Empirical means to validate skills models and assess the fit of a student model

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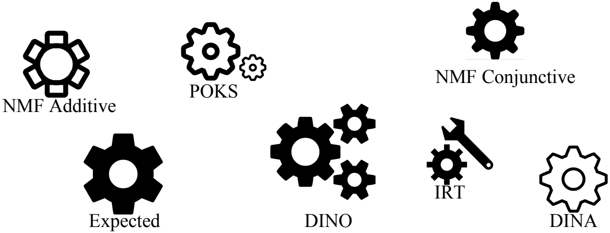
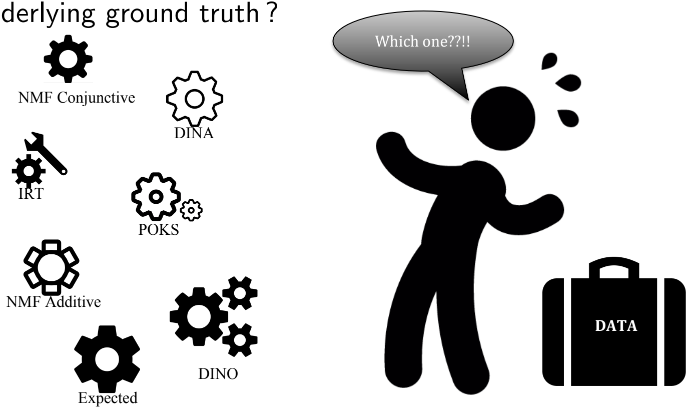
Problem Specification

- Student skills assessment models



Problem Specification

- Student skills assessment models
- How to decide which are the most representative of the underlying ground truth ?



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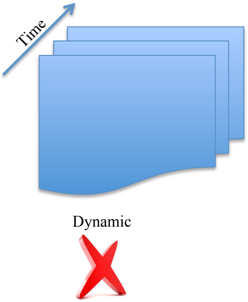
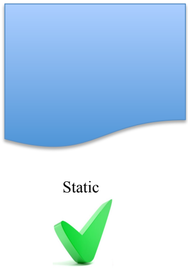
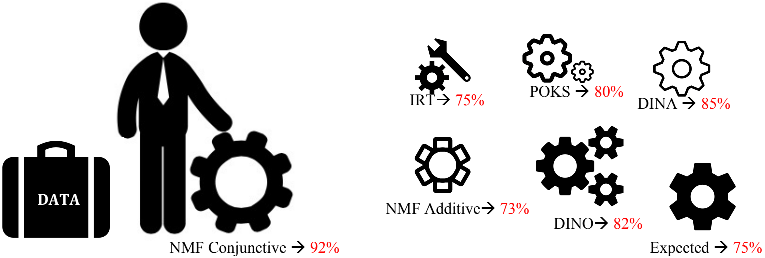
- Student skills assessment models
- How to decide which are the most representative of the underlying ground truth ?
- Static Vs. Dynamic





Problem Specification

- Student skills assessment models
- How to decide which are the most representative of the underlying ground truth ?
- Static Vs. Dynamic
- Model selection and goodness of fit
- A general answer : best performer



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- Student skills assessment models
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- Our contribution
  - To make a comprehensive comparison of educational data model performances
  - To propose a new approach to assessing model fit

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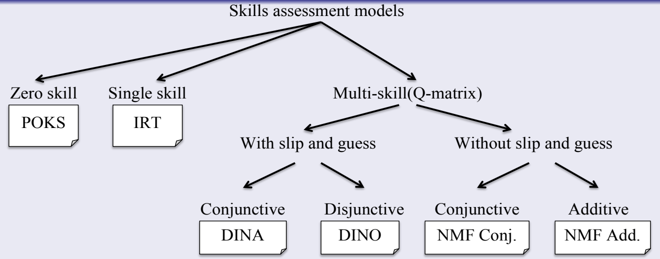
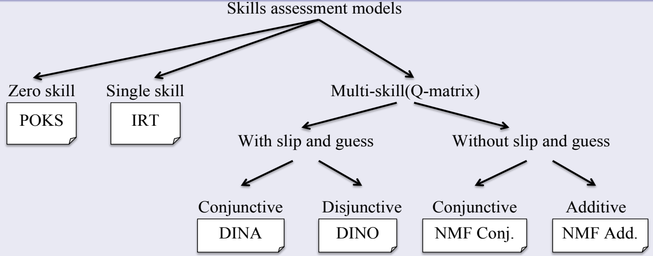
- A general answer : best performer
- Our contribution
  - To make a comprehensive comparison of educational data model performances
  - To propose a new approach to assessing model fit
- The proposed approach :
  - Assessing the fit of the model to the underlying ground truth using a methodology based on **synthetic data**

Student skills assessment models



- Number of Skills

Student skills assessment models



- Number of Skills
- Q-matrix

s_1 : fraction multiplication

s_2 : fraction addition

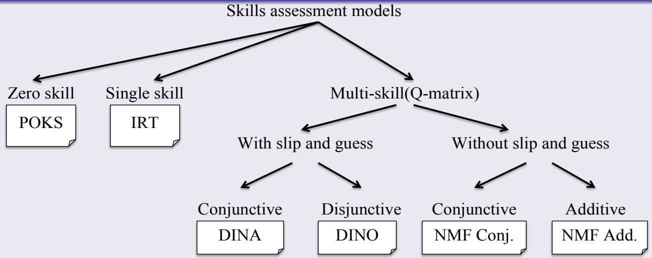
s_3 : fraction reduction

i_1	$\frac{4}{\frac{12}{3}} + \frac{3}{5} = \frac{8}{5}$
i_2	$\frac{4}{\frac{12}{3}} = \frac{4\times3}{12} = \frac{12}{12} = 1$
i_3	$1 + \frac{3}{5} = \frac{8}{5}$
i_4	$2\times\frac{1}{2} = 1$

Skills

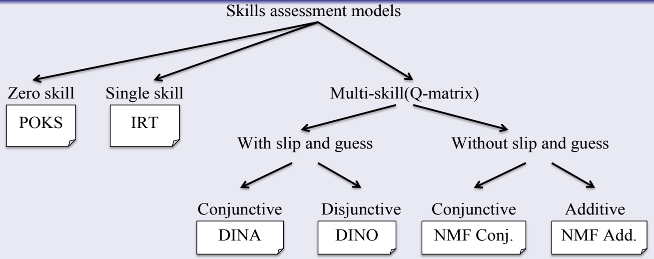
$$\text{Items}\quad \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

Student skills assessment models



- Number of Skills
- Q-matrix
- Slip and Guess

Student skills assessment models



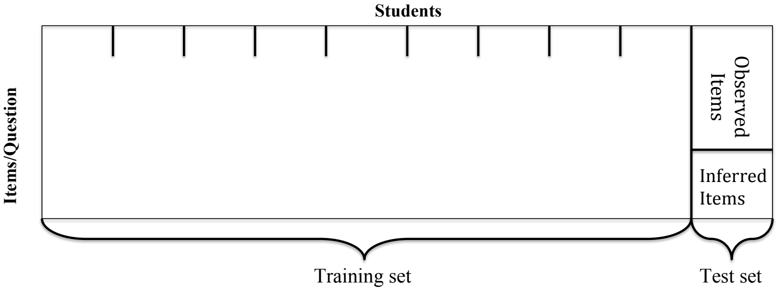
- Number of Skills
- **Q-matrices (types)**
- Slip and Guess

1. Conjunctive
2. Additive
3. Disjunctive

$$
\begin{array}{cc}
 & \text{Skills} \\
 & \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\
\text{Items} \begin{array}{c} i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} & \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}
\end{array}
$$

Presentation terms and concepts

- **Performance of a model** over a data set



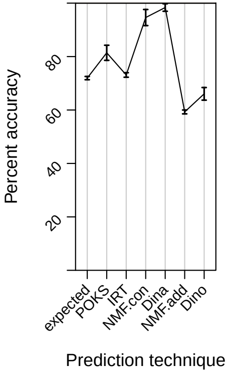
Presentation terms and concepts

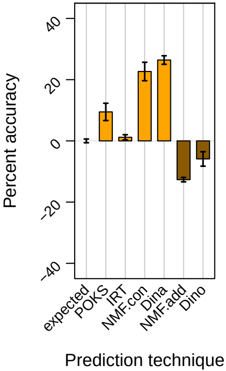
- **Performance of a model** over a data set
- Model parameters

<table>
<tr><th colspan="3"></th><th colspan="3">Parameters estimated from</th></tr>
<tr><th colspan="3">Skills Model</th><th colspan="2">Training set</th><th>Observed items</th></tr>
<tr><td rowspan="7">Contributed skills</td><td rowspan="4">Multiple</td><td>NMF Conj.</td><td rowspan="2"></td><td rowspan="4">• Q-matrix</td><td rowspan="4">• Students skills mastery matrix</td></tr>
<tr><td>NMF Add.</td></tr>
<tr><td>DINA</td><td rowspan="2">• Slip
• Guess</td></tr>
<tr><td>DINO</td></tr>
<tr><td rowspan="2">Single</td><td>IRT</td><td colspan="2">• Item difficulty
• Item discrimination</td><td>• Student Ability</td></tr>
<tr><td>Expected</td><td colspan="2">• Item Odds</td><td>• Student Odds</td></tr>
<tr><td>Zero</td><td>POKS</td><td colspan="2">• Initial Odds
• Odds ratio
• Partial order structure</td><td></td></tr>
</table>

Presentation terms and concepts

- **Performance of a model** over a data set
- Model parameters
- Performance vector
- Performance signature





Model	*Performace*
Expected	0.72%
POKS	0.80%
IRT	0.74%
NMF.Conj	0.94%
Dina	0.99%
NMF.Add	0.60%
Dino	0.65%

Presentation terms and concepts

- **Performance of a model** over a data set
- Model parameters
- Performance vector
- Performance signature
- Performance prototype

The *performance vector* associated with the synthetic data of a model class.

Presentation terms and concepts

- **Performance of a model** over a data set
- Model parameters
- Performance vector
- Performance signature
- Performance prototype
- Target performance vector

The *performance vector* of the real data set to classify.

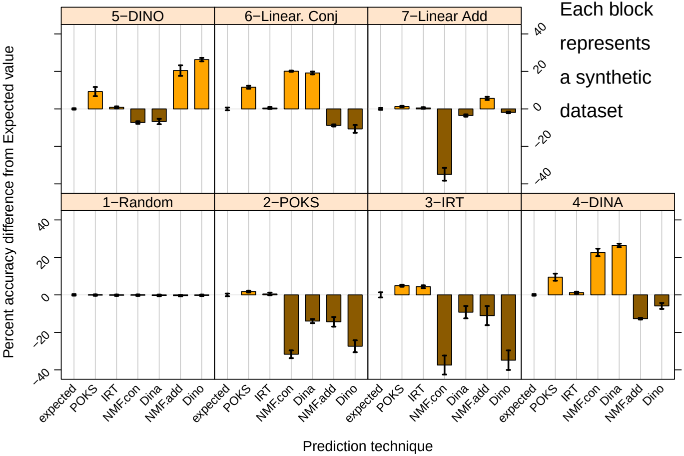
Research questions

1. ✓What is the *performance vector* of student skills assessment models over real and over synthetic data created using the same models ?
   - Experiment 1 : Predictive *performance vector* of models over real and synthetic data sets

Datasets

Data set	Number of Skills	Number of Items	Number of Students	Mean Score	Q-matrix
Synthetic					
1.Random	7	30	700	0.75	$\mathbf{Q}_{01}$
2.POKS	7	20	500	0.50	$\mathbf{Q}_{02}$
3.IRT-2PL	5	20	600	0.50	$\mathbf{Q}_{03}$
4.DINA	7	28	500	0.31	$\mathbf{Q}_{5}$
5.DINO	7	28	500	0.69	$\mathbf{Q}_{6}$
Linear (Matrix factorization)					
6. Conj.	8	20	500	0.24	$\mathbf{Q}_{1}$
7. Comp.	8	20	500	0.57	$\mathbf{Q}_{1}$
Real					
8.Fraction	8	20	536	0.53	$\mathbf{Q}_{1}$
9.Vomlel	6	20	149	0.61	$\mathbf{Q}_{4}$
10.ECPE	3	28	2922	0.71	$\mathbf{Q}_{3}$
Fraction subsets and variants of $\mathbf{Q}_1$					
11. 1	5	15	536	0.53	$\mathbf{Q}_{10}$
12. 2/1	3	11	536	0.51	$\mathbf{Q}_{11}$
13. 2/2	5	11	536	0.51	$\mathbf{Q}_{12}$
14. 2/3	3	11	536	0.51	$\mathbf{Q}_{13}$

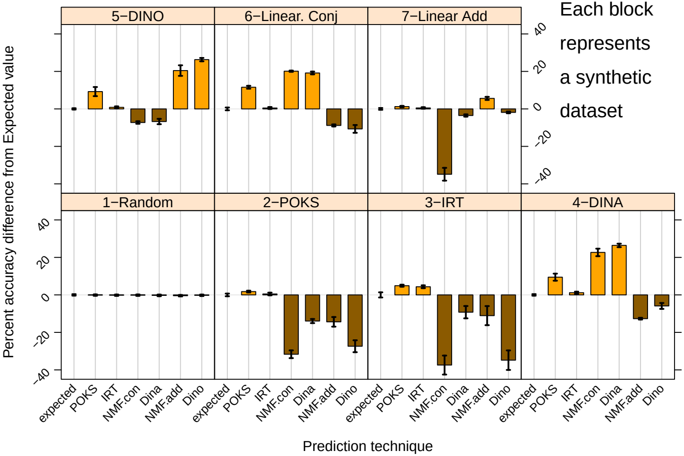
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

The random data set has a flat performance across techniques

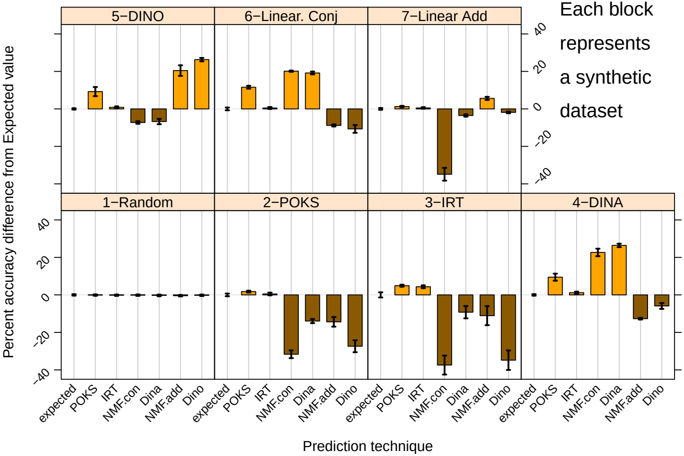
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

The highest performance is for the generative model behind the dataset

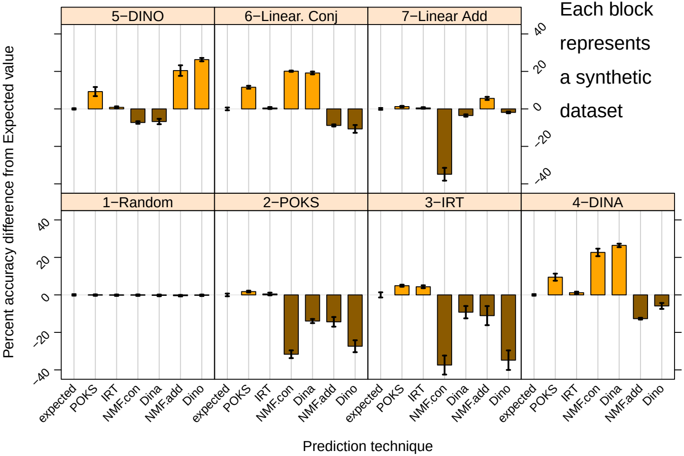
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

Data sets have unique pattern of performance vector across models

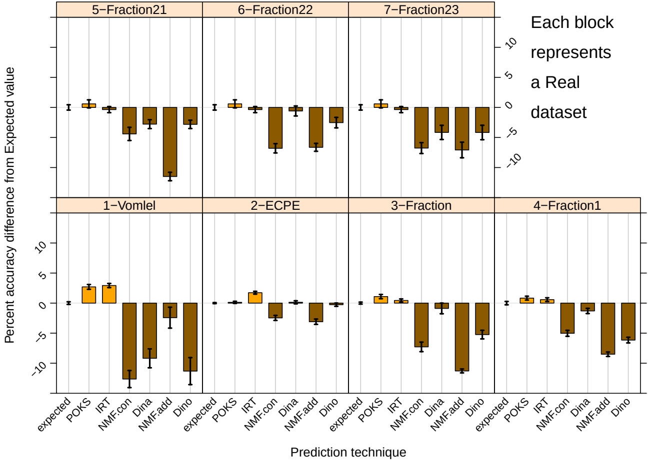
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

The capacity of recognizing a data set's true model relies on this uniqueness characteristic

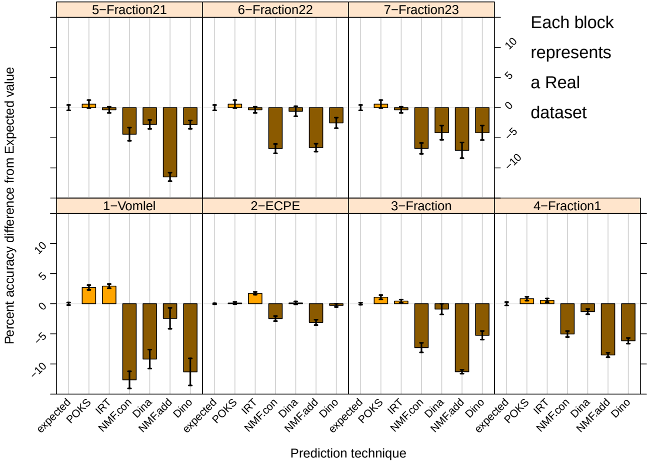
Predictive performance of models over real datasets



Each block represents a Real dataset

In most cases, the best performer is close to the baseline

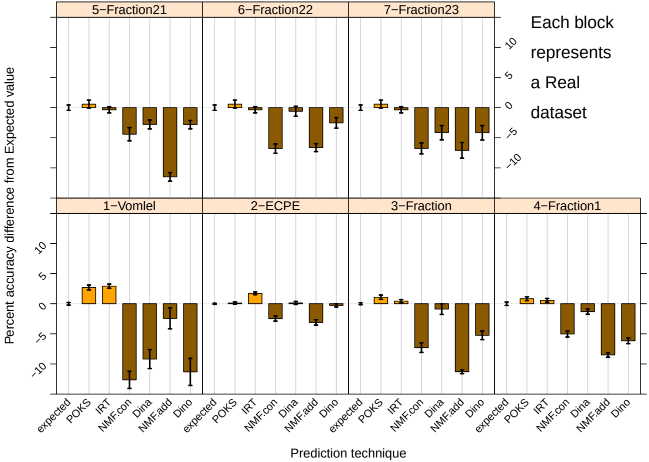
Predictive performance of models over real datasets



Each block represents a Real dataset

The pattern of the Fraction performance data set repeats over its subsets

Predictive performance of models over real datasets



Each block represents a Real dataset

None of the real data sets show the large variance and the differences found in the synthetic data sets models

Vector space of accuracy performances

- Performance vectors of datasets in columns(Data points in the performance space)

Model	Synthetic data set						
	Random	POKS	IRT	DINA	DINO	Linear .Conj	Linear .Comp
Expected	**0.75**	0.91	0.90	0.72	0.72	0.78	0.93
POKS	0.75	**0.94**	0.94	0.81	0.81	0.90	0.94
IRT	0.75	0.91	**0.95**	0.73	0.73	0.79	0.89
DINA	0.75	0.77	0.81	**1.00**	0.65	**0.98**	0.89
DINO	0.75	0.63	0.56	0.66	**1.00**	0.68	0.91
NMF.Conj	0.75	0.59	0.53	0.95	0.65	0.97	0.58
NMF.Comp	0.75	0.76	0.79	0.59	0.93	0.70	**0.98**

The diagonal generally displays the best performance

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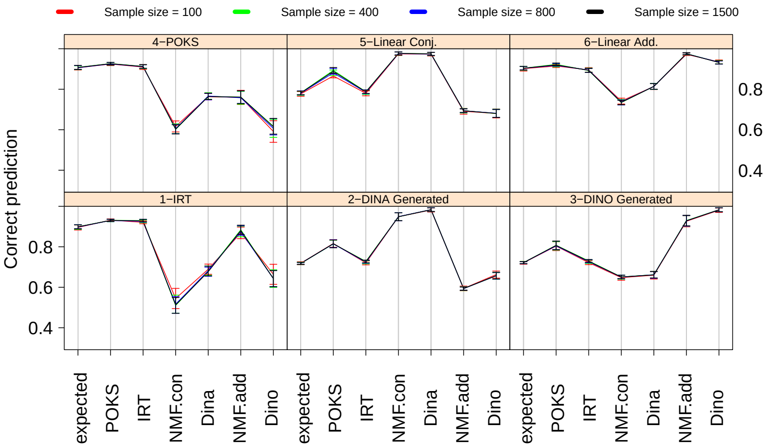
1. ✓What is the *performance vector* of student skills assessment models over real and over synthetic data created using the same models ?
   - Experiment 1 : Predictive performance of models over real and synthetic data sets
2. **Is the performance vector unique to each synthetic data type (data from the same ground truth model) ?**
   - Experiment 2 : Sensitivity of the Model performance over different data generation parameters

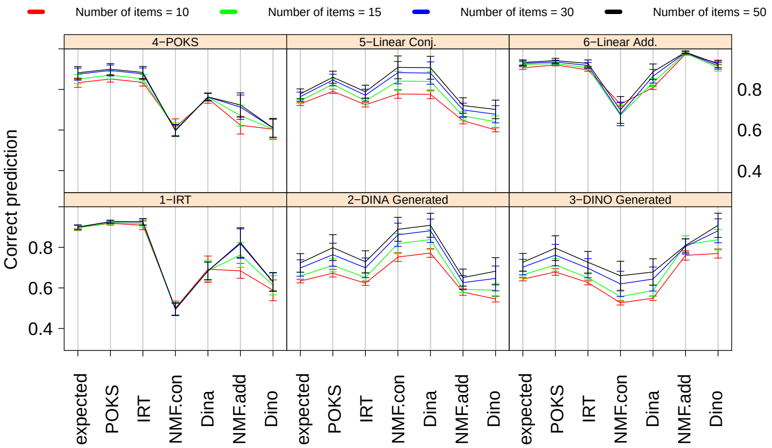
Are they stable in addition to be unique ?

Variation of sample size over synthetic data sets



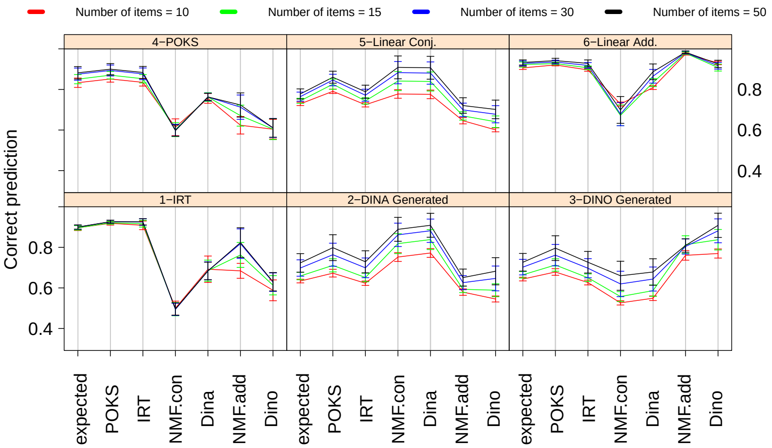
Obviously, the signature pattern did not change significantly for some parameters such as **Sample size**.

Variation of number of items over synthetic data sets



Even for synthetic data the performance of the ground truth model should not necessarily be close to 100%.

Variation of number of items over synthetic data sets



The Performance signature shifts down once the number of items degrades.

Data specific parameters

1. Sample size (Number of students)
2. Number of items
3. Number of latent skills
4. Item score variance
5. Student score variance
6. Average success rate

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Conclusion :

- Data specific parameters can potentially influence the performance of a model
- For better comparison of the results, we can also consider **data specific parameters** of the real data in the generation process

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3. **Can the performance vector be used to define a method to reliably identify the ground truth behind the synthetic data ?**
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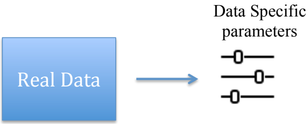
Signature framework

This approach relies on generating synthetic datasets



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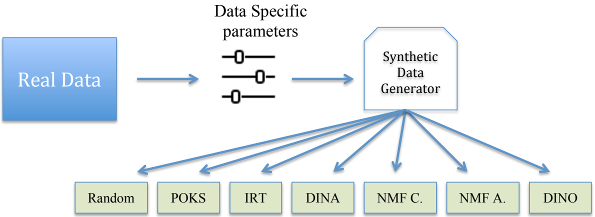

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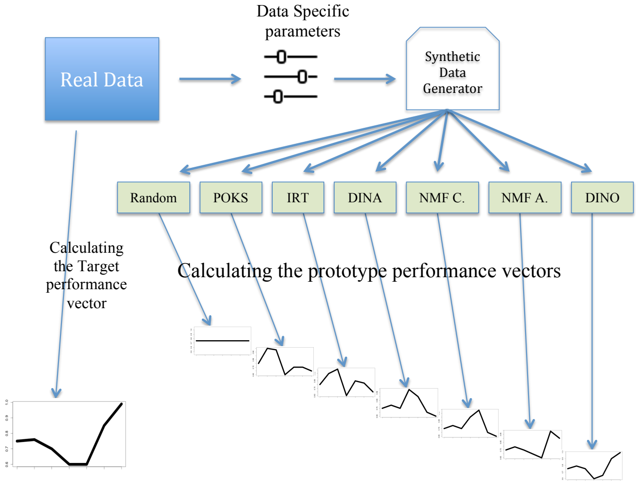
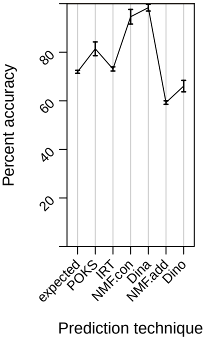


Recall



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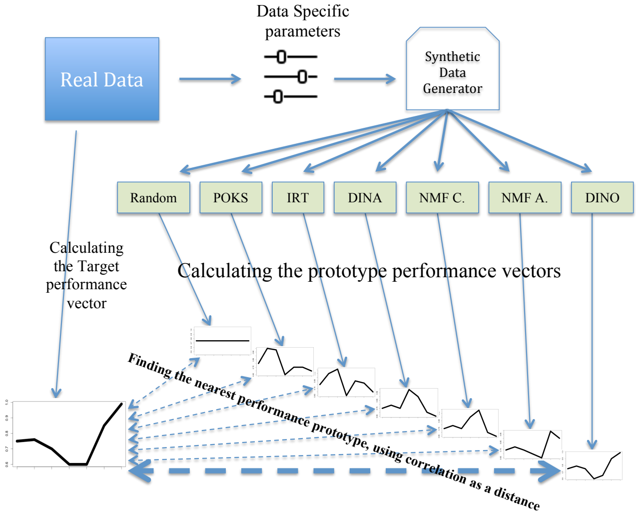
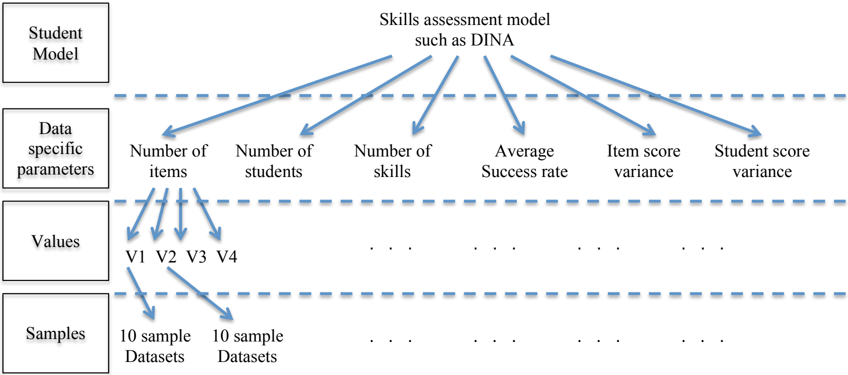
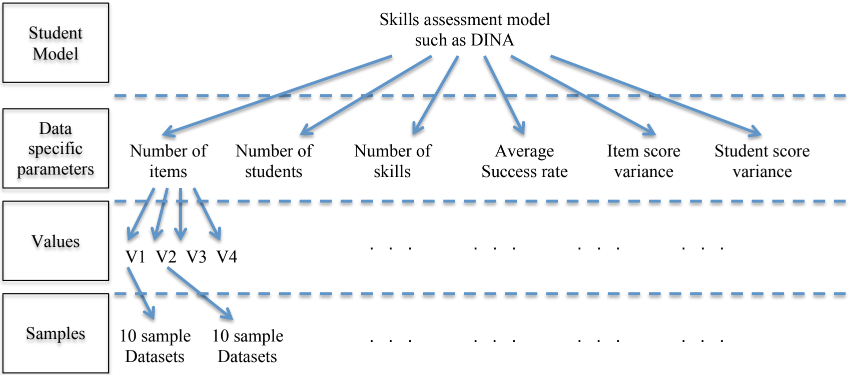


Pool of synthetic datasets



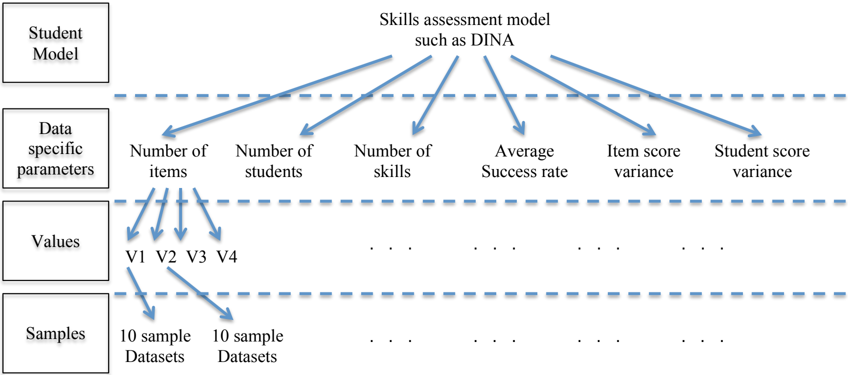
There exists 6 skills assessment models

Pool of synthetic datasets



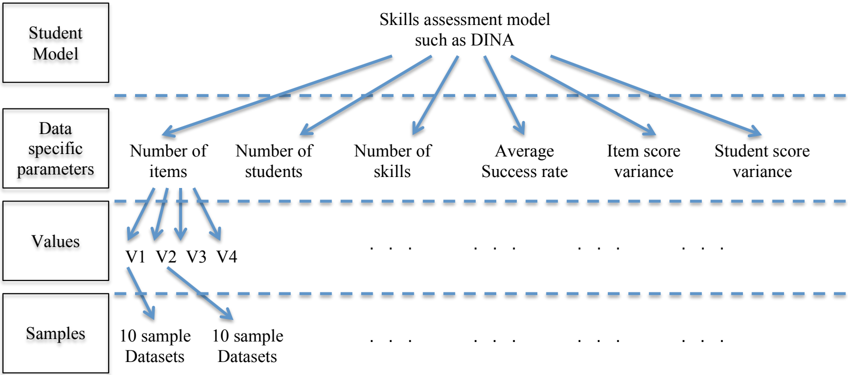
There exists 6 skills assessment models X 6 data specific parameters

Pool of synthetic datasets



There exists 6 skills assessment models X 6 data specific parameters X 4 values

Pool of synthetic datasets



There exists 6 skills assessment models X 6 data specific parameters X 4 values X 10 samples = 1440 samples in the pool

Degree of similarity between six synthetic datasets based on the correlation

Synthetic Datasets

Synthetic Datasets	POKS	IRT	NMF Conj.	DINA	NMF Add.	DINO
POKS	**0.96**					
IRT	0.86	**0.96**				
NMF Conj.	0.22	-0.20	**0.96**			
DINA	0.02	-0.40	0.94	**0.96**		
NMF Add.	0.44	0.75	-0.62	-0.73	**0.93**	
DINO	-0.15	0.20	-0.70	-0.69	0.63	**0.95**

- he diagonal shows high correlations because it compares the same model generated datasets.
- Datasets with similar ground truth also show a high correlation.

Degree of similarity between six synthetic datasets based on the correlation

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DINA	0.02	-0.40	0.94	**0.96**		
NMF Add.	0.44	0.75	-0.62	-0.73	**0.93**	
DINO	-0.15	0.20	-0.70	-0.69	0.63	**0.95**

In general, correlation similarity provides a very good measure of model fit.

Degree of similarity between six synthetic datasets and the ground truth based on the correlation

Real Datasets

Synthetic Datasets	Vomlel	ECPE	Fraction	Fraction subsets: 1	21	22	23
Random	0.58	**0.73**	0.61	0.43	0.24	0.61	0.57
IRT	**0.90**	0.42	0.72	0.88	0.60	0.77	0.61
DINA	-0.38	-0.09	0.23	0.30	0.56	0.06	0.38
DINO	0.34	0.15	-0.18	-0.31	0.10	-0.08	0.38
POKS	0.75	0.40	**0.83**	**0.95**	**0.70**	**0.83**	**0.80**
NMF Conj.	-0.05	0.54	0.51	0.55	0.66	0.33	0.57
NMF Add.	0.39	0.06	-0.04	-0.19	-0.03	0.13	0.28

Vomlel dataset shows a high correlation with IRT model

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Fraction with its subset datasets show similarity with POKS model.

Degree of similarity between six synthetic datasets and the ground truth based on the correlation

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As expected, ECPE has the highest correlation with random generated dataset.

Research questions

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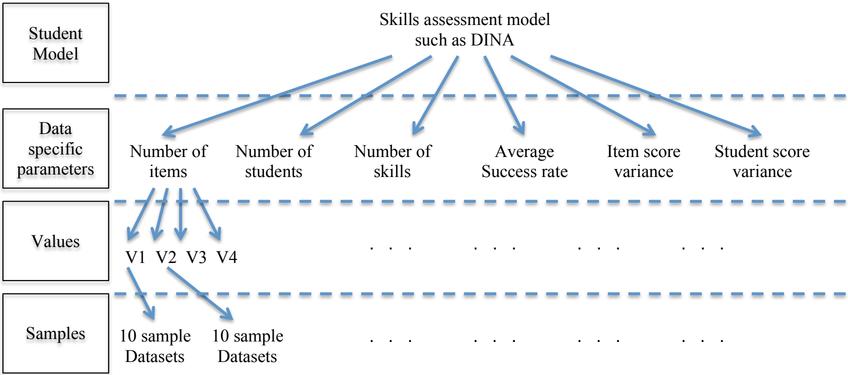
   - Experiment 1 : Predictive performance of models over real and synthetic data sets
2. ✓Is the *performance vector* unique to each synthetic data type (data from the same ground truth model) ?
   - Experiment 2 : Sensitivity of the Model performance over different data generation parameters
3. ✓Can the *performance vector* be used to define a method to reliably identify the ground truth behind the synthetic data ?
   - Experiment 3 : Model selection based on performance vector classification
4. **How does the method compare with the standard practice of using the model with the best performance ?**
   - Experiment 4 : Signature vs. best performer classification

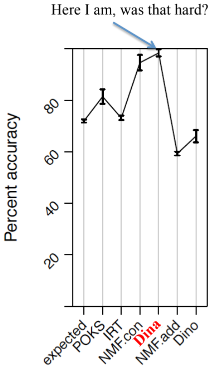
Problem specification

- Evaluating the ability of the Signature approach to identify the ground truth model **over datasets with different data specific parameters**



Problem specification

- Evaluating the ability of the Signature approach to identify the ground truth model **over datasets with different data specific parameters**
- Comparing the results with the best performer approach.



Problem specification

- Evaluating the ability of the Signature approach to identify the ground truth model **over datasets with different data specific parameters**
- Comparing the results with the best performer approach.
- Reporting the accuracy of these classification in terms of precision, recall, F1 measure

		Prediction outcome: Positive	Prediction outcome: Negative
Actual value	Positive	TP	FN
	Negative	FP	TN

$$Accuracy = \frac{TP + TN}{TP + TN + FP + FN}$$

$$Precision = \frac{TP}{TP + FP}$$

$$F_\beta = (1 + \beta^2).\frac{Precision.Recall}{\beta^2.Precision + Recall}$$

$$Recall = \frac{TP}{TP + FN}$$

Results of signature vs. best performer classification

	Performance							
	Best Performer				Nearest Neighbor			
Models	Precision	Recall	F-Measure	Accuracy	Precision	Recall	F-Measure	Accuracy
POKS	0.564	0.992	0.719	0.871	0.793	0.908	0.847	0.945
IRT	0.982	0.458	0.625	0.908	0.846	0.867	0.856	0.951
NMF Conj.	0.943	0.342	0.502	0.887	0.711	0.750	0.730	0.907
DINA	0.617	0.921	0.739	0.891	0.777	0.696	0.734	0.916
NMF Add.	0.938	0.875	0.905	0.969	0.946	0.879	0.911	0.971
DINO	1	0.929	0.963	0.988	0.996	0.946	0.970	0.990
Total accuracy	0.75%				0.84%			

- The F-measure increases when the signature approach is used for classification.

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- The F-measure increases when the signature approach is used for classification.
- In terms of individual scores per method, the accuracy increases when signature approach is used.
- The total accuracy considers true positive numbers over number of datasets regardless of individual models

Research questions

1. ✓What is the *performance vector* of student skills assessment models over real and over synthetic data created using the same models ?
   - Experiment 1 : Predictive performance of models over real and synthetic data sets
2. ✓Is the *performance vector* unique to each synthetic data type (data from the same ground truth model) ?
   - Experiment 2 : Sensitivity of the Model performance over different data generation parameters
3. ✓Can the *performance vector* be used to define a method to reliably identify the ground truth behind the synthetic data ?
   - Experiment 3 : Model selection based on performance vector classification
4. ✓How does the method compare with the standard practice of using the model with the best performance ?
   - Experiment 4 : Signature vs. best performer classification

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Conclusion

- Model fit of a data set is defined as the similarity between *prototype* and *target* performance vector
- The generative model does not always correspond to the best performer and our approach provides a more reliable means
- Datasets that share a common source have correlated performance vectors.
- It does not seem to substantially extend to data that shares the same domain

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Conclusion

- For real data sets, the performances are not better than the expected performance
- For synthetic data, datasets with different ground truths that share some concepts, show a high correlation.
- *performance vector* changes for some data specific parameters but it still shows a high correlation with datasets with the same ground truth.
- Best performer may not be the model that is most representative of the ground truth.

Future works

- Further studies with real and simulated data

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- generalize to dynamic data and skills assessment models

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- Candidate models and their complexity

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- Further studies with real and simulated data
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- Candidate models and their complexity
- Application in other fields of study

Thank you