Algebraic Fitting of Linear Constraints

- We form a system of linear equations given by,

$$[A]\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_8 \end{bmatrix} = 0, \text{ where } [A] = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} & \cdots & \cdots & A_{18} \\ A_{21} & A_{22} & A_{23} & A_{24} & \cdots & \cdots & A_{28} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n1} & A_{n2} & A_{n3} & A_{n4} & \cdots & \cdots & A_{n8}, \end{bmatrix}.$$

where $n < 5$ are number of exact linear constraints

- Singular Value Decomposition $[A] = [U][S][V]^T$
- Null space formed by right singular vectors corresponding to zero singular value is the solution space for optimization given by

$$\mathbf{p} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \ldots + \alpha_{8-n} \mathbf{v}_{8-n}$$

Constraints in the Null Space

- Unified Linear Form

$$\sum_{j=1}^{8} A_j p_j = 0,$$

- Candidate solution vector

$$\mathbf{p} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \ldots + \alpha_{8-n} \mathbf{v}_{8-n}$$

- Substitute **p** in the unified form of linear constraints

$$f_1(\alpha_i; i = 1 \ldots 8-n) = k_1 \alpha_1 + k_2 \alpha_2 + \ldots + k_{8-n} \alpha_{8-n} = 0,$$

- Substitute **p** in the non-linear constraint

$$A p_4^2 + B p_4 p_5 + C p_5^2 - D p_4 p_1 - E p_5 p_1 + F p_1^2 = 0$$

- To get

$$f_2(\alpha_i; i = 1 \ldots 8-n) = \sum_{i=1,j=1}^{i=8-n,j=8-n} k_{ij} \alpha_i \alpha_j + \sum_{i=1}^{i=8-n,} k_i \alpha_i + k_0 = 0$$