CSC356: Abstract machinery and Generative Processes

GEB Reading Assignment – tq- C- P- A- B- @ | - Problem Set

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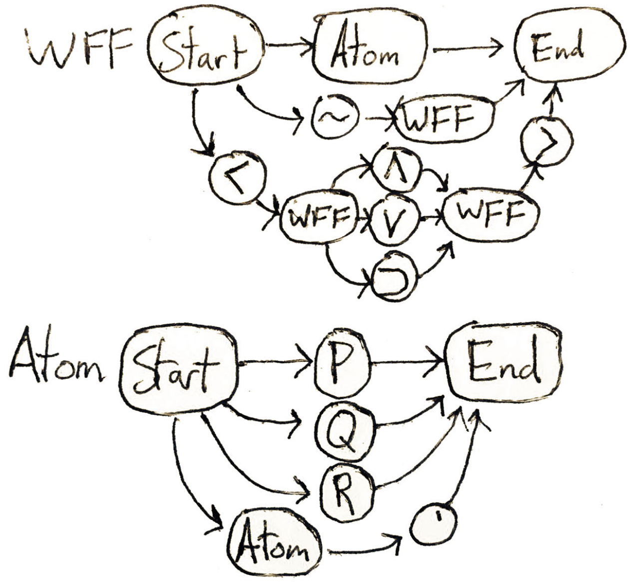
1. **Draw a set of recursive transition networks which define well-formed formulas in the propositional calculus.**



2. **Thinking of the propositional calculus in the terms that Hofstadter presents it, that is, as the formal system he constructs in the chapter:**

   (a) How many axioms in the formal system?

   There are infinitely many different axioms in the form of atoms with varying numbers of prime marks.

   (b) How many rules in the formal system

   There are five rules in this system.

   (c) What are the names he gives to these rules?

   He calls these formation rules.

   (d) What is the one rule you absolutely must use in order to derive a theorem in this system?

   All atoms are well formed.

3. **Write down each of the rules of the system,just as Hofstadter does on page 187.**

Joining Rule: If x and y are theorems, then $\langle x \wedge y \rangle$ is a theorem.

Separation Rule: If $\langle x \wedge y \rangle$ is a theorem, then both x and y are theorems.

Double-Tilde Rule: The string '$\sim\sim$' can be deleted from any theorem. It can also be inserted into any theorem, provided that the resulting string is itself well-formed.

Fantasy Rule: If y can be derived when x is assumed to be true, then $\langle x \supset y \rangle$ is a theorem.

Carry-Over Rule: Inside a fantasy, any theorem from the "reality" one level higher can be brought in and used.

Rule of Detachment: If x and $\langle x \supset y \rangle$ are theorems, then y is a theorem.

Contrapositive Rule: $\langle x \supset y \rangle$ and $\langle \sim x \supset \sim y \rangle$ are interchangeable.

DeMorgan's Rule: $\langle \sim x \wedge \sim y \rangle$ and $\sim \langle x \vee y \rangle$ are interchangeable.

Switcheroo Rule: $\langle x \vee y \rangle$ and $\langle \sim x \supset y \rangle$ are interchangeable.

4. **Derive:** $\langle\langle\langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle\rangle\rangle$

(1)	[	push
(2)	$\langle\langle P \wedge Q \rangle \wedge R \rangle$	premise
(3)	R	separation
(4)	⟨ P ∧ Q ⟩	separation
(5)	P	separation
(6)	Q	separation
(7)	$\langle Q \wedge R \rangle$	joining
(8)	$\langle P \wedge \langle Q \wedge R \rangle\rangle$	joining
(9)	]	pop
(10)	$\langle\langle\langle P \wedge Q \rangle \wedge R \rangle \supset \langle P \wedge \langle Q \wedge R \rangle\rangle\rangle$	fantasy rule

5. **Derive:** $\langle \langle \mathbf{P} \vee \mathbf{Q} \rangle \supset \langle \mathbf{Q} \vee \mathbf{P} \rangle \rangle$

6. **Derive a theorem in the propositional calculus that you think is a little bit interesting, one that neither I asked you to derive nor Hofstadter derived in his book.**

7. **As Hofstadter mentions mid-way through the chapter, there is a decision procedure for WFFs in the propositional calculus, the method of truth tables. Learn what this method entails, if you are not already clear on that, and write a description of the method that is clear and complete enough that one could easily apply it by referencing your description. That is, describe the process featuring truth tables by which one could determine whether or not a WFF is a theorem in the propositional calculus.**

   A truth table is a table with as many rows as there are different combinations of truth values for atoms, plus another row for column labels. The leftmost columns correspond to the atoms of the equation. Other columns correspond to various relevant logical combinations of these atoms. A truth table that is constructed to evaluate a compound logical form might have a column for not only that form, but for the forms from which it is made. The cells of the table take either a T or an F that represents the truth of that cell's column's logical form, given that rows truth values for atoms.

8. **Using the truth table based decision procedure, show that the heads will be cut off! Perhaps I should say a bit more. I'm referring to the section on Ganto's Ax. And I'm asking you to show by means of a truth table that the following WFF is a theorem:** $\langle \langle \langle \mathbf{P} \supset \mathbf{Q} \rangle \wedge \langle \tilde{}\mathbf{P} \supset \mathbf{Q} \rangle \rangle \supset \mathbf{Q} \rangle$

P	Q	$\langle P \supset Q \rangle$	$\langle \tilde{}P \supset Q \rangle$	$\langle \langle P \supset Q \rangle \wedge \langle \tilde{}P \supset Q \rangle \rangle$	$\langle \langle \langle P \supset Q \rangle \wedge \langle \tilde{}P \supset Q \rangle \rangle \supset Q \rangle$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

9. **Choose another interpretation for P and Q in Ganto's statement- one**

that doesn't involve heads or axes. Write down the words for your proposition P. Write down the words for your proposition Q. Write down a zentence corresponding to Ganto 's statement (what he says to the praying monks) under your interpretation.

$P \Rightarrow$ the car swerves.

$Q \Rightarrow$ the car will hit a pothole.

If the car swerves the car will hit a pothole; and if the car does not swerve, it will also hit a pothole.

10. **Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Shortcuts and Derived Rules.**

    Using shortcuts reduces the portability of ideas. Without a formalism, an idea cannot be communicated as effectively.

11. **Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Formalizing Higher Levels.**

    Separation of mechanical and intuitive processes are important in order to avoid mistakes in symbol manipulation.

12. **Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Reflections on the Strengths and Weaknesses of the System.**

    Propositional calculus is a generalize able system which can be applying to some extent to real world situations and can be built upon to create other systems.

13. **Write down in a meaningful manner, in no more that a few sentences, what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled Proofs vs Derivations.**

    A proof is the way by which someone might normally reach a conclusion, while a derivation is a formal representation of reasoning.

14. **Write down in a meaningful manner, in no more that a few sentences,**

what you think is the most salient idea that Hofstadter has embedded in the text contained within the section titled The Handling of Contradictions.

The propositional calculus is marred by it's reaction to contradictions. While some have tried correcting the calculus, it's value as a basis for other systems does not require such a correction.

15. **In one paragraph, write your reaction to this chapter.**

    This chapter shows some interesting characteristics of the propositional calculus in a very informal way. This informality makes the subject simpler to approach, but I think it also makes it a bit difficult to comprehend. All in all I liked the chapter quite a bit. I especially like the fantasy rule for building implications.