A Network Model for Dynamic Textual Communications with Application to Government Email Corpora

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Abstract

In this paper, we introduce the interaction-partitioned topic model (IPTM)—a probabilistic model of who communicates with whom about what, and when. Broadly speaking, the IPTM partitions time-stamped textual communications, such as emails, according to both the network dynamics that they reflect and their content. To define the IPTM, we integrate a dynamic version of the exponential random graph model—a generative model for ties that tend toward structural features such as triangles—and latent Dirichlet allocation—a generative model for topic-based content. The IPTM assigns each topic to an "interaction pattern"—a generative process for ties that is governed by a set of dynamic network features. Each communication is then modeled as a mixture of topics and their corresponding interaction patterns. We use the IPTM to analyze emails sent between department managers in two county governments in North Carolina; one of these email corpora covers the Outer Banks during the time period surrounding Hurricane Sandy. Via this application, we demonstrate that the IPTM is effective at predicting and explaining continuous-time textual communications.

1 Introduction

In recent decades, real-time digitized textual communication has developed into a ubiquitous form of social and professional interaction (see, e.g., Kanungo and Jain, 2008; Szóstek, 2011; Burgess et al., 2004; Pew, 2016). From the perspective of the computational social scientist, this has lead to a growing need for methods of modeling interactions that manifest as text exchanged in continuous time (e.g., e-mail messages). A number of models that build upon topic modeling through Latent Dirichlet Allocation (Blei et al., 2003) to incorporate link data as well as textual content have been developed recently (McCallum et al., 2005; Lim et al., 2013; Krafft et al., 2012). These models are innovative in their extensions that incorporate network tie information. However, none of the models that are currently available in the literature integrate the rich random-graph structure offered by state of the art models for network structure—in particular, the exponential random graph model (ERGM) (Robins et al., 2007; Chatterjee et al., 2013; Hunter et al., 2008). The ERGM is the canonical model for network structure, as it is flexible enough to specify a generative model that accounts for nearly any pattern of tie formation (e.g., tie reciprocation, clustering, popularity effects) (Desmarais and Cranmer, 2017). We build upon recent extensions of ERGM that model time-stamped ties (Perry and Wolfe, 2013; Butts, 2008), and develop the interaction-partitioned topic model (IPTM) to simultaneously model the network structural patterns that govern tie formation, and the content in the communications.

ERGM, and models based on ERGM, provide a framework for explaining or predicting ties between nodes using the network sub-structures in which the two nodes are embedded (e.g., an ERGM specification may predict ties between two nodes that have many shared partners). ERGM-style models have been used for many applications in which the ties between nodes are annotated with text. The text, despite providing rich information regarding the strength, scope, and character of the ties, has been largely excluded from these analyses, due to the inability of ERGM-style models to incorporate textual attributes of ties. These application domains include, among other applicaitons, the study of legislative networks in which networks reflect legislators' co-support of bills, but exclude bill text (Bratton and Rouse, 2011; Alemán and Calvo, 2013); the study of alliance networks in which networks reflect countries' co-signing of treaties, but exclude treaty text (Camber Warren, 2010; Cranmer et al., 2012b,a; Kinne, 2016); the study of scientific co-authorship networks that exclude the text of the co-authored papers (Kronegger et al., 2011; Liang, 2015; Fahmy and Young, 2016); and the study of text-based interaction on social media (e.g., users tied via 'mentions' on twitter) (Yoon and Park, 2014; Peng et al., 2016; Lai et al., 2017).

In defining and testing the IPTM we embed three core conceptual properties, in addition to modeling both text and network structure. First, we link the content component of the model, and network component of the model such that knowing who is communicating with whom at what time (i.e., the network component) provides information about the content of communication, and vice versa. Second, we fully specify the network dynamic component of the model such that, given the content of the communication and the history of tie formation, we can draw an exact, continuous-time prediction of when, by whom, and to whom the communication will be sent. Third, we formulate the network dynamic component of the model such that the model can represent, and be used to test hypotheses regarding, canonical processes relevant to network theory such as preferential attachment—the tendency for actors to prefer interacting with actors who have been popular in the past (Barabási and Albert, 1999; Vázquez, 2003; Jeong et al., 2003), reciprocity (Hammer, 1985; Rao and Bandyopadhyay, 1987), and transitivity—the tendency for the friends of friends to become friends (Louch, 2000; Burda et al., 2004). In what follows we (1) present the generative process for the IPTM, describing how it meets our theoretical criteria, (2) derive the sampling equations for Bayesian inference with the IPTM, and (3) illustrate the IPTM through application to email corpora of internal communications by county officials in North Carolina county governments.

2 IPTM: Model Definition and Derivation

To define and derive the IPTM, we begin by describing a probabilistic process by which documents are generated, where documents include a sender, recipients, contents, and timing. We provide a fully parametric definition of each component of the generative process, which enables the model to be used to simulate distributions of who communicates with whom about what, and when. We take a Bayesian approach to inference for the parameters of the IPTM. In the next section, we derive equations for sampling from the posterior distributions of the IPTM parameters conditional on data generated by the generative process that we define in the current section.

The data generated under the IPTM consists of D unique documents. A single email, indexed by $d \in \{1, ..., D\}$, is represented by the four components $(i^{(d)}, J^{(d)}, t^{(d)}, \boldsymbol{w}^{(d)})$. The first two are the sender and recipients of the email: an integer $i^{(d)} \in \{1, ..., A\}$ indicates the identity of the sender out of A actors (or nodes) and an integer vector $J^{(d)} = \{j_r^{(d)}\}_{r=1}^{|J^{(d)}|}$, which indicates the identity of the receiver (or receivers) out of $A-1$ actors, where $|J^{(d)}| \in \{1, ..., A-1\}$ denotes the total number of receivers. Next, $t^{(d)}$ is the timestamp of the email d. Lastly, $\boldsymbol{w}^{(d)} = \{w_n^{(d)}\}_{n=1}^{N^{(d)}}$ is a set of tokens, or word type instances, that comprise the text of the email, where $N^{(d)}$ denotes the total number of words in a document.

In this section, we illustrate how the words $\boldsymbol{w}^{(d)}$ are generated according to latent Dirichlet allocation (Blei et al., 2003), and then how the other components, $(i^{(d)}, J^{(d)}, t^{(d)})$, are generated conditional on the document content. For simplicity, we assume that documents are ordered by time such that $t^{(d)} < t^{(d+1)}$ for all $d = 1, ..., D$.

2.1 Content Generating Process

The content generating process follows from the generative process of Latent Dirichlet Allocation Blei et al. (2003). First we generate the global (corpus-wide) variables. Each topic k is associated with a cluster, or interaction pattern, assignment c_k, where c_k can take one of $c = \{1, 2, ..., C\}$ values. There are two main sets of global variables—those that describe the content via topics and those that describe how people interact (interaction patterns). These variables are linked by a third set of variables that associate each topic with the pattern that best describes how people interact when talking about that topic.

There are K topics. Each topic k is a discrete distribution over V word types.

1. $\boldsymbol{\phi}^{(k)} \sim \text{Dirichlet}(\beta, \mathbf{u})$ **[See Algorithm 1]**
   - A topic k is characterized by a discrete distribution over V word types with probability vector $\phi^{(k)}$. We specify a symmetric Dirichlet prior $\mathbf{u}$ with the concentration parameter β for the probability vector $\phi^{(k)}$.

There are C interaction patterns. Each interaction pattern consists of a vector of coefficients $\boldsymbol{b}^{(c)}$ in $\boldsymbol{R}^P$ and a vector of P-dimensional dynamic network statistics for directed edge (i, j) at time t $\boldsymbol{x}_t^{(c)}(i, j)$. The inner product of $\boldsymbol{b}^{(c)}$ and $\boldsymbol{x}_t^{(c)}(i, j)$ is used to generate both the recipient vector for a document and the timing of the document.

2. $\boldsymbol{b}^{(c)} \sim \text{Multivariate Normal}(\mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}})$ [**See Algorithm 2**]:
   - The vector of coefficients depends on the interaction pattern c. This means that there is variation across interaction patterns in the degree to which document timing and recipients depend upon the dynamic network statistics. The prior for $\boldsymbol{b}^{(c)}$ is a P-variate multivariate Normal with mean vector $\mu_{\boldsymbol{b}}$ and covariance matrix $\Sigma_{\boldsymbol{b}}$.

The topics and interaction patterns are tied together via a set of K categorical variables.

3. $c_k \sim \text{Uniform}(1, C)$ [**See Algorithm 3**]:
   - Each topic is associated with a single interaction pattern, and topics under same interaction pattern share the network properties via $\boldsymbol{b}^{(c)}$.

We have now defined all of the variables that make up the generative process of the IPTM. We assume the following generative process for each document d in a corpus D [**See Algorithm 4**]:

4-1. Choose the number of words $\bar{N}^{(d)} = \max(1, N^{(d)})$, where $N^{(d)}$ is known.

4-2. Choose document-topic distribution $\boldsymbol{\theta}^{(d)} \sim \text{Dir}(\alpha, \boldsymbol{m})$

4-3. For $n = 1$ to $\bar{N}^{(d)}$:

   (a) Choose a topic $z_n^{(d)} \sim \text{Multinomial}(\boldsymbol{\theta}^{(d)})$

   (b) if $N^{(d)} > 0$, choose a word $w_n^{(d)} \sim \text{Multinomial}(\phi^{(z_n^{(d)})})$

2.2 Stochastic Intensity

In this section, we illustrate how a set of dynamic network features and topic-interaction assignments jointly identify the stochastic intensity of a document, which plays a key role in the tie generating process in Section 2.4. Assume that each document $d \in \{1, ..., D\}$ is associated with an $A \times A$ stochastic intensity matrix $\boldsymbol{\lambda}^{(d)}(t)$, where the $(i, j)^{th}$ element $\lambda_{ij}^{(d)}(t)$ can be interpreted as the likelihood of document d being sent from node i to node j at time t.

First, content of a document is reflected to the stochastic intensity via the distribution of interaction patterns, $\{p_c^{(d)}\}_{c=1}^{C}$. To calculate the distribution of interaction patterns within a document, we estimate the proportion of words in document d which are assigned the topics corresponding to the interaction pattern c from Section 2.1:

$$p_c^{(d)} = \frac{\sum\limits_{k:c_k=c} N^{(k|d)}}{N^{(d)}}, \tag{1}$$

where $N^{(k|d)}$ is the number of times topic k appears in the document d and $N^{(d)}$ is the total number of words, as defined earlier. By definition, $\sum\limits_{c=1}^{C} p_c^{(d)} = 1$.

Now, we define the $(i, j)^{th}$ element of the stochastic intensity matrix $\boldsymbol{\lambda}^{(d)}(t)$ in the form of continuous-time ERGM:

$$\lambda_{ij}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T} \boldsymbol{x}_t^{(c)}(i, j)\Big\}, \tag{2}$$

where $p_c^{(d)}$ is as defined in Equation (1), $\lambda_0^{(c)}$ is the baseline intensity for the interaction pattern c, $\boldsymbol{b}^{(c)}$ is an unknown vector of coefficients in $\boldsymbol{R}^p$ corresponding to the interaction pattern c, and $\boldsymbol{x}_t^{(c)}(i, j)$ is a vector of the p-dimensional dynamic network statistics for directed edge (i, j) at time t corresponding to the interaction pattern c. Detailed specifications of the dynamic network statsitics are demonstrated in Section 2.3.

2.3 Dynamic Network Statistics

We develop a suite of eight different effects to be used as the components of $\boldsymbol{x}_t^{(c)}(i, j)$, (intercept, outdegree, indegree, send, receive, 2-send, 2-receive, sibling, and cosibling), which are incorporated as in Equation (2). These statistics capture common network properties such as popularity, centrality, reciprocity, and transitivity. Each network statistic is calculated for each interaction pattern $c = 1, ..., C$, which means that each interaction pattern can be understood in terms of the ways that network dynamics shape tie formation within the interaction pattern. Below are the specifications of the degree, dyadic, and triadic network statistics we use in this paper.

We follow Perry and Wolfe (2013) we define each network feature to have potentially different effects within a number of intervals of recency in the formation of the ties that contribute to the network feature. We partition the interval $[-\infty, t)$ into $L = 4$ sub-intervals with equal length in the log-scale, by setting $\Delta_l = (6 \text{ hours}) \times 4^l$ for $l = 1, ..., L-1$ such that Δ_l takes the values 24 hours (=1 day), 96 hours (=4 days), 384 hours (=16 days):

$$\begin{aligned}
[-\infty, t) &= [-\infty, t - \Delta_3) \cup [t - \Delta_3, t - \Delta_2) \cup [t - \Delta_2, t - \Delta_1) \cup [t - \Delta_1, t - \Delta_0) \\
&= [-\infty, t - 384h) \cup [t - 384h, t - 96h) \cup [t - 96h, t - 24h) \cup [t - 24h, t - 0) \\
&= I_t^{(4)} \cup I_t^{(3)} \cup I_t^{(2)} \cup I_t^{(1)},
\end{aligned}$$

where $\Delta_0 = 0$ and $I_t^{(l)}$ is the half-open interval $[t - \Delta_l, t - \Delta_{l-1})$.

In the application of the IPTM below, we do not include the last interval $I_t^{(4)}$, history before 16 days ago, since the time intervals covered by our datasets are only eight and twelve weeks in length. Although the specification of these dynamic network covariates could be reformulated based on the objectives of each study, in this paper, we define the degree and dyadic efffects for each $l = 1, ..., L-1$ and $c = 1, ..., C$ as

1. $\textbf{outdegree}_{t,l}^{(c)}(i) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \rightarrow \forall j\}$

2. $\textbf{indegree}_{t,l}^{(c)}(j) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{\forall i \rightarrow j\}$

3. $\textbf{send}_{t,l}^{(c)}(i, j) = \sum\limits_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{i \rightarrow j\}$

4. $\textbf{receive}_{t,l}^{(c)}(i,j) = \sum_{d:t^{(d)} \in I_t^{(l)}} p_c^{(d)} \cdot I\{j \to i\}$

Next, we define four triadic statistics involving pairs of messages, which are analogous to 2-path statistics commonly used in the network science literature. While Perry and Wolfe (2013) adapted full sets of triadic statistics for each combination of time intervals (e.g. $3 \times 3 = 9$), we maintain 3 intervals per each statistic, by defining 3×3 time windows and sum the combination-specific statistics based on the interval where the triads are closed. (Refer to Figure 1.) As a result, our interval-adjusted definitions of triadic effects become

5. $\textbf{2-send}_{t,l}^{(c)}(i,j) = \sum_{(l_1=l \text{ or } l_2=l)} \sum_{h \neq i,j} \Big(\sum_{d:t^{(d)} \in I_t^{(l_1)}} p_c^{(d)} \cdot I\{i \to h\} \Big) \cdot \Big(\sum_{d':t^{(d')} \in I_t^{(l_2)}} p_c^{(d')} \cdot I\{h \to j\} \Big)$
6. $\textbf{2-receive}_{t,l}^{(c)}(i,j) = \sum_{(l_1=l \text{ or } l_2=l)} \sum_{h \neq i,j} \Big(\sum_{d:t^{(d)} \in I_t^{(l_1)}} p_c^{(d)} \cdot I\{h \to i\} \Big) \cdot \Big(\sum_{d':t^{(d')} \in I_t^{(l_2)}} p_c^{(d')} \cdot I\{j \to h\} \Big)$
7. $\textbf{sibling}_{t,l}^{(c)}(i,j) = \sum_{(l_1=l \text{ or } l_2=l)} \sum_{h \neq i,j} \Big(\sum_{d:t^{(d)} \in I_t^{(l_1)}} p_c^{(d)} \cdot I\{h \to i\} \Big) \cdot \Big(\sum_{d':t^{(d')} \in I_t^{(l_2)}} p_c^{(d')} \cdot I\{h \to j\} \Big)$
8. $\textbf{cosibling}_{t,l}^{(c)}(i,j) = \sum_{(l_1=l \text{ or } l_2=l)} \sum_{h \neq i,j} \Big(\sum_{d:t^{(d)} \in I_t^{(l_1)}} p_c^{(d)} \cdot I\{i \to h\} \Big) \cdot \Big(\sum_{d':t^{(d')} \in I_t^{(l_2)}} p_c^{(d')} \cdot I\{j \to h\} \Big)$,

where $l_1 \in \{1, ..., 3\}$ and $l_2 \in \{1, ..., 3\}$.

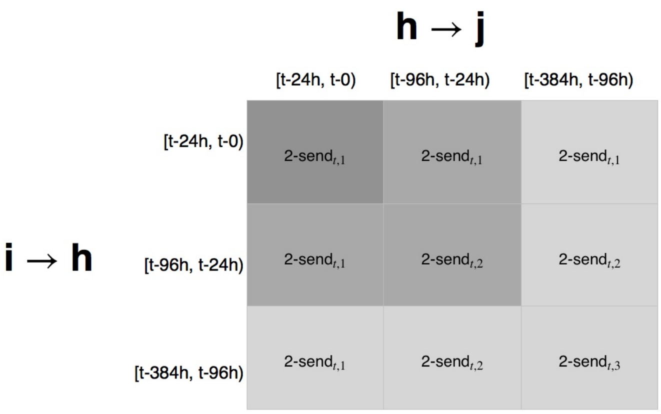

Figure 1: Example of 2-send statistic defined for each interval $l = 1, ..., 3$. Cells with same shades sum up to one statistic, based on when the triads are "closed".

2.4 Tie Generating Process

The tie generating process determines the sender, recipients, and timing $(i^{(d)}, J^{(d)}, t^{(d)})$ of the document. We assume the following tie generating process for each document d in a corpus of D documents:

1. For each sender $i \in \{1, ..., A\}$, we create binary receiver vector of length $A-1$, $J_i^{(d)}$, by applying the non-empty Gibbs measure (Fellows and Handcock, 2017) to every $j \in \mathcal{A}_{\setminus i}$, since we exclude self-loop.

$$\mathrm{P}(J_i^{(d)}) = \frac{1}{Z(\delta, \log(\lambda_i^{(d)}))} \exp\Big\{ \log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\setminus i}} J_{ij}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\setminus i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}, \quad (3)$$

where δ is real-valued intercept that controls the overall recipient size or the length of $J_i^{(d)}$, with its prior distribution specified as Normal$(\mu_\delta, \sigma_\delta^2)$. As defined in Section 2.2, $\lambda_{ij}^{(d)}$ is a positive dyad-specific stochastic inensity included in the model, and we use $\lambda_i^{(d)} = \{\lambda_{ij}^{(d)}\}_{j\in\mathcal{A}_{\setminus i}}$ to denote the vector of dyadic weights in which i is the sender. Note that we omitted the notation (t) from Equation (2) and used $\lambda_{ij}^{(d)}$ instead, since the stochastic intensity $\lambda_{ij}^{(d)}$ is always evaluated at time $t_+^{(d-1)}$, implying that λ_{ij} for d^{th} document is obtained using the history of interactions up to and including the time when the previous document was sent, $t^{(d-1)}$.

To assure that the probabilities sum to unity, we use the normalizing constant $Z(\delta, \log(\lambda_i^{(d)}))$, which is the sum of $P(J_i^{(d)})$ over the entire support, and it can be simplified as:

$$Z(\delta, \log(\lambda_i^{(d)})) = \Big(\prod_{j\in\mathcal{A}_{\setminus i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1. \tag{4}$$

Details on how the normalizing constant ends up with this functional form are shown in AppendixA.

2. For every sender $i \in \mathcal{A}$, generate the time increments given the latent ties from previous step:

$$\Delta T_{iJ_i} \sim \text{Exponential}(\lambda_{iJ_i}^{(d)}), \tag{5}$$

where the mean parameter $\lambda_{iJ_i}^{(d)}$ is computed by taking the average of network effect terms $\boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)$ across the chosen receivers $J_i^{(d)}$:

$$\lambda_{iJ_i}^{(d)}(t) = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \frac{1}{|J_i^{(d)}|}\sum_{j\in J_i} \boldsymbol{b}^{(c)T}\boldsymbol{x}_t^{(c)}(i,j)\Big\}. \tag{6}$$

Note that Equation (6) reduces to the stochastic intensity $\lambda_{ij}^{(d)}$ in Equation (2) in case of single receiver documents (i.e. $|J_i^{(d)}| = 1$), so we can interpret this mean parameter as weighted stochastic intensity across the chosen recievers. When there are multiple chosen receivers (i.e. $|J_i^{(d)}| > 1$), we call it as multicast interactions—those involving a single sender but multiple receivers.

3. Set the observed sender, recipient, and time of the document simultaneously by choosing the sender who generated the minimum time in step 2 and the corresponding receipient and time increment (NOTE: $t^{(0)} = 0$):

$$\begin{aligned} i^{(d)} &= i_{\min(\Delta T_{iJ_i})}, \\ J^{(d)} &= J_{i^{(d)}}, \\ t^{(d)} &= t^{(d-1)} + \min(\Delta T_{iJ_i}). \end{aligned} \tag{7}$$

The intuition behind this choice is that all possible senders $i \in \mathcal{A}$ are competing against each other to send the document to their chosen receivers $\{J_i^{(d)}\}_{i=1}^{A}$, and the one with highest urgency (or highest importance) becomes the observed sender, jointly determining the observed recipient and timestamp of d^{th} document.

2.5 Joint Generative Process

The algorithms we present in this section form the generative process for D documents. This generative process integrates Sections 2.1 through 2.4.

Algorithm 1 Topic Word Distributions

for k=*1 to* K **do**
  draw $\boldsymbol{\phi}^{(k)} \sim \text{Dirichlet}(\beta, \mathbf{u})$
end

Algorithm 2 Interaction Pattern Parameters

for *c=1 to C* **do**
  draw $\boldsymbol{b}^{(c)} \sim$ Multivariate Normal$(\mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}})$
end

Algorithm 3 Topic Interaction Pattern Assginments

for *k=1 to K* **do**
  draw $c_k \sim$ Uniform(1, C)
end

Algorithm 4 Recipient Size Parameter

draw $\delta \sim$ Normal$(\mu_\delta, \sigma_\delta^2)$

Algorithm 5 Document Generating Process

for *d=1 to D* **do**
  set $\bar{N}^{(d)} = \max(1, N^{(d)})$
  draw $\boldsymbol{\theta}^{(d)} \sim$ Dirichlet$(\alpha, \boldsymbol{m})$
  for *n=1 to* $\bar{N}^{(d)}$ **do**
    draw $z_n^{(d)} \sim$ Multinomial$(\boldsymbol{\theta}^{(d)})$
    if $N^{(d)} > 0$ **then**
      draw $w_n^{(d)} \sim$ Multinomial$(\boldsymbol{\phi}^{(z_n^{(d)})})$
    end
  end
  for *c=1 to C* **do**
    set $p_c^{(d)} = \frac{\sum_{k:c_k=c} N^{(k|d)}}{N^{(d)}}$
  end
  for *i=1 to A* **do**
    for *j=1 to A* **do**
      if $j \neq i$ **then**
        calculate $\boldsymbol{x}^{(c)}_{t_+^{(d-1)}}(i,j)$
        set $\lambda_{ij}^{(d)} = \sum_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T}\boldsymbol{x}^{(c)}_{t_+^{(d-1)}}(i,j)\Big\}$
      end
    end
    draw $J_i^{(d)} \sim$ Gibbs measure$(\{\lambda_{ij}^{(d)}\}_{j=1}^{A}, \delta)$
    draw $\Delta T_{iJ_i} \sim$ Exponential$(\lambda_{iJ_i}^{(d)})$
  end
  set $i^{(d)} = i_{\min(\Delta T_{iJ_i})}$, $J^{(d)} = J_{i^{(d)}}$, and $t^{(d)} = t^{(d-1)} + \min(\Delta T_{iJ_i})$
end

3 Inference

We take Bayesian approach to inferring the latent variables (i.e., parameters) in the IPTM. The likelihood function is implied by the generative process in Section 2.5. In this section, we derive the joint distribution over the variables $\Phi = \{\boldsymbol{\phi}^{(k)}\}_{k=1}^{K}, \Theta = \{\boldsymbol{\theta}^{(d)}\}_{d=1}^{D}, \mathcal{Z} = \{\boldsymbol{z}^{(d)}\}_{d=1}^{D}, \mathcal{C} = \{c_k\}_{k=1}^{K}, \mathcal{B} =$

$\{\boldsymbol{b}^{(c)}\}_{c=1}^{C}, \delta, \mathcal{J}_{\text{a}} = \{\{J_i^{(d)}\}_{i \neq i_o^{(d)}}\}_{d=1}^{D}$, and $\mathcal{T}_{\text{a}} = \{\{t_{iJ_i}^{(d)}\}_{i \neq i_o^{(d)}}\}_{d=1}^{D}$, and $\mathcal{P} = \{(i, J, t)^{(d)}\}_{d=1}^{D}$ given the observed four components $\mathcal{W} = \{\boldsymbol{w}^{(d)}\}_{d=1}^{D}$, $\mathcal{I}_{\text{O}} = \{i_o^{(d)}\}_{d=1}^{D}$, $\mathcal{J}_{\text{O}} = \{J_o^{(d)}\}_{d=1}^{D}$, and $\mathcal{T}_{\text{O}} = \{t^{(d)}\}_{d=1}^{D}$, and the hyperparameters $(\beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2)$.

After integrating out Φ and Θ using Dirichlet-multinomial conjugacy (Griffiths and Steyvers, 2004) we sample the remaining unobserved variables from their joint posterior distribution using Markov chain Monte Carlo methods. Additionally, we integrate out the latent time-increments $\mathcal{T}_{\text{a}}$ using the property of the minimum of Exponential random variables, as shown in B.1. Our inference goal is to draw samples from the posterior distribution

$$\begin{aligned}
&P(\mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta, \mathcal{J}_{\text{a}} | \mathcal{W}, \mathcal{I}_{\text{O}}, \mathcal{J}_{\text{O}}, \mathcal{T}_{\text{O}}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2) \\
&\propto P(\mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{J}_{\text{a}}, \mathcal{I}_{\text{O}}, \mathcal{J}_{\text{O}}, \mathcal{T}_{\text{O}} | \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2) \\
&= P(\mathcal{Z}|\alpha, \boldsymbol{m}) P(\mathcal{C}) P(\mathcal{B}|\mathcal{C}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}) P(\delta|\mu_\delta, \sigma_\delta^2) P(\mathcal{W}|\mathcal{Z}, \beta, \boldsymbol{u}) P(\mathcal{J}_{\text{a}}, \mathcal{I}_{\text{O}}, \mathcal{J}_{\text{O}}, \mathcal{T}_{\text{O}} | \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta).
\end{aligned} \tag{8}$$

The detailed derivation of sampling equations can be found in Appendix B.

To summarize the inference procedure outlined above, we provide pseudocode for Markov Chain Monte Carlo (MCMC) sampling. For better performance and interpretability of the topics we infer, we run n_1 iterations of the hyperparameter optimization technique called "new fixed-point iterations using the Digamma recurrence relation" in Wallach (2008), for every outer iteration o. Also, while we update the categorical variables $\mathcal{Z}$ and $\mathcal{C}$ once per outer iteration, we specify a larger number of inner iterations (n_2 and n_3) for the continuous variables $\mathcal{B}$ and δ, respectively. The continuous variables converge slower than the discrete variables since we sample the categorical variables using Gibbs sampling and the continuous variables using Metropolis-Hastings.. When summarizing model results, we only use the samples from the last (i.e., O^{th}) outer loop.

Algorithm 6 MCMC

```
set initial values Z^(0), C^(0), and (B^(0), δ^(0))
for o=1 to O do
    for n=1 to n_1 do
        optimize α and m using hyperparameter optimization in Wallach (2008)
    end
    for d=1 to D do
        for i ∈ A_{\i_o^(d)} do
            sample the augmented data J_i^(d) following Section B.2
        end
        for n=1 to N^(d) do
            draw of z_n^(d) ~ Multinomial(p^Z) following Section B.3
        end
    end
    for k=1 to K do
        draw c_k ~ Multinomial(p^C) following Section B.4
    end
    for n=1 to n_2 do
        sample B using Metropolis-Hastings following Section B.5
    end
    for n=1 to n_3 do
        sample δ using Metropolis-Hastings following Section B.6
    end
end
Summarize the results with:
last sample of C, last sample of Z, last n_2 length chain of B, last n_3 length chain of δ
```

4 Getting It Right (GiR) Test

Software development is integral to the objective of applying IPTM to real world data. Code review is a valuable process in any research computing context, and the prevalence of software bugs in statistical software is well documented (e.g., Altman et al., 2004; McCullough, 2009). With highly complex models such as IPTM, there are many ways in which software bugs can be introduced and go unnoticed. As such, we present a joint analysis of the integrity of our generative model, sampling equations, and software implementation.

Geweke (2004) introduced the "Getting it Right" (GiR) test—a joint distribution test of posterior simulators which can detect errors in sampling equations as well as coding errors. The test involves comparing the distributions of variables simulated from two joint distribution samplers, which we call "forward" and "backward" samples. The forward sampler draws unobservable variables from the prior and then generate the observable data conditional on unobservables. The backward sampler alternates between the inference and an observables simulator, by running the inference code on observable data to obtain posterior estimates of the unobervable variables and then re-generate the observables given the inferred unobservables. The backward sampler is initialized through an iteration of inference on observables drawn directly from the prior. Since the only information on which both the forward and backward samplers are based is the prior, if the sampling equations are correct and the code is implemented without bugs, each variable should have the same distribution in the forward and backward samples.

In the forward samples, both observable and unobservable variables are generated using Algorithm 5. In the backward samples, unobservable variables are generated using the sampling equations for inference, which we derived in Section 3. In order to generate observable variables in the backward samples (sender, recipients, timestamp), we use the collapsed-time generative process, which we presented in Section C.1. For each forward and backward sample that consists of D number of documents, we save the statistics below:

1. Mean of network effect parameters $(\boldsymbol{b}_p^{(1)}, ..., \boldsymbol{b}_p^{(C)})$ for every $p = 1, ..., P$,
2. Network statistic 'send' calculated for the last D^{th} document for every $l = 1, ..., 3$
3. δ value used to generate the samples
4. Mean of the recipient size $|J^{(d)}|$ across $d = 1, ..., D$,
5. Mean of time-increments $t^{(d)} - t^{(d-1)}$ across $d = 1, ..., D$,
6. Mean topic-interaction pattern assignment c_k across $k = 1, ..., K$,
7. Number of tokens in topics assigned to each interaction pattern $c = 1, ..., C$,
8. Number of tokens assigned to each topic $k = 1, ..., K$,
9. Number of tokens assigned to each unique word type $w = 1, ..., W$.

To keep the computational burden of re-running thousands of rounds of inference manageable, we run GiR using a relatively small artificial sample, consisting of 5 documents, 4 tokens per document, 4 actors, 5 unique word types, 2 interaction patterns, and 4 topics per each forward or backward samples. For detailed settings including the prior specifications, see Appendix C.4. We generated 5×10^4 sets of forward and backward samples, and then calculated 1,000 quantiles for each of the network effect statistics (1.), and 50 quantiles for the rest of the statistics. We also calculated t-test and Mann-Whitney test p-values in order to test for differences in the distributions generated in the forward and backward samples. Before we calculated these statistics, we thinned our samples by taking every 40th sample starting at the 10,000th sample for a resulting sample size of 1,000. Thinning reduces the autocorrection in the Markov chains. In each case, if we observe a large p-value, this gives us evidence that the distributions generated under forward and backward sampling have the same locations. We depict the GiR results using probability-probability (PP) plots. To compare two samples with a PP-plot we calculate the empirical quantile in each sample of a set of values observed across the two samples, then plot the sets of quantiles in the two samples against each other. If the two samples are from equivalent distributions, the quantiles should line up on a line with zero y-intercept, and unit slope (i.e., a 45-degree line). The GiR results are depicted in Figure 2.

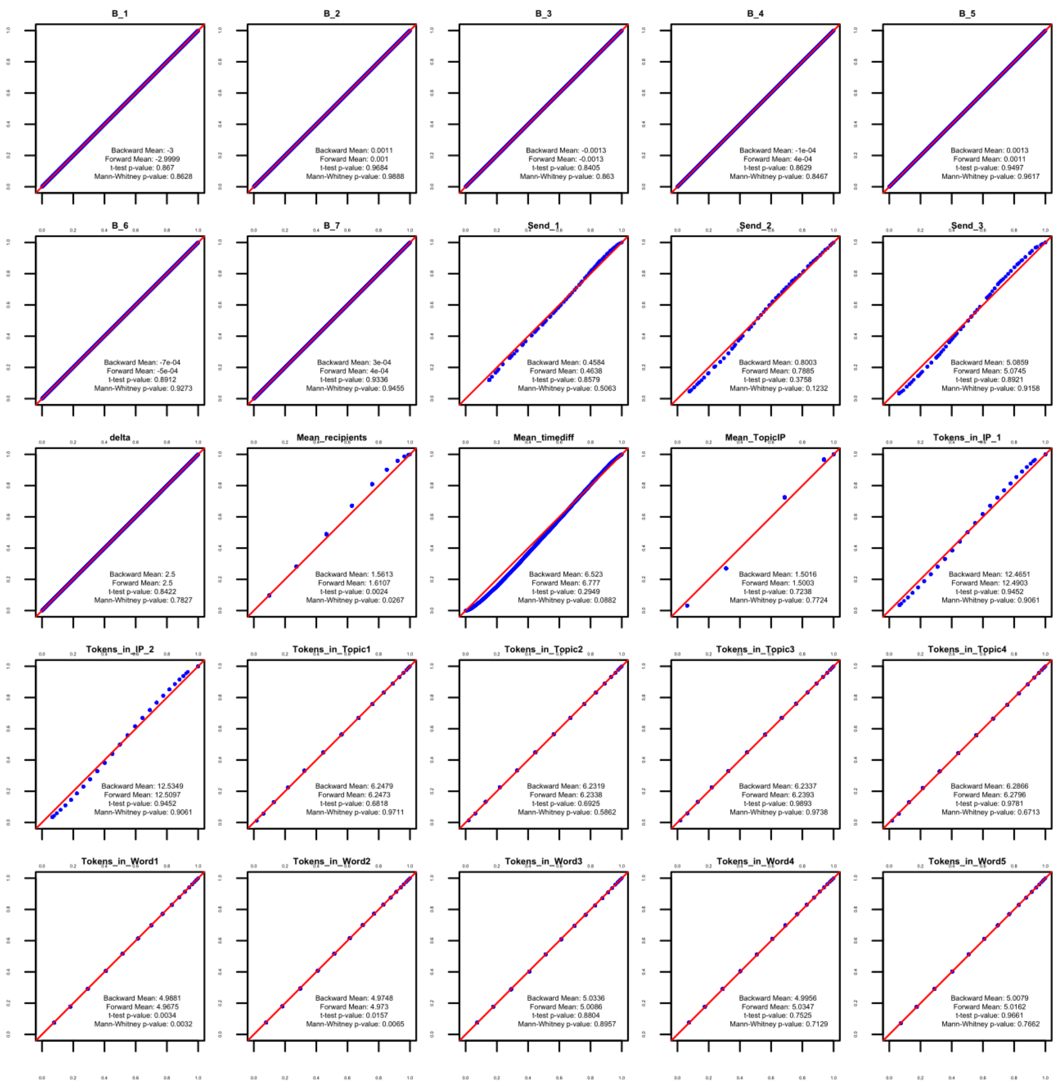

Figure 2: Probability-Probability plot for the 25 GiR test statistics, with topic-interaction pattern being inferred.

The GiR results indicate that all of the LDA-related (i.e., tokens and topics) statistics passed the test in terms of the PP-plot as well as the p-values from the two different tests. However, several tie-related statistics, (2.), (4.), (5.), (6.) and (7.), did not generate consistent distributions across forward and backward samplers. Some of these had high p-values, but the PP dots do not line up on the diagonal line. We investigated possible bugs and realized that the failure comes from non-convergence of the topic-interaction pattern assignment $\mathcal{C}$ in the inference. Incorrect assignment of a topic's interaction pattern c_k results in considerable bias in $\{p_c^{(d)}\}_{d=1}^{D}$. Since all of the statistics which failed the test are generated using the stochastic intensity $\{\boldsymbol{\lambda}^{(d)}\}_{d=1}^{D}$, which is a function of $p_c^{(d)}$, it is intuitive that the tie variables would fail when the wrong value of $\mathcal{C}$ is inferred. We tried running the GiR test again without inferring the topic-interaction pattern assignment; instead, we used the same fixed set of $\{c_k\}_{k=1}^{K}$ for forward and backward samples such that we guarantee zero bias from $\{p_c^{(d)}\}_{d=1}^{D}$. As shown in Figure 3, now we pass the test on every statistic.

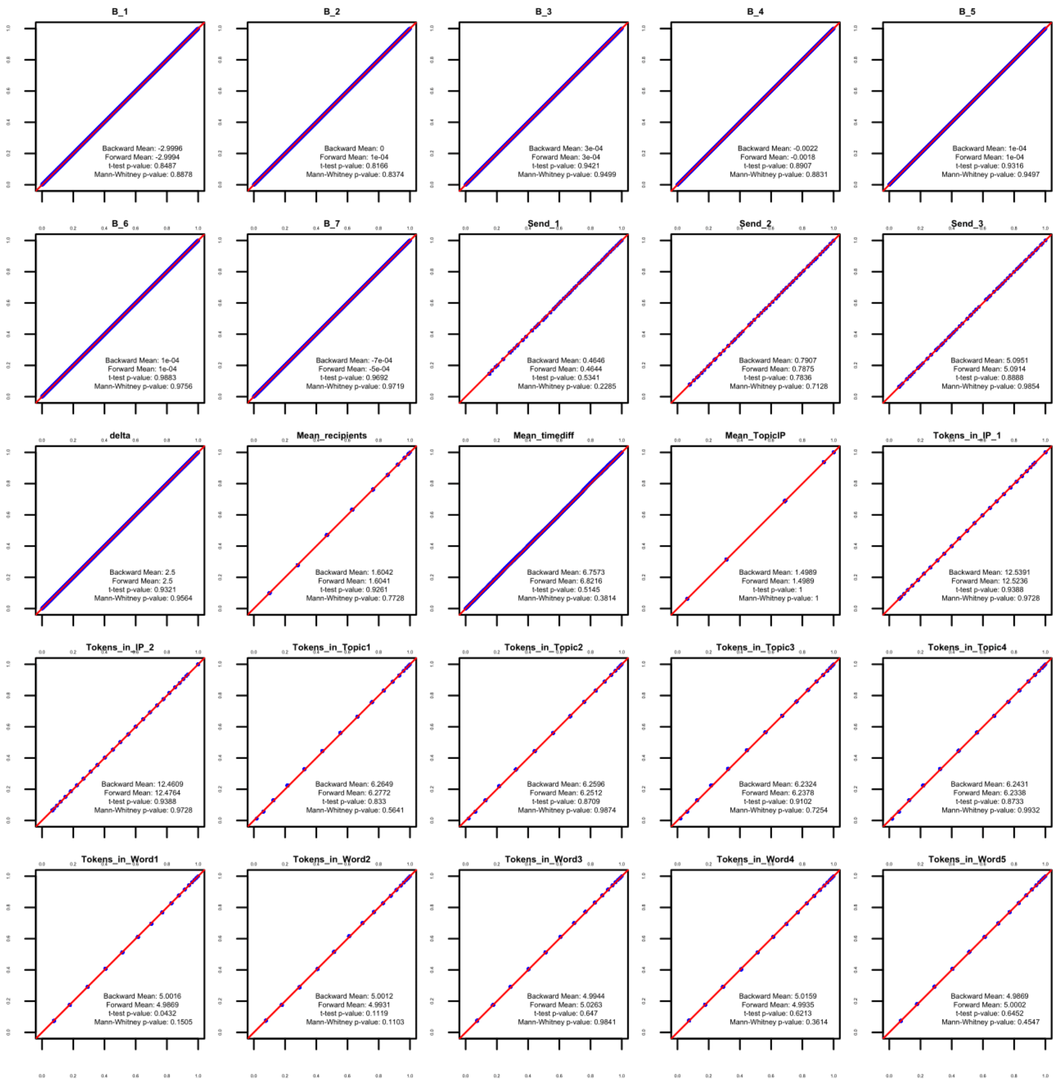

Figure 3: Probability-Probability plot for the 25 GiR test statistics, with topic-interaction pattern not being inferred.

The next step in our work will be to either repair any bugs that exist in our $\mathcal{C}$ inference code or change the model such that the interaction patterns of topics are fixed. Since we do not want to present empirical results based on problematic code, in the applications that follow we use the version of the model in which interaction patterns are fixed (i.e., the version that has passed GiR). In this version of the IPTM, we assume there are K topics in each interaction pattern, and the interaction pattern of each topic is fixed.

5 Appliction: North Carolina County Government Email Communication During Hurricane Sandy

In our application of the IPTM, we use a subset of the North Carolina county government e-mail dataset collected by ben Aaron et al. (2017). This dataset includes internal e-mail corpora covering the inboxes and outboxes of managerial-level employees of North Carolina county governments. Each county corpus covers a three-month span in 2012. The full dataset covers over twenty counties, but we focus on two counties for which the time span included a notable national emergency—Hurricane Sandy (October 26, 2012—October 30, 2012). We chose these two counties, (1) in order to see whether and how communication networks surrounding Hurricane Sandy differed from those surrounding other governmental functions, and (2) to limit the scope of this initial application. Studying these two counties also presents a case study of how the geographic exposure to the storm relates to the prevalence of internal governmental communication regarding the storm. From Figure 4, we see that Dare County covers a large coastal region, including the Outer Banks, one of the most critically affected area by the hurricane. Vance County is much further inland than Dare. In this section we apply IPTM to both county corpora.

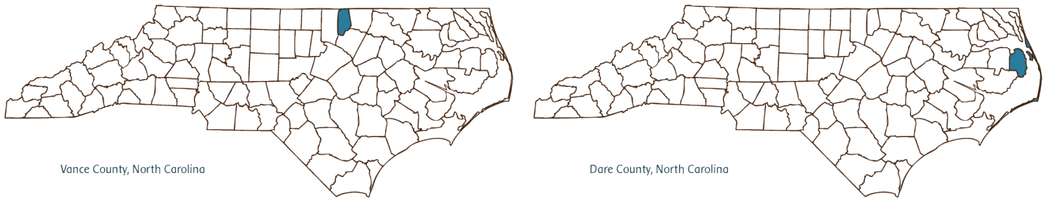

Figure 4: Geographical location of Vance County (left) and Dare County (right) in North Carolina

5.1 Exploratory Data Analysis

Before discussing the IPTM results, we present a set of exploratory analyses in which we examine characteristics of the data that are relevant to the prevalence of Hurricane Sandy in the two county government email networks. Based on this preliminary exploration, we see that Sandy received much more attention in Dare County. However, this basic descriptive finding does not shed light on whether, in one or both of the counties, the network structure of communication surrounding Sandy differed from the typical communication patterns.

5.1.1 Vance County

The Vance County email dataset spans September 4th to November 30th. It is also the smallest corpus in the NC overall dataset, containing $D = 183$ emails sent between $A = 17$ actors from 17 departments. The corpus has a vocabulary size of $W = 620$. In our first descriptive analysis we compare the email exchange patterns between the non-Sandy period (not overlapping October 26—30) and the Sandy period (October 26—30).

To visualize temporal trends in sending and receiving behavior in email exchanges, we plotted the number of emails sent and received between the county government managers based on their departments, which are shown in Figure 5. Since the data size is quite small, there are not many emails exchanged per day. Three departments—social services, planning, and emergency services—were the most active over the three months. We do not see any distinctive difference during hurricane Sandy in Vance County.

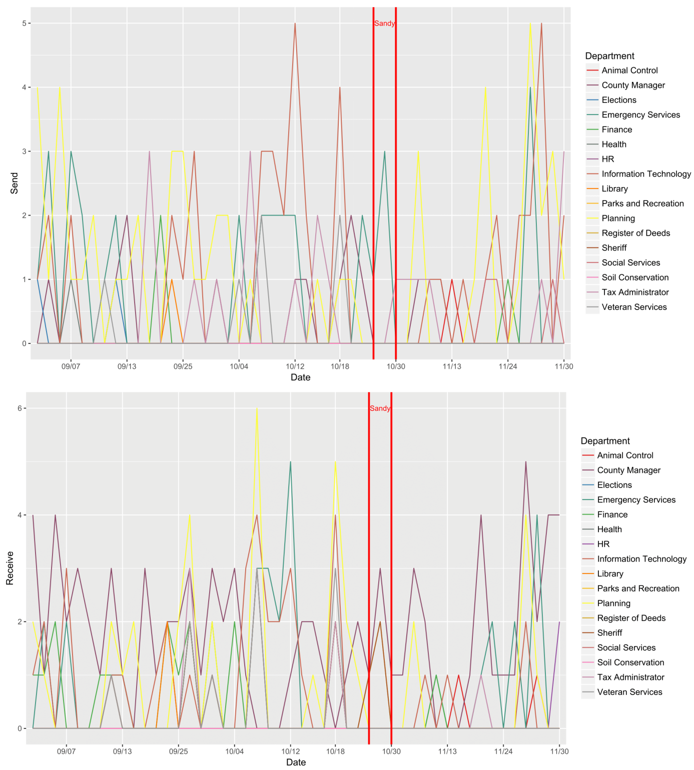

Figure 5: Number of emails sent from (upper) and received by (lower) each department in Vance County

To examine shifts in the patterns of connectivity surrounding Sandy we created network plots for three separate time windows: pre-Sandy, Sandy, and post-Sandy. The Sandy period includes October 19th to November 2nd, which covers the one week before and after the official period of Sandy. Figure 6 shows a similar pattern as that seen in Figure 5, in that the same three department managers are placed in the center of each network. One noticeable thing is that during the Sandy period, social services did not exchange any email, while it went back to its normal status after Sandy. In addition, the emergency service department interacted slightly more during Sandy, compared to the other two time windows.

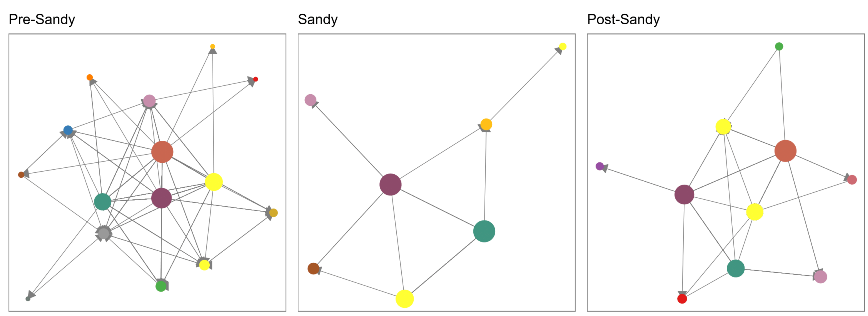

Figure 6: Network plot for three time windows: before Sandy (September 4—October 18), during Sandy (October 19—November 2), and after Sandy (November 3—November 30), in Vance County

Lastly, we moved to the content aspects of data and looked at how many times the words ‘hurricane’ and ‘Sandy’ appeared. As shown in Figure 7, the two words were only used 4 times each during the Sandy period, and not used in any other times. This is not surprising given that Vance County is quite far from the shore, and was not in the main path of the storm.

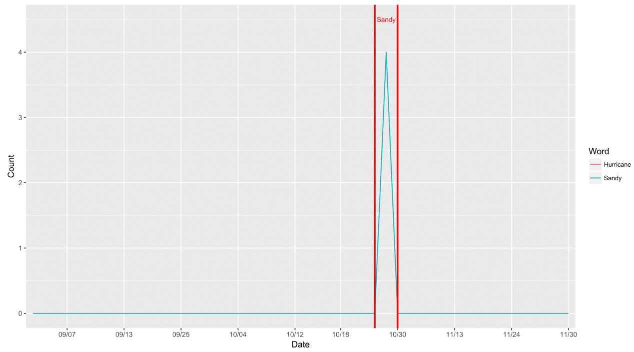

Figure 7: Frequency plot counting how many times the words 'hurricane' and 'Sandy' appeared.

5.1.2 Dare County

We ran the same exploratory analyses using the Dare County data, which spans October 1st to November 30th containing $D = 1,456$ emails between $A = 27$ actors from 22 departments, and with a vocabulary of size $W = 2907$. As we did with Vance County, we looked at three different plots—sending and receiving counts, networks, and word counts—to visualize the networks and content of the email data, with emphasis on the changes during hurricane Sandy.

In Dare County we saw considerable change in email sending/receiving behaviors during hurricane Sandy (See Figure 8). As the hurricane approaches on October 26th, the manager from the emergency services department sent significantly more emails than before, and at the same time there was dramatic rise in the receiving counts for almost every department. Further analysis demonstrated that emergency services department sent a lot of 'multicast' emails with a large number of receivers during the Sandy period.

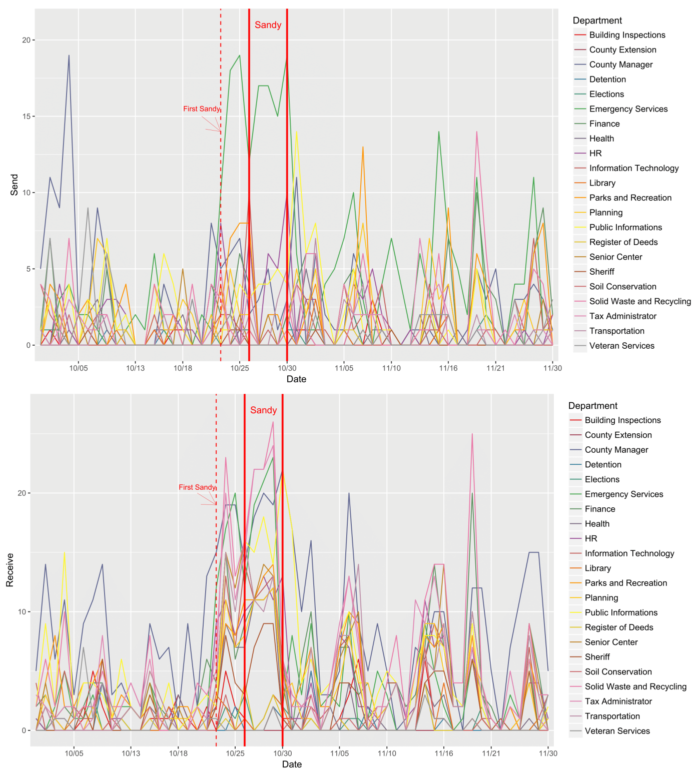

Figure 8: The number of emails sent to (upper) and received by (lower) department in Dare County

The network plots in Figure 9 illustrated same patterns we found in Figure 8. Again, the manager from the emergency services department became highly central in the network during the Sandy period, and it maintained this pattern after Sandy. Hurricane related conversations continued after Sandy passed the county, since there remained post-hurricane issues from the damage.

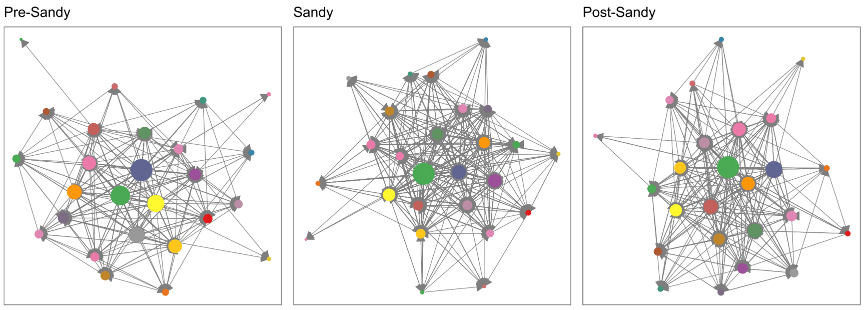

Figure 9: Network plot for three time windows: before Sandy (October 1—October 18), during Sandy (October 19—November 2), and after Sandy (November 3—November 30), in Dare County

Figure 10 reflects the hurricane's effects on email exchanges as well, and it matches our interpretations from the network aspects. Usage of the two words, 'hurricane' and 'Sandy', exploded starting a few days before Sandy arrived in Dare County, and multiple emails used the words again in November, implying the continuous discussions on hurricane-related topics.

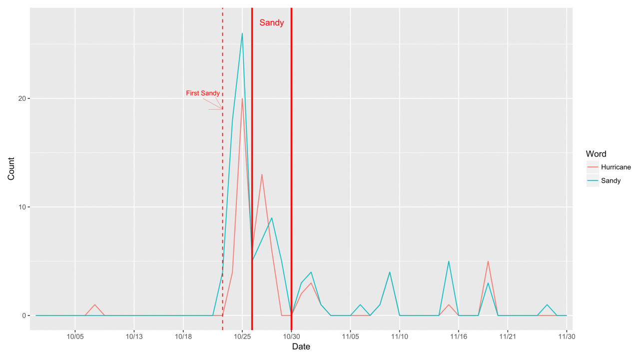

Figure 10: Frequency plot counting how many times the word 'hurricane' and 'sandy' appeared

5.2 IPTM Results

In this section we present the IPTM results. Researchers who use the IPTM can test hypotheses regarding network structure, which is a common—perhaps the most common—use of ERGM-stye models for networks. However, with the IPTM those hypotheses can be conditioned on the content area of communication. To provide an example, we articulate expectations regarding the content-conditional structural properties of the county government email networks. Information diffuses more efficiently in networks characterized by a lack of loops (Lin et al., 2010; Iribarren and Moro, 2011) and closed triangles (Roca et al., 2010; Tadić and Thurner, 2004). Assuming that the county governments' internal communication networks are characterized by efficient communication structure, we expect to see communication regarding the everyday business of the county characterized by negative reciprocity (i.e., 2-loop) effects, and negative triadic effects. The "receive" term captures reciprocity, and triadic effects are captured by 2-send, 2-receive, sibling and cosibling. Reciprocity and closed triangles are, however, common structural properties of social networks. We expect to see communication regarding personal/social matters to be characterized by positive reciprocity and triadic effects. Lastly, we have limited expectations regarding how communication surrounding Hurricane Sandy will be structured. We take an exploratory approach to the question of whether or not discussion surrounding Sandy forms an efficient communication network structure.

5.2.1 Vance County

In the IPTM analysis of the Vance County data we set $C = 2$, $K = 6$ and $O = 500$. Adding more interaction patterns and/or topics did not appear to change or improve results significantly, but in future work we plan to develop principled methods for selecting these hyperparameters. MCMC sampling was implemented based on the scheme illustrated in Section 3. We applied hyperparameter optimization with $n_1 = 5$, and the inner iterations for $\mathcal{B}$ and δ were set as $n_2 = 5500$ and $n_3 = 550$, respectively. First 500 and 50 iterations were discarded as a burn-in for inference on $\mathcal{B}$ and δ, and every 10^{th} sample was taken in order to thin the chain for $\mathcal{B}$. Below are the summary of interaction pattern-topic-word assignments. Each interaction pattern is summarized with (a) Table 1: the top 15 most likely words in the topics within the interaction pattern, and (b) Figure 11: boxplots visualizing posterior estimates of dynamic network effects $\boldsymbol{b}^{(c)}$ within each interaction pattern

In general, we do not see any major differences between interaction patterns from examining Table 1 and Figure 11. The posterior distributions of the relational parameters overlap considerably for nearly all parameters. Furthermore, most of the posterior distributions of the relational parameters are nearly centered at zero, which offers little evidence of complexity in tie formation in the Vance County e-mail networks. Topic-token assignments for two interaction patterns in Table 1 also do not reveal obvious contrast, because the topics all contain words used for typical conversations between government officials. We also do not find any topics that are relevant to Hurricane Sandy.

IP	**1**	**1**	**1**	**2**	**2**	**2**
Topic	**1** (0.078)	**3** (0.224)	**5** (0.141)	**2** (0.139)	**4** (0.369)	**6** (0.038)
Word	directory	message	operations	phase	dropbox	phones
	switch	electronic	emergency	description	cecd	october
	network	ncgs	office	planning	henderson-vance	will
	address	chapter	communications	board	box	polycom
	extension	response	center	water	commission	intstructions
	tax	public	lines	taps	economic	training
	latest	manager	fax	keep	unemployment	conference
	department	attachments	enp	phone	licensed	three
	henderson	siemens	cem	compliance	rural	finalized
	january	pursuant	suite	signups	development	room
	young	subject	good	meter	reduced	cutover
	wireless	review	asap	suite	financial	contact
	installation	records	henderson	fax	private	thursday
	cutting	jail	street	meeting	labor	folks
	rest	hereto	church	tuesday	force	finishing

Table 1: Summary of topic-token assignments from Vance County data: top 15 words assigned to each topic, corresponding to interaction pattern assignments

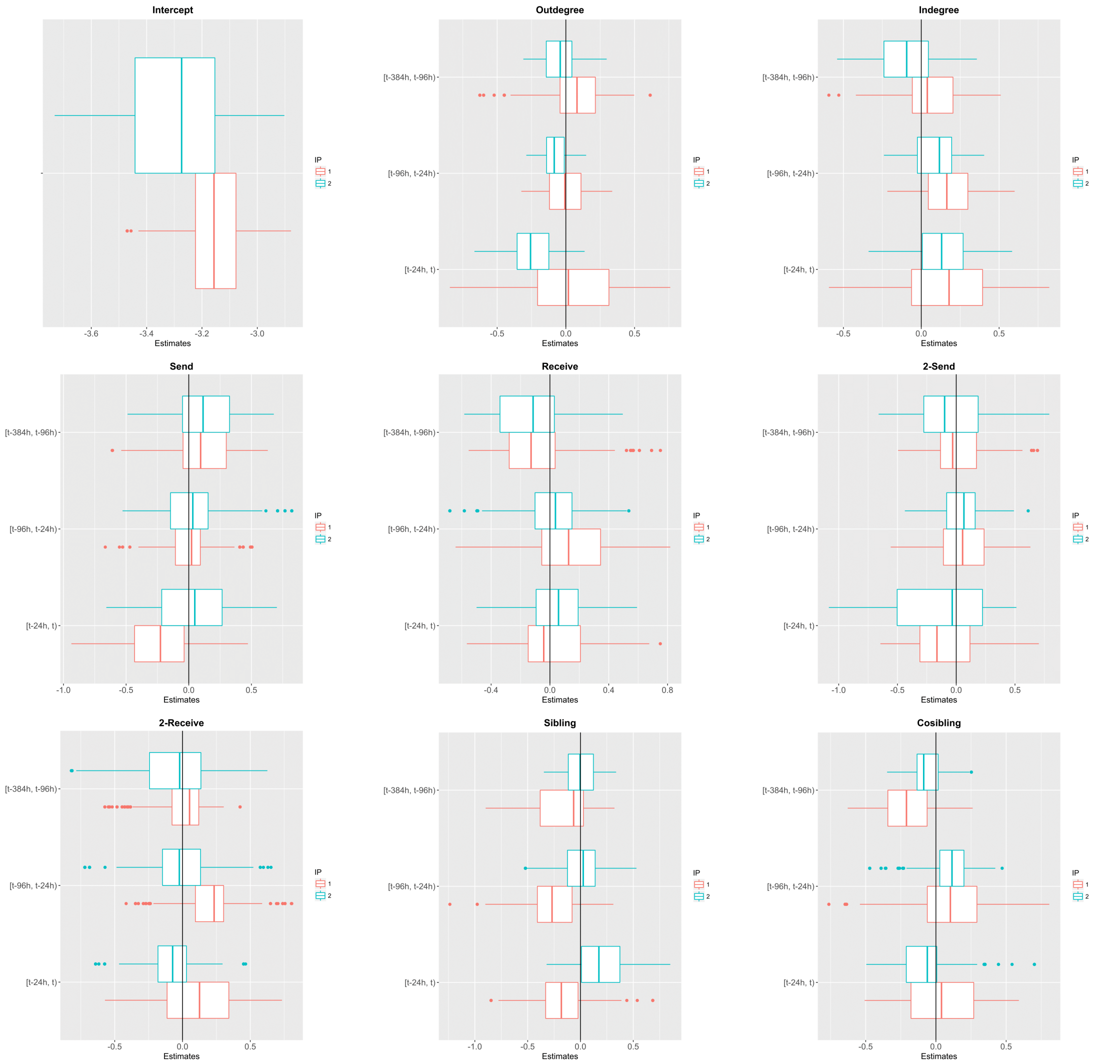

Table 2: 95% credible intervals of posterior estimates of the network effects $\boldsymbol{b}^{(c)}$: $c = 1$ (red) and $c = 2$ (green), using Vance County data

5.2.2 Dare County

For the IPTM application to the Dare County email data we used $C = 2$, $K = 20$, and $O = 100$[1]. Again, we applied hyperparameter optimization with $n_1 = 5$, while the inner iterations for $\mathcal{B}$ and δ were set as $n_2 = 15000$ and $n_3 = 1500$, respectively. This time, first 10000 and 500 iterations were discarded as a burn-in for inference on $\mathcal{B}$ and δ, and every 10^{th} and 5^{th} samples were taken as a thinning process for $\mathcal{B}$ and δ, respectively.

Same as Section 5.2.2, each interaction pattern is summarized with (a) Table 2: the top 15 most likely words to be generated in the topics within the interaction pattern, and (b) Figure 12: boxplots visualizing posterior estimates of dynamic network effects $\boldsymbol{b}^{(c)}$ within each interaction pattern.

We see significant differences in the contents related to each interaction pattern. Overall, 55.2% of the words were assigned to the topics in interaction pattern 1, and 44.8% were assigned to the topics in interaction pattern 2. Table 4 demonstrates 5 examples of topics from each interaction pattern, where each topic is summarized by the top 15 words. Similarly as what we saw in Vance County, interaction pattern 1 seems to represent topics commonly used by government managers. On the other hand, it is

[1]Preliminary results with small number of outer iterations. Results subject to change.

interesting that interaction pattern 2 included several topics (e.g. topic 2 and topic 18) with hurricane related words, such as ‘storm’, ‘impacts’ ‘damage’ and ‘ocean’. In addition, one more impressive point is that topic 12 contained words related politics or election, e.g. ‘survey’, ‘parties’, and ‘elections’. In summary, interaction pattern 1 represents usual administrative communications between managers in county government, and interaction pattern 2 to reflects temporary conversations driven by events or emergencies occurring in the County.

IP	**1**	**1**	**1**	**1**	**1**
Topic	**3** (0.078)	**5** (0.065)	**19** (0.064)	**13** (0.057)	**15** (0.053)
Word	water	planning	phone	questions	contact
	relocation	meter	collins	board	info
	location	room	drive	december	problem
	hills	asked	marshall	call	release
	utilities	needed	director	sheets	check
	mustian	sure	human	agenda	weather
	hydrant	afternoon	resources	nov	priority
	department	cheryl	manteo	hope	readings
	skyco	johnson	phr	item	rodanthe
	kill	issues	fax	weekly	top
	devil	case	box	management	collection
	road	letter	timesheets	internet	located
	lane	antennas	-lsb-	told	health
	tank	inspection	wanted	care	heads
	map	keep	touch	comp	ahead
IP	**2**	**2**	**2**	**2**	**2**
Topic	**14** (0.058)	**12** (0.047)	**2** (0.045)	**6** (0.044)	**18** (0.036)
Word	time	survey	road	library	status
	hours	voice	mirlo	week	system
	leave	copy	storm	working	area
	monday	discovery	beach	place	south
	administrative	regional	high	best	forecast
	employees	parties	coastal	start	track
	empoyee	disclosed	impacts	visit	pay
	work	elections	saturday	year	move
	day	pin	dot	albemarle	assessment
	friday	sending	night	librarian	opens
	october	prior	winds	web	damage
	storm	editor	hold	learning	well
	tomorrow	students	bridge	east	ocean
	hour	cost	expressed	holiday	operation
	question	residents	normal	system	addition

Table 3: Summary of topic-token assignments from Dare County data: top 15 words assigned to each topic, corresponding to interaction pattern assignments

In the Dare County analysis the two interaction patterns exhibited quite different network effects. The effects in interaction pattern 1 were generally greater in magnitude than those in interaction pattern 2, implying that topics related to interaction pattern 2 are less affected by previous email exchanges than those in interaction pattern 1. Most effects are greater in magnitude for the time interval $[t-24h, t)$ and the effect is diminishing as we move to older time intervals $[t-96h, t-24h)$ and $[t-384h, t-96h)$. On the contrary, the statistic ‘send’ had strongest effect in the time interval farthest in the past $[t-384h, t-96h)$. Furthermore, negative reciprocity, which we noted is expected in an efficient communication network, are found in the ‘receive’ statistics effect (except $[t-96h, t-24h)$), in interaction pattern 1. Most of the posterior distributions of network effects in interaction pattern 2 are centered close to zero, providing little evidence of complex network dynamics. The differences between interaction patterns can be explained by the content differences conveyed through Table 3. Since interaction pattern 2 consists of highly time-sensitive topics, ties are less likely to be effected by previous interactions.

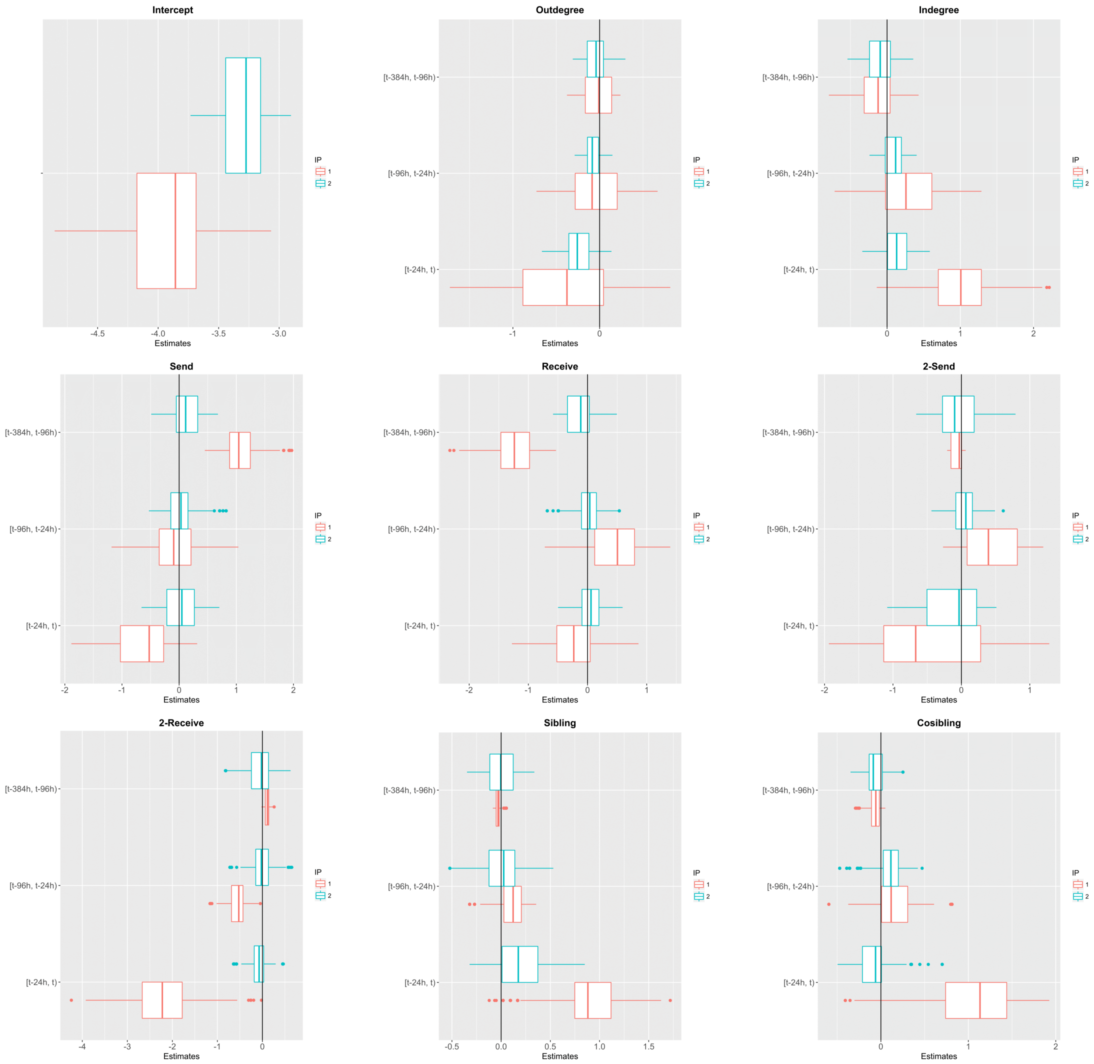

Table 4: 95% credible intervals of posterior estimates of the network effects $\boldsymbol{b}^{(c)}$: $c = 1$ (red) and $c = 2$ (green), using Dare County data

6 Posterior Predictive Experiments

We use a set of posterior predictive experiments to evaluate the performance of the IPTM as compared to alternative modeling approaches, and with respect to alternative parameterizations of the IPTM. For documents $d = \{M, M+1, \ldots, D-1\}$, we fit the IPTM to the first d documents, then use the inferred posterior distributions to generate a distribution of predicted tie data $(i^{(d+1)}, J^{(d+1)}, t^{(d+1)})$ for document $d+1$ conditional on the content in document $d+1$, $(\boldsymbol{w}^{(d+1)})$. A reasonable choice for M would be $D/2$, to assure a sufficient size training set. The variables that need to be sampled are the token topic assignments, $\mathcal{Z}^{d+1}$, and the tie data $(i^{(d+1)}, J^{(d+1)}, t^{(d+1)})$.

Algorithm 7 Predicting tie data for the next document

Input

1. O, number of outer iterations of inference from which to generate predictions
2. d, the last document to use in inference
3. R, the number of iterations to sample predicted data within each outer iteration

Run burnin iterations

for *o=1 to O* **do**

  run an outer iteration of inference on documents 1 through d

  initialize values for $i^{(d+1)}$, $J^{(d+1)}$, $t^{(d+1)}$, and $\mathcal{Z}^{d+1}$

  for *r=1 to R* **do**

    sample $i^{(d+1)}$, $J^{(d+1)}$, and $t^{(d+1)}$ conditional on $\mathcal{Z}^{d+1}$, via the generative process

    sample $\mathcal{Z}^{d+1}$ via Equation 24

  end

  store $i^{(d+1)}$, $J^{(d+1)}$, $t^{(d+1)}$, and $\mathcal{Z}^{d+1}$

end

We have not implemented this posterior predictive experiments yet, so it will go to the next step of this project.

7 Conclusion

The IPTM is, to our knowledge, the first model to be capable of jointly modeling sender, receivers, time and contents in time stamped text valued networks. The IPTM incorporates innovative components, including the modeling of multicast tie formation and the conditioning of ERGM style network generative features on topic-based content. The application to North Carolina county government email data demonstrates, among other capabilities, the effectiveness at the IPTM in separating out both the content and relational structure underlying the normal day-to-day function of an organization and the management of a highly time-sensitive event—Hurricane Sandy. The IPTM is applicable to a variety of networks in which ties are attributed with textual documents. These include, for example, economic sanctions sent between countries and legislation attributed with sponsors and co-sponsors.

Three major tasks remain in finalizing our work with the IPTM. First, we will either correct the code and/or sampling equations for the full model, in which interaction pattern assignments are inferred, or revise the model to reflect fixed interaction pattern assignments. Second, we will run posterior predictive experiments to evaluate model fit in comparison with other methods. Third, we will complete our implementation of IPTM as a publicly distributed R package.

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Appendix

A Normalizing constant of non-empty Gibbs measure

In Section 2.4, we define the non-empty Gibbs measure such that the probability of sender i selecting the binary receiver vector of length $(A-1)$, $J_i^{(d)}$ is given by

$$\mathrm{P}(J_i^{(d)}) = \frac{1}{Z(\delta, \log(\lambda_i^{(d)}))} \exp\Big\{\log\big(\mathrm{I}(\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) + \sum_{j\in\mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}.$$

To use this distribution efficiently, we need to derive a closed-form expression for $Z(\delta, \log(\lambda_i^{(d)}))$ that does not require brute-force summation over the support of $J_i^{(d)}$. We begin by recognizing that if $J_i^{(d)}$ were drawn via independent Bernoulli distributions in which $\mathrm{P}(J_{ij}^{(d)}{=}1)$ was given by $\mathrm{logit}\Big(\delta + \lambda_{ij}^{(d)}\Big)$, then

$$\mathrm{P}(J_i^{(d)}) \propto \exp\Big\{\sum_{j\in\mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}.$$

This is straightforward to verify by looking at

$$\begin{aligned}\mathrm{P}(J_{ij}^{(d)} = 1 | J_{i,-j}) &= \frac{\exp(\delta + \log(\lambda_{ij}^{(d)}))\exp\Big\{\sum_{h\neq i,j}(\delta + \log(\lambda_{ih}^{(d)}))J_{ih}^{(d)}\Big\}}{\exp(\delta + \log(\lambda_{ij}^{(d)}))\exp\Big\{\sum_{h\neq i,j}(\delta + \log(\lambda_{ih}^{(d)}))J_{ih}^{(d)}\Big\} + \exp(0)\exp\Big\{\sum_{h\neq i,j}(\delta + \log(\lambda_{ih}^{(d)}))J_{ih}^{(d)}\Big\}}, \\ &= \frac{\exp(\delta + \log(\lambda_{ij}^{(d)}))}{\exp(\delta + \log(\lambda_{ij}^{(d)})) + 1}.\end{aligned}$$

We denote the logistic-Bernoulli normalizing constant as $Z^l(\delta, \lambda_i^{(d)})$, which is defined as

$$Z^l(\delta, \log(\lambda_i^{(d)})) = \sum_{J_i\in[0,1]^{(A-1)}} \exp\Big\{\sum_{j\neq i}(\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}.$$

Now, since

$$\exp\Big\{\log\big(\mathrm{I}(\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij} > 0)\big) + \sum_{j\in\mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\} = \exp\Big\{\sum_{j\in\mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\},$$

except when $\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij}^{(d)} = 0$, in which case the left-hand side

$$\exp\Big\{\log\big(\mathrm{I}(\sum_{j\in\mathcal{A}_{\backslash i}} J_{ij} > 0)\big) + \sum_{j\in\mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\} = 0.$$

As such, we note that

$$\begin{aligned}Z(\delta, \log(\lambda_i^{(d)})) &= Z^l(\delta, \log(\lambda_i^{(d)})) - \exp\Big\{\sum_{j\in\mathcal{A}_{\backslash i}, J_{ij}^{(d)}=0} (\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\} \\ &= Z^l(\delta, \log(\lambda_i^{(d)})) - 1.\end{aligned}$$

We can therefore derive a closed form expression for $Z(\delta, \log(\lambda_i^{(d)}))$ via a closed form expression for $Z^l(\delta, \log(\lambda_i^{(d)}))$. This can be done by looking at the probability of the zero vector under the logistic-Bernoulli model:

$$\frac{\exp\Big\{\sum_{j\neq i, J_{ij}^{(d)}=0}(\delta+\log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}}{Z^l(\delta,\log(\lambda_{ij}^{(d)}))} = \prod_{j\in\mathcal{A}_{\backslash i}} \frac{\exp\{(-(\delta+\log(\lambda_{ij}^{(d)})))\}}{\exp\{(-(\delta+\log(\lambda_{ij}^{(d)})))\}+1},$$

$$\frac{1}{Z^l(\delta,\log(\lambda_{ij}^{(d)}))} = \prod_{j\in\mathcal{A}_{\backslash i}} \frac{\exp\left(-(\delta+\log(\lambda_{ij}^{(d)}))\right)}{\exp\left(-(\delta+\log(\lambda_{ij}^{(d)}))\right)+1},$$

$$Z^l(\delta,\log(\lambda_{ij}^{(d)})) = \frac{1}{\prod_{j\in\mathcal{A}_{\backslash i}} \frac{\exp\left(-(\delta+\log(\lambda_{ij}^{(d)}))\right)}{\exp\left(-(\delta+\log(\lambda_{ij}^{(d)}))\right)+1}}.$$

The closed form expression for the normalizing constant under the non-empty Gibbs measure is therefore

$$Z(\delta,\lambda_i^{(d)}) = \Big(\prod_{j\in\mathcal{A}_{\backslash i}}\Big(\exp\{\delta+\log(\lambda_{ij}^{(d)})\}+1\Big)\Big)-1.$$

B Sampling Equations

B.1 Joint distribution of latent and observed tie variables

As mentioned earlier in Section 2.4, we use data augmentation in the tie generating process. Since we should include both the observed and augmented data to make inferences on the related latent variables, the derivation steps for the contribution of tie data is not as smiple as other variables. Therefore, here we provide the detailed derivation steps for the last term of joint posterior distribution in Equation (8), starting from the likelihood before integriting out the latent time $\mathcal{T}_{\mathrm{a}}$:

$$\begin{aligned}
&P(\mathcal{J}_{\mathrm{a}},\mathcal{T}_{\mathrm{a}},\mathcal{I}_{\mathrm{O}},\mathcal{J}_{\mathrm{O}},\mathcal{T}_{\mathrm{O}}|\mathcal{Z},\mathcal{C},\mathcal{B},\delta)\\
&=\prod_{d=1}^{D}P(\mathcal{J}_{\mathrm{a}}^{(d)},\mathcal{T}_{\mathrm{a}}^{(d)},i_{\mathrm{O}}^{(d)},J_{\mathrm{O}}^{(d)},t_{\mathrm{O}}^{(d)}|\mathcal{I}_{\mathrm{O}}^{(-d)},\mathcal{J}_{\mathrm{O}}^{(-d)},\mathcal{T}_{\mathrm{O}}^{(-d)},\mathcal{Z},\mathcal{C},\mathcal{B},\delta)\\
&=\prod_{d=1}^{D}P(\mathcal{J}_{\mathrm{a}}^{(d)},\mathcal{T}_{\mathrm{a}}^{(d)},i_{\mathrm{O}}^{(d)},J_{\mathrm{O}}^{(d)},t_{\mathrm{O}}^{(d)}|\mathcal{I}_{\mathrm{O}}^{(<d)},\mathcal{J}_{\mathrm{O}}^{(<d)},\mathcal{T}_{\mathrm{O}}^{(<d)},\mathcal{Z},\mathcal{C},\mathcal{B},\delta).
\end{aligned}\tag{9}$$

Note that the conditional probability only depends on the past documents $(\mathcal{I}_{\mathrm{O}}^{(<d)},\mathcal{J}_{\mathrm{O}}^{(<d)},\mathcal{T}_{\mathrm{O}}^{(<d)})$, but not on the future ones $(\mathcal{I}_{\mathrm{O}}^{(>d)},\mathcal{J}_{\mathrm{O}}^{(>d)},\mathcal{T}_{\mathrm{O}}^{(>d)})$, since the network covariates $\boldsymbol{x}_t^{(c)}$ is calculated only based on the past interaction history.

Now we tackle the problem by deriving $P(\mathcal{J}_{\mathrm{a}}^{(d)},\mathcal{T}_{\mathrm{a}}^{(d)},i_{\mathrm{O}}^{(d)},J_{\mathrm{O}}^{(d)},t_{\mathrm{O}}^{(d)}|\mathcal{I}_{\mathrm{O}}^{(<d)},\mathcal{J}_{\mathrm{O}}^{(<d)},\mathcal{T}_{\mathrm{O}}^{(<d)},\mathcal{Z},\mathcal{C},\mathcal{B},\delta)$ for d^{th} document. There are three steps involved. First is the generation of the latent receivers J_i for each i; second is the generation of the observed time increment $\Delta T^{(d)}=t^{(d)}-t^{(d-1)}$ from the observed sender-receiver pairs $(i_{\mathrm{O}}^{(d)},J_{\mathrm{O}}^{(d)})$; and the last part is the simultaneous selection process of the observed sender, receivers, and timestamp, implying that the latent time increments generated from the latent sender-receiver pairs were greater than the observed time increment. Reflecting the three steps, the joint distribution is:

$$
\begin{aligned}
&P(\mathcal{J}_{\text{a}}^{(d)}, \mathcal{T}_{\text{a}}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)} | \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&= P\Big(\text{latent receivers generation}\Big) \times P\Big(\text{latent time generation}\Big) \times P\Big(\text{choose the observed}\Big) \\
&= \prod_{i \in \mathcal{A}} \Big(J_i^{(d)} \sim \text{Gibbs measure}(\{\lambda_{ij}^{(d)}\}_{j=1}^{A}, \delta)\Big) \times \prod_{i \in \mathcal{A}} \Big(\Delta T_{iJ_i}^{(d)} \sim \text{Exp}(\lambda_{iJ_i}^{(d)})\Big) \times \prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} P\Big(\Delta T_{iJ_i}^{(d)} > \Delta T_{i_o^{(d)} J_o^{(d)}}\Big) \\
&= \Big(\prod_{i \in \mathcal{A}} \frac{1}{Z(\delta, \log(\lambda_i^{(d)}))} \exp\Big\{\log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)}\Big\}\Big) \\
&\qquad \times \Big(\prod_{i \in \mathcal{A}} \lambda_{iJ_i}^{(d)} e^{-\Delta T_{iJ_i}^{(d)} \lambda_{iJ_i}^{(d)}}\Big) \times \Big(\prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_i^{(d)}}^{(d)}}\Big) \\
&\propto \Big(\prod_{i \in \mathcal{A}} \frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{\sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)}\Big\}\Big) \\
&\qquad \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}}\Big) \times \Big(\prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} \lambda_{iJ_i}^{(d)} e^{-(\Delta T_{iJ_i}^{(d)} + \Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)}) \lambda_{iJ_i}^{(d)}}\Big),
\end{aligned}
\tag{10}
$$

We can simplify this further by integreting out the latent time $\mathcal{T}_{\text{a}}^{(d)} = \{\Delta T_{iJ_i}^{(d)}\}_{i \in \mathcal{A}_{\backslash i_o^{(d)}}}$ in the last term:

$$
\begin{aligned}
&\int_0^\infty \cdots \int_0^\infty \Big(\prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} \lambda_{iJ_i}^{(d)} e^{-(\Delta T_{iJ_i}^{(d)} + \Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)}) \lambda_{iJ_i}^{(d)}}\Big) d\Delta T_{1J_1}^{(d)} \cdots d\Delta T_{AJ_A}^{(d)} \\
&= \prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_i}^{(d)}} \Big(\int_0^\infty \lambda_{iJ_i}^{(d)} e^{-\Delta T_{iJ_i}^{(d)} \lambda_{iJ_i}^{(d)}} d\Delta T_{iJ_i}^{(d)}\Big) \\
&= \prod_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{iJ_i}^{(d)}} \Big(\Big[-e^{-\Delta T_{iJ_i}^{(d)} \lambda_{iJ_i}^{(d)}}\Big]_{\Delta T_{iJ_i}^{(d)}=0}^{\infty}\Big) \\
&= e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \sum_{i \in \mathcal{A}_{\backslash i_o^{(d)}}} \lambda_{iJ_i}^{(d)}},
\end{aligned}
\tag{11}
$$

where $\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)}$ is the observed time difference between d^{th} and $(d-1)^{th}$ document. Therefore, we can simplify Equation (11) as below:

$$
\begin{aligned}
&P(\mathcal{J}_{\text{a}}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)} | \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&\propto \Big(\prod_{i \in \mathcal{A}} \frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{\sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)}\Big\}\Big) \\
&\qquad \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \sum_{i \in \mathcal{A}} \lambda_{iJ_i^{(d)}}^{(d)}}\Big),
\end{aligned}
\tag{12}
$$

where this joint distribution can be interpreted as 'probability of latent and observed edges from non-empty Gibbs measure × probability of the observed time-increment comes from Exponential distribution × probability of all latent time greater than the observed time, given that the latent time-increments also come from Exponential distribution.' Finally for implementation, we need to compute these equations in log space to prevent underflow:

$$
\begin{aligned}
&\log\Big(P(\mathcal{J}_{\text{a}}^{(d)}, i_{\text{o}}^{(d)}, J_{\text{o}}^{(d)}, t_{\text{o}}^{(d)} | \mathcal{I}_{\text{o}}^{(<d)}, \mathcal{J}_{\text{o}}^{(<d)}, \mathcal{T}_{\text{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta)\Big) \\
&\propto \Big(\sum_{i \in \mathcal{A}} \Big(-\log\Big(\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1\Big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)}\Big)\Big) \\
&\qquad + \Big(\log(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)})\Big) - \Big(\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \sum_{i \in \mathcal{A}} \lambda_{iJ_i^{(d)}}^{(d)}\Big).
\end{aligned}
\tag{13}
$$

B.2 Resampling $\mathcal{J}_{\mathrm{a}}$

First of all, for each document d, we sample the latent sender-receiver(s) pairs as in pseudocode (Algorithm 6). That is, given the observed sender of the document $i_o^{(d)}$, we sample the latent receivers for each sender $i \in \mathcal{A}_{\backslash i_o^{(d)}}$. Here we illustrate how each sender-receiver pair in the document d is updated.

Define $\mathcal{J}_i^{(d)}$ be the $(A-1)$ length random vector of indicators with its realization being $J_i^{(d)}$, representing the latent receivers corresponding to the sender i in the document d. For each latnet sender i, we are going to resample $J_{ij}^{(d)}$, which is the j^{th} element of the receiver vector $J_i^{(d)}$, one at a time with random order. The full conditional probability of $J_{ij}^{(d)}$ is:

$$P(\mathcal{J}_{ij}^{(d)} = J_{ij}^{(d)} | \mathcal{J}_{i\backslash j}^{(d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{J}_{\mathrm{a},-i}, \mathcal{I}_{\mathrm{O}}, \mathcal{J}_{\mathrm{O}}, \mathcal{T}_{\mathrm{O}}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2), \tag{14}$$

which we can drop some independent terms and move to

$$\begin{aligned}
&P(\mathcal{J}_{ij}^{(d)} = J_{ij}^{(d)} | \mathcal{J}_{i\backslash j}^{(d)}, i_{\mathrm{O}}^{(d)}, J_{\mathrm{O}}^{(d)}, t_{\mathrm{O}}^{(d)}, \mathcal{I}_{\mathrm{O}}^{(<d)}, \mathcal{J}_{\mathrm{O}}^{(<d)}, \mathcal{T}_{\mathrm{O}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&\propto P(\mathcal{J}_{ij}^{(d)} = J_{ij}^{(d)}, \mathcal{J}_{i\backslash j}^{(d)}, i_{\mathrm{O}}^{(d)}, J_{\mathrm{O}}^{(d)}, t_{\mathrm{O}}^{(d)} | \mathcal{I}_{\mathrm{O}}^{(<d)}, \mathcal{J}_{\mathrm{O}}^{(<d)}, \mathcal{T}_{\mathrm{O}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&\propto \Big(\frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{ \log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}\Big) \\
&\qquad \times \Big(\lambda^{(d)}_{i_o^{(d)} J_o^{(d)}}\Big) \times \Big(e^{-\Delta T^{(d)}_{i_o^{(d)} J_o^{(d)}} \lambda^{(d)}_{i J_i^{(d)}}}\Big) \\
&\propto \Big(\exp\Big\{ \log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}\Big) \times \Big(e^{-\Delta T^{(d)}_{i_o^{(d)} J_o^{(d)}} \lambda^{(d)}_{i J_i^{(d)}}}\Big),
\end{aligned} \tag{15}$$

where we replace typical use of $(-d)$ to $(< d)$ on the right hand side, due to the fact that $d^{(th)}$ document only depends on the past documents. The last line of Equation (16) is obtained by dropping the terms that do not include $J_{ij}^{(d)}$, such as the normalizing constant of Gibbs measure.

To be more specific, since $J_{ij}^{(d)}$ could be either 1 or 0, we divide into two cases as below:

$$\begin{aligned}
&P(\mathcal{J}_{ij}^{(d)} = 1 | \mathcal{J}_{i\backslash j}^{(d)}, i_{\mathrm{O}}^{(d)}, J_{\mathrm{O}}^{(d)}, t_{\mathrm{O}}^{(d)}, \mathcal{I}_{\mathrm{O}}^{(<d)}, \mathcal{J}_{\mathrm{O}}^{(<d)}, \mathcal{T}_{\mathrm{O}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&\propto \exp\Big(\log(1) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{i[+j]}^{(d)} - \Delta T^{(d)}_{i_o^{(d)} J_o^{(d)}} \lambda^{(d)}_{i J_{i[+j]}^{(d)}} \Big) \\
&\propto \exp\Big(\delta + \log(\lambda_{ij}^{(d)}) - \Delta T^{(d)}_{i_o^{(d)} J_o^{(d)}} \lambda^{(d)}_{i J_{i[+j]}^{(d)}} \Big),
\end{aligned} \tag{16}$$

where $J_{i[+j]}^{(d)}$ meaning that the j^{th} element of $J_i^{(d)}$ is fixed as 1 (thus making $\log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big) = 0$ for sure). On the other hand,

$$\begin{aligned}
&P(\mathcal{J}_{ij}^{(d)} = 0 | \mathcal{J}_{i\backslash j}^{(d)}, i_{\mathrm{O}}^{(d)}, J_{\mathrm{O}}^{(d)}, t_{\mathrm{O}}^{(d)}, \mathcal{I}_{\mathrm{O}}^{(<d)}, \mathcal{J}_{\mathrm{O}}^{(<d)}, \mathcal{T}_{\mathrm{O}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&\propto \exp\Big(\log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{i[-j]}^{(d)} > 0)\big) + \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{i[-j]}^{(d)} - \Delta T^{(d)}_{i_o^{(d)} J_o^{(d)}} \lambda^{(d)}_{i J_{i[-j]}^{(d)}} \Big) \\
&\propto \exp\Big(\log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{i[-j]}^{(d)} > 0)\big) - \Delta T^{(d)}_{i_o^{(d)} J_o^{(d)}} \lambda^{(d)}_{i J_{i[-j]}^{(d)}} \Big),
\end{aligned} \tag{17}$$

where $J_{i[-j]}^{(d)}$ meaning similarly that the j^{th} element of $J_i^{(d)}$ is fixed as 0. In this case, we cannot guarantee that $\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)$ is 0 or 1, so we have to leave the term. When it is zero, $\exp\{\log\big(\mathrm{I}(\sum_{j \in \mathcal{A}_{\backslash i}} J_{ij}^{(d)} > 0)\big)\} = 0$, thus we will sample 1 with probability 1. From this property of non-empty Gibbs measure, we prevent from the instances where the sender has no recipients to send the document. Now we can use multinomial sampling using the two probabilities, Equation (17) and Equation (18), which is equivalent to Bernoulli sampling with probability $\frac{P(\mathcal{J}_{ij}^{(d)}=1)}{P(\mathcal{J}_{ij}^{(d)}=0)+P(\mathcal{J}_{ij}^{(d)}=0)}$.

B.3 Resampling $\mathcal{Z}$

Second, we resample the topic assignments, one words in a document at a time. The new values of $z_n^{(d)}$ are sampled using the conditional posterior probability of being topic k, and we derive the sampling equation by starting from the conditional distribution used in Latent Dirichlet allocation (Blei et al., 2003):

$$\begin{aligned} &P(\boldsymbol{w}^{(d)}, \boldsymbol{z}^{(d)}|\mathcal{W}_{\backslash d}, \mathcal{Z}_{\backslash d}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}) \\ &\propto \prod_{n=1}^{N^{(d)}} P(z_n^{(d)} = k, w_n^{(d)} = w|\mathcal{W}_{\backslash d,n}, \mathcal{Z}_{\backslash d,n}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}). \end{aligned} \tag{18}$$

To obtain the Gibbs sampling equation, we need to obtain an expression for $P(z_n^{(d)} = k, w_n^{(d)} = w|\mathcal{W}_{\backslash d,n}, \mathcal{Z}_{\backslash d,n}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m})$, From Bayes' theorem and Gamma identity $\Gamma(k+1) = k\Gamma(k)$,

$$\begin{aligned} &P(z_n^{(d)} = k, w_n^{(d)} = w|\mathcal{W}_{\backslash d,n}, \mathcal{Z}_{\backslash d,n}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}) \\ &\propto \frac{P(\mathcal{W}, \mathcal{Z}|\beta, \boldsymbol{u}, \alpha, \boldsymbol{m})}{P(\mathcal{W}_{\backslash d,n}, \mathcal{Z}_{\backslash d,n}|\beta, \boldsymbol{u}, \alpha, \boldsymbol{m})} \\ &\propto \frac{\prod_{k=1}^{K} \frac{\prod_{w=1}^{W} \Gamma(N_{wk}^{WK} + \beta u_w)}{\Gamma(\sum_{w=1}^{W} N_{wk}^{WK} + \beta)} \times \prod_{k=1}^{K} \frac{\Gamma(N_{k|d} + \alpha m_k)}{\Gamma(N_{\cdot|d} + \alpha)}}{\prod_{k=1}^{K} \frac{\prod_{w=1}^{W} \Gamma(N_{wk,\backslash d,n}^{WK} + \beta u_w)}{\Gamma(\sum_{w=1}^{W} N_{wk,\backslash d,n}^{WK} + \beta)} \times \prod_{k=1}^{K} \frac{\Gamma(N_{k|d,\backslash d,n} + \alpha m_k)}{\Gamma(N_{\cdot|d,\backslash d,n} + \alpha)}} \\ &\propto \frac{N_{wk,\backslash d,n}^{WK} + \frac{\beta}{W}}{\sum_{w=1}^{W} N_{wk,\backslash d,n}^{WK} + \beta} \times \frac{N_{k|d,\backslash d,n} + \alpha m_k}{N^{(d)} - 1 + \alpha}. \end{aligned} \tag{19}$$

Now, considering the modeling framework of IPTM, we re-derive the sampling equation reflecting the network effects as well:

$$\begin{aligned} &P(z_n^{(d)} = k|\mathcal{Z}_{\backslash d,n}, \mathcal{C}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{J}_{\text{a}}, \mathcal{I}_{\text{O}}, \mathcal{J}_{\text{O}}, \mathcal{T}_{\text{O}}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2) \\ &\propto P(z_n^{(d)} = k, w_n^{(d)}, \mathcal{J}_{\text{a}}^{(\geq d)}, i_{\text{O}}^{(\geq d)}, J_{\text{O}}^{(\geq d)}, t_{\text{O}}^{(\geq d)}|\mathcal{Z}_{\backslash d,n}, \mathcal{C}, \mathcal{B}, \delta, \mathcal{W}_{\backslash d,n}, \mathcal{I}_{\text{O}}^{(<d)}, \mathcal{J}_{\text{O}}^{(<d)}, \mathcal{T}_{\text{O}}^{(<d)}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}) \\ &\propto P(z_n^{(d)} = k|\mathcal{Z}_{\backslash d,n}, \alpha, \boldsymbol{m}) P(w_n^{(d)}|z_n^{(d)} = k, \mathcal{W}_{\backslash d,n}, \mathcal{Z}_{\backslash d,n}, \beta, \boldsymbol{u}) \times \prod_{d=d}^{D} P(\mathcal{J}_{\text{a}}^{(d)}, i_{\text{O}}^{(d)}, J_{\text{O}}^{(d)}, t_{\text{O}}^{(d)}|z_n^{(d)} = k, \mathcal{Z}_{\backslash d,n}, \mathcal{C}, \mathcal{B}, \delta), \end{aligned} \tag{20}$$

where the subscript "$\backslash d, n$" denotes the exclsuion of position n in d^{th} document. Note that since selecting a topic for any token influences the histories acting on all documents from d on, we use the product from d through D for the tie contribution part. From Equation (20), we know that:

$$P(z_n^{(d)} = k|\mathcal{Z}_{\backslash d,n}, \alpha, \boldsymbol{m}) = \frac{N_{\backslash d,n}^{(k|d)} + \alpha \boldsymbol{m}_k}{N^{(d)} - 1 + \alpha} \tag{21}$$

which is the well-known form of collapsed Gibbs sampling equation for LDA. We also know that

$$P(w_n^{(d)}|z_n^{(d)} = k, \mathcal{W}_{\backslash d,n}, \mathcal{Z}_{\backslash d,n}, \beta, \boldsymbol{u}) = \frac{N_{\backslash d,n}^{(w_n^{(d)}|k)} + \frac{\beta}{W}}{N_{\backslash d,n}^{(k)} + \beta}, \tag{22}$$

where $N^{(w_n^{(d)}|k)}$ is the number of tokens assigned to topic k whose type is the same as that of $w_n^{(d)}$, excluding $w_n^{(d)}$ itself, and $N_{\backslash d,n}^{(k)} = \sum_{w=1}^{W} N_{\backslash d,n}^{(w_n^{(d)}|k)}$. We already have shown in Section B.1 that

$$\begin{aligned} &P(\mathcal{J}_{\text{a}}^{(d)}, i_{\text{O}}^{(d)}, J_{\text{O}}^{(d)}, t_{\text{O}}^{(d)}|z_n^{(d)} = k, \mathcal{Z}_{\backslash d,n}, \mathcal{C}, \mathcal{B}, \delta) \\ &= \Big(\prod_{i \in \mathcal{A}} \frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{\sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)}\Big\}\Big) \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{i J_i^{(d)}}^{(d)}}\Big), \end{aligned} \tag{23}$$

where every part includes $\lambda_{ij}^{(d)}$ such that we cannot simplify any further.

Therefore, if $N^{(d)} > 0$, the conditional probability of n^{th} word in document d being topic k is:

$$P(z_n^{(d)} = k|\mathcal{Z}_{\backslash d,n}, \mathcal{C}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2)$$
$$\propto (N_{\backslash d,n}^{(k|d)} + \alpha \boldsymbol{m}_k) \times \frac{N_{\backslash d,n}^{(w_n^{(d)}|k)} + \frac{\beta}{W}}{N_{\backslash d,n}^{(k)} + \beta} \times$$
$$\prod_{d=d}^{D} \Bigg(\Big(\prod_{i \in \mathcal{A}} \frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{ \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}\Big) \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{i J_i^{(d)}}^{(d)}}\Big)\Bigg), \tag{24}$$

and if $N^{(d)} = 0$, then the first term becomes $\alpha \boldsymbol{m_k}$ and disappears because it is a constant. The second term disappears since there are no tokens, thus we just have the term remaining as below.

$$P(z_1^{(d)} = k|\mathcal{Z}_{\backslash d,1} = \emptyset, \mathcal{C}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2)$$
$$\propto \prod_{d=d}^{D} \Bigg(\Big(\prod_{i \in \mathcal{A}} \frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{ \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}\Big) \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)} e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{i J_i^{(d)}}^{(d)}}\Big)\Bigg). \tag{25}$$

B.4 Resampling $\mathcal{C}$

The next variable to resample is the topic-interaction pattern assignments, one topic at a time. We derive the posterior conditional probability for the interaction pattern $\mathcal{C}$ for k^{th} topic as below:

$$\begin{aligned} &P(c_k = c|\mathcal{Z}, \mathcal{C}_{\backslash k}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2) \\ &\propto P(c_k = c, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}|\mathcal{Z}, \mathcal{C}_{\backslash k}, \mathcal{B}, \delta) \\ &\propto P(c_k = c) P(\mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}|\mathcal{Z}, c_k = c, \mathcal{C}_{\backslash k}, \mathcal{B}, \delta) \end{aligned} \tag{26}$$

where $P(c_k = c) = \frac{1}{C}$ so this term disappears. Therefore, throughout $c_k = c$:

$$\begin{aligned} &P(c_k = c|\mathcal{Z}, \mathcal{C}_{\backslash k}, \mathcal{B}, \delta, \mathcal{W}, \mathcal{I}_\mathrm{a}, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2) \\ &\propto P(\mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}|\mathcal{Z}, c_k = c, \mathcal{C}_{\backslash k}, \mathcal{B}, \delta) \\ &= \prod_{d=1}^{D} \Bigg(\Big(\prod_{i \in \mathcal{A}} \frac{1}{\Big(\prod_{j \in \mathcal{A}_{\backslash i}} \Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1} \exp\Big\{ \sum_{j \in \mathcal{A}_{\backslash i}} (\delta + \log(\lambda_{ij}^{(d)})) J_{ij}^{(d)} \Big\}\Big) \\ &\qquad \times \Big(\lambda_{i_o^{(d)} J_o^{(d)}}^{(d)}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)} J_o^{(d)}}^{(d)} \lambda_{i J_i^{(d)}}^{(d)}}\Big)\Bigg). \end{aligned} \tag{27}$$

B.5 Resampling $\mathcal{B}$

Next, we update $\mathcal{B} = \{\boldsymbol{b}^{(c)}\}_{c=1}^C$. For this, we use the Metropolis-Hastings algorithm with a proposal density Q being the multivariate Gaussian distribution, with a diagonal covariance matrix multiplied by σ_Q^2 (proposal distribution variance parameters set by the user), centered on the current values of $\mathcal{B} = \{\boldsymbol{b}^{(c)}\}_{c=1}^C$. Under the symmetric proposal distribution, we cancel out Q-ratio and then accept the new proposed value $\mathcal{B}' = \{\boldsymbol{b}'^{(c)}\}_{c=1}^C$ with probability equal to:

$$\text{Acceptance Probability} = \begin{cases} \frac{P(\mathcal{B}'|\mathcal{Z}, \mathcal{C}, \delta, \mathcal{W}, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2)}{P(\mathcal{B}|\mathcal{Z}, \mathcal{C}, \delta, \mathcal{W}, \mathcal{J}_\mathrm{a}, \mathcal{I}_\mathrm{O}, \mathcal{J}_\mathrm{O}, \mathcal{T}_\mathrm{O}, \beta, \boldsymbol{u}, \alpha, \boldsymbol{m}, \mu_{\boldsymbol{b}}, \Sigma_{\boldsymbol{b}}, \mu_\delta, \sigma_\delta^2)} & \text{if} < 1 \\ 1 & \text{else} \end{cases} \tag{28}$$

After factorization, we get

$$
\begin{aligned}
&\frac{P(\mathcal{B}'|\mathcal{Z},\mathcal{C},\delta,\mathcal{W},\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O},\beta,\boldsymbol{u},\alpha,\boldsymbol{m},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}},\mu_\delta,\sigma_\delta^2)}{P(\mathcal{B}|\mathcal{Z},\mathcal{C},\delta,\mathcal{W},\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O},\beta,\boldsymbol{u},\alpha,\boldsymbol{m},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}},\mu_\delta,\sigma_\delta^2)}\\
=&\frac{P(\mathcal{Z},\mathcal{C},\mathcal{B}',\delta,\mathcal{W},\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\beta,\boldsymbol{u},\alpha,\boldsymbol{m},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}},\mu_\delta,\sigma_\delta^2)}{P(\mathcal{Z},\mathcal{C},\mathcal{B},\delta,\mathcal{W},\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\beta,\boldsymbol{u},\alpha,\boldsymbol{m},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}},\mu_\delta,\sigma_\delta^2)}\\
=&\frac{P(\mathcal{B}'|\mathcal{C},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}})P(\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\mathcal{Z},\mathcal{C},\mathcal{B}',\delta)}{P(\mathcal{B}|\mathcal{C},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}})P(\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\mathcal{Z},\mathcal{C},\mathcal{B},\delta)},
\end{aligned}
\tag{29}
$$

where $P(\mathcal{B}|\mathcal{C},\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}})$ is calculated from the product of $\boldsymbol{b}^{(c)} \sim$ Multivariate Normal$(\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}})$ over the interaction patterns $c \in \{1,...,C\}$ (as defined in Section 2) and $P(\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\mathcal{Z},\mathcal{C},\mathcal{B},\delta)$ is the same as Equation (28). Again, we take the log and obtain the log of acceptance ratio:

$$
\begin{aligned}
&\sum_{c=1}^{C}\log\Big(\mathcal{N}(\boldsymbol{b}'^{(c)};\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}})\Big)-\sum_{c=1}^{C}\log\Big(\mathcal{N}(\boldsymbol{b}^{(c)};\mu_{\boldsymbol{b}},\Sigma_{\boldsymbol{b}})\Big)\\
&+\sum_{d=1}^{D}\Bigg(\Big(\sum_{i\in\mathcal{A}}\Big(-\log\Big(\Big(\prod_{j\in\mathcal{A}_{\setminus i}}\Big(\exp\{\delta+\log(\lambda_{ij}^{(d)})\}+1\Big)\Big)-1\Big)+\sum_{j\in\mathcal{A}_{\setminus i}}(\delta+\log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big)\\
&\qquad+\Big(\log(\lambda^{(d)}_{i_o^{(d)}J_o^{(d)}})-\Delta T^{(d)}_{i_o^{(d)}J_o^{(d)}}\sum_{i\in\mathcal{A}}\lambda^{(d)}_{iJ_i^{(d)}}\Big)\text{ given }\boldsymbol{b}'\Bigg)\\
&-\sum_{d=1}^{D}\Bigg(\Big(\sum_{i\in\mathcal{A}}\Big(-\log\Big(\Big(\prod_{j\in\mathcal{A}_{\setminus i}}\Big(\exp\{\delta+\log(\lambda_{ij}^{(d)})\}+1\Big)\Big)-1\Big)+\sum_{j\in\mathcal{A}_{\setminus i}}(\delta+\log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big)\\
&\qquad+\Big(\log(\lambda^{(d)}_{i_o^{(d)}J_o^{(d)}})-\Delta T^{(d)}_{i_o^{(d)}J_o^{(d)}}\sum_{i\in\mathcal{A}}\lambda^{(d)}_{iJ_i^{(d)}}\Big)\text{ given }\boldsymbol{b}\Bigg),
\end{aligned}
\tag{30}
$$

where $\mathcal{N}$ is the multivariate normal density. Then the log of acceptance ratio we have is:

$$\log(\text{Acceptance Probability}) = \min(\text{Equation (31), 0}). \tag{31}$$

Use the log of acceptance ratio, if the log of a sample from Uniform(0,1) is less than the log-acceptance probability (31), we accept the proposal $\boldsymbol{b}'$. Otherwise, we reject.

B.6 Resampling δ

Finally we move on to the updates of δ, which is very similar to the steps illustrated in Section B.5. Again we use Metropolis-Hastings algorithm with Normal proposal distribution such that we can cancel out the Q-ratio. We may change the proposal variance σ_δ^2 to ensure appropriate level of acceptance rate. Then, it follows that the simplified version of acceptance probability is

$$
\text{Acceptance Probability} = \begin{cases}\frac{P(\delta'|\mu_\delta,\sigma_\delta^2)P(\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\mathcal{Z},\mathcal{C},\mathcal{B},\delta')}{P(\delta|\mu_\delta,\sigma_\delta^2)P(\mathcal{J}_\mathrm{a},\mathcal{I}_\mathrm{O},\mathcal{J}_\mathrm{O},\mathcal{T}_\mathrm{O}|\mathcal{Z},\mathcal{C},\mathcal{B},\delta)} & \text{if} < 1\\ 1 \quad \text{else}\end{cases}
\tag{32}
$$

By taking the log, we obtain the log of acceptance ratio:

$$
\begin{aligned}
&\log\Big(\mathcal{N}(\delta';\mu_\delta,\sigma_\delta^2)\Big)-\log\Big(\mathcal{N}(\delta;\mu_\delta,\sigma_\delta^2)\Big)\\
&+\sum_{d=1}^{D}\Bigg(\sum_{i\in\mathcal{A}}\Big(-\log\Big(\Big(\prod_{j\in\mathcal{A}_{\setminus i}}\Big(\exp\{\delta'+\log(\lambda_{ij}^{(d)})\}+1\Big)\Big)-1\Big)+\sum_{j\in\mathcal{A}_{\setminus i}}(\delta'+\log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big)\\
&\qquad-\sum_{i\in\mathcal{A}}\Big(-\log\Big(\Big(\prod_{j\in\mathcal{A}_{\setminus i}}\Big(\exp\{\delta+\log(\lambda_{ij}^{(d)})\}+1\Big)\Big)-1\Big)+\sum_{j\in\mathcal{A}_{\setminus i}}(\delta+\log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big)\Bigg),
\end{aligned}
\tag{33}
$$

and determine whether to accept or reject using the log of acceptance ratio

$$\log(\text{Acceptance Probability}) = \min(\text{Equation (34), 0}). \tag{34}$$

C Details on Getting It Right Test

C.1 Collapsed-time Tie Generating Process

Considering that we integrated out latent time $\mathcal{T}_{\mathrm{a}}$ in the inference, we develop the new generative process for tie data with the latent time variable integrated out. Note that this is built upon the property of the minimum of independent Exponential random variables, where the probability ΔT_{iJ_i} being the minimum is $\frac{\lambda_{iJ_i}^{(d)}}{\sum_{i=1}^{A}\lambda_{iJ_i}^{(d)}}$. Details are illustrated in Algorithm 8.

Algorithm 8 Collapsed-time Tie Generating Process

for *d=1 to D* **do**
  for *i=1 to A* **do**
    for *j=1 to A* **do**
      if $j \neq i$ **then**
        calculate $\boldsymbol{x}^{(c)}_{t_+^{(d-1)}}(i,j)$, the network statisitcs evaluated at time $t_+^{(d-1)}$
        set $\lambda_{ij}^{(d)} = \sum\limits_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T}\boldsymbol{x}^{(c)}_{t_+^{(d-1)}}(i,j)\Big\} \cdot 1\{j \in \mathcal{A}_{\backslash i}\}$
      end
    end
    draw $J_i^{(d)} \sim$ Gibbs measure$(\{\lambda_{ij}^{(d)}\}_{j=1}^{A}, \delta)$
  end
  draw $i^{(d)} \sim$ Multinomial$(\{\frac{\lambda_{iJ_i}^{(d)}}{\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\}_{i=1}^{A})$
  set $J^{(d)} = J_{i^{(d)}}$
  draw $\Delta T_{i^{(d)}J^{(d)}} \sim$ Exponential$(\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)})$
  set $t^{(d)} = t^{(d-1)} + \Delta T_{i^{(d)}J^{(d)}}$
end

With this generative process, the joint likelihood (comparable to Equation (10)) becomes:

$$
\begin{aligned}
&P(\mathcal{J}_{\mathrm{a}}^{(d)}, i_{\mathrm{o}}^{(d)}, J_{\mathrm{o}}^{(d)}, t_{\mathrm{o}}^{(d)} | \mathcal{I}_{\mathrm{o}}^{(<d)}, \mathcal{J}_{\mathrm{o}}^{(<d)}, \mathcal{T}_{\mathrm{o}}^{(<d)}, \mathcal{Z}, \mathcal{C}, \mathcal{B}, \delta) \\
&= P\Big(\text{latent receivers generation}\Big) \times P\Big(\text{choose the sender}\Big) \times P\Big(\text{observed minimum time generation}\Big) \\
&= \prod_{i\in\mathcal{A}}\Big(J_i^{(d)} \sim \text{Gibbs measure}(\{\lambda_{ij}^{(d)}\}_{j=1}^{A}, \delta)\Big) \times \Big(i_o^{(d)} \sim \text{Multinom}(\{\frac{\lambda_{iJ_i}^{(d)}}{\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\}_{i=1}^{A})\Big) \times \Big(\Delta T_{i^{(d)}J^{(d)}}^{(d)} \sim \text{Exp}(\sum_{i\in\mathcal{A}}\lambda_{i_o^{(d)}J_o^{(d)}}^{(d)})\Big) \\
&= \Big(\prod_{i\in\mathcal{A}}\frac{1}{Z(\delta, \log(\lambda_i^{(d)})}\exp\Big\{\sum_{j\in\mathcal{A}_{\backslash i}}(\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}\Big) \times \Big(\frac{\lambda_{i_o^{(d)}J_o^{(d)}}^{(d)}}{\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\Big) \times \Big((\sum_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)})e^{-\Delta T_{i_o^{(d)}J_o^{(d)}}^{(d)}\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\Big) \\
&= \Big(\prod_{i\in\mathcal{A}}\frac{1}{Z(\delta, \log(\lambda_i^{(d)})}\exp\Big\{\sum_{j\in\mathcal{A}_{\backslash i}}(\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}\Big) \times \Big(\frac{\lambda_{i_o^{(d)}J_o^{(d)}}^{(d)}}{\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\Big) \times \Big((\sum_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)})e^{-\Delta T_{i_o^{(d)}J_o^{(d)}}^{(d)}\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\Big) \\
&\propto \Big(\prod_{i\in\mathcal{A}}\frac{1}{\Big(\prod_{j\in\mathcal{A}_{\backslash i}}\Big(\exp\{\delta + \log(\lambda_{ij}^{(d)})\} + 1\Big)\Big) - 1}\exp\Big\{\sum_{j\in\mathcal{A}_{\backslash i}}(\delta + \log(\lambda_{ij}^{(d)}))J_{ij}^{(d)}\Big\}\Big) \\
&\qquad\qquad \times \Big(\lambda_{i_o^{(d)}J_o^{(d)}}^{(d)}\Big) \times \Big(e^{-\Delta T_{i_o^{(d)}J_o^{(d)}}^{(d)}\sum\limits_{i\in\mathcal{A}}\lambda_{iJ_i}^{(d)}}\Big),
\end{aligned}
\tag{35}
$$

which is exactly the same as Equation (12), thus we will use this collpased-time generative process as a forward/backward generative process in Geweke's "Getting it Right" test in Section C.2.

C.2 Backward Generating Process

For backward sampling, we let NKV be a $V \times K$ dimensional matrix where each entry will record the count of the number of tokens of word-type v that are currently assigned to topic k. Also let NK be a K dimensional vector recording the total count of tokens currently assigned to topic k. Word-assignments are implemented via collapsed Gibbs sampling (Griffiths, 2002), while the generation of tie data directly follows the generating process in Section 2.4, only with the latent time integrated out as following Algorithm 8 (in order to save computing time). This "backward" version of the generative process is detailed below in Algorithm 9.

Algorithm 9 Generate data with backward sampling

Input:
1) token topic assignments $\{\{z_n^{(d)}\}_{n=1}^{N^{(d)}}\}_{d=1}^{D}$,
2) topic interaction pattern assignments, $\{C_k\}_{k=1}^{K}$,
3) interaction pattern parameters $\{\boldsymbol{b}^{(c)}\}_{c=1}^{C}$,
4) receiver size parameter δ.

for *d=1 to D* **do**
  set $NKV = 0$ and $NK = 0$
  for *n=1 to* $\bar{N}^{(d)}$ **do**
    for *v=1 to V* **do**
      token-word-type-distribution$_n^{(d)}[v] = \frac{NKV_{v,z_n^{(d)}} + \beta \boldsymbol{u}_v}{NK_{z_n^{(d)}} + \beta}$
    end
    draw $w_n^{(d)} \sim$ (token-word-type-distribution$_n^{(d)}$)
    $NKV_{w_n^{(d)}, z_n^{(d)}} += 1$
    $NK_{z_n^{(d)}} += 1$
  end
  for *i=1 to A* **do**
    for *j=1 to A* **do**
      if $j \neq i$ **then**
        calculate $\boldsymbol{x}^{(c)}_{t_+^{(d-1)}}(i, j)$, the network statisitcs evaluated at time $t_+^{(d-1)}$
        set $\lambda_{ij}^{(d)} = \sum\limits_{c=1}^{C} p_c^{(d)} \cdot \exp\Big\{\lambda_0^{(c)} + \boldsymbol{b}^{(c)T} \boldsymbol{x}^{(c)}_{t_+^{(d-1)}}(i, j)\Big\} \cdot 1\{j \in \mathcal{A}_{\setminus i}\}$
      end
    end
    draw $J_i^{(d)} \sim$ Gibbs measure$(\{\lambda_{ij}^{(d)}\}_{j=1}^{A}, \delta)$
  end
  draw $i^{(d)} \sim$ Multinomial$(\{\frac{\lambda_{iJ_i}^{(d)}}{\sum_{i \in \mathcal{A}} \lambda_{iJ_i}^{(d)}}\}_{i=1}^{A})$
  set $J^{(d)} = J_{i^{(d)}}$
  draw $\Delta T_{i^{(d)} J^{(d)}} \sim$ Exponential$(\sum_{i \in \mathcal{A}} \lambda_{iJ_i}^{(d)})$
  set $t^{(d)} = t^{(d-1)} + \Delta T_{i^{(d)} J^{(d)}}$
end

C.3 Intitialization of History $\boldsymbol{x}_t^{(c)}$

Considering that our network statistics $\boldsymbol{x}_t^{(c)}$ are generated as a function of the network history, it is necessary to use the same initial value of $\boldsymbol{x}_t^{(c)}$ across the forward and backward samples. If not, when we generate fixed number of documents, we cannot guarantee the same number of documents used for the inference, since only the documents with its timestamp greater than 384 hours are used in the inference. In the extreme cases, we may end up with two types of failure:

1. Zero document generated after 384 hours (i.e. $t^{(10)} < 384$), making no documents to be used for inference,
2. Zero document generated before 384 hours (i.e. $t^{(1)} > 384$), making the estimate of $\mathcal{B}$ totally biased since $\forall \boldsymbol{x}_t^{(c)}(i, j) = 0$.

Therefore, we fix the initial state of $\boldsymbol{x}_t^{(c)}$ over the entire GiR process. Specifically, we fix some baseline documents where the timestamps are all smaller than 384 and use as an input for forward sampling, backward sampling, and the inference. Then, in the forward and backward generative process, we set the starting point of the timestamp as $t^{(0)} = 384$ and generate fixed number of documents given the initial $\boldsymbol{x}_{t^{(0)}=384}^{(c)}$ so that we can acheive consistency in the generated number of documents with $t^{(d)} > 384$.

C.4 GiR Implementation Details

While we tried a number of different parameter combinations in the course of testing, we outline our standard setup. We selected the following parameter values:

- D (number of documents) = 5
- $N^{(d)}$ (tokens per document) = 4
- A (number of actors)= 4
- W (unique word types) = 5
- C (number of interaction patterns) = 2
- K (number of topics) = 4
- α (Dirichlet concentration prior) = 2
- $\boldsymbol{m}$ (Dirichlet base prior) = $\boldsymbol{u}$
- β (Dirichlet concentration prior)= 2
- $\boldsymbol{n}$ (Dirichlet base prior) = $\boldsymbol{u}$
- netstat = "intercept" and "dyadic"
- prior for $\boldsymbol{b}^{(c)}$: $\mu_{\boldsymbol{b}^{(c)}} = (-3, \mathbf{0}_6), \Sigma_{\boldsymbol{b}^{(c)}} = 0.005 \times I_7$
- prior for δ: $\mu_\delta = 0$, $\sigma^2_\delta = 0.1$
- I (outer iteration) = 3
- n_1 (hyperparamter optimization) = 0
- n_2 (M-H sampling iteration of $\mathcal{B}$) = 330
- burn (M-H sampling burn-in of $\mathcal{B}$)= 30
- thin (M-H sampling thinning of $\mathcal{B}$)= 3
- σ^2_{Q1} (proposal variance for $\mathcal{B}$) = 0.04
- n_3 (M-H sampling iteration of δ) = 10
- σ^2_{Q2} (proposal variance for δ) = 2