Geometric Constraints on Planar Mechanisms

- Pose Constraint on Coupler

Pose having parameters $(Z_{1p}, Z_{2p}, Z_{3p}, Z_{4p})$

$$p_1(Z_{1p}^2 + Z_{2p}^2) + p_2(Z_{1p}Z_{3p} - Z_{2p}Z_{4p}) + p_3(Z_{2p}Z_{3p} + Z_{1p}Z_{4p})$$
$$+p_4(Z_{1p}Z_{3p} + Z_{2p}Z_{4p}) + p_5(Z_{2p}Z_{3p} - Z_{1p}Z_{4p}) + p_6 Z_{3p}Z_{4p}$$
$$+p_7(Z_{3p}^2 - Z_{4p}^2) + p_8(Z_{3p}^2 + Z_{4p}^2) = 0,$$

Geometric Constraints on Planar Mechanisms

Curve Constraints on Fixed Pivots

For a conic section given by

$$AX^2 + BXY + CY^2 + DX + EY + F = 0$$

constraint equation is,

$$Ap_4^2 + Bp_4p_5 + Cp_5^2 - Dp_4p_1 - Ep_5p_1 + Fp_1^2 = 0$$

$$X = X_f = -p_4/p_1,$$
$$Y = Y_f = -p_5/p_1$$

- When $A = B = C = 0$

  Line with parameters (L_1, L_2, L_3)

  $$-L_1p_4 - L_2p_5 + L_3p_1 = 0$$

- When $A = B = C = D = 0$ or $A = B = C = E = 0$

  Point with coordinates (X_f, Y_f)

  $$X_fp_1 + p_4 = 0,$$
  $$Y_fp_1 + p_5 = 0$$