Semi-Classical Limit in Computational Space Theory

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Abstract

This paper explores the semi-classical limit of Computational Space Theory (CST), where space is treated as an active computational medium. We derive the transition from the quantum computational regime to classical General Relativity (GR) by considering limits where either Planck's constant ($\hbar$) or Newton's gravitational constant (G) approaches zero. We discuss implications for quantum gravity and propose a novel computational framework for emergent spacetime.

1 Introduction

Computational Space Theory (CST) postulates that spacetime is a discrete computational network, where physical laws emerge from information processing. In this work, we investigate the semi-classical transition, analyzing how CST reproduces classical gravity when quantum effects vanish.

2 Mathematical Framework

In CST, the total energy is defined as:

$$E = \hbar R(x) S + \frac{1}{2} m \left(\frac{dS}{dt} \right)^2 - G \int \rho(x) R(x) \, dV, \tag{1}$$

where $R(x)$ represents the local computational rate of space, S is an information state function, and $\rho(x)$ is the local energy/mass density.

3 Semi-Classical Limits

3.1 Classical Limit ($\hbar \to 0$)

Setting $\hbar \to 0$, the quantum computational term vanishes, yielding:

$$E \approx \frac{1}{2} m \left(\frac{dS}{dt} \right)^2 - G \int \rho(x) R(x) \, dV. \tag{2}$$

This recovers classical mechanics and General Relativity, where information-processing effects become negligible.

3.2 Quantum Limit ($G \to 0$)

Setting $G \to 0$, the gravitational term disappears, leading to:

$$E \approx \hbar R(x)S + \frac{1}{2}m\left(\frac{dS}{dt}\right)^2. \tag{3}$$

This corresponds to pure quantum mechanics, where spacetime remains flat and gravity does not influence quantum states.

4 Implications for Quantum Gravity

For finite $\hbar$ and G, CST naturally describes quantum gravity as an emergent phenomenon. Gravitational time dilation results from computational resource constraints, and quantum uncertainty arises from finite information-processing capacity.

Furthermore, CST provides a natural link to black hole thermodynamics. The entropy of a black hole, given by the Bekenstein-Hawking formula:

$$S_{BH} = \frac{k_B A}{4\ell_P^2}, \tag{4}$$

suggests that the number of computational degrees of freedom in CST scales with the horizon area, reinforcing the idea that information and computation are fundamental to gravitational dynamics.

5 Conclusion

This work demonstrates that CST recovers both classical gravity and quantum mechanics as limiting cases. These results suggest a computational approach to quantum gravity, bridging the gap between quantum field theory and General Relativity. The connection with Bekenstein-Hawking entropy further supports the role of information processing in the structure of spacetime.