Optimal Model

Minimize $f(\alpha_i) = \sum_{i=1}^{i=u} {f_{1_i}}^2 \omega_{1i} + \sum_{j=1}^{j=v} {f_{2_j}}^2 \omega_{2j}$

$$\text{subject to} \quad h_1 = 0,\ h_2 = 0, \ldots, h_{(m+2)} = 0$$
$$g \leq 0$$

where

h_1 and h_2 are the two quadratic constraints on G-manifolds,

while *g* is an inequality constraint.

- Objective function represents algebraic fitting error of relaxed geometric constraints.

Minimization using Lagrange Multipliers

- Lagrange Objective function is formed,

$$\text{Maximize } F(\alpha_i, \lambda_i) = -f(\alpha_i) - \lambda_1 h_1 - \lambda_2 h_2 - \ldots \lambda_{(m+2)} h_{(m+2)} - \mu g$$

- Taking partial derivative we form the system of polynomial equations given by,

$$\frac{\partial}{\partial \alpha_2}(F) = 0, \frac{\partial}{\partial \alpha_3}(F) = 0, \ldots, \frac{\partial}{\partial \alpha_{(8-n)}}(F) = 0,$$

$$\frac{\partial}{\partial \lambda_1}(F) = 0, \frac{\partial}{\partial \lambda_2}(F) = 0, \ldots, \frac{\partial}{\partial \lambda_{(m+2)}}(F) = 0.$$

along with Karush-Kuhn-Tucker conditions

$$\mu \frac{\partial}{\partial \mu}(F) = 0, \text{feasibility: } g \leq 0, \text{optimality: } \mu \geq 0$$