

# Geometric Constraints on Planar Mechanisms

## Curve Constraints on Fixed Pivots

For a conic section given by

$$AX^2 + BXY + CY^2 + DX + EY + F = 0$$

constraint equation is,

$$Ap_4^2 + Bp_4p_5 + Cp_5^2 - Dp_4p_1 - Ep_5p_1 + Fp_1^2 = 0$$

$$\begin{aligned} X &= X_f = -p_4/p_1, \\ Y &= Y_f = -p_5/p_1 \end{aligned}$$

- When  $A = B = C = 0$

Line with parameters  $(L_1, L_2, L_3)$

$$-L_1p_4 - L_2p_5 + L_3p_1 = 0$$

- When  $A = B = C = D = 0$  or  $A = B = C = E = 0$

Point with coordinates  $(X_f, Y_f)$

$$\begin{aligned} X_fp_1 + p_4 &= 0, \\ Y_fp_1 + p_5 &= 0 \end{aligned}$$

# Linear Constraints in the Coefficient Space

$$\begin{aligned} p_1(Z_{1p}^2 + Z_{2p}^2) + p_2(Z_{1p}Z_{3p} - Z_{2p}Z_{4p}) + p_3(Z_{2p}Z_{3p} + Z_{1p}Z_{4p}) \\ + p_4(Z_{1p}Z_{3p} + Z_{2p}Z_{4p}) + p_5(Z_{2p}Z_{3p} - Z_{1p}Z_{4p}) + p_6Z_{3p}Z_{4p} \\ + p_7(Z_{3p}^2 - Z_{4p}^2) + p_8(Z_{3p}^2 + Z_{4p}^2) = 0, \end{aligned}$$

$$-L_1p_4 - L_2p_5 + L_3p_1 = 0, \quad -l_1p_2 - l_2p_3 + l_3p_1 = 0$$

$$\begin{aligned} X_fp_1 + p_4 &= 0, \\ Y_fp_1 + p_5 &= 0 \end{aligned}$$

$$\begin{aligned} x_mp_1 + p_2 &= 0, \\ y_mp_1 + p_3 &= 0, \end{aligned}$$

Unified Form

$$\sum_{j=1}^8 A_j p_j = 0,$$