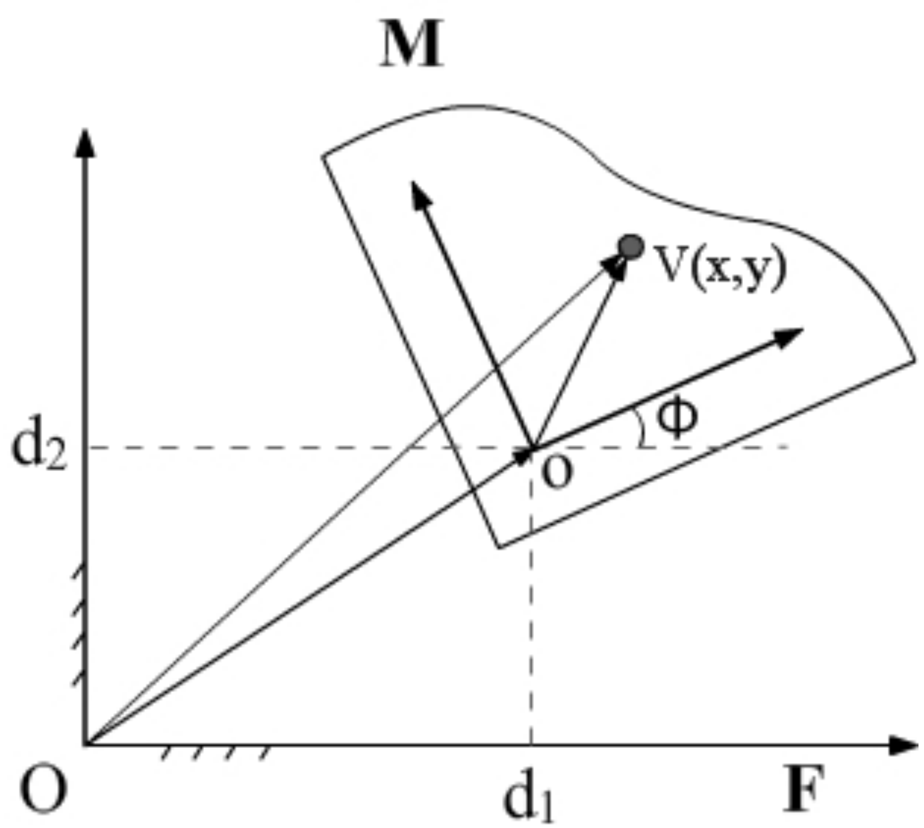
Review: Image Space Approach

Planar-Quaternions and -Kinematic Mapping



$$Z_1 = \frac{1}{2}(d_1 \sin\frac{\phi}{2} - d_2 \cos\frac{\phi}{2}),$$
$$Z_2 = \frac{1}{2}(d_1 \cos\frac{\phi}{2} + d_2 \sin\frac{\phi}{2}),$$
$$Z_3 = \sin\frac{\phi}{2},$$
$$Z_4 = \cos\frac{\phi}{2},$$

$$\begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & d_1 \\ \sin\phi & \cos\phi & d_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \mathrm{X} = [\mathrm{H}]\mathrm{x} \qquad L = [\overline{H}]l$$

Constraints on Homogeneous Form

$$
\begin{aligned}
p_1(Z_1^2 + Z_2^2) + p_2(Z_1Z_3 - Z_2Z_4) &+ p_3(Z_2Z_3 + Z_1Z_4) \\
&+ p_4(Z_1Z_3 + Z_2Z_4) + p_5(Z_2Z_3 - Z_1Z_4) + p_6Z_3Z_4 \\
&+ p_7(Z_3^2 - Z_4^2) + p_8(Z_3^2 + Z_4^2) = 0,
\end{aligned}
$$

- Above is defined by a set of 8 homogeneous coordinates ($p_1 \ldots p_8$)
- However, there are only 5 independent mechanism parameters related by

$$
\begin{aligned}
&p_1 = -a_0, \quad p_2 = a_0 x \quad p_3 = a_0 y, \quad p_4 = a_1, \quad p_5 = a_2, \\
&p_6 = -a_1 y + a_2 x, \quad p_7 = -(a_1 x + a_2 y)/2, \\
&p_8 = (a_3 - a_0(x^2 + y^2))/4,
\end{aligned}
$$

- There are two quadratic conditions on ($p_1 \ldots p_8$)

$$p_1p_6 + p_2p_5 - p_3p_4 = 0,$$

$$2p_1p_7 - p_2p_4 - p_3p_5 = 0.$$