Inference on the Effects of Observed Features
in Latent Space Models for Networks

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Abstract

Due to the complex interdependence found in relational data, political networks scholars draw upon an increasingly sophisticated toolkit for statistical inference. The latent space model (LSM) for network data combines a generalized linear model with a latent spatial embedding of the network. It is assumed that the latent spatial embedding can control for unmeasured confounding structure that is related to the values of edges in the network. There has been little to no research that considers the LSM's performance in adjusting for unmeasured structure. We investigate the LSM's performance via a simulation study. In the presence of an unmeasured covariate that can be modeled using a latent space, estimation and inferential error remain high under even moderate confounding. However, the prediction error of the LSM when unmeasured network structure is present is substantially lower in most cases. We conclude that the LSM is most appropriate for exploration or prediction.[1]

1 Introduction

Inferential analysis of political network data has grown increasingly sophisticated in recent years. Political networks scholars are well versed in the risks associated with ignoring unmodeled network structure. Dependencies such as reciprocity, transitivity, and homophily—if not accounted for—can lead to biased estimates and errors in hypothesis testing, much in the way that omitted variable bias can affect results in conventional regression models (Ward, Siverson and Cao 2007; Kinne 2014; Cranmer and Desmarais Accepted; Hays, Kachi and Franzese 2010). A number of statistical modeling frameworks have been proposed to account for confounding structure in network data that cannot be modeled with observed covariates. These include the exponential random graph model (ERGM) (e.g., Lazer, Rubineau, Chetkovich, Katz and Neblo 2010; Cranmer and Desmarais 2011; Desmarais and Cranmer 2012), the latent space model (LSM) (e.g., Ward, Siverson and Cao 2007; Ward and Hoff 2007; Kirkland 2012), and the stochastic actor oriented model (SAOM) (e.g., Berardo and Scholz 2010; Kinne 2014).

Despite their growing popularity, few studies exist that investigate the performance of these models in adjusting for confounding network structure. The approach to adjusting for dependencies in the two other models commonly used for network data—ERGM and SAOM—is quite

similar to adjusting for confounding covariates in regression modeling. The researcher specifies a set of dependencies that (s)he hypothesizes to be important in the generative model for the network. These dependencies are then explicitly included in a model that simultaneously represents the effects of observed covariates (Cranmer and Desmarais 2011). The LSM takes a different approach, which involves the incorporation of latent variables to model network structure. The LSM has an advantage over ERGM and SAOM in that researchers need not develop a set of hypothesized dependencies in order to model network structure that is not reflected in observed covariates. However, this advantage hinges upon the capacity for the LSM to discover unmodeled structure that could otherwise be attributed to the observed covariates. In the current study, we focus on the LSM, examining its performance in reducing estimation and inferential errors regarding the effects of observed covariates via adjustment for confounding network structure.

1.1   Central Problem

Latent variable inference, generally conceived, presents the possibility of representing unmeasured data in statistical models. The LSM, introduced by Hoff, Raftery and Handcock (2002), is used to estimate the effect of covariates in the presence of latent network structure. Here the distance function $|z_i - z_j|$ represents latent network structure as homophily with respect to latent variables. The distance function is additively combined with a regression on observed dyadic covariates, x_{ij}, to form a linear predictor for tie prediction. As with a GLM, a link function, g^{-1}, maps this linear predictor to the appropriate edge distribution.

$$\mathbb{E}(y_{ij}|x_{ij}) = g^{-1}(\alpha + \beta x_{ij} - |z_i - z_j|)$$

However, it is unclear that the introduction of a latent space decreases the expected error for the parameter(s) of interest when aspects of the network structure (e.g., homophily) that are unmeasured, are correlated with measured covariates. There are two reasons that using the LSM may lead to increased error. First, the latent configurations inferred may result in a representation of the network wherein a node's position in the latent space is spuriously correlated with the observed covariates. Inferring latent variables that are correlated with observed covariates would lead to reduced efficiency in estimating observed covariate effects. Second, if the unobserved (i.e., latent) network structure is truly correlated with the observed covariates, the unobserved structure that can be correlated with the observed variable may be attributed to the observed variable. Under this condition the latent space parameters are used to model sources of variation that cannot be attributed to the observed covariate, and inferences regarding the observed covariates remain subject to omitted variable bias.

2   Applications and Development of the Latent Space Model

The LSM has seen use in a variety of fields in which network data is common, particularly the social sciences. The apparent appeal of the LSM appears to be driven primarily by the LSM's

usefulness in modeling transitivity (i.e., clustering) and homophily, which are ubiquitous in social networks. In political science the LSM and variations on the form developed in Hoff, Raftery and Handcock (2002) have been used to estimate the effect of democracy on the probability of a militarized interstate dispute (Ward, Siverson and Cao 2007), the amount of portfolio investment between states (Cao and Ward 2013), and the effect of multimember districts on the probability of collaboration between state legislators in the United States (Kirkland 2012). Variations of the LSM developed for networks measured over time have been applied to the study of international trade, wherein the effects of various features of trading partners are estimated (Ward, Ahlquist and Rozenas 2013). In ecology the LSM has been used to study the sociality of elephants (Vance, Archie and Moss 2009) and orcas (Fearnbach, Durban, Ellifrit, Waite, Matkin, Lunsford, Peterson, Barlow and Wade 2014), birds (Nomano, Browning, Savage, Rollins, Griffith and Russell 2015), to discover ecological communities (Fletcher, Acevedo, Reichert, Pias and Kitchens 2011; Fletcher Jr, Revell, Reichert, Kitchens, Dixon and Austin 2013), and to study food webs (Chiu and Westveld 2011). In epidemeology it has been used to identify clusters of infected persons for later isolation (Zhang, Wang, Wang and Fang 2015) and to study patterns of interaction amongst physicians (Paul, Keating, Landon and O'Malley 2014). In marketing and business research it has been used to study inter-group trust (Dass and Kumar 2011), optimal bundling and pricing of goods and brands for retailers (Dass and Kumar 2012). It has been used to describe topic-specific patterns of interaction in e-mail communication networks (Krafft, Moore, Desmarais and Wallach 2012). Lastly, in neuroscience it has been proposed as a method for modelling fMRI data (Simpson, Bowman and Laurienti 2013).

Although the LSM is arguably most useful as an exploratory or predictive model (see Shmueli (2010) for a discussion of the differences between predictive and explanatory modeling), it has been applied in some cases to reduce estimation and/or inferential error with respect to the effects of observed covariates. For example Ward, Siverson and Cao (2007), in a high profile example, argue that the the LSM improves inference about the effects of democracy, international trade, and participation in international organizations on the probability of inter-state conflict.

> "The history of international disputes, and consequently the extant data on militarized interstate disputes, is replete with ...dependencies. We formally incorporate and estimate the extent of these ...dependencies in our model of the Kantian peace in order to more precisely determine the effects of the Kantian tripod on international conflict" (Ward, Siverson and Cao 2007, p. 585)

Likewise, similar claims are made in Dorff and Ward (2013); Cao and Ward (2013); Ward, Ahlquist and Rozenas (2013); Kirkland (2012), and Cao and Ward (2016).

> "For the most part, however, most dyadic research in international relations ignores the essential features of dyads in that they fail to satisfy the assumption of independence or, by construction, have missing data but ignore its effects: both of these bias the results in a fundamental way" (Dorff and Ward 2013, p. 2).

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"This approach combines a network analysis with a standard-looking regression to permit us to access the importance of our explanatory factors without having them biased by the interdependencies in the network we are studying" (Cao and Ward 2013, p. 15).

"The presence of . . . dependence implies misspecification and a high likelihood of bias in most current applications. Building on the latent space framework we model the world trading system without assuming a particular network structure or the sufficiency of particular network statistics" (Ward, Ahlquist and Rozenas 2013, p. 20).

". . . the latent space model allows for the assessment of distance between two unconnected actors while simultaneously controlling for the interdependence inherent in network data. This interdependence in latent space positions allows the model to control for common network effects like reciprocity or transitivity that would ordinarily bias results" (Kirkland 2012, p. 336).

"Ignoring third-order dependence in dyadic data and treating dyads Germany-France, France-Italy, and Germany-Italy as independent observations can cause bias in parameter estimates (Hoff 2005). The statistical literature has proposed a series of latent space to control for autocorrelation among dyadic observations (Hoff et al. 2002; Hoff 2005). Countries? unobserved characteristics are captured by latent vectors..." (Cao and Ward 2016, p. 17).

The above examples are drawn from political science, but the results of our analysis have implications in other fields. We see a similar logic for using the LSM articulated in recent work in ecology by Nomano et al. (2015, p. 989).

". . . can create artificially exaggerated synchrony rates regardless of motivation for signaling, which can be modeled with a term known as "transitivity" in the social network literature. A latent space model (Krivitsky et al. 2009) was used to examine the propensities for synchrony between helper males and the primary male while accounting for the variability deriving from the transitivity."

Though this list of quotations is by no means complete, the last example to which we point comes from the business literature, and uses the bilinear form of the latent space model (Dass and Kumar 2011, p. 7).

"This model controls for higher order team dynamics using the bilinear component $z_i' z_j$ and also allows both trustor ($x_{tor,i}$) and trusted ($x_{ted,j}$) characteristics to be investigated along with the dyadic covariates ($x_{d,i,j}$)."

If the LSM does reduce inferential/estimation error under certain conditions, it will serve as a valuable general model for explanatory analysis of network data, especially since its use does

not require the researcher to specify a set of network dependencies from theory. However, as far as we know, there has been no research into the performance of the LSM in adjusting for confounding network structure. Additionally, the predictive performance of the LSM has only seen limited evaluation (Hoff, Raftery and Handcock 2002), despite having been used for model selection (Ward, Ahlquist and Rozenas 2013; Fletcher et al. 2011; Fletcher Jr et al. 2013; Chiu and Westveld 2011). Hence, finding the conditions under which the LSM reduces prediction error also may affect future use of the LSM.

The LSM has seen a substantial amount of subsequent development and extension. The latent space has been represented as a k-dimensional Euclildean space and by latent factors (Hoff, Raftery and Handcock 2002; Hoff 2009). Additional structure has been introduced by adding random effects, which, for example, may involve sender or receiver specific effects for directed networks which capture differential activity rates amongst nodes (Hoff 2003). Within this framework Westveld and Hoff (2011) models dynamic network data by treating the latent space as a stochastic process. Handcock, Raftery and Tantrum (2007) enable the LSM to model clustering that is not representable as homophily (i.e., stochastic equivalence) by combining the LSM with latent cluster models. Hoff (2008) shows that the LSM and latent cluster models are special cases of an "eigenmodel." That is, an eigendecomposition of a symmetric sociomatrix can be used to represent both latent space and cluster models, but not vice-versa. Most of these developments have represented new forms for the latent space and/or the further development of the LSM as a Bayesian hierarchical model. In the next section we propose an approach to specifying the Bayesian prior in order to improve inference regarding covariate parameters.

3   Bayesian Inference and Priors in the Latent Space Model

Maximum likelihood estimation (MLE) is problematic in the LSM since the value of the likelihood function is invariant to rotation of the latent space (Hoff, Raftery and Handcock 2002) (i.e., only the distances between points matter; not their absolute positions). Invariance to rotation means that the ML estimate is unidentified. As such, inference in the LSM is conducted using a Bayesian approach. Latent positions and other parameters are sampled using a Metropolis-Hastings algorithm. After sampling, latent space positions are rotated to match a common set of positions, via Procrustes transformation (Borg and Groenen 2005). Procrustes transformation variance that is due to rotation of the latent space. Hoff, Raftery and Handcock (2002) propose the use of diffuse independent normal priors for the latent positions and regression parameters. They also recognize the fact that isolates' (i.e., nodes with no ties) finite positions are only identified through the prior in the LSM. In one application they actually just exclude the isolates. Excluding isolates seems fine if the inferential purpose is to estimate positions in the latent space (for, e.g., visualization). However, if the goal is to conduct explanatory analysis that is representative of the full set of nodes, excluding isolates would amount to selecting on the dependent variable. Unlike in many practical applications of Bayesian inference, it is not possible to use an improper "flat" prior (Tibshirani 1989) with the LSM, as an informative prior is necessary to assure finite latent positions. Since

it is necessary to use an informative prior with the LSM, we ask whether it is possible to specify the prior in a way that improves inference regarding covariate parameters. We propose to calibrate the prior to be on a scale comparable to the linear predictor estimated from a GLM. This scaling should discourage the latent space from replicating the linear predictor, encouraging the discovery of structure that remains in the data after identifying the effects of measured covariates.

Our scaling rule is expressed as the prior variance for the latent positions, denoted ν^2. We focus on a one-dimensional latent space $\mathbf{z}$ with a single dyadic covariate $\mathbf{x}$. The rule we derive generalizes naturally to more than one dimension, as the prior variance for each dimension could simply be set to ν^2/d where d is the number of dimensions. If we assume that the $\mathbf{z}$ are independent and distributed $\mathcal{N}(0, \sigma^2)$, it is straightforward to derive the distribution of $|z_i - z_j|$.[2] Since it is a special case of a "normal difference distribution", we know that $z_i - z_j \sim \mathcal{N}(0, 2\sigma^2)$ (Devore and Berk 2012). The normality of $z_i - z_j$ implies that $|z_i - z_j|$ has a folded normal distribution (Leone, Nelson and Nottingham 1961). Following from this result, we know that the variance of $|z_i - z_j|$ is $\sigma^2(2 - 4/\pi)$. Let θ^2 be the empirical variance of $\mathbf{x}$. Then the variance in the linear predictor is $\beta^2\theta^2$. Given an estimate $\hat{\beta}$, we can set the variance of the latent position prior to

$$\nu^2 = \hat{\beta}^2\theta^2/(2 - 4/\pi).$$

This prior actively penalizes latent distance distributions that match or exceed the variance of the observed linear predictor.

3.1 Simulation Study

We use a simulation study to evaluate the LSM's performance at inferring covariate parameters in the situation in which we have some observed covariates as well as omitted network structure which can be represented using a Euclidean latent space. We are particularly interested in whether the LSM can adjust for confounding when the confounding variable is unmeasured. In the simulation design, we study how the level of confounding affects the performance of the LSM relative to the GLM. We evaluate performance in terms of bias, inferential error, and prediction.

3.2 Simulation Design

For simplicity and computational efficiency we consider a unidimensional latent space. We expect our results to generalize to higher dimensional LSMs. To generate an observed covariate that exhibits a controllable degree of colinearity with the latent network structure we follow a three-step process. First, we simulate unidimensional positions for each node, drawing from a normal distribution, and calculate the Euclidean distance $\mathbf{d}$ between each pair of positions. Second, given a target covariance matrix (Σ) among the covariates and distances, $\langle \mathbf{x}, \mathbf{d} \rangle$, we derive the conditional mean vector and covariance, assuming that $\mathbf{x}$ has a normal distribution given $\mathbf{d}$ (see (Eaton 1983, pp. 116–117) for the conditional normal derivation). Third, we simulate x as a normal random

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[2]The assumption that $\mathbf{z} \sim \mathcal{N}(0, \sigma^2)$ is consistent with $\mathbf{z}$ being drawn from the prior that is conventionally used in the LSM.

Figure 1: The distribution of the absolute value of the correlation between the observed covariate and the latent distances.

variable with the respective conditional means and covariance. Finally we standardize x to have zero mean and unit variance. To generate the covariance matrix Σ, which controls the dependence between the omitted network structure and the observed covariate $\mathbf{x}$ we utilize the C-vine method of Lewandowski, Kurowicka and Joe (2009).

We consider three exponential family distributions from which the adjacency matrix entries are drawn: Gaussian, Bernoilli, and Poisson. We also vary the size of the network under study, considering networks with $n = 25, 50$, and 100 nodes. To vary the degree of confounding attributable to the latent positions, we consider three values of the collinearity parameter η, 1, 100, and $1,000,000$, where 1 corresponds to a standard uniform distribution of the correlation (i.e., moderate collinearity) between the observed covariate and the latent distances, and $1,000,000$ to independence between the observed covariate and the latent distances. See Figure 1 for the distribution of the absolute value of the correlation generated at each value of η.

We consider the LSM with several different priors on the coefficient for the observed covariate and the latent space. We set the prior variance of β to be either 1 or 10 and alternatively use a diffuse normal prior on the latent space or scale it by $\hat{\beta}^2 \theta^2 / (2 - 4/\pi)$, where θ^2 is the empirical variance of x.

The LSM is estimated using the canonical implementation in the `latentnet` package in R (Krivitsky and Handcock 2008). For each iteration of the simulation, we run 10,000 burn in iterations, followed by 1,000,000 iterations of the sampler. Every 100th post burn in iteration is saved. Convergence in the log probability of the model is assessed using the Geweke diagnostic in the `coda` package (Plummer, Best, Cowles and Vines 2006; Geweke et al. 1991). If the convergence criterion is satisfied, the simulation continues to the next set of arguments, otherwise the number of iterations is doubled. If the convergence criterion is still not satisfied, then the aforementioned step in the simulation is flagged for review. However, this was not necessary in any cases. At each

point in the simulation's parameter space, we execute 1,000 Monte Carlo iterations. [3]

We compute the MLE estimated by iteratively reweighted least squares via the `glm` function in R (R Core Team 2015). For the LSM we use the posterior mode as our point estimate. In the cases where edges are Bernoulli and there is omitted network structure, we scale the estimates using the reciprocal of the bias of β when $\mathbf{x}$ and $\mathbf{d}$ are independent: $\sqrt{\frac{3.28+\beta^2\mathrm{Var}(\mathbf{d})}{3.29}}$, where 3.29 is the variance of a standard logistic distribution. We do this to adjust for bias in the coefficient estimate that arises due to the lack of a scale coefficient in logistic regression (See the derivation of the bias under an independent but omitted covariate in Mood (2010)).

For each combination of simulation conditions we evaluate the mean square prediction error of the model on new edges drawn condtional on $\mathbf{x}$ and $\mathbf{d}$, the bias of the estimated coefficient for the measured covariate, and the Type-1 and 2 error rates. Inference regarding Credible intervals for β are defined by using the region of highest posterior density which covers 95% of the marginal posterior distribution Turkkan and Pham-Gia (1993). We consider β to be statistically significant at the 0.05 level if the 95% credible interval excludes 0.

3.3 Results

To evaluate estimation error we compute the bias. Figures 2 and 3 shows the results. In Figure ?? we can see that with a uniform distribution over the correlation between the distances and the observed covariate, all methods (except the true model wherein the distances are included via an observed covariate) show substantial bias. In most cases the LSM performs better than the GLM, sometimes by as much as 50%. However, in absolute terms, the amount of bias is large. Our results demonstrate that using the LSM will not permit the discovery of and adjustment for the true latent positions when the observed covariates and latent distances are confounded. For both the GLM and the LSM, bias decays rapidly as the degree of correlation between the distances and the measured covariate decreases. In 3 we can see that when there is no omitted network structure, the LSM exhibits bias greater than that of the GLM, though it does appear that scaling the prior of the latent positions to match the scale of the linear predictor decreases this tendency.

Figures 4 and 5 show the inferential error rates of the LSM with different priors for β and the latent positions, as well as a GLM. Type-1 error rates shown in Figure 4 for the LSM are in general somewhat lower than those of a GLM when an unmeasured covariate exists. Under uniform correlation between the omitted network structure and the observed covariate, the Type-1 error rate is often more than 10 times greater than the nominal rate of 0.05. This illustrates that the LSM cannot be considered a viable substitute to measuring the omitted structure, as the true model exhibits a Type-1 error rate that matches the nominal rate. Scaling the prior of the latent space to match that of the observed covariate appears to make the LSM's error rate comparable to that of the GLM. Type-2 error rates, shown in Figure 5 are also substantially above their nominal rate when there is a uniform distribution on the strength of confounding, though the inflation is not as bad as

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[3]One of the primary difficulties in executing the above simulation design is the computational cost of estimating the LSM. We utilize the `BatchExperiments` R package to construct and execute our computational experiments on a Torque cluster Bischl, Lang, Mersmann, Weihs et al. (2015).
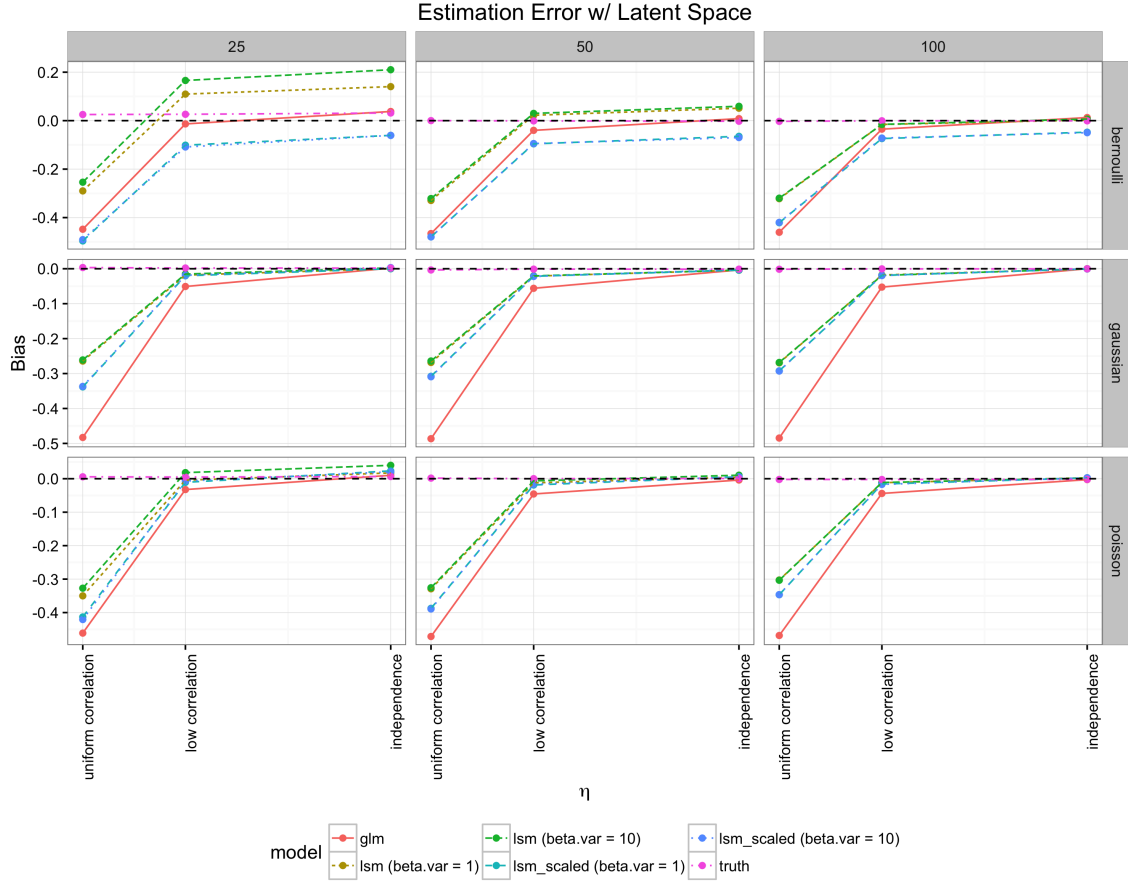
Figure 2: The bias of estimates of the effect of the observed covariate **x** when there is an ommitted variable. The x-axis gives the value of the parameter η which controls the degree of dependence between **x** and ommitted covariate. Lower values of η indicate higher levels of dependence between the observed and ommitted covariate. The y-axis gives a Monte Carlo estimate of the bias. The number of nodes are indicated in the top panels, while the distributional family of the edges is shown on the right panel. Each panel represents 4 values of η with 1,000 Monte Carlo iterations executed at each point.
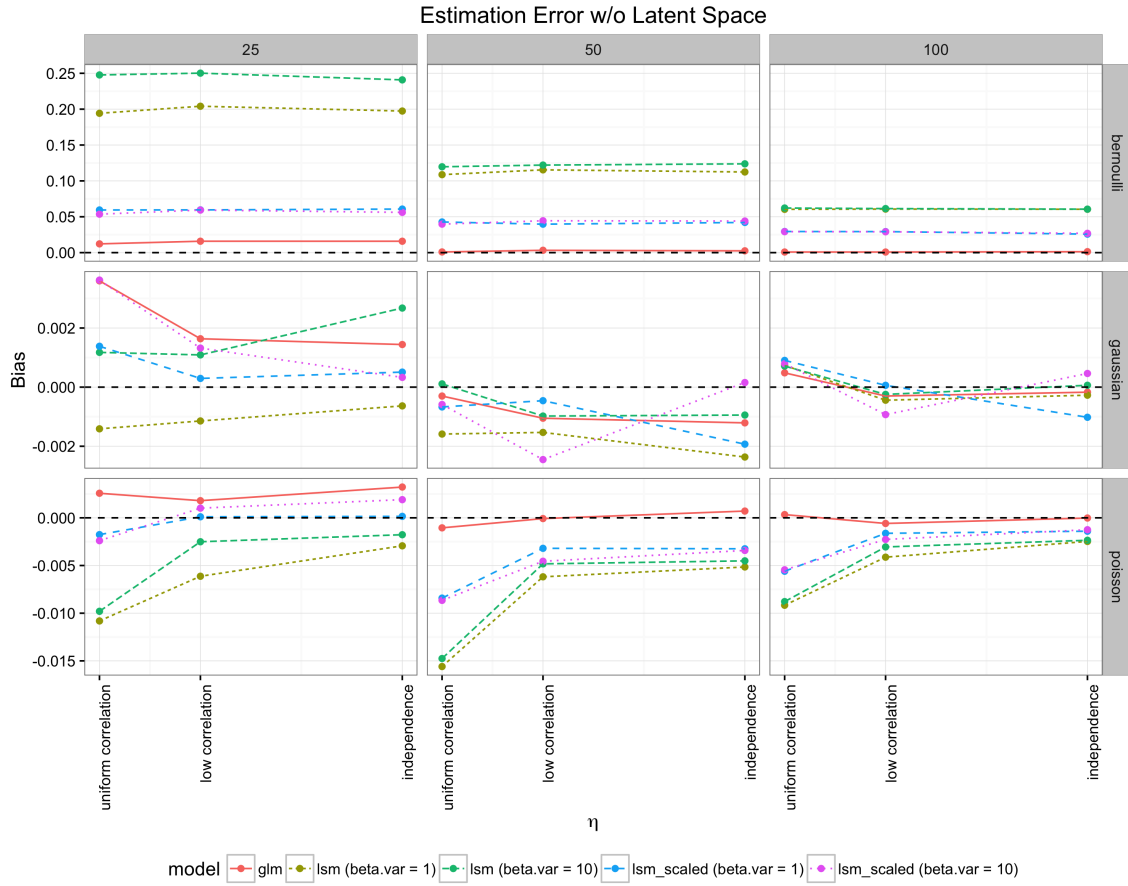
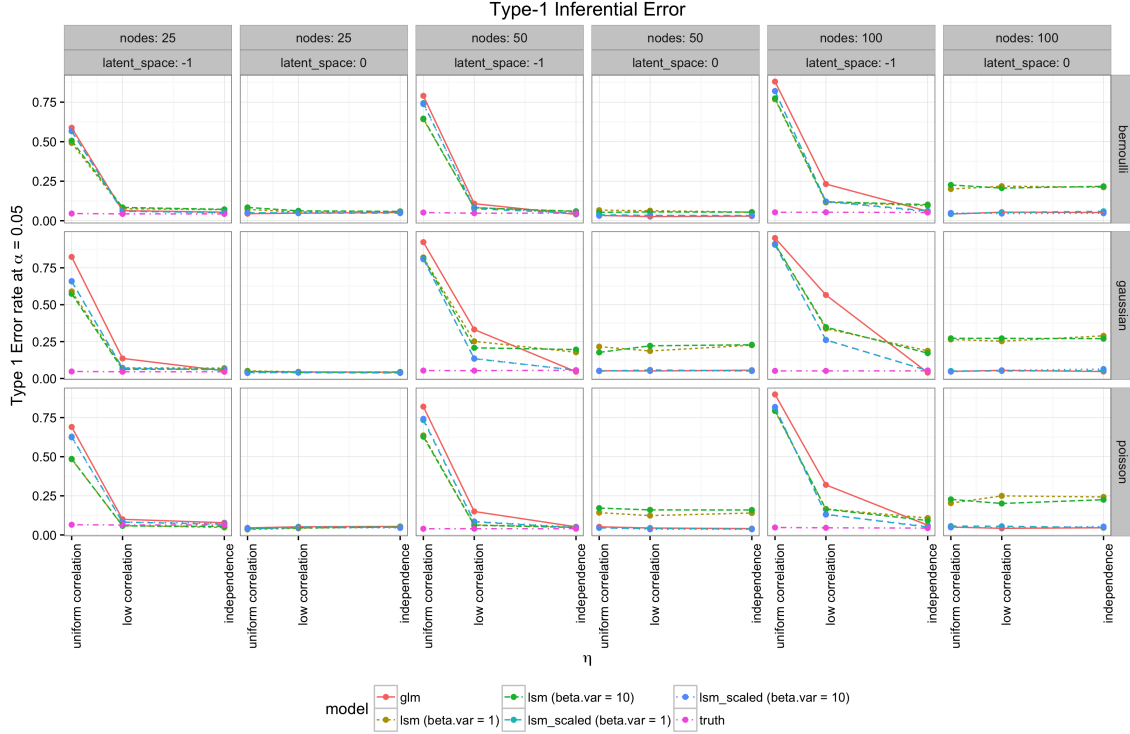Figure 3: The bias of estimates of the effect of the observed covariate **x** when there is no ommitted variable.

Figure 4: Monte Carlo estimates of the Type-1 error regarding the effect of the observed covariate **x** are shown on the y-axis. Here, $\beta = 0$ and the error rate shown in each panel in that row gives 1 minus the probability of a 95% confidence region (for the LSM) or interval (for the GLM) including 0, giving the probability that a true null hypothesis of $\beta = 0$ is falsely rejected.

that of the Type-1 rates. These results indicate that, in the presence of unobserved confounders that can be represented through the LSM, the LSM does not adequately correct the inferential errors that would arise due to omitted structure among the dyads in the network.

The last property we consider in comparing the GLM and LSM is generalization error—the performance exhibited by the models in predicting new data that was drawn from the same model that generated the data used for estimation (i.e., out of sample predictive performance). We estimate generalization error as the expected prediction error on new edges generated using a fixed set of latent positions for the nodes and a fixed observed covariate. Generalization error results are shown in Figure 6. In nearly all cases the LSM substantially outperforms the GLM, especially when the omitted covariate is not highly collinear with the observed covariate. In nearly all cases it performs as well as the true model.
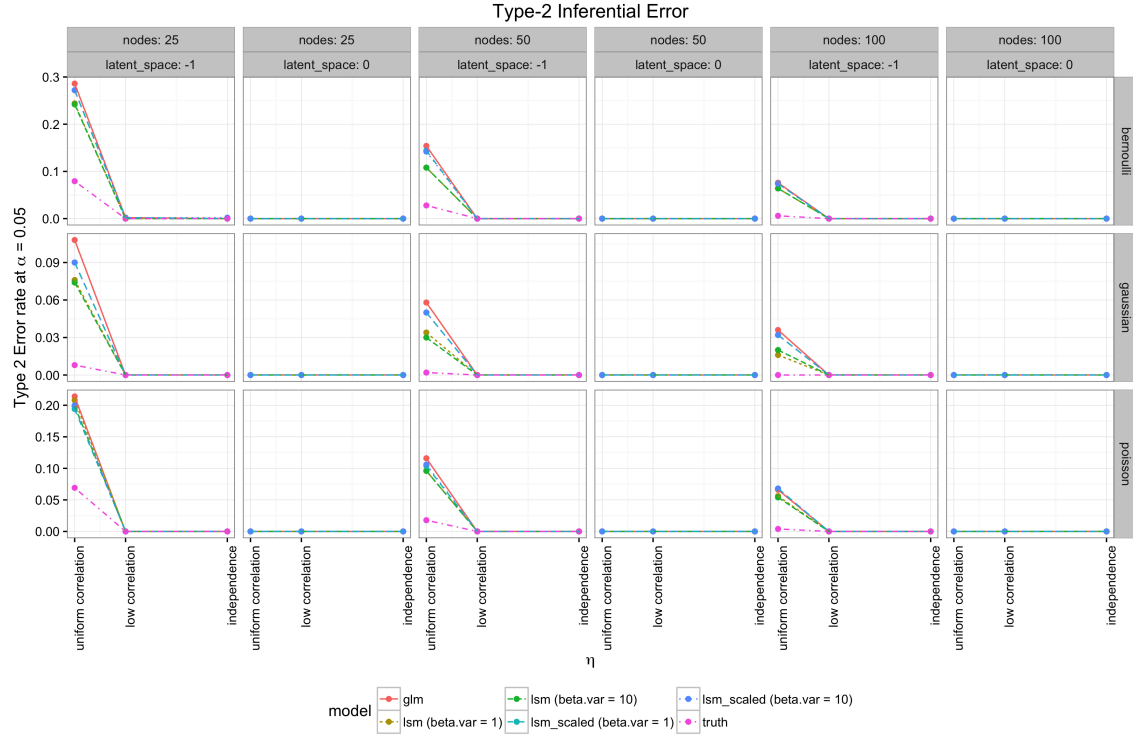
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Figure 5: Monte Carlo estimates of the Type-2 error regarding the effect of the observed covariate **x** are shown on the $y - axis$. Here, $\beta = 1$, and the error rate shown in each panel in each row gives the probability of the probability/confidence intervals covering 0, giving the probability of accepting a false null hypothesis.
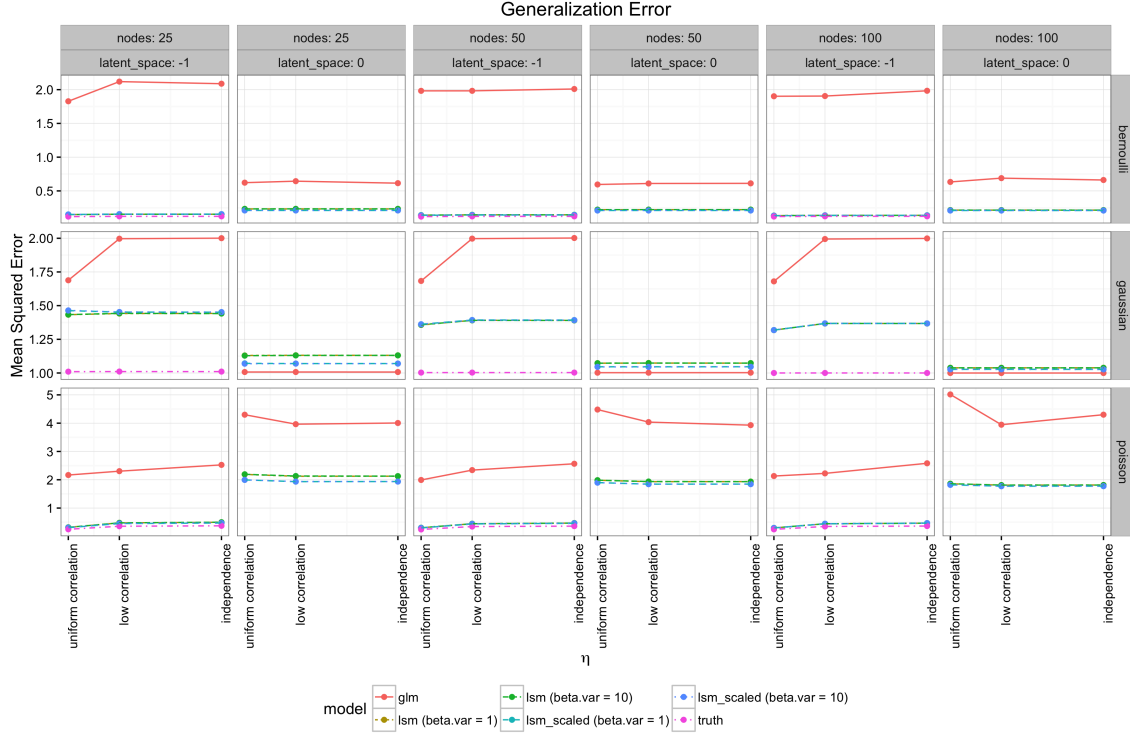
Figure 6: The x-axis shows a Monte Carlo estimate of the mean square error for edge values drawn from the appropriate distribution with the observed covariate $\mathbf{x}$ and the latent positions $\mathbf{z}$ fixed. In the Bernoulli case the Brier score is computed and in the poisson and normal cases the mean-square-error.

4   Conclusion

Based on our simulation study, we conclude that the primary advantage of using the LSM—relative to a dyadic regression model in which the network is fit using observed covariates only—is that the latent space provides an efficient complement to observed covariates when it comes to fitting and predicting the network. Considering both (1) that the LSM does not substantially reduce bias or inferential error relative to the GLM, and (2) that the LSM exhibits strong predictive performance, we conclude that the latent positions are used to explain the systematic variation that cannot be explained through the observed covariates. This is of considerable use in research focused on developing a predictive model, or fitting and exploring latent positions. However, since the latent positions are inferred to complement the observed covariates—explaining residual variation—the LSM is ill-suited to adjust for confounding network structure, as adjusting for confounding would require that the latent positions explain variation that could otherwise be attributed to the observed covariates.

The key recommendation arising from our results is that researchers not use the LSM as an alternative to measuring potential confounding variables and/or explicitly modeling the endogenous network structure. The LSM cannot control for unmeasured confounders. This property is, perhaps, not surprising. Given that all priors are centered at zero, the parameter regions that exhibit high posterior probability will be those that explain the observed network (i.e., high likelihood) using minimal departures from zero (i.e., high prior probability). Since explaining the observed network using a covariate requires moving one parameter value away from zero, whereas explaining the network using a latent dimension requires moving many latent coordinates away from zero, the LSM exhibits an inherent preference to explain network structure using observed covariates over latent coordinates. One way to address this issue would be to use a highly diffuse prior for the latent positions, but as we discuss above, that is not possible with the LSM. If the LSM is to be used as a tool for drawing inferences regarding observed covariate effects while adjusting for confounding network structure, further development is needed to enable the LSM to avoid bias and inferential error under confounding.

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