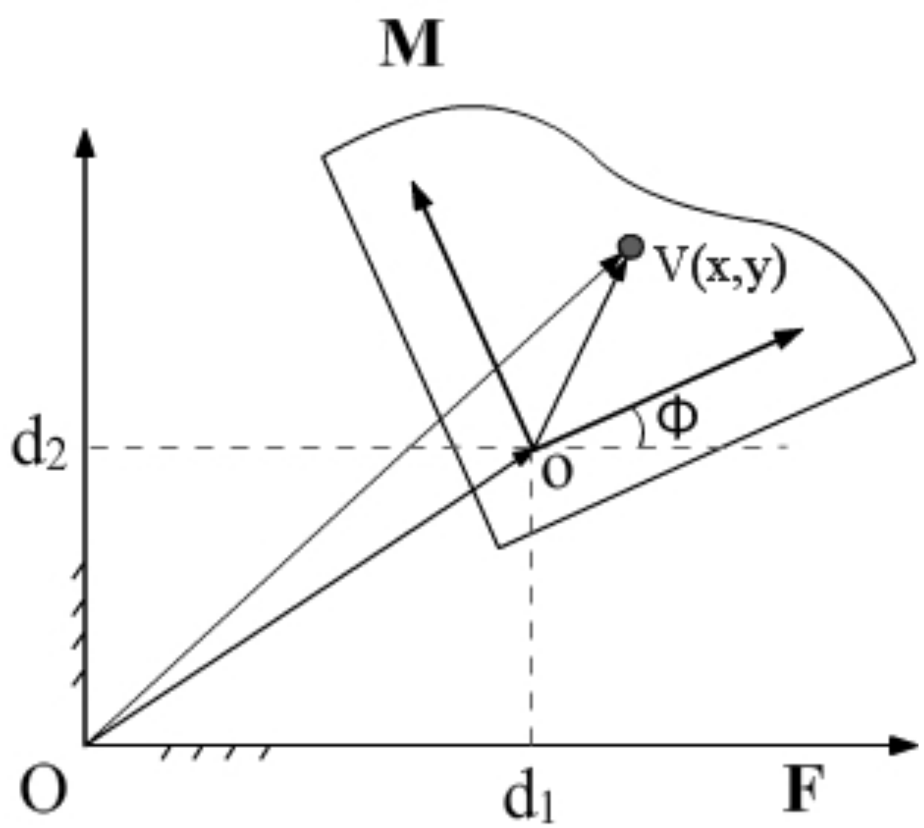
Review: Image Space Approach

Planar-Quaternions and -Kinematic Mapping



$$Z_1 = \frac{1}{2}\left(d_1 \sin\frac{\phi}{2} - d_2 \cos\frac{\phi}{2}\right),$$

$$Z_2 = \frac{1}{2}\left(d_1 \cos\frac{\phi}{2} + d_2 \sin\frac{\phi}{2}\right),$$

$$Z_3 = \sin\frac{\phi}{2},$$

$$Z_4 = \cos\frac{\phi}{2},$$

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & d_1 \\ \sin\phi & \cos\phi & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \mathrm{X} = [\mathrm{H}]\mathrm{x} \qquad L = [\overline{H}]l$$

Constraints on Homogeneous Form

$$\begin{aligned} &p_1(Z_1^2+Z_2^2)+p_2(Z_1Z_3-Z_2Z_4)+p_3(Z_2Z_3+Z_1Z_4) \\ &\quad +p_4(Z_1Z_3+Z_2Z_4)+p_5(Z_2Z_3-Z_1Z_4)+p_6Z_3Z_4 \\ &\quad +p_7(Z_3^2-Z_4^2)+p_8(Z_3^2+Z_4^2)=0, \end{aligned}$$

- Above is defined by a set of 8 homogeneous coordinates ($p_1 \dots p_8$)
- However, there are only 5 independent mechanism parameters related by

$$\begin{aligned} &p_1=-a_0, \quad p_2=a_0x \quad p_3=a_0y, \quad p_4=a_1, \quad p_5=a_2, \\ &p_6=-a_1y+a_2x, \quad p_7=-(a_1x+a_2y)/2, \\ &p_8=(a_3-a_0(x^2+y^2))/4, \end{aligned}$$

- There are two quadratic conditions on ($p_1 \dots p_8$)

$$p_1p_6+p_2p_5-p_3p_4=0,$$

$$2p_1p_7-p_2p_4-p_3p_5=0.$$