COG356: Generative Processes and

Logical Forms +

Ace DeSiena - Apr 2015

1. Write a paragraph which says something significant about Jacques Herbrand and his contributions to symbolic logic.

Jacques Herbrand was a French logician who died at a very young age after making great contributions to the field of logic. One of his legacies is Herbrand's theorem, a proof that any first-order logical expression is provable if and only if it can be derived propositionally. He also determined the consistency of a weaker form of mathematics than the form used by Godel in his first incompleteness theorem.

2. Write a paragraph which says something significant about Alfred Horn and his contributions to symbolic logic.

Alfred Horn was an American mathematician who described the horn clauses which are used in logic programming.

3. Write a paragraph which says something significant about John Alan Robinson and his contributions to symbolic logic.

John Alan Robinson is a mathematician and Professor Emeritus at SU. He developed the resolution principle and unification algorithm which are used in logic programming.

4. define disjunctive normal form.

Disjunctive normal form is a series of disjoined expressions which are all either literals or conjunctions of literals.

## 5. Transform the following into disjunctive normal form:

(a) 
$$(\sim P \land Q) \to R$$
  
 $\Rightarrow \sim (\sim P \land Q) \lor R$ 

(b) 
$$\sim (P \lor \sim Q) \land (S \to T)$$
  
 $\Rightarrow \sim (P \lor \sim Q) \land (\sim S \lor T)$   
 $\Rightarrow \sim \sim (\sim P \land \sim \sim Q) \land \sim (\sim \sim S \land \sim T)$   
 $\Rightarrow \sim P \land Q \land \sim S \land \sim T$ 

(c) 
$$(P \to Q) \to R$$
  

$$\Rightarrow (\sim P \lor Q) \to R$$

$$\Rightarrow \sim (\sim P \lor Q) \lor R$$

$$\Rightarrow \sim \sim (\sim \sim P \land \sim Q) \lor R$$

$$\Rightarrow (P \land \sim Q) \lor R$$

## 6. Define conjunctive normal form.

Conjunctive normal form is a series of conjoined expressions which are all either literals or disjunctions of literals.

## 7. Transofrm the following into conjunctive normal form:

(a) 
$$P \lor (\sim P \land Q \land R)$$
  
 $\Rightarrow (P \lor \sim P) \land (P \lor Q) \land (P \lor R)$   
 $\Rightarrow (P \lor Q) \land (P \lor R)$ 

(b) 
$$\sim (P \to Q)$$
  
 $\Rightarrow \sim (\sim P \lor Q)$   
 $\Rightarrow P \land \sim Q$ 

(c) 
$$(P \to Q) \to R$$
  
 $\Rightarrow \sim (P \to Q) \lor R$   
 $\Rightarrow \sim (\sim P \lor Q) \lor R$   
 $\Rightarrow \sim \sim (\sim \sim P \land \sim Q) \lor R$   
 $\Rightarrow (P \land \sim Q) \lor R$   
 $\Rightarrow (P \lor R) \land (\sim Q \lor R)$ 

- 8. State the resolution principle.
- 9. Define what is meant by the resolution principle.
- 10. Show by means of resolution that the formula U is a logical consequence of these three formulae:  $(P \to S), (S \to U), \text{ and } P.$
- 11. Show thy means of the inconsistency truth table approach that the formula U is a logical consequent of the three formulae  $(P \to S), (S \to U),$  and P.
- 12. Define the horn clause.
- 13. Can the formula  $(P \land Q \land R) \to S$  be converted to a horn clause?