared with Pure Random Search.

The major difference is that each run in this case consisted of 1500 (=300*n) function evaluations only. In the Hide & Seek method, the known value of global minimum (zero) was used to calculate the cooling schedule, rather than the estimated value. The results are shown in Fig. 8.

Performance of Hide & Seek With Powell's Method

Hide & Seek algorithm was employed for 200*n iterations and then Powell's method of conjugate directions (Press et al., 1988) was used for local optimization. The method was modified to incorporate linear constraints necessary for keeping positive link lengths, limiting the maximum link-length ratio and obtaining a crank-rocker mechanism. Figure 9 shows the sequence of points accepted by the Hide and Seek algorithm (the first 1500 function evaluations) followed by the points found by successive line minimizations of Powell's method, in three different cases.

Shape Analogous Points

The minimization of Fourier deviation gives relative lengths and angles of a mechanism (r_1/r_2, r_3/r_2, r_4/r_2, r_7/r_2, g) with a coupler curve similar in shape to the desired curve but with no reference to the size, position and orientation of the mechanism which are determined by r_2, r_0, θ_0, and θ_1 respectively. Values of these four variables can be chosen such that two points of the solution curve coincide with two analogous points of the desired curve. If the shapes of the two curves were exactly the same, this would ensure that the two curves coincide fully. In practice, a second step is necessary in which the distance between a set of analogous points is minimized. This is a process similar to minimization of the structural error except for two differences: (i) the variables affecting the shape of the curve are not included in the optimization, only the size, location and orientation are varied and (ii) analogous points on the two curves are chosen by utilizing special properties of Fourier descriptors explained below (rather than by prescribed timing, etc.)

If C and C^* represent clockwise curves of the same shape but different starting points Z_0 and Z_0^* respectively, then the harmonic phase angles for C and C^* are related by

$$\alpha_k' = \alpha_k + k\Delta\alpha \quad k = 1, 2, \ldots$$

$$\Delta\alpha = \frac{-2\pi\Delta l}{L}$$

where Δl is the clockwise arc length along curve C from Z_0 to Z_0^* and L is the total length of the curve C.

The above two equations may be solved for Δl to get:

$$\Delta l = \frac{(\alpha_k - \alpha_k')L}{2\pi k}$$

Using this relation, we can locate a point on the first curve that is analogous to the starting point of the second curve, when the harmonic phase angles of the two curves and the total length of the first curve are known. Since phase angles do not depend upon size, orientation or location of the curves, these need not be the same. The curves must, however, be prescribed in the same sense—either clockwise or counter-clockwise. Although the relation is valid strictly for curves of the same shape, experience indicate that it remains fairly good for curves that are similar in shape. The starting point of a curve can, of course, be shifted easily, allowing us to identify analogous points on a candidate curve for any point on the desired curve.

Design Examples

Results of two synthesis examples are presented to illustrate the effectiveness of the method. In each case, the optimization algorithm was run five times, starting from random points. In each run, both branches of the coupler curve were searched in turn and both directions of crank travel were tried. Each run comprised 1000 function evaluations in Hide & Seek and typically a further 200 to 300 function evaluations to convergence in Powell's method. The objective function was taken as: Fdev = Ampdev + 0.1*Angdev

The first desired curve is triangular in shape with rounded ends and is described using 27 points. The five solutions obtained are listed in Table 1 along with values of Fourier Deviation. The desired curve and the best solution found are depicted in Fig. 10. Points marked by '*' were determined by the algorithm as analogous to specified points on the desired curve (marked by 'o' in Fig. 10). Note that the objective function value of 0.44 for the "best" solution includes differences in shape, size, orientation, and position between the desired trajectory and the "best" solution. However, mechanisms with objective function values that are much less 0.05 were discovered in stage one when comparisons were made on shape alone. Proper choice of analogous points can improve the final solution further. For detailed discussion, the reader is encouraged to refer to (Ullah, 1995).

Results for the second example are presented in Table 2 and Fig. 11. Note that in two specific cases (rows 3 and 5 in Table 2), the solutions obtained have the same values for shape-determining variables, except that the copuler angle is $2\pi - \gamma$ and the branch is changed. These are mirror images of the previous solutions and are considered as admissible solutions since the desired curve is symmetrical.

Conclusions

Optimal synthesis procedures reported in the literature usually employ local optimization algorithms, which converge to the local minimum nearest to the starting mechanism design. The results are thus dependent on how good a starting design is provided. Furthermore, the search is limited to the vicinity of the starting point. A new approach to synthesis of mechanisms for path generation is presented in this paper. The two key features that are central to success with this approach are: (1) curve shape is optimized separately from the size, orientation and location of the curve, and (2) an effective objective function based on Fourier descriptors is used to evaluate curve shape deviation. The search for optimal shape is carried out

Table 2 Results of synthesis example 2

r1/r2	r3/r2	r4/r2	r7/r2	γ	θ_1	x0	y0	r2	Branch	Fdev
5.2690	1.6628	5.2864	2.9060	3.5548	0.4073	0.3288	12.297	5.9521	1	0.7339
2.3394	2.1410	3.4560	1.9049	0.6173	4.5433	2.4268	7.0800	6.4326	1	0.8167
1.3160	3.1801	3.4543	2.4571	0.3527	4.9638	3.4137	7.8002	6.4777	1	0.4161
5.2690	1.6628	5.2864	2.9060	2.7284	2.7343	0.3288	12.297	5.9521	2	0.7339
1.3160	3.1801	3.4543	2.4571	5.9305	4.4610	3.4137	7.8002	6.4777	2	0.4161

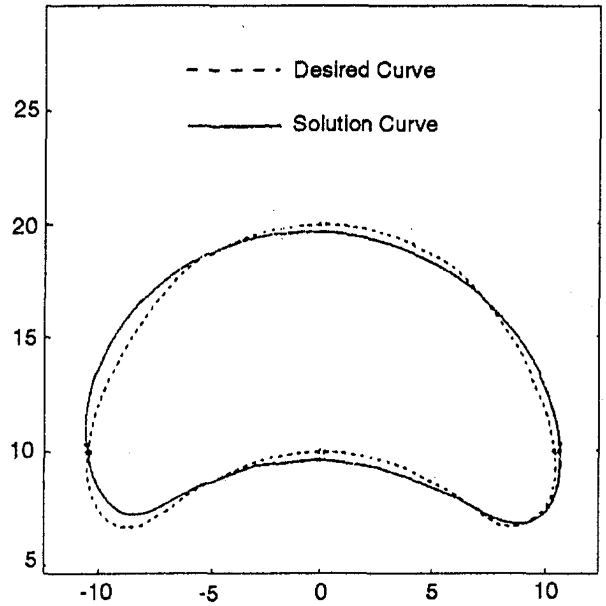

Fig. 11 Solution of the second synthesis example

using a stochastic global optimization algorithm. Considering the nature of Fourier deviation function and the requirements of the synthesis problem, an efficient global search method that does not use gradients is needed. The proposed method is to apply Hide & Seek for the global search, followed by Powell's method of conjugate directions for efficient local convergence. Hide & Seek works much more efficiently compared with genetic algorithms, and, unlike most two-phase methods, does not require the function to be continuous. Powell's method also does not require gradients to exist. Together, these two methods provide efficient convergence to good minima starting from randomly generated solutions.

By taking advantage of certain properties of Fourier descriptors, a set of points are located on the solution curve that are 'shape analogous' to points specified on the desired curve. Minimizing the distance between analogous points on the two curves enabled us to determine the optimal size, orientation and position of the solution mechanism. Design examples presented in this paper illustrate the effectiveness of the synthesis approach in identifying near-global optimal solutions consistently without requiring the user to provide any initial guess.

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APPENDIX

Hide & Seek Method

Hide and Seek, the global optimization algorithm used in this research, is an extension of the original simulated annealing to continuous spaces (Romeijn and Smith, 1992). Beginning with any feasible interior point x_1, in each iteration the algorithm generates first a direction vector from the uniform distribution over a unit hypersphere around the current point.

Then a line segment is determined going through the current point along the chosen direction and with end points determined by bounds on the problem variables. (In mechanism synthesis, bounds on variables ensure positive link lengths and limit link length ratio.) A candidate point x is chosen uniformly from this line segment, rejecting points that are infeasible due to constraints other than variable bounds (e.g., those imposed to obtain crank-rocker type of four bars). The candidate point is accepted as the next iteration point according to the Metropolis criterion, i.e., with probability

$$p = \min(1, \exp\{(f(x_1) - f(x))/T\})$$

where $f(x_1)$ is the function value at the current point, $f(x)$ the function value at the candidate point and T a (positive) control parameter. T starts with a high value and decreases toward zero as iterations proceed. In Hide & Seek, the sequence of decreasing values of temperature (called the cooling schedule) is adapted according to the progress of minimization as below:

$$T = \frac{2(f(x_1) - f^*)}{\chi^2_{1-p}(n)}$$

where $f(x_1)$ is the current function value, f^* the global minimum, n the dimension of the search space, and $\chi^2_{1-p}(n)$ is the $100(1 - p)$ percentile point of the chi-squared distribution with n degrees of freedom. For experiments in this paper, the parameter p was fixed at 0.1. Temperature is updated every time a point with lower function value is found. The temperature update formula involves the global minimum function value f^*. In most cases this is not known. Romeijn and Smith (1992) present the following relation (reformulated here for the minimization problem) for an estimate $\hat{f}$ of the global minimum, with reference to De Haan (1981):

$$\hat{f} = f(x_1) - \frac{f(x_2) - f(x_1)}{(1 - p)^{-n/2} - 1}$$

where $f(x_1)$ is the current objective function value, and $f(x_2)$ the previous (higher) value.