

Constraints in the Null Space

- Unified Linear Form

$$\sum_{j=1}^8 A_j p_j = 0,$$

- Candidate solution vector

$$\mathbf{p} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \dots + \alpha_{8-n} \mathbf{v}_{8-n}$$

- Substitute \mathbf{p} in the unified form of linear constraints

$$f_1(\alpha_i; i = 1 \dots 8 - n) = k_1 \alpha_1 + k_2 \alpha_2 + \dots + k_{8-n} \alpha_{8-n} = 0,$$

- Substitute \mathbf{p} in the non-linear constraint

$$Ap_4^2 + Bp_4p_5 + Cp_5^2 - Dp_4p_1 - Ep_5p_1 + Fp_1^2 = 0$$

- To get

$$f_2(\alpha_i; i = 1 \dots 8 - n) = \sum_{i=1, j=1}^{i=8-n, j=8-n} k_{ij} \alpha_i \alpha_j + \sum_{i=1}^{i=8-n} k_i \alpha_i + k_0 = 0$$

Constraints in the Null Space

- Two quadratic Conditions

$$p_1 p_6 + p_2 p_5 - p_3 p_4 = 0,$$

$$2p_1 p_7 - p_2 p_4 - p_3 p_5 = 0.$$

- In the null space

$$h_1(\alpha_i; i = 1 \dots 8 - n) = \sum_{i=1, j=1}^{i=8-n, j=8-n} k_{1ij} \alpha_i \alpha_j = 0,$$

$$h_2(\alpha_i; i = 1 \dots 8 - n) = \sum_{i=1, j=1}^{i=8-n, j=8-n} k_{2ij} \alpha_i \alpha_j = 0$$