APPENDIX 8-C. LIFETIME DISTRIBUTIONS

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APPENDIX 8-C. LIFETIME DISTRIBUTIONS

8-C.1 INTRODUCTION

For each product class, DOE characterized the product lifetime using a Weibull probability distribution that ranged from the minimum to maximum lifetime estimates described in chapter 8, Life Cycle Cost and Payback Period Analyses. The Weibull distribution is recommended for application to lifetime data because it can be shaped to match low, average, and high values while still allowing some probability of exceeding the high value.[1, 2]

8-C.2 DERIVATION OF WEIBULL DISTRIBUTION PARAMETERS

DOE utilized an approach for calculating the Weibull distribution using product lifetime data to determine low, average, high value, and percentile of high value. A similar approach is described in the Crystal Ball Technical Note, which uses the most likely value instead of an average.[3] Available data is used to assign low, average, and high values to a random variable with unknown distribution parameters.

The Weibull distribution can be defined as:

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x-L}{\alpha}\right)^{\beta-1} \exp^{-\left(\frac{x-L}{\alpha}\right)^{\beta}}$$

Where:

L = location
α = scale
β = shape

Therefore, the cumulative distribution is:

$$F(x) = 1 - \exp^{-\left(\frac{x-L}{\alpha}\right)^{\beta}}$$

Based on available data, Weibull distribution parameters are specified in the following manner:

1. The output deviates must be greater than the expert opinion for low value,
2. The average X_{avg}, must be equal to the average value from the available data,
3. The high value, xb, must correspond to some particular percentile point (e.g., 95 percent, or 90 percent)

The values for the parameters in the above equations are determined using the approach outlined in the Crystal Ball Technical Note.[3]

This solution can be checked using Crystal Ball by specifying a Weibull distribution with the calculated parameters (location, scale, and shape) in an assumption cell and generating a forecast equal to the assumption. The forecast histogram and statistics verify that the Weibull distribution matches the desired shape.

8-C.3 WATER HEATER LIFETIME DISTRIBUTIONS

Table 8-C.3.1 shows the average, minimum, and maximum lifetime, and maximum percentile values used to determine the Weibull distribution parameters alpha and beta for water heaters. The analysis utilized a maximum percentile of 95 to 99 percent.

Table 8-C.3.1 Water Heaters

	Expert Opinion Values				**Weibull Parameters**	
Product Class	**Minimum** *(years)*	**Average** *(years)*	**Maximum** *(years)*	**Maximum percentile** *(%)*	**Alpha (scale)**	**Beta (shape)**
Gas-Fired/LPG Storage	6	13	20	95	7.8903	1.9134
Electric Storage	6	13	20	95	7.8903	1.9134
Oil-Fired Storage	6	13	20	95	7.8903	1.9134
Gas-Fired Instantaneous	8	20	30	99	13.4203	3.0897

Figures 8-C.3.1 to 8-C.3.8 show the Weibull distributions for each water heater product class.

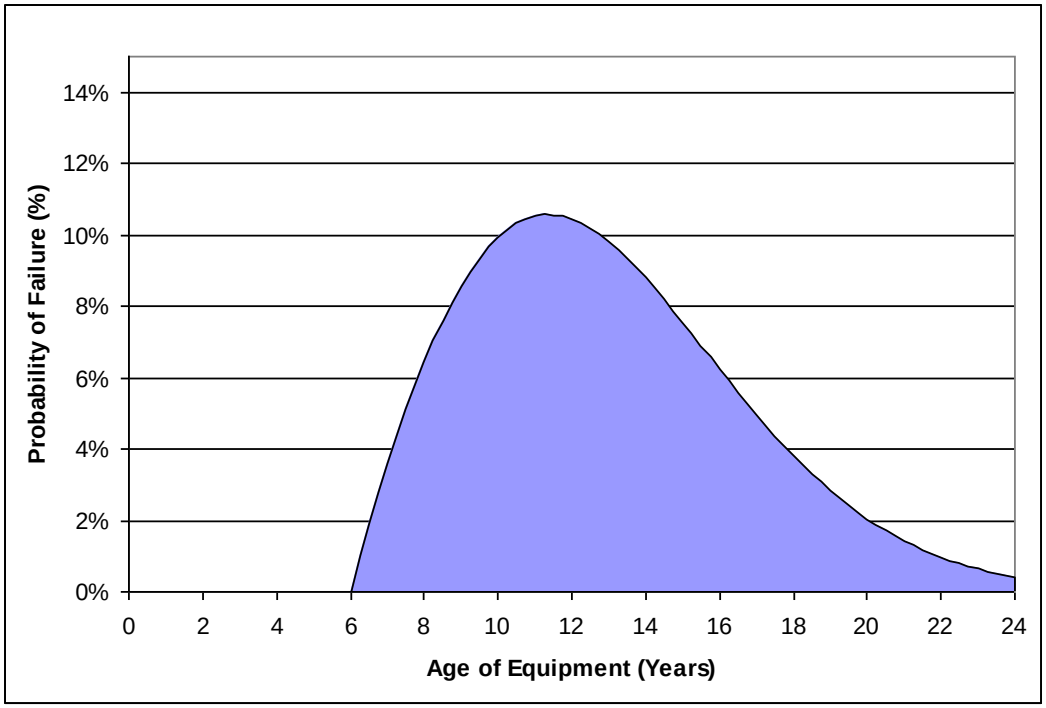

Figure 8-C.3.1 Fraction of the Gas-Fired Storage Water Heaters Failing

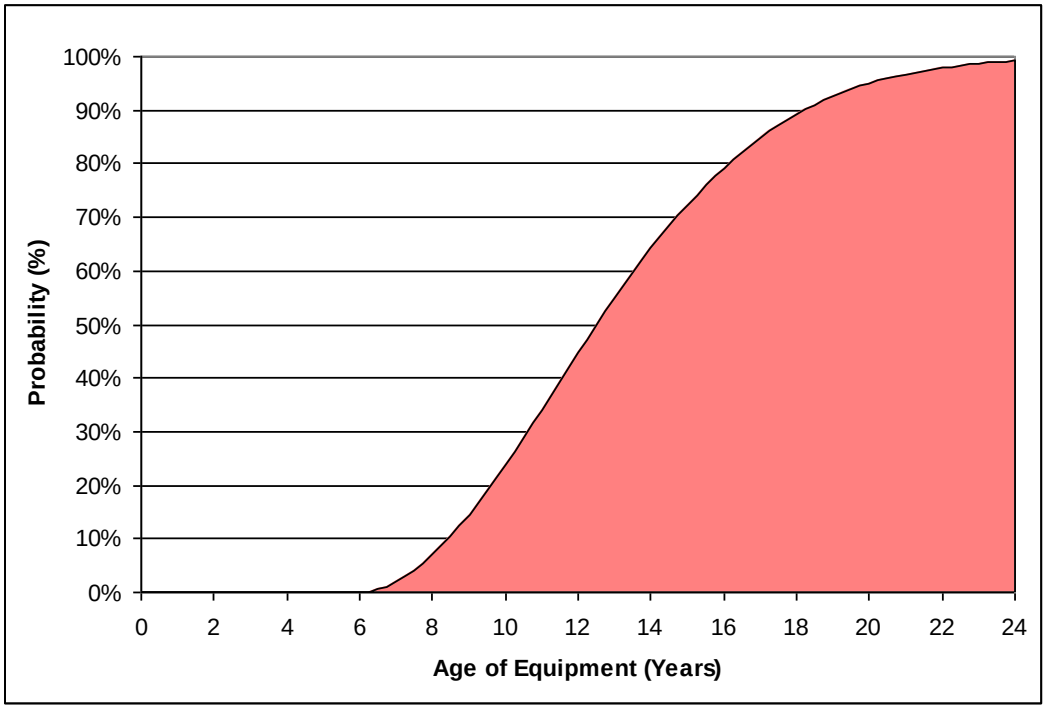

Figure 8-C.3.2 Cumulative Lifetime Length of Gas-Fired Storage Water Heaters

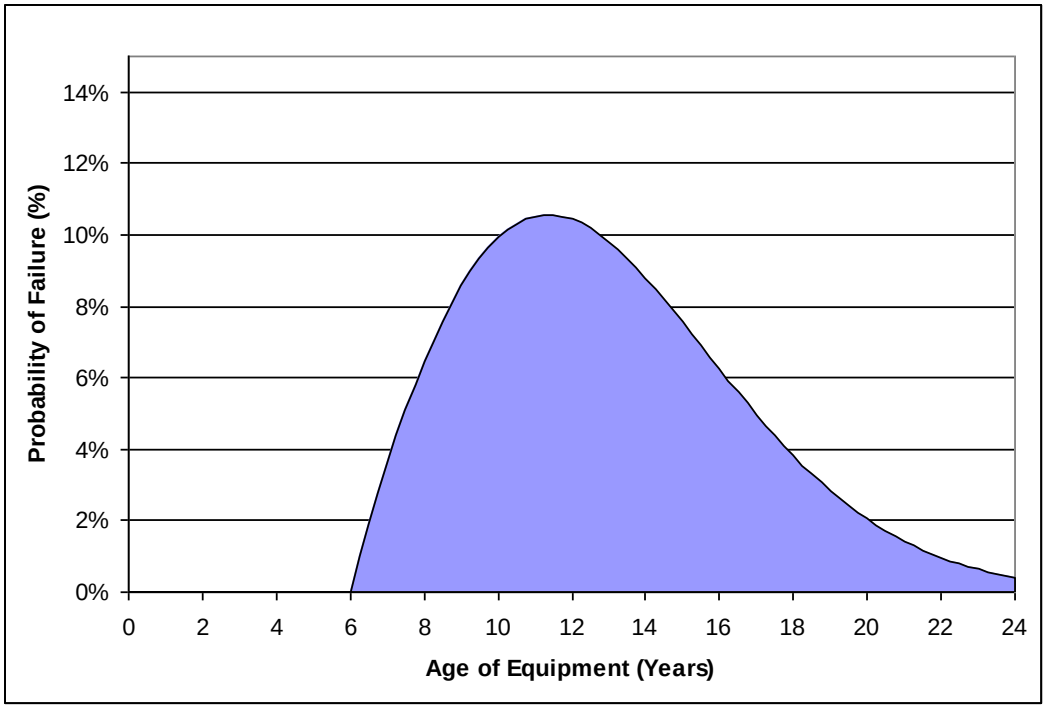

Figure 8-C.3.3 Fraction of the Electric Storage Water Heaters Failing

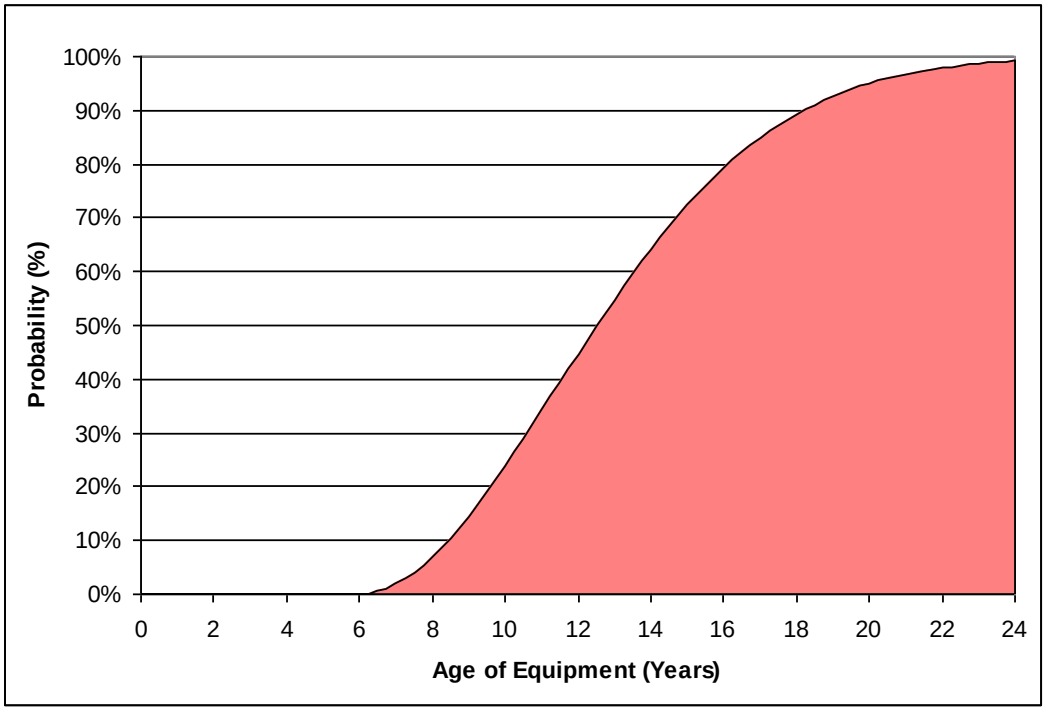

Figure 8-C.3.4 Cumulative Lifetime Length of Electric Storage Water Heaters

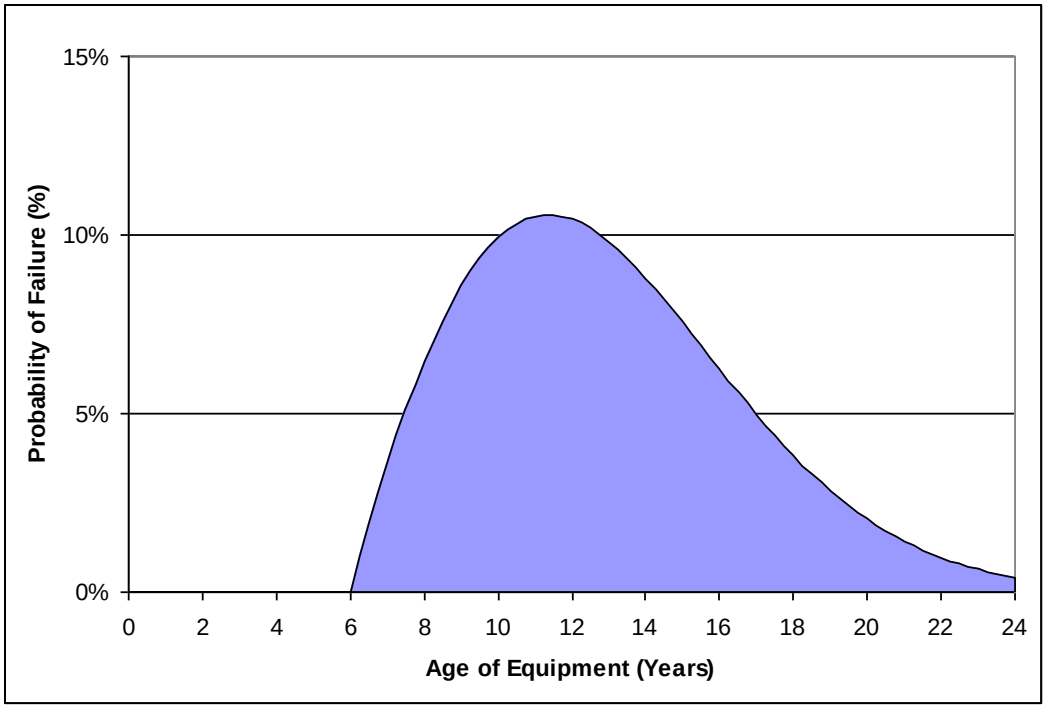

Figure 8-C.3.5 Fraction of the Oil-Fired Storage Water Heaters Failing

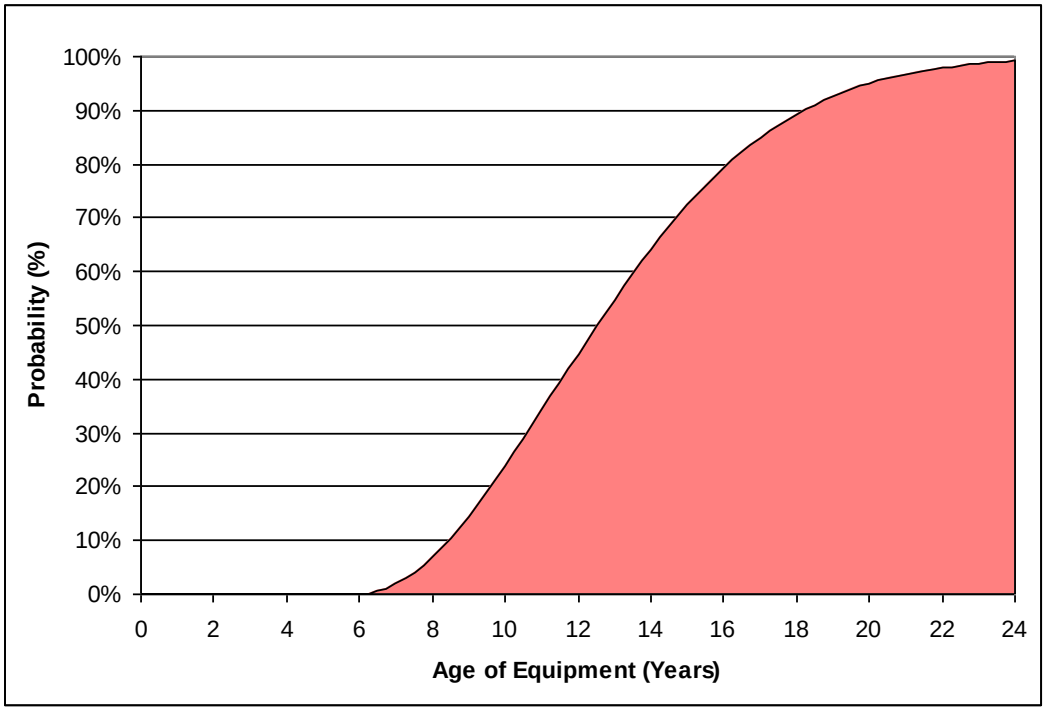

Figure 8-C.3.6 Cumulative Lifetime Length of Oil-Fired Storage Water Heaters

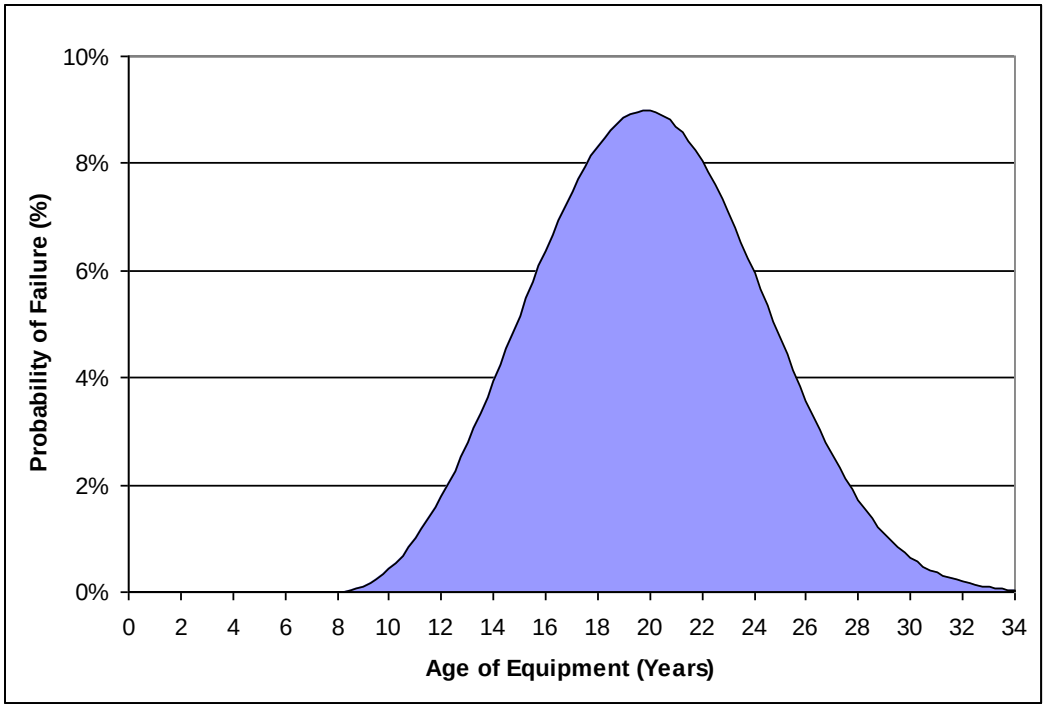

Figure 8-C.3.7 Fraction of the Gas-Fired Instantaneous Water Heaters Failing

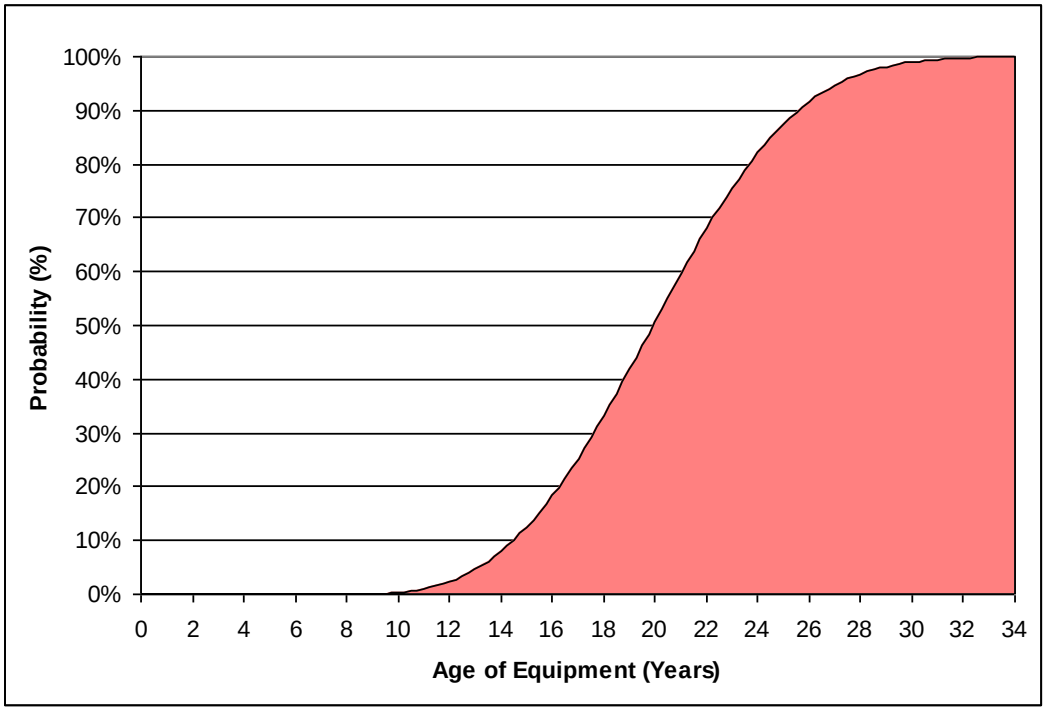

Figure 8-C.3.8 Cumulative Lifetime Length of Gas-Fired Instantaneous Water Heaters

8-C.4 GAS-FIRED POOL HEATERS LIFETIME DISTRIBUTIONS

Table 8-C.4.1 shows the average, minimum, and maximum lifetime, and maximum percentile values used to determine the Weibull distribution parameters alpha and beta for gas-fired pool heaters. The analysis utilized a maximum percentile of 99.5 percent.

Table 8-C.4.1 Gas-Fired Pool Heaters

	Expert Opinion Values				Weibull Parameters	
Product Class	**Minimum** *(years)*	**Average** *(years)*	**Maximum** *(years)*	**Maximum percentile** *(%)*	**Alpha (scale)**	**Beta (shape)**
Gas-Fired Pool Heaters	3	10	20	99.5	5.4466	1.3417

Figure 8-C.4.1 and Figure 8-C.4.2 show the Weibull distributions for pool heaters.

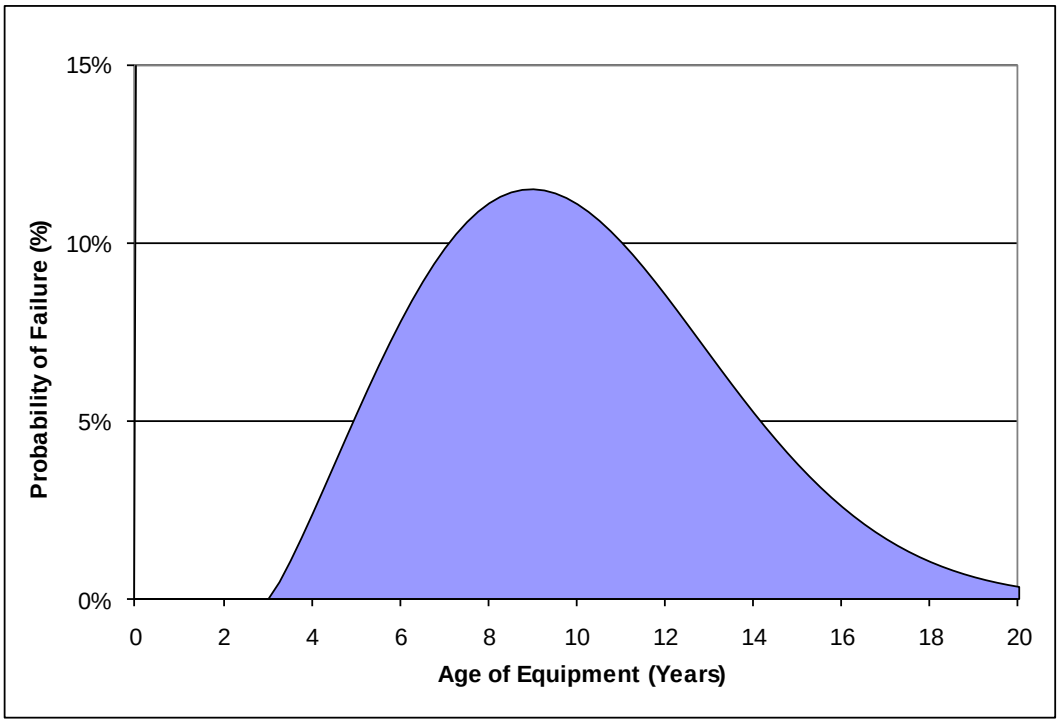

Figure 8-C.4.1 Fraction of the Gas-Fired Pool Heater Failing

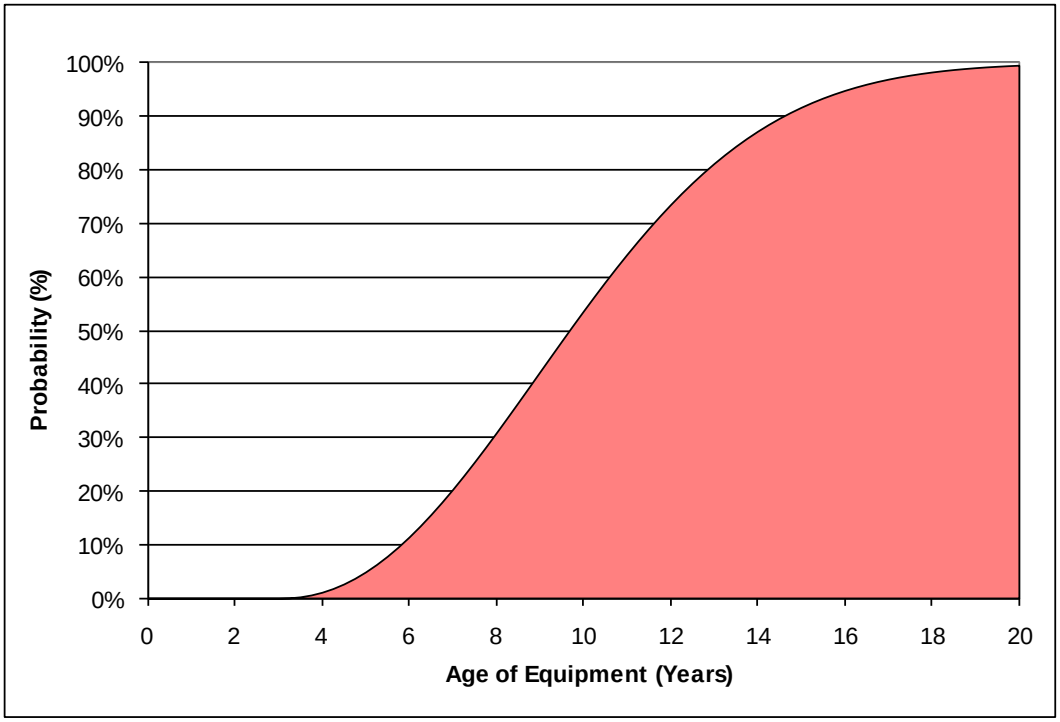

Figure 8-C.4.2 Cumulative Lifetime Length of Gas-Fired Pool Heaters

8-C.5 DIRECT HEATING EQUIPMENT LIFETIME DISTRIBUTIONS

Table 8-C.5.1 shows the average, minimum, maximum lifetime, and maximum percentile values used to determine the Weibull distribution parameters alpha and beta. The analysis utilized a maximum percentile of 99 percent.

Table 8-C.5.1 Direct Heating Equipment

	Expert Opinion Values				Weibull Parameters	
Product Class	**Minimum** *(years)*	**Average** *(years)*	**Maximum** *(years)*	**Maximum percentile** *(%)*	**Alpha (scale)**	**Beta (shape)**
All DHE Equipment	10	15	20	99	5.6257	2.6548

Figure 8-C.5.1 and Figure 8-C.5.2 show the Weibull distributions for direct heating equipment.

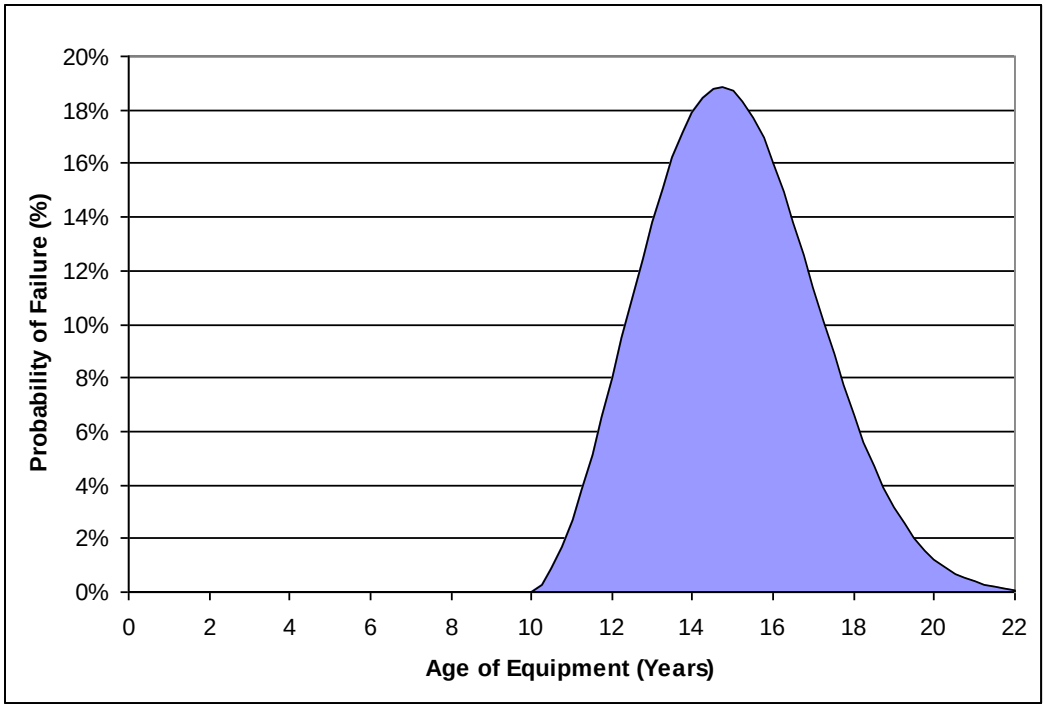

Figure 8-C.5.1 Fraction of the Direct Heating Equipment Failing

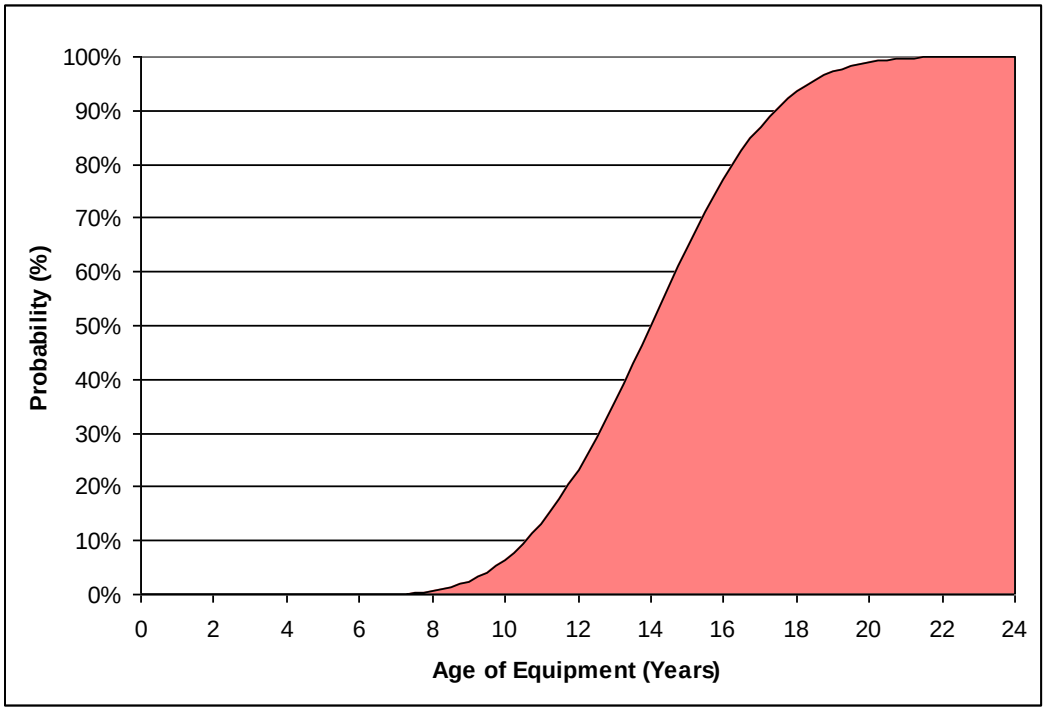

Figure 8-C.5.2 Cumulative Lifetime Length of Direct Heating Equipment

REFERENCES

1. Barnes, P. R., J. W. Van Dyke, B. W. McConnell, S. M. Cohn, and S. L. Purucker, *The Feasibility of Replacing or Upgrading Utility Distribution Transformers During Routine Maintenance*, 1995. Oak Ridge National Laboratory. Oak Ridge, TN. Report No. Report No. ORNL-6804/R1. <http://www.ornl.gov/~webworks/cpr/v823/rpt/78562.pdf>

2. Karr, T., Making the Most of Life Test Data. *Appliance Magazine*, 2003. <http://www.appliancemagazine.com/print.php?article=197&zone=1&first=1>

3. Crystal Ball, *Technote: Derivation of Weibull Distribution Parameters When the Minimum, Most Likely Value, and a Percentile are Known*, 2007. <http://www.crystalball.com/support/simulation/cbl_gen_021A.html>