

# Analyzing the Supreme Court Citation Network

January 2, 2018

## Abstract

## 1 Introduction

Majority opinions written by the United States Supreme Court exercise their authority and influence, in part, through their roles as precedents in future Supreme Court decisions and opinions. The findings regarding the extent and exact nature of the influences of precedent have been mixed, but the balance of the literature finds that past decisions exert some form of influence on the justices' decision making (Knight and Epstein, 1996; Gillman, 2001; Richards and Kritzer, 2002; Hansford and Spriggs, 2006; Bailey and Maltzman, 2008, 2011). Despite a considerable body of research that focuses on the way in which relevant precedents shape decision making on the Court, relatively little work has focused on understanding which past opinions are cited by an opinion. Our focus in this paper is to provide what is, to our knowledge, the first comprehensive analysis of exactly which cases are cited by a case. We follow an emerging body of work on legal citations, and treat the system of citations as a network (e.g., Fowler et al., 2007; Fowler and Jeon, 2008; Bommarito II, Katz and Zelner, 2009; Lupu and Voeten, 2012; Pelc, 2014).

We are not the first to ask what predicts the citations in US Supreme Court Opinions. Indeed, a voluminous body of work has sought to explain how many times an opinion is cited (e.g., Cross, 2010; Benjamin and Desmarais, 2012), when in a lifecycle an opinion is cited (e.g., Black and Spriggs, 2013; Spriggs and Hansford, 2001), and how many cases are cited by an opinion (e.g., Lupu and Fowler, 2013)—all focused on the US Supreme Court. One common feature of the research design in all of these studies is that the observations are at the case or case-time level. The outcome variables in these analyses are defined as measures of the number of citations to a case over a period of time, the number of citations to a case at a particular time, or a measurement on the cases cited by a case. None of these studies treats citation in its micro-level

form—as a relationship between two opinions, the citing opinion and the cited opinion. We are aware of one prior study, Clark and Lauderdale (2010), in which a statistical model is used to analyze dyadic citations between cases. However, citetclark2010locating uses a dyadic latent variable model in order to estimate ideal points for Supreme Court Opinions, but does not use any explanatory variables to predict the formation of citation ties between opinions. We build upon this literature both methodologically and substantively. Methodologically, we develop a novel statistical framework for modeling directed dyadic citations. Second, we apply this methodology to a half-century of directed dyadic citations between US Supreme Court opinions.

There exist two broad reasons why empirical analyses of citations are best defined on the directed dyad level, not the case level. The first is that directed dyadic analyses can test both dyadic and case-level hypotheses. For example, case-level analyses can model whether opinions supported by a liberal majority coalition are more likely than those supported by a conservative majority coalition to be cited heavily in the future, but they cannot precisely model whether liberal cases will be cited more by liberal cases than by conservative cases. Thus, the first reason for analyzing citations at the dyadic level is to expand the set of hypotheses that can be represented in the model. The second reason for studying citations at the directed dyadic level is that, as articulated in the growing literature on legal citation networks, citations form complex networks in which a citation at one point in time may influence future citations. This phenomenon of complex dependence is very common in networks of many types, but processes specific to Supreme Court citations create interdependence in citations. For example, if opinion  $i$  relies heavily on opinion  $j$  as precedent, opinion  $i$  is likely to discuss the legal basis for opinion  $j$ , and as a consequence cite some of the opinions cited by opinion  $j$ . Suppose opinion  $k$  is cited by opinion  $j$ . Opinion  $k$  is more likely to be cited by opinion  $i$  because opinion  $i$  relies heavily on  $j$ , and opinion  $j$  cites  $k$ . This is a special case of a very common process on networks referred to as “triad closure”. Complex dependence is theoretically interesting on its own merits, but the effects of covariates cannot be reliably identified—either in terms of coefficient values or standard errors—without accounting for the interdependence inherent in networks (Desmarais and Cranmer, 2017).

In what follows we develop a theoretical case that citations on the US Supreme Court are characterized by forms of complex dependence that are common in networks. We then develop an extension of a model—the exponential random graph model—that can incorporate both exogenous covariates and complex forms of interdependence into a directed dyadic analysis of citations. Finally, we develop and estimate a specification of this model in an analysis of US Supreme Court citations between 1937 and 2001. We find robust support for the inherent complexity underpinning the formation of citation ties, and show that incorporating complex dependence into the model of citation formation significantly improves the model’s predictive performance.

## 2 Complex Interdependence in Supreme Court Citations

When it comes to the development and testing of theory, the defining feature of networks is that the fundamental element under study—the relationship between two units (i.e., the citation from one opinion to another) is a piece of a complex web of relations. The formation (or lack thereof) of that relationship cannot be fully understood without considering how the relationship fits into the complex web. Analytical designs that account only for covariates in explaining tie formation are incomplete theoretically, and, as a consequence, are subject to a form of omitted variable bias [CITE ISQ]. Citations in legal opinions are unique in terms of the windows into network dependencies offered by the texts of the opinions. A number of common structural dependencies that are found in networks are likely to apply to citations in Supreme Court opinions. In this section we present these dependence forms, and document the mechanisms by which they arise through quotations in archetypal example opinions.

**Transitivity:** In a network of directed relations (e.g., A cites B, but B doesn’t cite A) transitivity refers to the tendency for A to send a tie to C if A sends a tie to B and B sends a tie to C (Holland and Leinhardt, 1971). In undirected networks, transitivity is simply the process by which friends of friends become friends (i.e., a friend of a friend is a friend). The term, “transitive closure” refers to a tie forming from A to C in response to extant ties from A to B and B to C. When writing opinions, Supreme Court justices present the legal bases for their rulings, which often involves discussing the most primary/relevant precedents underpinning these legal bases, but also the precedents and legal rules on which the primary precedents were based. This process of presenting several layers/levels of precedent in an opinion follows the structure of transitive closure exactly—opinion A cites opinion B as a primary precedent, and then cites opinion C because opinion B cites opinion C. The two examples presented below illustrate this process.

In the first example, a passage from *Kansas v. Marsh* (548 U.S. 163, 2006)—a case considering the constitutionality of a death sentence statute in Kansas. In this example, the case *Stringer v. Black* is cited by *Kansas v. Marsh* as a case that is quoted by *Sochor v. Florida*. The primary precedent under discussion in this passage of the opinion is *Sochor v. Florida*, but *Stringer v. Black* is cited as a result of its role in the *Sochor v. Florida* opinion.

The statute thus addresses the risk of a morally unjustifiable death sentence, not by minimizing it as precedent unmistakably requires, but by guaranteeing that in equipoise cases the risk will be realized, by “placing a ‘thumb [on] death’s side of the scale,’ ” *Sochor v. Florida*, 504 U. S. 527, 532 (1992) (quoting *Stringer v. Black*, 503 U. S. 222, 232 (1992); alteration in original).

The second example is a passage from *Seminole Tribe of Fla. v. Florida*, (517 U.S. 44 1996)—a case addressing the rights of groups and citizens to sue states in federal court. In this example,

*Pennsylvania v. Union Gas Co* (491 U.S. 1, 1989) is the primary precedent under discussion, and several cases are cited and discussed in terms of their roles as precedents in the *Union Gas* opinion.

Never before the decision in *Union Gas* had we suggested that the bounds of Article III could be expanded by Congress operating pursuant to any constitutional provision other than the Fourteenth Amendment. Indeed, it had seemed fundamental that Congress could not expand the jurisdiction of the federal courts beyond the bounds of Article III. *Marbury v. Madison*, 1 Cranch 137 (1803). The plurality's citation of prior decisions for support was based upon what we believe to be a misreading of precedent. See *Union Gas*, 491 U. S., at 40-41 (SCALIA, J., dissenting). The plurality claimed support for its decision from a case holding the unremarkable, and completely unrelated, proposition that the States may waive their sovereign immunity, see *id.*, at 14-15 (citing *Parden v. Terminal Railway of Ala. Docks Dept.*, 377 U. S. 184 (1964)), and cited as precedent propositions that had been merely assumed for the sake of argument in earlier cases, see 491 U. S., at 15 (citing *Welch v. Texas Dept. of Highways and Public Transp.*, 483 U. S., at 475-476, and *n. 5*, and *County of Oneida v. Oneida Indian Nation of N. Y.*, 470 U. S., at 252).<sup>7</sup>

**Reciprocity:** Reciprocity (also referred to as mutuality) is the tendency for node B to send a tie to node A in response to or coordination with A sending a tie to B (?). It is typically not possible for reciprocated ties to form in legal opinions. Most citations reference past opinions that were issued before the citing opinion's case was even argued before the Court. However, opinions written within the same Supreme Court term are often drafted in tandem, and can cite each other reciprocally. Opinion A citing opinion B within the same term represents a signal that opinion B is relevant to the legal reasoning underpinning opinion A. Unlike the citations themselves, the applicability of legal rules or lines of reasoning across cases is not directed—if A is relevant to B, B is highly likely to be relevant to A. We expect opinions written within the same term to exhibit a high degree of reciprocity. Below we provide two example passages from opinions that illustrate the phenomenon of within-term reciprocity.

The first case in our example reciprocal dyad is a passage from *Western Air Lines v. Criswell* (472 U.S. 400, 1985)—a case considering mandatory retirement in the context of age discrimination laws. The second case in the dyad, *Johnson v. Mayor of Baltimore* (472 U.S. 353, 1985) is another case considering whether mandatory retirement violates the Age Discrimination in Employment Act. These cases addressed very similar legal questions, which increased the likelihood that they would inform each other, and the opinions were written within the same term, which made it possible for them to cite each other.

*From Western Air Lines:* On a more specific level, Western argues that flight engi-

neers must meet the same stringent qualifications as pilots, and that it was therefore quite logical to extend to flight engineers the FAA's age 60 retirement rule for pilots. Although the FAA's rule for pilots, adopted for safety reasons, is relevant evidence in the airline's BFOQ defense, it is not to be accorded conclusive weight. *Johnson v. Mayor and City Council of Baltimore*, ante at 472 U. S. 370-371. The extent to which the rule is probative varies with the weight of the evidence supporting its safety rationale and "the congruity between the . . . occupations at issue." Ante at 472 U. S. 371. In this case, the evidence clearly established that the FAA, Western, and other airlines all recognized that the qualifications for a flight engineer were less rigorous than those required for a pilot.

*From Johnson:* The city, supported by several amici, argues for affirmance nonetheless. It asserts first that the federal civil service statute is not just a federal retirement provision unrelated to the ADEA, but in fact establishes age as a BFOQ for federal firefighters based on factors that properly go into that determination under the ADEA, see *Western Air Lines, Inc. v. Criswell*, post p. 472 U. S. 400. Second, the city asserts, a congressional finding that age is a BFOQ for a certain occupation is dispositive of that determination with respect to nonfederal employees in that occupation.

**Popularity** Popularity, also termed "preferential attachment" is the tendency for ties to be sent to nodes to which many ties have already been sent [CITE FROM SYLLABUS]. Citations to an opinion signal both the Court's awareness of the legal reasoning of the case and the Court's evaluation that the opinion is an authoritative precedent. The more citations, the stronger this signal. Landmark cases, or those that establish new legal rules, are particularly authoritative and accrue citations from most future opinions that follow the respective line of reasoning. The passage below, from XXXX, in which an authoritative opinion is referenced, and even discussed in terms of the number of other cases by which it was followed. We consider each of these contentions in turn. Based on this popularity phenomenon, we expect that new citations will be disproportionately directed at cases to which many past citations have accrued.

"The first case in which this Court struck down a statute under the Equal Protection Clause of the Fourteenth Amendment was *Strauder v. West Virginia*, 100 U. S. 303, decided in the 1879 Term. [Footnote 2/1] In the 1961 Term, we squarely held that the manner of apportionment of members of a state legislature raised a justiciable question under the Equal Protection Clause, *Baker v. Carr*, 369 U. S. 186. That case was followed by numerous others, e.g.: that one person could not be given twice or 10 times the voting power of another person in a state-wide election merely because he lived in a rural area..."

<https://supreme.justia.com/cases/federal/us/400/112/case.html>

**Sociality** Sociality is the tendency for ties sent to beget more ties sent [CITE]. Structurally, this means that there are few nodes that send a moderate number of ties—the number of ties sent is either small or fairly large. In terms of Supreme Court opinions, for each opinion discussed, that discussion is likely to raise other tangential issues on which the justices will want to draw upon past opinions. Furthermore, for each opinion that applies to and is cited by the current opinion, there is often a case/opinion that needs to be discussed in terms of why it does not apply. Justices often clarify not only those legal principles that apply, but often those that do not. The passages below represent examples of this process of citation proliferation, or sociality.

"1. The "overbreadth" doctrine is not applicable here. There is nothing in the record to indicate that §28.04 will have any different impact on any third parties' interests in free speech than it has on appellees' interests, and appellees have failed to identify any significant difference between their claim that §28.04 is invalid on overbreadth grounds and their claim that it is unconstitutional when applied to their signs during a political campaign. Thus, it is inappropriate to entertain an overbreadth challenge to §28.04. Pp. 466 U. S. 796-803."

<https://supreme.justia.com/cases/federal/us/466/789/>

"No allegations of racial or class-based invidiously discriminatory animus are required to establish a cause of action under the first part of 1985(2). The statutory provisions now codified at §1985 were originally enacted as §2 of the Civil Rights Act of 1871, and the substantive meaning of the 1871 Act has not been changed. The provisions relating to institutions and processes of the Federal Government (including the first part of §1985(2)) – unlike those encompassing activity that is usually of primary state concern (including the second part of §1985(2) and the part of ¶1985(3) involved in Griffin, supra – contain no language requiring that the conspirators act with intent to deprive their victims of the equal protection of the laws. Thus, the reasoning of Griffin is not applicable here, and, given the structure of §2 of the 1871 Act, it is clear"

<https://supreme.justia.com/cases/federal/us/460/719/>

### 3 Network Approaches to Studying Citations

Researchers from several fields have used network analysis to analyze citations—legal, patent, and scientific. Before describing our approach to modeling the Supreme Court citations, we review the methods researchers have previously used to study citations. A citation is a directed link between two documents that indicates the citing document attributes the cited to be relevant to the evidentiary basis of the citing document. Collective patterns of citations within a certain domain make up citation networks. When raw citation data is formally represented as a network, the nodes are usually the documents themselves, and each directed arc is the existence of one or more cites from the sending to the receiving document. Given the permanent nature of documents in citations networks, these networks are acyclic (Leicht et al., 2007; Karrer and

Newman, 2009). Relatedly, citation networks grow as new documents and citations enter the network, but established arcs persist in all but exceptional cases.

[HERE] In its raw form a citation is a link between two documents, but it is the result of human judgment about the utility of the cite (Fowler and Jeon, 2008). Studies of citation networks can be broadly categorized according to whether they treat this set of observed behavioral patterns as information that can be used to understand different aspects of the system, or treat them as outcomes to be modeled. We discuss these in turn.

First, as these arcs are observed patterns of behaviour, they can be used to infer latent characteristics of nodes or the relations between nodes that produced them (Batagelj, 2003). Moreover, since each judgment is embedded in the broader network of citations, which influences the judgment beyond the characteristics of the two directly linked documents, inferences about the neighborhood of these two documents are possible as well. A common application of this general logic is to obtain local or global network indices, such as Kleinberg centrality at the node level (Kleinberg, 1999; Fowler et al., 2007) or global hierarchy (Mones, Pollner and Vicsek, 2014), then use them to draw conclusions about networks or compare networks across contexts such as time periods (e.g. Vazquez, 2001; Fowler and Jeon, 2008; Greenberg, 2009; Lupu and Voeten, 2012; Lupu and Fowler, 2013; Dawson et al., 2014; Jaffe and de Rassenfosse, 2017).

Still within the category of using observed network constellation to make inferences about components of the system, identification of communities using methods for clustering, such as modularity maximization (e.g. Kajikawa et al., 2007; Shibata et al., 2011; Chen and Redner, 2010) and stochastic block modeling (Jo et al., 2009); and subnetwork identification (e.g. Batagelj, Ferligoj and Squazzoni, 2017) have been used to analyze relevance of topics within an academic field or to determine trends in technological advancement (Verspagen, 2007; Érdi et al., 2013). Main path analysis (Hummon and Dereian, 1989), which determines the main path through an acyclic network, is a particularly useful way to examine trends as it allows researchers identify structures of knowledge flow. Other methods include classifying documents into groups based on similarities in their citation profiles of cites over time, which can be examined as a function of time to see if there are temporal patterns (Leicht et al., 2007). The identification of communities is often coupled with classification of these communities by the researcher using historical knowledge.

Researchers are also interested in understanding the mechanisms that drive citations between documents. Work in this area generally proceed by specifying a set of citation behaviour, then proposing a model to capture the combination of these behavioural rules (Simkin and Roychowdhury, 2007). Assessment of the model is based on how well it fit citation distributions. Researchers working in this area tend to focus on modeling the growth of the citation network as governed by a degree-based mechanism such as preferential attachment (Barabási and Albert, 1999) or the recursive search (Vazquez, 2001), and the age of the paper (Jeong, Néda and

Barabási, 2003; Eom and Fortunato, 2011; Wang, Song and Barabási, 2013). Regarding age, the previous standard approach was to treat the probability of citation as a separable function of degree and age (Hajra and Sen, 2005, 2006), then examine the distribution of citations, scaled to the rate of new publications, by age. More recently, work has been done in relaxing the assumption that the effect of degree is static, instead allowing it to vary with time (Wang, Yu and Yu, 2008).

Another approach to understanding the generative process of citation networks is to examine the existence of network motifs, or subnetwork structures, that can be interpreted as measuring different generative mechanisms. One way of doing so is to compare the observed network statistics to a null distribution. This null model must also be characterized by features of citation networks discussed earlier, meaning that it must be a directed acyclic graph with unweighted arcs (Carstens, 2016). Two sets of uniform random graph models have been proposed for directed acyclic graphs. In the former, Karrer and Newman (2009) introduce two uniform random graph models based respectively on fixed-degree sequence and fixed-expected degree sequence. Both models preserve the requisite characteristics of citation networks with the exception that they can introduce weighted edges with low probability, which leads to networks from the null distribution to be sampled with nonequal probability. Carstens (2014, 2016) introduces a model that preserves the requisite characteristics and also do not introduce weighted arcs. Loosely related is an egocentric framework, introduced by Vu et al. (2011), that models continuous-time network data through a multivariate counting process, which can be used to count network motifs.

## 4 Extra Notes

This section contains some of the extra notes I cut out from the draft that might be useful.

### 4.1 Broader definitions of citation networks

More broadly, citation networks can have as nodes the document producers, including scientists (Ji, Jin et al., 2016), judges (Landes, Lessig and Solimine, 1998), journals (Rice, Borgman and Reeves, 1988), or aggregated units (Gelter and Siems, 2012; Pan, Kaski and Fortunato, 2012). In these cases, which are essentially exercises in aggregation, the resulting network can take on different characteristics such as weighted or cyclic edges. Other networks derived from the raw citation network is the co-citation or co-cited-by networks whereby two documents are connected if they co-cite or are co-cited by another document (Van Raan, 2005).



## 4.2 Weighted edges and multiplexity

Recognition, and subsequent quantification, of different levels of complexity in citation arcs allows for a host of advancements in citation network analysis. Weights can indicate relevance (Liu et al., 2014) or be used to incorporate the temporal dimension into citation networks (Fujita et al., 2014). Moving to multiplexity leads to further advancements. For example, Greenberg (2009) accounted for level of support and through simulations based on arc typed switching demonstrated how citation bias against critical articles yielded differences in beliefs within a medical discipline. Bommarito et al. (2010) argued that articles contain different types of information and accounting for the specific piece of information that resulted in the cite can yield clustering that are more interpretable. In all cases however, substantial work has to be done into reclassifying the weightedness or multiplexity of citation arcs (Zhang and Koppaka, 2007).

## 4.3 Misc. notes

- To control for age, Clough et al. (2015) propose transitive reduction of citation networks (i.e. removal of redundant information ties) as it will primarily remove citation arcs that are disparate in age.
- Most citation networks are directed acyclic graphs, but not all. Different versions of the same document can also cause problems (such as strong network components). The “preprint transformation” is a solution to small strong components (Batagelj, Ferligoj and Squazzoni, 2017).
- Date of publication can be assigned as level to nodes in acyclic networks (Batagelj, Ferligoj and Squazzoni, 2017). Depending on the citation network at hand, temporal ordering can be difficult, as documents can share publication dates. To overcome this, begin each iteration by sampling from one of the possible orderings. (Carstens, 2016).

## 5 The Citation Temporal Exponential Random Graph Model

We build upon existing methods for network analysis to define a model that fits the constraints to which Supreme Court citation networks are subject. The Supreme Court citation network is nearly a directed acyclic graph—a graph in which there can be no loops/cycles (e.g., ties cannot be reciprocated, which would constitute a two-node loop). If two cases are decided in different terms, the later case can cite the earlier case, but not the reverse. However, as noted above, if two opinions are written in the same term, they can cite each other reciprocally. We implement a version of the temporal exponential random graph model (TERGM)—a model that is commonly

used for longitudinal network data [CITES]—that is subject to the constraints specific to the Supreme Court citation network. The TERGM is a model that can simultaneously incorporate structural dependencies such as transitivity and reciprocity and exogenous covariates (e.g. the issue area of the citing case, the ideological distance between the opinion authors of the two cases in the dyad) to explain tie formation. If the ties do not depend upon each other, and the coefficients associated with the structural dependencies are either assumed or estimated to be zero, the TERGM reduces to a logistic regression in which the observation is a directed dyad [CITE OLD PA]. In this section we describe the citation TERGM (c-TERGM), and explain estimation methods.

Let  $C$  be the case-to-case adjacency matrix such that  $C_{ij} \in \{0, 1\}$  is a binary indicator of whether case  $i$  cites  $j$ . Furthermore, let

$$\mathcal{C} = \{C \in \{0, 1\}^{(N \cdot (N-1))/2} : C_{ij} \in \{0, 1\}\}$$

be the set of all possible adjacency matrices among  $N$  cases. Note that the cardinality of  $\mathcal{C}$  increases exponentially for every newly added case. The probability function of the c-TERGM is defined as

$$P_{\theta}(C) = \frac{\exp(\theta^T \cdot h(C))}{\sum_{Z^* \in \mathcal{C}} \exp(\theta^T \cdot h(Z^*))} \quad (1)$$

where  $\theta \in \mathbb{R}^q$  is a  $q$ -dimensional vector of parameters,  $h : \mathcal{C} \rightarrow \mathbb{R}^q$  is a  $q$ -dimensional vector of different statistics and  $\kappa(\theta) := \sum_{Z^* \in \mathcal{C}} \exp(\theta^T \cdot h(Z^*))$  is a normalization constant that ensures that Equation (1) defines a probability function on  $\mathcal{C}$ .

The c-TERGM is specified through the decision regarding which network statistics  $h(\cdot)$  to incorporate. We include the following statistics for the Supreme Court citation network, which are derived from the interdependence hypotheses described in the previous section:

$$h_{edges} : \mathcal{C} \rightarrow \mathbb{R} \quad , \quad C \rightarrow \sum_{ij} C_{ij},$$

the number of citations.  $h_{edges}$  performs the function of an intercept, and models the expected value of any given edge.

$$h_{outstar} : \mathcal{C} \rightarrow \mathbb{R} \quad , \quad C \rightarrow \sum_{i=1}^N \binom{\sum_{j \neq i} C_{ij}}{2}$$

the number of out-two-stars. An out-two-star is a configuration in which one node sends to two other nodes. The number of these configurations grows quadratically as the origins of ties concentrate on a few highly active/social senders. The out-two-stars configuration is commonly

used to model sociality in ERGMs.

$$h_{instar} : \mathcal{C}_t \rightarrow \mathbb{R} \quad , \quad C \rightarrow \sum_{j=1}^N \binom{\sum_{i \neq j} c_{ij}}{2}$$

the number of in-two-stars. An in-two-star is a configuration in which one node receives ties from two other nodes. The number of these configurations grows quadratically as the destinations of ties concentrate on a few highly popular recipients. The out-two-stars configuration is commonly used to model popularity in ERGMs.

$$\begin{aligned} h_{triangle} : \mathcal{C} \rightarrow \mathbb{R} \quad , \quad C \rightarrow & \sum_{j < i < k} C_{ij} \cdot C_{jk} \cdot C_{ki} + C_{ji} \cdot C_{jk} \cdot C_{ki} + C_{ij} \cdot C_{kj} \cdot C_{ki} \\ & + C_{ji} \cdot C_{kj} \cdot C_{ki} + C_{ij} \cdot C_{jk} \cdot C_{ik} + C_{ji} \cdot C_{jk} \cdot C_{ik} \\ & + C_{ij} \cdot C_{kj} \cdot C_{ik} + C_{ji} \cdot C_{kj} \cdot C_{ik}, \end{aligned}$$

The number of triangles in the network. Triangles measure triad closure, as discussed above.

$$h_{covariate} : \mathcal{C} \rightarrow \mathbb{R} \quad , \quad C \rightarrow \sum_{ij} C_{ij} X_{ij},$$

the effect of an exogenous covariate ( $X$ ). We include several exogenous covariates based on this standard statistic formulation. These covariates are discussed below.

The structure of this model is very similar to a conventional (T)ERGM setup. The main difference is that  $\mathcal{C}$  excludes adjacency matrices in which there are loops that include edges sent at different time points. Loops can exist in the Supreme Court citation network, but only among cases decided in the same term. We estimate the c-TERGM via bootstrapped pseudolikelihood, as introduced by [CITE PHYSICA].

## 6 Correlates of Legal Citations

Theorizing that legal citations arise through network interdependence processes represents a potential contribution to the literature. To demonstrate the value of our contribution we must also incorporate established non-network processes that have been found to affect legal citations. We incorporate these processes through the specification of  $h_{covariate}$  terms to be included in the c-TERGM.

We incorporate a statistic to model the degree to which cases cite those that are similar in terms of the ideological positions of the justices who supported the decision. We account for this effect following citetspriggs2001explaining, who find that cases are more likely to be overruled when the Court is ideologically distant from the median justice in the majority coalition that

decided the case. Clark and Lauderdale (2010) estimates a latent coordinate model of Supreme Court opinions based on the network of case-to-case citations. They find that the majority opinion falls at the ideal point of the median member of the majority coalition in the case. We include a covariate term in which  $X_{ij}$  is the absolute difference between the Martin-Quinn [CITE MQ] scores of the median justices in the majority coalitions for cases  $i$  and  $j$ . We expect this variable to have a negative effect, which would correspond to cases citing those to which they are ideologically similar.

We include two sets of dummy variables that account for the issue areas of cases. In one set of dummy variable—sender intercepts—the variable  $X_{ij}$  indicates the issue area of the sending case ( $i$ ). In the set of receiver intercepts  $X_{ij}$  indicates the issue area of the receiver case. Issue area data comes from the Supreme Court Database (SCDB) [CITE SCDB]. We include these variables because Cross (2010) find that the number of citations to Supreme Court opinions depends heavily on the issue area of the case.

We model the way in which citations to a case depend upon the age of a case. For this we use a second-order polynomial in which one covariate  $X_{ij}$  is defined as the age of case  $j$  at the time that case  $i$  is decided, and another term in which  $X_{ij}$  is the squared age of case  $j$  at the time that case  $i$  is decided. We include these covariates because Black and Spriggs (2013) find that the number of citations to a Supreme Court case over time depends significantly on the age of the case, characterized by a sharp drop off and leveling out with age.

Benjamin and Desmarais (2012) Study the propensity for cases to be overruled and cited in other negative ways. They find that cases with majority coalitions that are large and ideologically broad are less likely to be cited negatively. In our data we do not differentiate between negative and positive citations, but since the overwhelming majority of citations are positive, we hypothesize that the effects they found will be reversed in our analysis. We include one covariate in which  $X_{ij}$  is the number of justices in the majority coalition for case  $j$ . We also include another covariate in which  $X_{ij}$  is the absolute difference between the maximum and minimum ideal points of the justices in the majority coalition for case  $j$ . We expect both of these covariates to have positive effects.

## 7 Results

### Descriptive Results

The supreme court citation network from 1937 – 2005 consists of 8817 cases which got voted at 2116 different time points. The network has a total of 93,263 ties, of which 452 are mutual. The number of triangles in the network is 211,855. The in- and outdegree distribution is visualized in figures ?? and ?. The maximum indegree is 190 and the maximum outdegree is 159.

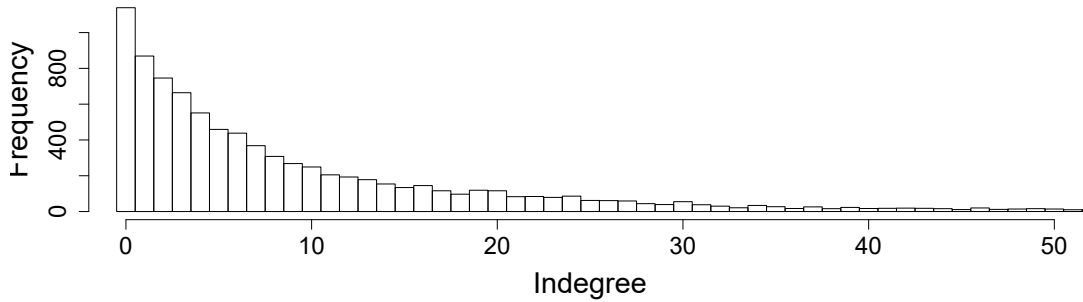
	Terms	Total Number Cases	Cases/Term
CE Hughes*	1937 - 1941	629	125.5
HF Stone	1942 - 1946	766	153.2
FM Vinson	1946 - 1953	788	98.5
E Warren	1954 - 1969	2159	127.0
WE Burger	1970 - 1986	2805	155.8
W Rehnquist**	1987 - 2001	1670	83.5

Table 1: For the time range of interest (1937 - 2001) this table displays the chief justices, the time range they served as chief justice, the number of cases in their time range as well as the average number of cases per year.

\* CE Hughes served as chief justice from 1930 - 1941.

\*\* W Rehnquist served as chief justice from 1987 - 2005.

## Indegree Distribution



## Outdegree Distribution

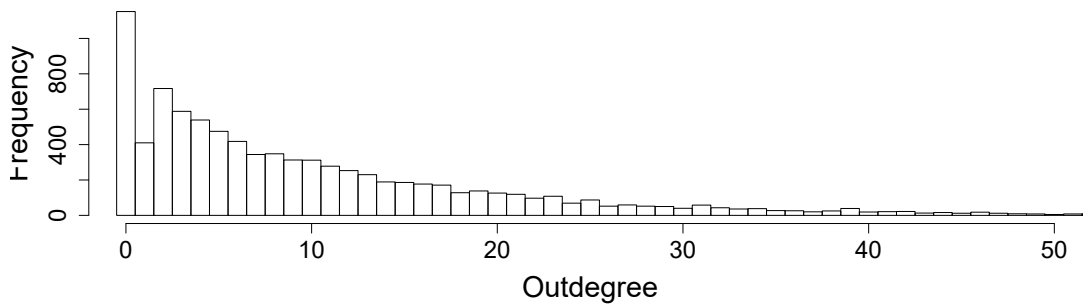


Figure 1: The in- and outdegree distribution of the Supreme Court Citation Network from 1937 - 2001. There are cases with an indegree  $>50$ , but they are not captured in this figure.

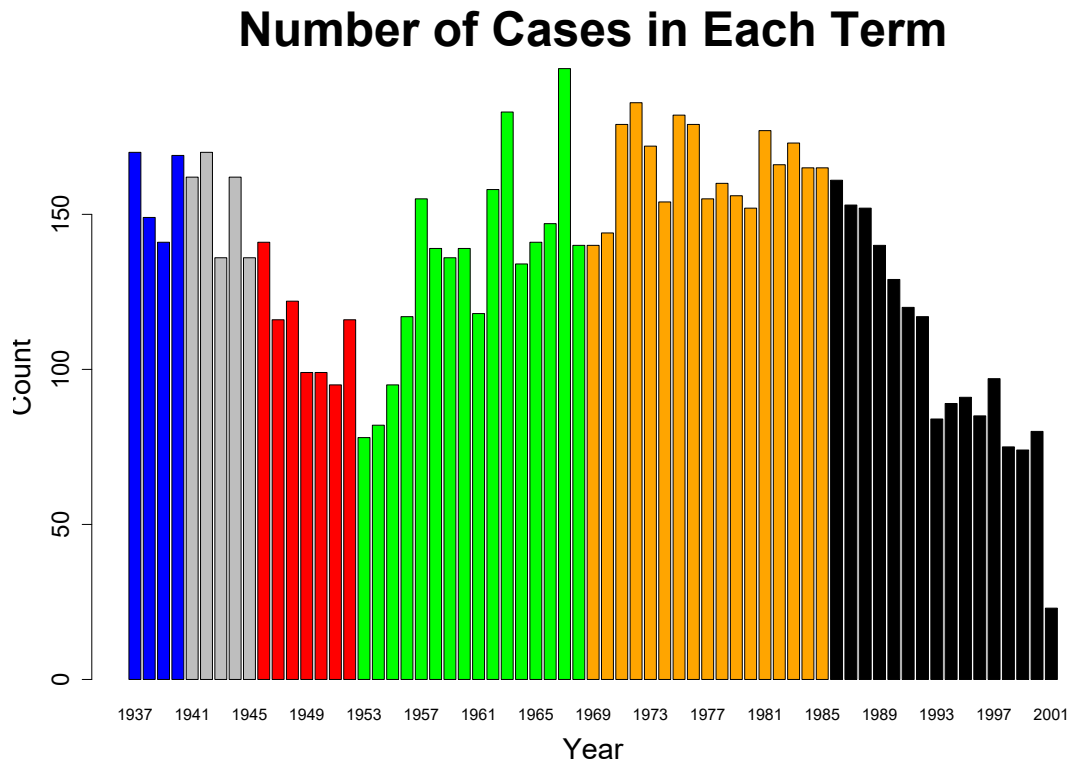


Figure 2: Number of cases in each term. Different colors indicate different chief justices.

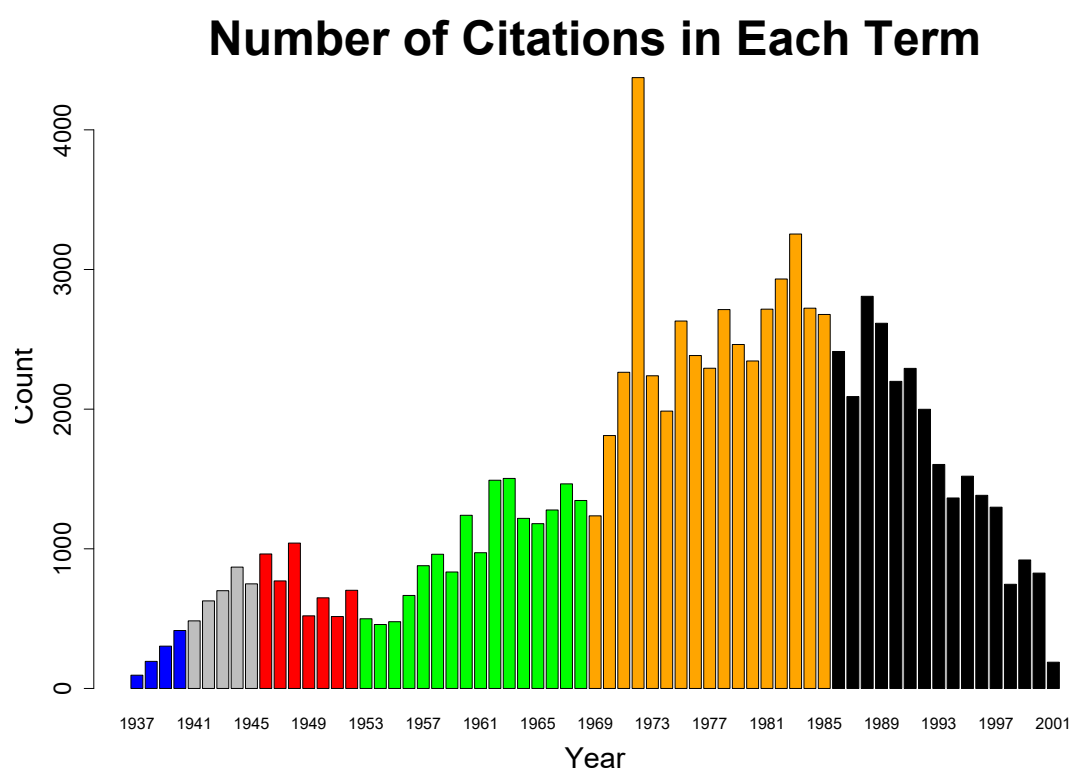


Figure 3: Number of citations for the 1937-2001 time period. Citations for cases prior 1937 are not considered in this figure. Different colors indicate different chief justices.

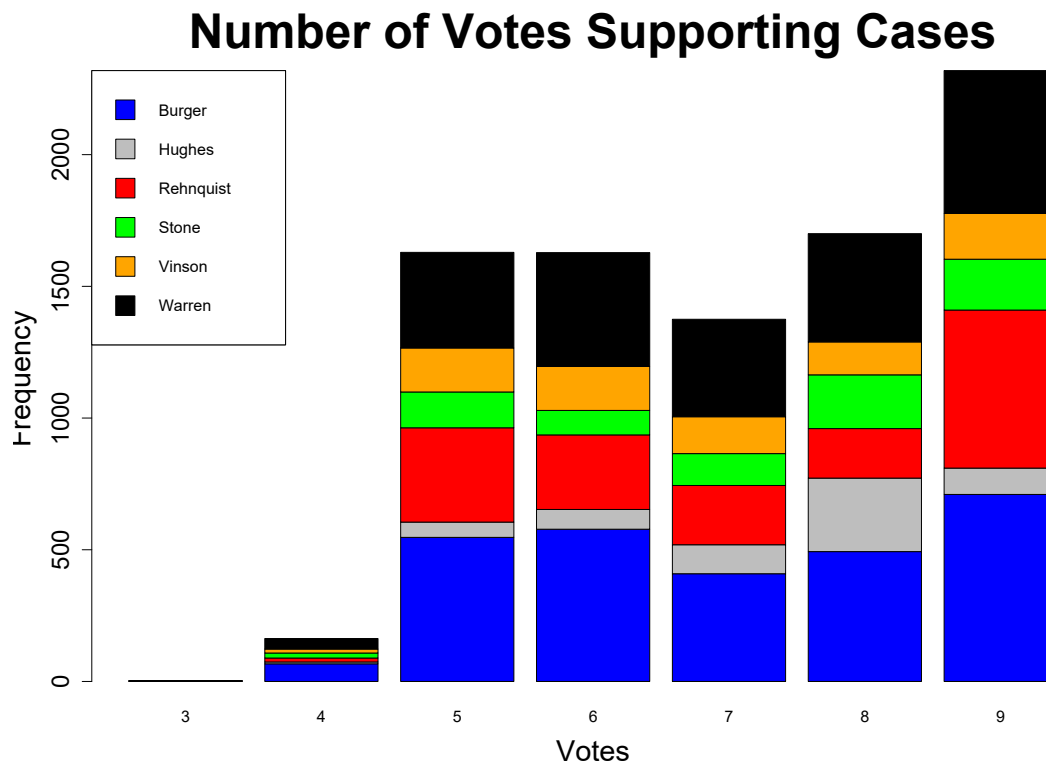


Figure 4: Number of Votes that were supporting cases between 1937-2001. Different colors indicate terms with different chief justices.



## 7.1 Inferential Results

### Model updates

- Add sender issue area and receiver issue area node covariates (i.e., nodeifactor and nodeofactor applied to issue area).
- In the GLM, multiply each variable by the sender time covariate.
- Write the bootstrap coefficients to a file, send to Bruce so he can write up a code for summarizing the over-time trends in effects.
- Add a receiver variable (i.e., nodeicov) that equals the absolute difference between the maximum MQ score of a justice in the majority and a minimum MQ score of a justice in the majority.
- Add a receiver variable (nodeicov) equivalent to the number of justices in the majority

## 7 Results

	Estimate	Lower Bound	Upper Bound	Significance
Edges	-5.533	-5.733	-5.388	*
Instar(2)	0.031	0.027	0.036	*
Outstar(2)	0.022	0.020	0.032	*
Mutual	3.316	2.622	3.983	*
Triangle	1.490	1.410	1.560	*
Martin Quinn Score	0.080	0.022	0.126	*
Same Issue Area Homophily	1.378	1.313	1.451	*
Year Difference	-0.077	-0.090	-0.064	*
(Year Difference) <sup>2</sup>	0.0029	0.0025	0.0032	*
Receiver Abs Diff MQ Score in Majority	-0.030	-0.048	-0.014	*
Receiver Number Justices in Majority	-0.115	-0.135	-0.089	*
Receiver Sender Year	0.0007	0.0006	0.0009	*
Sender Same Issue Area 2	0.162	0.105	0.217	*
Sender Same Issue Area 3	-0.317	-0.403	-0.236	*
Sender Same Issue Area 4	0.497	0.426	0.570	*
Sender Same Issue Area 5	0.332	0.174	0.495	*
Sender Same Issue Area 6	0.527	0.402	0.635	*
Sender Same Issue Area 7	0.327	0.251	0.394	*
Sender Same Issue Area 8	0.076	0.021	0.129	*
Sender Same Issue Area 9	0.261	0.209	0.315	*
Sender Same Issue Area 10	0.369	0.301	0.432	*
Sender Same Issue Area 11	-0.046	-0.233	0.151	
Sender Same Issue Area 12	0.0073	-0.075	0.089	
Sender Same Issue Area 13	0.381	0.162	0.564	*
Sender Same Issue Area 14	0.151	-0.019	0.300	
Receiver Same Issue Area 2	0.231	0.166	0.300	*
Receiver Same Issue Area 3	-0.101	-0.209	-0.006	*
Receiver Same Issue Area 4	0.516	0.425	0.612	*
Receiver Same Issue Area 5	0.351	0.183	0.534	*
Receiver Same Issue Area 6	0.461	0.284	0.628	*
Receiver Same Issue Area 7	0.361	0.281	0.437	*
Receiver Same Issue Area 8	0.153	0.097	0.221	*
Receiver Same Issue Area 9	0.279	0.221	0.346	*
Receiver Same Issue Area 10	0.492	0.413	0.570	*
Receiver Same Issue Area 11	0.247	0.056	0.411	*
Receiver Same Issue Area 12	0.339	0.246	0.430	*
Receiver Same Issue Area 13	1.060	0.845	1.239	*
Receiver Same Issue Area 14	0.557	0.255	0.793	*
Instar(2) $\times$ Sender Year	-0.00025	-0.00034	-0.00017	*
Outstar(2) $\times$ Sender Year	-0.00036	-0.00056	-0.00028	*
Mutual $\times$ Sender Year	-0.010	-0.043	0.029	
Triangle $\times$ Sender Year	-0.0004	-0.0021	0.0016	
Martin Quinn Score $\times$ Sender Year	-0.0025	-0.0035	-0.0013	*
Same Issue Area $\times$ Sender Year	-0.0058	-0.0076	-0.0043	*
Year Difference $\times$ Sender Year	0.0004	0.0002	0.0007	*
Year Difference <sup>2</sup> $\times$ Sender Year	-0.000037	-0.000044	-0.000030	*
MQ Score in Majority $\times$ Sender Year	0.0013	0.0009	0.0017	*
Justices in Majority $\times$ Sender Year	0.0029	0.0023	0.0035	*

Table 2: Bootstrapped MPLE Results for the time period 1937 – 2001. A ‘\*’ indicates that the 2.5th and 97.5th quantile of the variable does not include ‘0’ and as a result is statistically significant.

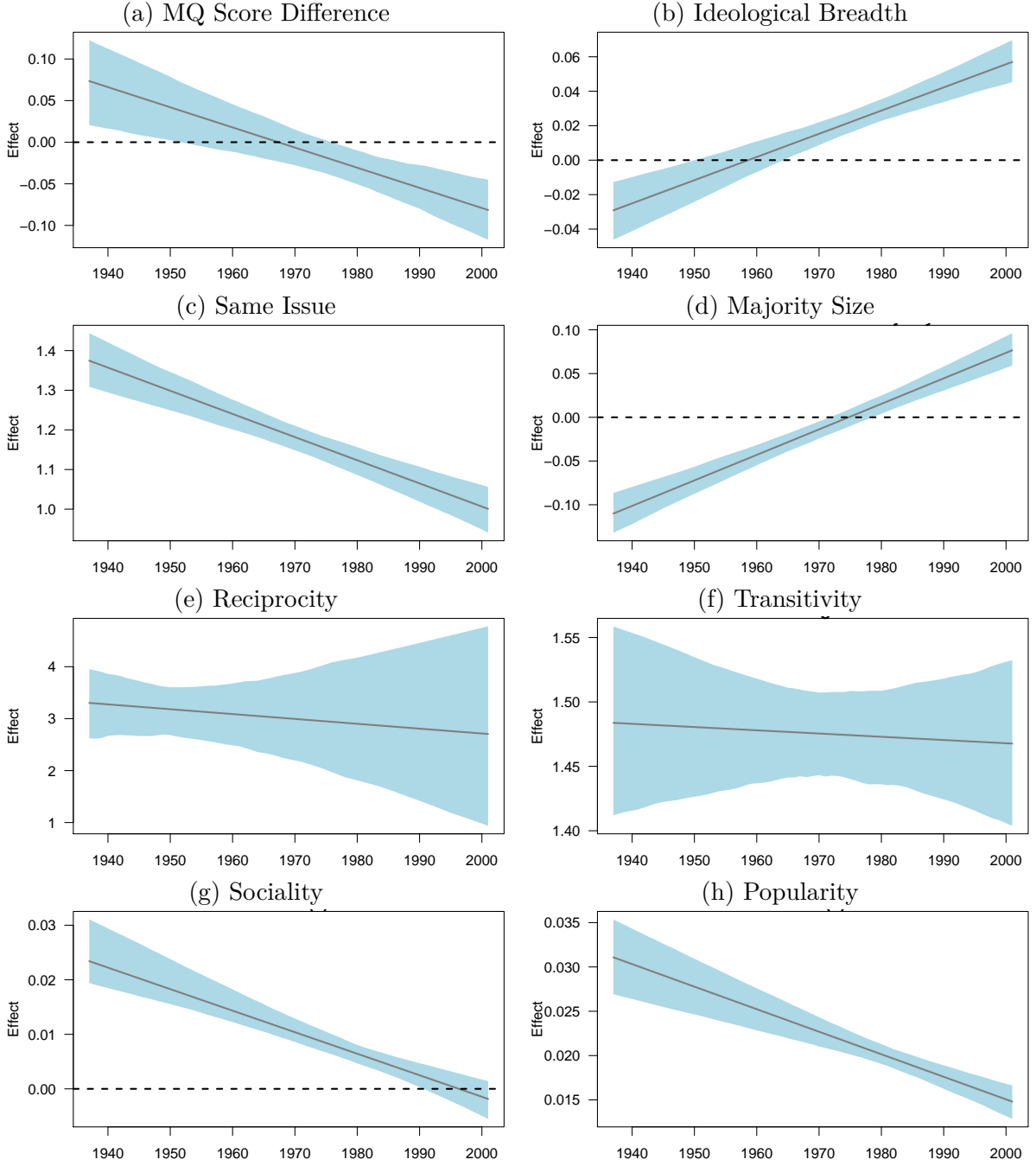


Figure 5: Effects

## 7.2 Predictive Performance

Our case for studying legal citations at the directed dyadic level hinges upon the contribution to modeling offered by incorporating network dependence. To quantify this contribution, we

use out-of-sample prediction [CITES]. Predicting out-of-sample offers an unbiased and general purpose way to evaluate the contribution, in terms of model fit, of one or more terms/parameters in a model. Unlike in-sample measures of model fit, out-of-sample methods are highly robust in avoiding overfitting, and work when we cannot accurately calculate the value of the likelihood function, as in the current case. Out-of-sample prediction is a common way to evaluate methods for modeling ties in networks, and has recently been applied to TERGMs in particular [CITE PA AND IEEE].

In our prediction experiment we randomly split the directed dyads into an 80% training set and a 20% test set. The parameters of the model are estimated using the directed dyads in the training set, and the parameters are used to form the conditional probability of a tie for all of the directed dyads in the test set. Directed dyads for which the conditional probability of a tie exceeds 0.5 are predicted to be citations. The experiment is run with the full model, and with a model that excludes all of the dependence terms (i.e., all terms involving reciprocity, in and out stars, and triangles)—the independent dyads model. We run this experiment for 10 iterations. Predictive performance is evaluated with three common and related measures—precision (i.e., the proportion of predicted citations that are actually citations), recall (the proportion of actual citations that are predicted to be citations), and the F1 score (the harmonic mean of precision and recall). All three measures are bounded between 0 and 1, with higher scores indicating better performance.

	Independent Model		Full Model	
	mean	range	mean	range
precision	0.5499	(0.5384, 0.5619)	0.8605	(0.8526, 0.8666)
recall	0.0827	(0.0811, 0.0843)	0.5858	(0.5817, 0.5891)
F1 score	0.1438	(0.141, 0.1463)	0.6971	(0.6941, 0.7008)

Table 3: The predictive performance of the directed dyadic models, over ten 80/20 train/test splits.

We see from Table 3 that, based on all three measures, the predictive performance of the model improves dramatically from adding the network dependence terms. The recall of the full model is particularly impressive, indicating that it recovers over half of the actual citations in the test set. This provides clear evidence that the full model, which includes covariates and network dependence terms, represents a more accurate and complete model of the process of citation formation in US Supreme Court opinions.

## 8 conclusion

MAJOR LIMITATION—DO NOT CONSIDER SIGN OF CASES

## Bibliography

- Bailey, Michael A and Forrest Maltzman. 2008. “Does legal doctrine matter? Unpacking law and policy preferences on the US Supreme Court.” *American Political Science Review* 102(3):369–384.
- Bailey, Michael A and Forrest Maltzman. 2011. *The constrained court: Law, politics, and the decisions justices make*. Princeton University Press.
- Barabási, Albert-László and Réka Albert. 1999. “Emergence of scaling in random networks.” *science* 286(5439):509–512.
- Batagelj, Vladimir. 2003. “Efficient algorithms for citation network analysis.” *arXiv preprint cs/0309023*.
- Batagelj, Vladimir, Anuška Ferligoj and Flaminio Squazzoni. 2017. “The emergence of a field: a network analysis of research on peer review.” *Scientometrics* 113(1):503–532.
- Benjamin, Stuart Minor and Bruce A Desmarais. 2012. “Standing the test of time: The breadth of majority coalitions and the fate of us supreme court precedents.” *Journal of Legal Analysis* 4(2):445–469.
- Black, Ryan C and James F Spriggs. 2013. “The Citation and Depreciation of US Supreme Court Precedent.” *Journal of Empirical Legal Studies* 10(2):325–358.
- Bommarito II, Michael J, Daniel Katz and Jon Zelner. 2009. Law as a seamless web?: comparison of various network representations of the united states supreme court corpus (1791-2005). In *Proceedings of the 12th international conference on artificial intelligence and law*. ACM pp. 234–235.
- Bommarito, Michael J, Daniel Martin Katz, Jonathan L Zelner and James H Fowler. 2010. “Distance measures for dynamic citation networks.” *Physica A: Statistical Mechanics and its Applications* 389(19):4201–4208.
- Carstens, CJ. 2014. “A uniform random graph model for directed acyclic networks and its effect on motif-finding.” *Journal of Complex Networks* 2(4):419–430.
- Carstens, J. 2016. “Topology of complex networks: models and analysis.”
- Chen, P and Sidney Redner. 2010. “Community structure of the physical review citation network.” *Journal of Informetrics* 4(3):278–290.
- Clark, Tom S and Benjamin Lauderdale. 2010. “Locating Supreme Court opinions in doctrine space.” *American Journal of Political Science* 54(4):871–890.

- Clough, James R, Jamie Gollings, Tamar V Loach and Tim S Evans. 2015. “Transitive reduction of citation networks.” *Journal of Complex Networks* 3(2):189–203.
- Cross, Frank B. 2010. “Determinants of citations to Supreme Court opinions (and the remarkable influence of Justice Scalia).” *Supreme Court Economic Review* 18(1):177–202.
- Dawson, Shane, Dragan Gašević, George Siemens and Srecko Joksimovic. 2014. Current state and future trends: A citation network analysis of the learning analytics field. In *Proceedings of the fourth international conference on learning analytics and knowledge*. ACM pp. 231–240.
- Desmarais, Bruce A and Skyler J Cranmer. 2017. Statistical Inference in Political Networks Research. In *The Oxford Handbook of Political Networks*.
- Eom, Young-Ho and Santo Fortunato. 2011. “Characterizing and modeling citation dynamics.” *PloS one* 6(9):e24926.
- Érdi, Péter, Kinga Makovi, Zoltán Somogyvári, Katherine Strandburg, Jan Tobochnik, Péter Volf and László Zalai. 2013. “Prediction of emerging technologies based on analysis of the US patent citation network.” *Scientometrics* 95(1):225–242.
- Fowler, James H and Sangick Jeon. 2008. “The authority of Supreme Court precedent.” *Social networks* 30(1):16–30.
- Fowler, James H, Timothy R Johnson, James F Spriggs, Sangick Jeon and Paul J Wahlbeck. 2007. “Network analysis and the law: Measuring the legal importance of precedents at the US Supreme Court.” *Political Analysis* 15(3):324–346.
- Fujita, Katsuhide, Yuya Kajikawa, Junichiro Mori and Ichiro Sakata. 2014. “Detecting research fronts using different types of weighted citation networks.” *Journal of Engineering and Technology Management* 32:129–146.
- Gelter, Martin and Mathias Siems. 2012. “Networks, Dialogue or One-Way Traffic: An Empirical Analysis of Cross-Citations between Ten of Europe’s Highest Courts.” *Utrecht L. Rev.* 8:88.
- Gillman, Howard. 2001. “What’s law got to do with it? Judicial behavioralists test the ?legal model? of judicial decision making.” *Law & Social Inquiry* 26(2):465–504.
- Greenberg, Steven A. 2009. “How citation distortions create unfounded authority: analysis of a citation network.” *Bmj* 339:b2680.
- Hajra, Kamalika Basu and Parongama Sen. 2005. “Aging in citation networks.” *Physica A: Statistical Mechanics and its Applications* 346(1):44–48.

- Hajra, Kamalika Basu and Parongama Sen. 2006. “Modelling aging characteristics in citation networks.” *Physica A: Statistical Mechanics and its Applications* 368(2):575–582.
- Hansford, Thomas G and James F Spriggs. 2006. *The politics of precedent on the US Supreme Court*. Princeton University Press.
- Holland, Paul W and Samuel Leinhardt. 1971. “Transitivity in Structural Models of Small Groups.” *Small Group Research* 2(2):107–124.
- Hummon, Norman P and Patrick Dereian. 1989. “Connectivity in a citation network: The development of DNA theory.” *Social networks* 11(1):39–63.
- Jaffe, Adam B and Gaétan de Rassenfosse. 2017. “Patent citation data in social science research: Overview and best practices.” *Journal of the Association for Information Science and Technology* 68(6):1360–1374.
- Jeong, Hawoong, Zoltan Néda and Albert-László Barabási. 2003. “Measuring preferential attachment in evolving networks.” *EPL (Europhysics Letters)* 61(4):567.
- Ji, Pengsheng, Jiashun Jin et al. 2016. “Coauthorship and citation networks for statisticians.” *The Annals of Applied Statistics* 10(4):1779–1812.
- Jo, Sung Jun, Chang-Wook Jeung, Sunyoung Park and Hea Jun Yoon. 2009. “Who is citing whom: Citation network analysis among HRD publications from 1990 to 2007.” *Human Resource Development Quarterly* 20(4):503–537.
- Kajikawa, Yuya, Junko Ohno, Yoshiyuki Takeda, Katsumori Matsushima and Hiroshi Komiyama. 2007. “Creating an academic landscape of sustainability science: an analysis of the citation network.” *Sustainability Science* 2(2):221.
- Karrer, Brian and Mark EJ Newman. 2009. “Random graph models for directed acyclic networks.” *Physical Review E* 80(4):046110.
- Kleinberg, Jon M. 1999. “Authoritative sources in a hyperlinked environment.” *Journal of the ACM (JACM)* 46(5):604–632.
- Knight, Jack and Lee Epstein. 1996. “The norm of stare decisis.” *American Journal of Political Science* pp. 1018–1035.
- Landes, William M, Lawrence Lessig and Michael E Solimine. 1998. “Judicial influence: A citation analysis of federal courts of appeals judges.” *The Journal of Legal Studies* 27(2):271–332.

- Leicht, Elizabeth A, Gavin Clarkson, Kerby Shedden and Mark EJ Newman. 2007. "Large-scale structure of time evolving citation networks." *The European Physical Journal B-Condensed Matter and Complex Systems* 59(1):75–83.
- Liu, John S, Hsiao-Hui Chen, Mei Hsiu-Ching Ho and Yu-Chen Li. 2014. "Citations with different levels of relevancy: Tracing the main paths of legal opinions." *Journal of the Association for Information Science and Technology* 65(12):2479–2488.
- Lupu, Yonatan and Erik Voeten. 2012. "Precedent in international courts: A network analysis of case citations by the European court of human rights." *British Journal of Political Science* 42(2):413–439.
- Lupu, Yonatan and James H Fowler. 2013. "Strategic citations to precedent on the US Supreme Court." *The Journal of Legal Studies* 42(1):151–186.
- Mones, Enys, Péter Pollner and Tamás Vicsek. 2014. "Universal hierarchical behavior of citation networks." *Journal of Statistical Mechanics: Theory and Experiment* 2014(5):P05023.
- Pan, Raj Kumar, Kimmo Kaski and Santo Fortunato. 2012. "World citation and collaboration networks: uncovering the role of geography in science." *Scientific reports* 2.
- Pelc, Krzysztof J. 2014. "The politics of precedent in international law: A social network application." *American Political Science Review* 108(3):547–564.
- Rice, Ronald E, Christine L Borgman and Byron Reeves. 1988. "Citation networks of communication journals, 1977–1985 cliques and positions, citations made and citations received." *Human communication research* 15(2):256–283.
- Richards, Mark J and Herbert M Kritzer. 2002. "Jurisprudential regimes in Supreme Court decision making." *American Political Science Review* 96(2):305–320.
- Shibata, Naoki, Yuya Kajikawa, Yoshiyuki Takeda, Ichiro Sakata and Katsumori Matsushima. 2011. "Detecting emerging research fronts in regenerative medicine by the citation network analysis of scientific publications." *Technological Forecasting and Social Change* 78(2):274–282.
- Simkin, Mikhail V and Vwani P Roychowdhury. 2007. "A mathematical theory of citing." *Journal of the Association for Information Science and Technology* 58(11):1661–1673.
- Spriggs, James F and Thomas G Hansford. 2001. "Explaining the overruling of US Supreme Court precedent." *The Journal of Politics* 63(4):1091–1111.
- Van Raan, Anthony FJ. 2005. "Reference-based publication networks with episodic memories." *Scientometrics* 63(3):549–566.



- Vazquez, Alexei. 2001. “Statistics of citation networks.” *arXiv preprint cond-mat/0105031* .
- Verspagen, Bart. 2007. “Mapping technological trajectories as patent citation networks: A study on the history of fuel cell research.” *Advances in Complex Systems* 10(01):93–115.
- Vu, Duy Q, Arthur U Asuncion, David R Hunter and Padhraic Smyth. 2011. Dynamic ego-centric models for citation networks. In *Proceedings of the 28th International Conference on International Conference on Machine Learning*. Omnipress pp. 857–864.
- Wang, Dashun, Chaoming Song and Albert-László Barabási. 2013. “Quantifying long-term scientific impact.” *Science* 342(6154):127–132.
- Wang, Mingyang, Guang Yu and Daren Yu. 2008. “Measuring the preferential attachment mechanism in citation networks.” *Physica A: Statistical Mechanics and its Applications* 387(18):4692–4698.
- Zhang, Paul and Lavanya Koppaka. 2007. Semantics-based legal citation network. In *Proceedings of the 11th international conference on Artificial intelligence and law*. ACM pp. 123–130.