

Constraints on Homogeneous Form

$$\begin{aligned} p_1(Z_1^2 + Z_2^2) + p_2(Z_1 Z_3 - Z_2 Z_4) + p_3(Z_2 Z_3 + Z_1 Z_4) \\ + p_4(Z_1 Z_3 + Z_2 Z_4) + p_5(Z_2 Z_3 - Z_1 Z_4) + p_6 Z_3 Z_4 \\ + p_7(Z_3^2 - Z_4^2) + p_8(Z_3^2 + Z_4^2) = 0, \end{aligned}$$

- Above is defined by a set of 8 homogeneous coordinates $(p_1 \dots p_8)$
- However, there are only 5 independent mechanism parameters related by

$$\begin{aligned} p_1 &= -a_0, & p_2 &= a_0 x & p_3 &= a_0 y, & p_4 &= a_1, & p_5 &= a_2, \\ p_6 &= -a_1 y + a_2 x, & p_7 &= -(a_1 x + a_2 y)/2, \\ p_8 &= (a_3 - a_0(x^2 + y^2))/4, \end{aligned}$$

- There are two quadratic conditions on $(p_1 \dots p_8)$

$$\begin{aligned} p_1 p_6 + p_2 p_5 - p_3 p_4 &= 0, \\ 2p_1 p_7 - p_2 p_4 - p_3 p_5 &= 0. \end{aligned}$$

Geometric Constraints on Planar Mechanisms

- Pose Constraint on Coupler

Pose having parameters $(Z_{1p}, Z_{2p}, Z_{3p}, Z_{4p})$

$$\begin{aligned} p_1(Z_{1p}^2 + Z_{2p}^2) + p_2(Z_{1p}Z_{3p} - Z_{2p}Z_{4p}) + p_3(Z_{2p}Z_{3p} + Z_{1p}Z_{4p}) \\ + p_4(Z_{1p}Z_{3p} + Z_{2p}Z_{4p}) + p_5(Z_{2p}Z_{3p} - Z_{1p}Z_{4p}) + p_6Z_{3p}Z_{4p} \\ + p_7(Z_{3p}^2 - Z_{4p}^2) + p_8(Z_{3p}^2 + Z_{4p}^2) = 0, \end{aligned}$$