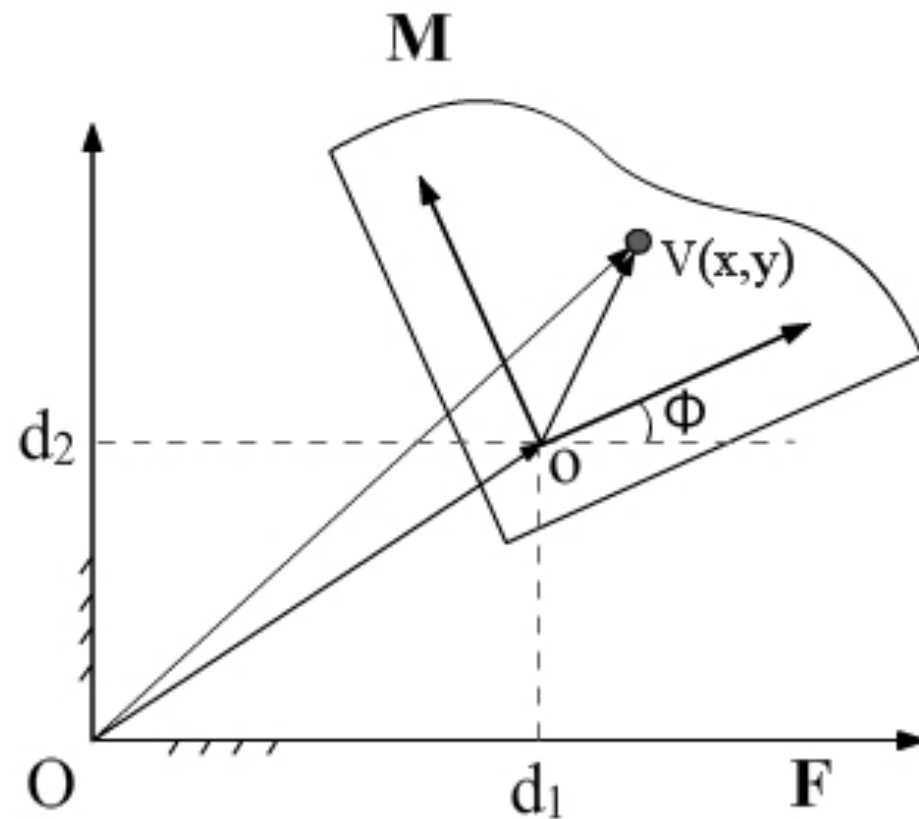


Review: Image Space Approach

Planar-Quaternions and -Kinematic Mapping



$$Z_1 = \frac{1}{2} \left(d_1 \sin \frac{\phi}{2} - d_2 \cos \frac{\phi}{2} \right),$$

$$Z_2 = \frac{1}{2} \left(d_1 \cos \frac{\phi}{2} + d_2 \sin \frac{\phi}{2} \right),$$

$$Z_3 = \sin \frac{\phi}{2},$$

$$Z_4 = \cos \frac{\phi}{2},$$

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \phi & -\sin \phi & d_1 \\ \sin \phi & \cos \phi & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad X = [H]_x \quad L = [\overline{H}]l$$

Constraints on Homogeneous Form

$$\begin{aligned} p_1(Z_1^2 + Z_2^2) + p_2(Z_1Z_3 - Z_2Z_4) + p_3(Z_2Z_3 + Z_1Z_4) \\ + p_4(Z_1Z_3 + Z_2Z_4) + p_5(Z_2Z_3 - Z_1Z_4) + p_6Z_3Z_4 \\ + p_7(Z_3^2 - Z_4^2) + p_8(Z_3^2 + Z_4^2) = 0, \end{aligned}$$

- Above is defined by a set of 8 homogeneous coordinates $(p_1 \dots p_8)$
- However, there are only 5 independent mechanism parameters related by

$$\begin{aligned} p_1 &= -a_0, & p_2 &= a_0x & p_3 &= a_0y, & p_4 &= a_1, & p_5 &= a_2, \\ p_6 &= -a_1y + a_2x, & p_7 &= -(a_1x + a_2y)/2, \\ p_8 &= (a_3 - a_0(x^2 + y^2))/4, \end{aligned}$$

- There are two quadratic conditions on $(p_1 \dots p_8)$

$$\begin{aligned} p_1p_6 + p_2p_5 - p_3p_4 &= 0, \\ 2p_1p_7 - p_2p_4 - p_3p_5 &= 0. \end{aligned}$$