Modern Portfolio Theory for Cryptocurrencies

Eunice Ofori-Addo
MATH 530
Eastern Washington University

INTRODUCTION

Modern Portfolio Theory (MPT), was pioneered by Harry Markowitz in his paper “Portfolio Selection” published in 1952 by the Journal of Finance.

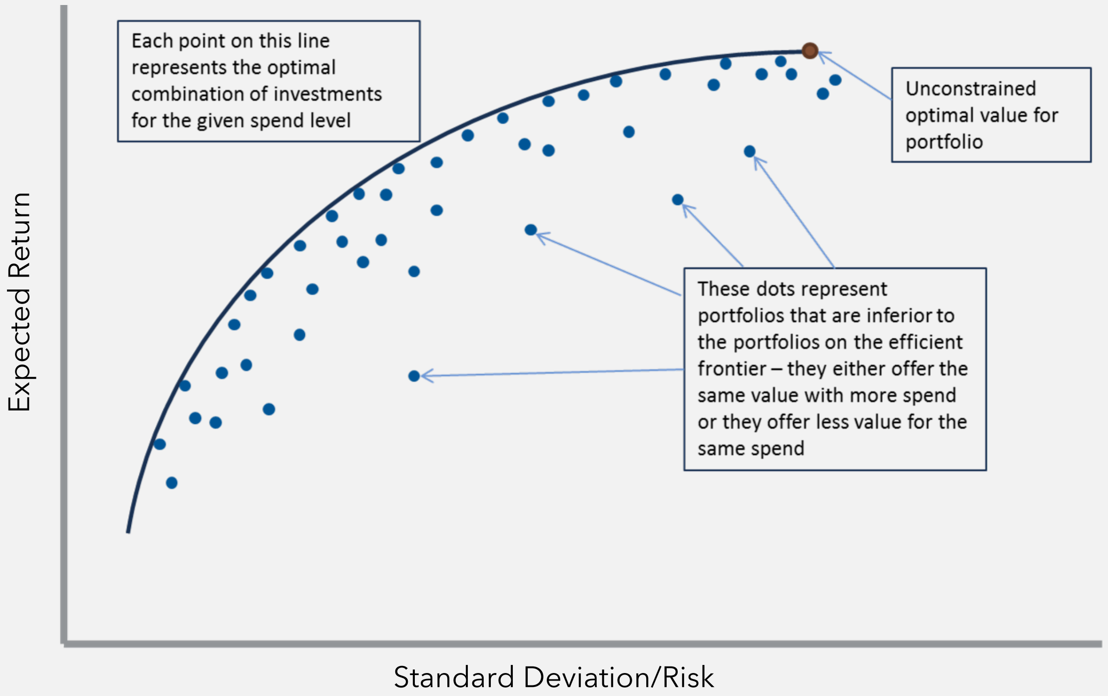
MPT is a mathematical framework for constructing the ideal portfolio that maximizes the expected return and simultaneously reduces the volatility(risk) of the portfolio.

MODERN PORTFOLIO THEORY

The theory assumes that investors are risk-averse.

And employs the core idea of diversification.

Portfolio Frontier/Efficient Frontier

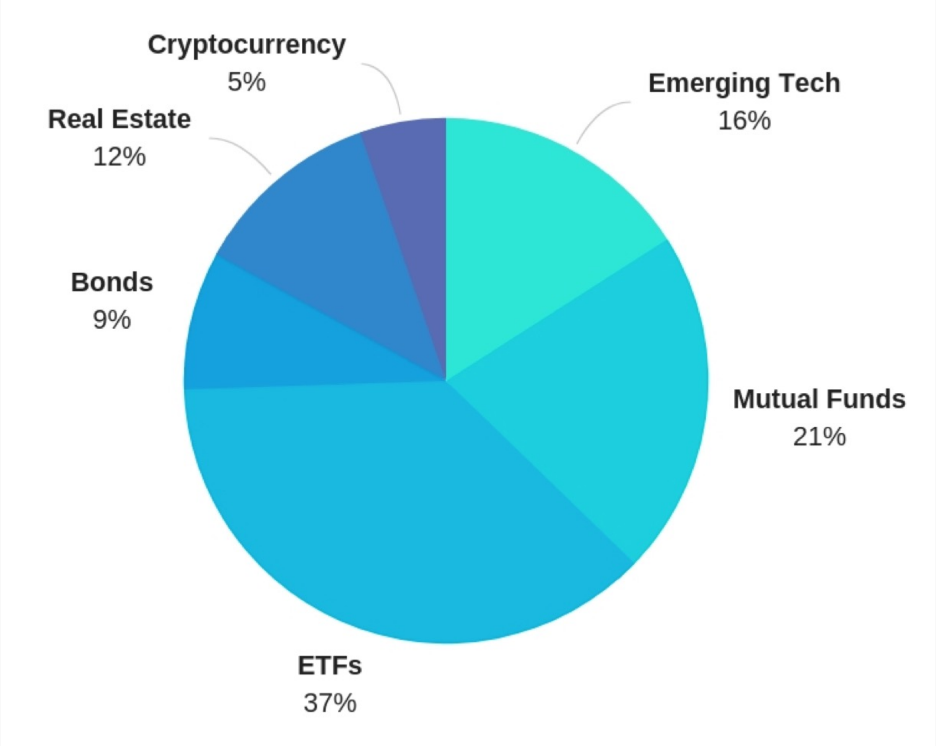


Portfolio Selection

Goals:
- Maximize returns
- Minimize risk
- Stay within budget

Inputs:
- Historical price data
- Budget
- Risk tolerance

Output:
- A portfolio representing a list of investments and the expected return



A good portfolio is more than a long list of good stocks and bonds. It is a balanced whole, providing the investor with protections and opportunities with respect to a wide range of contingencies. – Harry Markowitz

MATHEMATICAL MODEL

The classical mean-variance optimization model can be formulated as:

$$min_w \frac{1}{2} w^T \Sigma\, w$$

Subject to a set of constraints.

$$\sum_{i=1}^{n} r_i w_i \geq r$$

$$\sum_{i=1}^{n} w_i = 1$$

$$0 \leq w_i \leq 1, \qquad i = 1, \dots, n$$

Since the objective function is quadratic, and the constraints are linear, the resulting optimization problem is a quadratic problem.

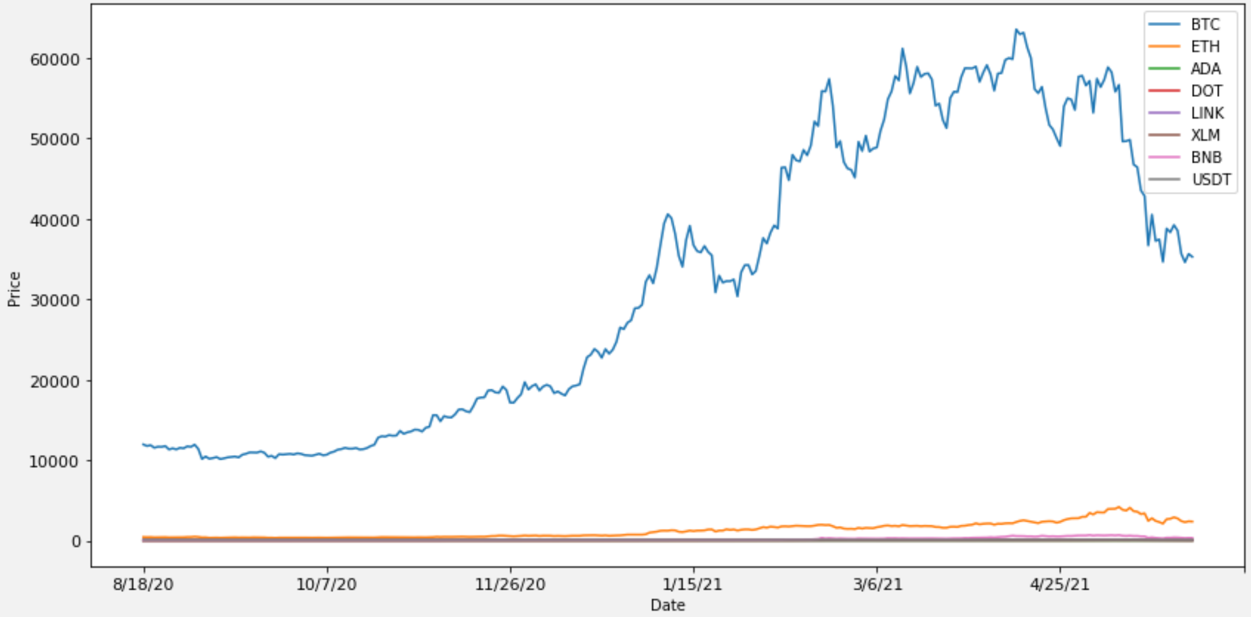
Solve this problem as a convex problem and thus use Lagrangian to find the weights.

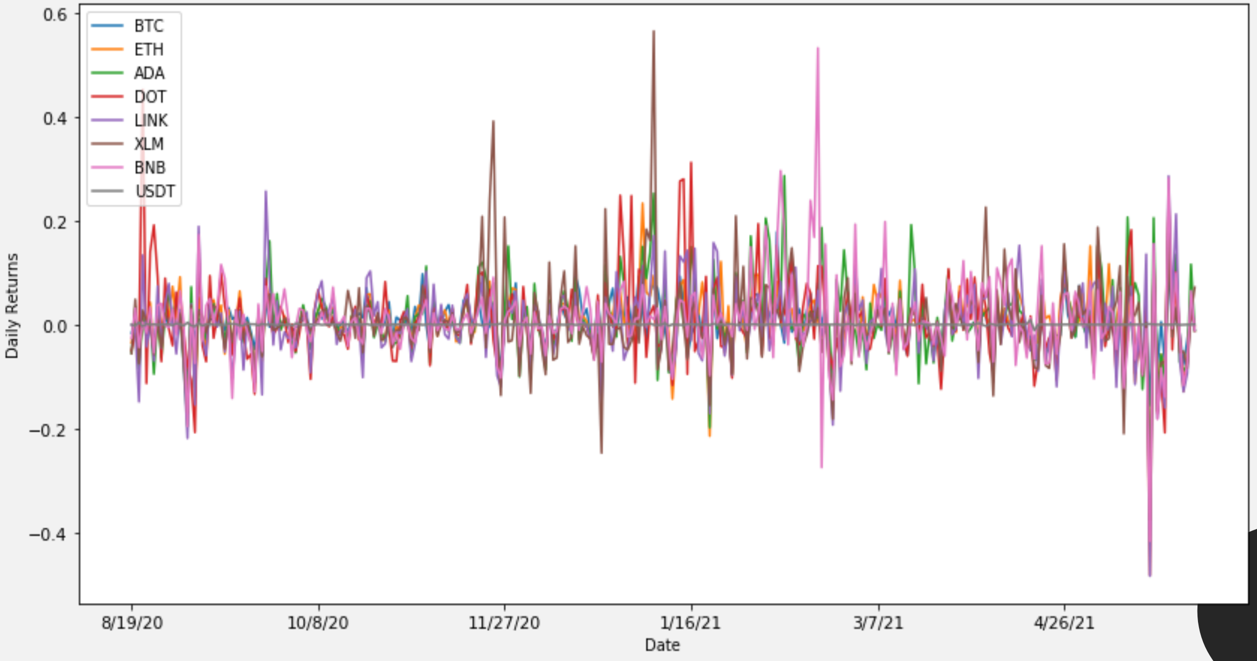
METHODOLOGY

By choosing cryptocurrencies from top 20 list.

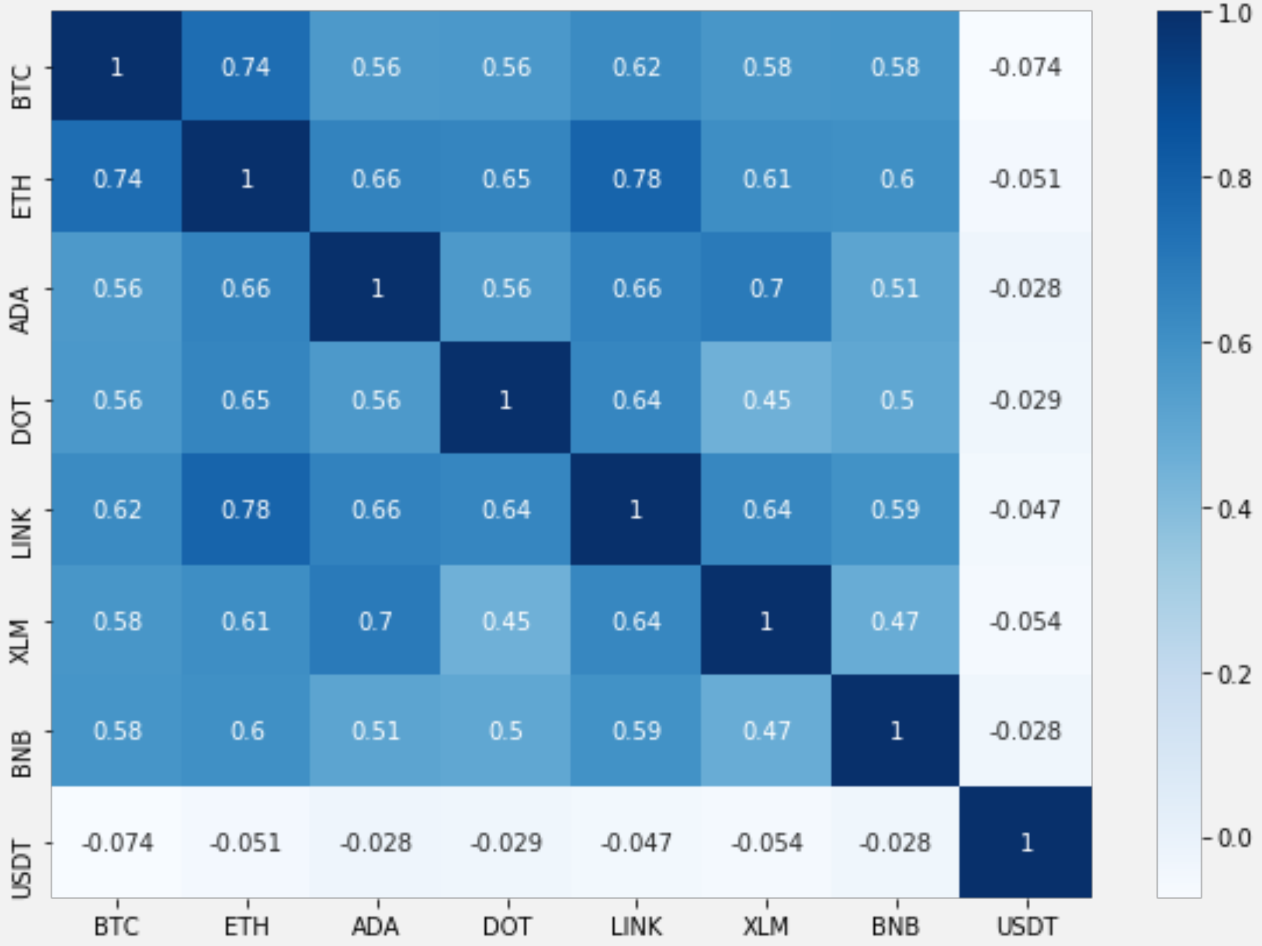
Assets: Bitcoin (*BTC*), Ethereum (*ETH*), Cardona (*ADA*), Polkadot (*DOT*), Chainlink (*LINK*), Stellar (*XLM*), Binance Coin (*BNB*) and Tether (*USDT*).

Data: Historical closing price data. Computed the expected returns, standard deviation and correlation.





Rate of Return Correlation and Covariance Matrix



	BTC	ETH	ADA	DOT	LINK	XLM	BNB	USDT
BTC	0.001679	0.001797	0.001692	0.001952	0.002032	0.001955	0.001908	-0.000004
ETH	0.001797	0.003469	0.002842	0.003220	0.003675	0.002984	0.002822	-0.000004
ADA	0.001692	0.002842	0.005407	0.003488	0.003874	0.004230	0.002990	-0.000003
DOT	0.001952	0.003220	0.003488	0.007119	0.004324	0.003168	0.003350	-0.000003
LINK	0.002032	0.003675	0.003874	0.004324	0.006321	0.004200	0.003758	-0.000005
XLM	0.001955	0.002984	0.004230	0.003168	0.004200	0.006830	0.003112	-0.000006
BNB	0.001908	0.002822	0.002990	0.003350	0.003758	0.003112	0.006346	-0.000003
USDT	-0.000004	-0.000004	-0.000003	-0.000003	-0.000005	-0.000006	-0.000003	0.000002

IMPLEMENTATION

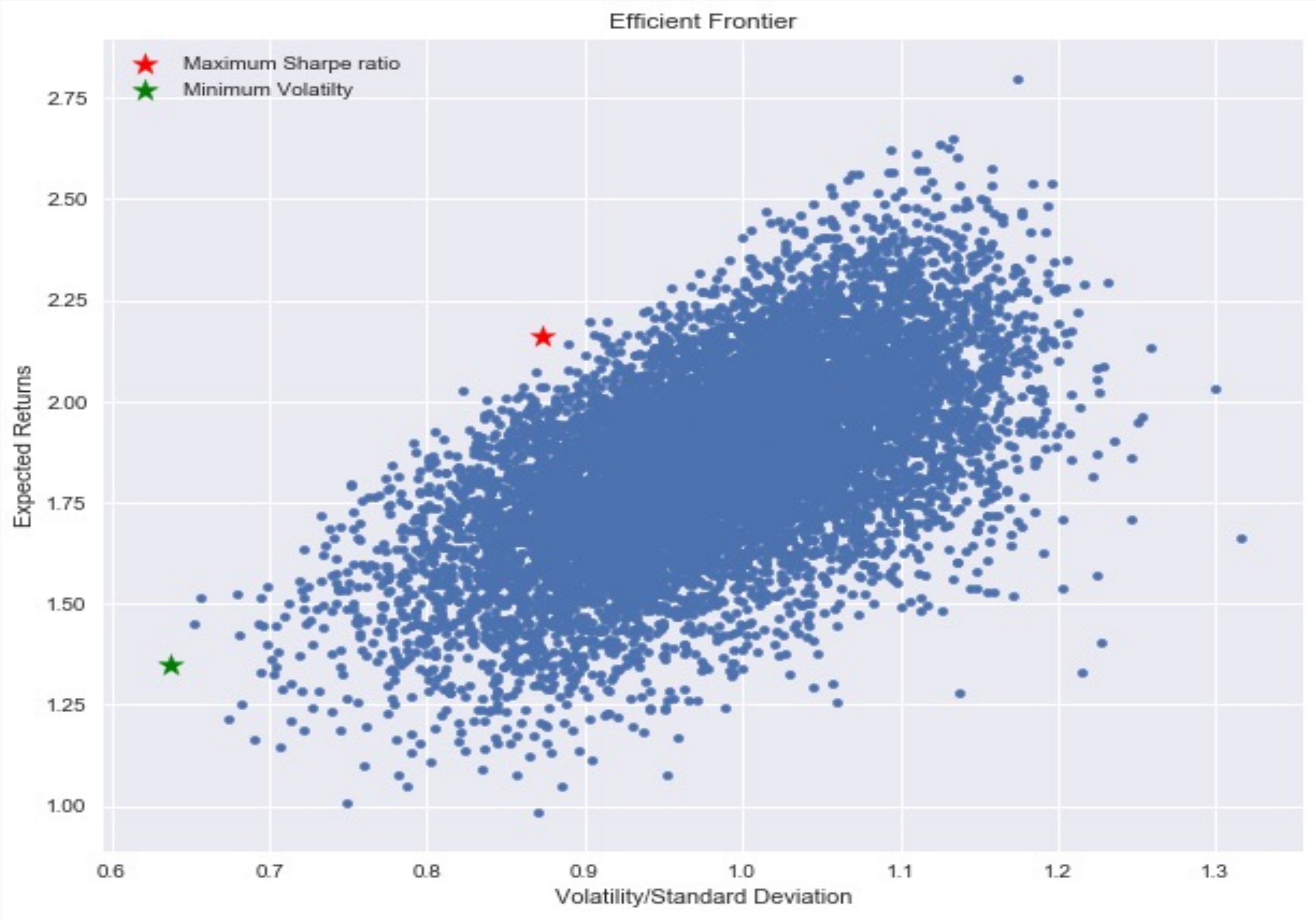
By coding a simulation in python, historical data is used to simulate the modeling of 10,000 different portfolios by random generation of asset weights.

Optimal Portfolio: Portfolio with the maximum Sharpe Ratio.

Minimum Volatility Portfolio: Portfolio with minimum standard deviation.

	Returns	Volatility	Sharpe Ratio	BTC weight	ETH weight	ADA weight	DOT weight	LINK weight	XLM weight	BNB weight	USDT weight
0	1.932547	1.021557	1.891571	0.245803	0.076195	0.197609	0.139965	0.166302	0.056053	0.082264	0.035809
1	1.942314	0.967662	2.007016	0.242714	0.040284	0.009369	0.235118	0.071736	0.071928	0.225111	0.103740
2	1.932486	1.025233	1.884730	0.010170	0.190177	0.139464	0.078747	0.183944	0.090136	0.181719	0.125642
3	1.489055	0.924090	1.611158	0.141631	0.040543	0.182929	0.143331	0.211727	0.082819	0.005997	0.191023
4	1.565053	0.955179	1.638281	0.006648	0.007378	0.274672	0.038381	0.255900	0.094678	0.081887	0.240456

Efficient Frontier



RESULTS

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Maximum Sharpe Ratio Portfolio Allocation

Annualised Return: 2.160293306670845
Annualised Volatility: 0.8725066035511487


Allocation:

 BTC weight     0.118305
ETH weight     0.023159
ADA weight     0.286227
DOT weight     0.049655
LINK weight    0.013580
XLM weight     0.011716
BNB weight     0.270454
USDT weight    0.226902
Name: 8516, dtype: float64
----------------------------------------------------------------------
Minimum Volatility Portfolio Allocation

Annualised Return: 1.3468223949020957
Annualised Volatility: 0.6370392312435751


Allocation:

 BTC weight     0.101083
ETH weight     0.016179
ADA weight     0.009373
DOT weight     0.186172
LINK weight    0.007681
XLM weight     0.059611
BNB weight     0.175495
USDT weight    0.444405
Name: 9733, dtype: float64
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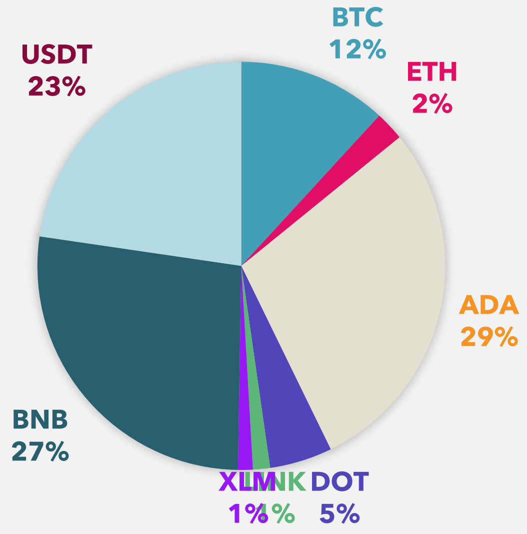
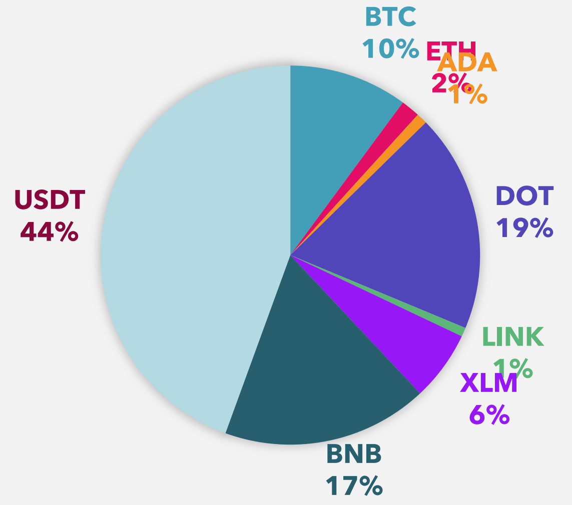
OPTIMAL PORTFOLIO

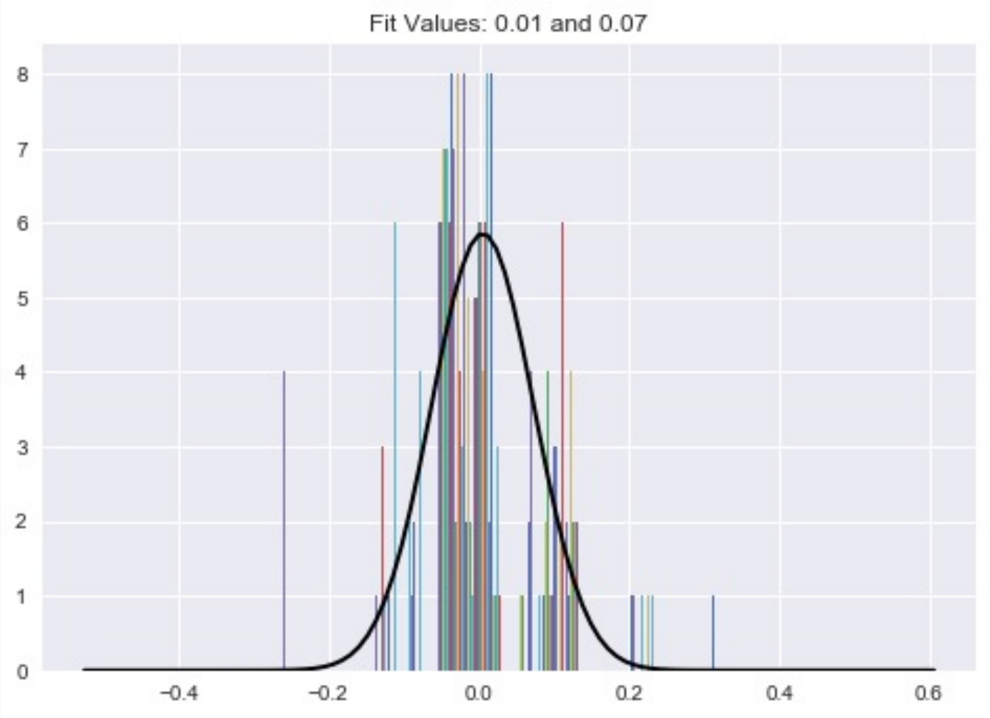


MINIMUM RISK PORTFOLIO



FUTURE WORK – VOLATILITY MODELS

Returns are assumed to be normally distributed.
However, some return distributions have fat tails. Standard deviation may not be a perfect measure of risk/volatility.
Therefore, we could explore other volatility models to represent risk.



FUTURE WORK - QUBO

Writing optimization problem as QUBO (Quadratic Unconstrained Binary Optimization) and solving with quantum annealing optimizer.

Thank You!