Empirical means to validate skills models and assess the fit of a student model

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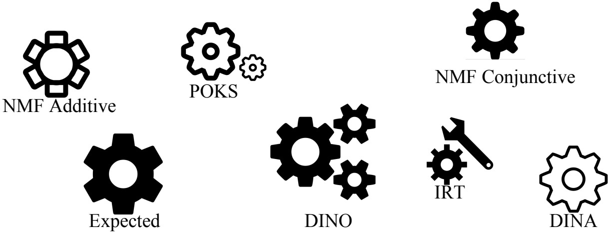
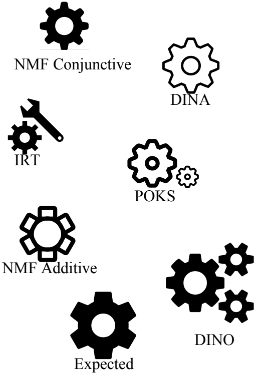
Problem Specification

- Student skills assessment models



Problem Specification

- Student skills assessment models
- How to decide which are the most representative of the underlying ground truth ?



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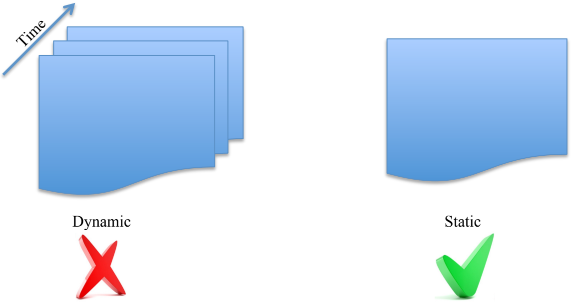
- Student skills assessment models
- How to decide which are the most representative of the underlying ground truth ?
- Static Vs. Dynamic



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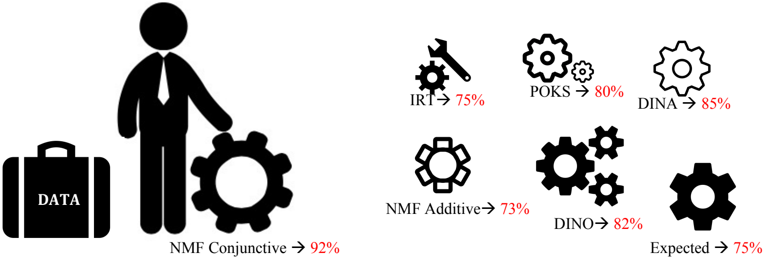
- Student skills assessment models
- How to decide which are the most representative of the underlying ground truth ?
- Static Vs. Dynamic
- Model selection and goodness of fit
- A general answer : best performer



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    - To make a comprehensive comparison of educational data model performances
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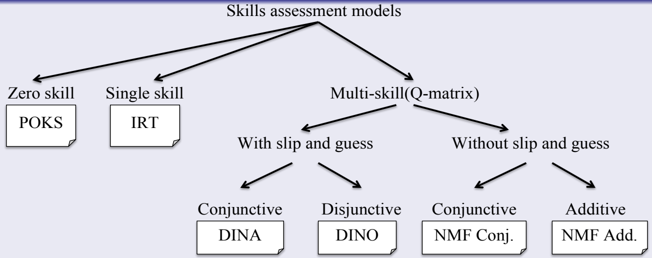
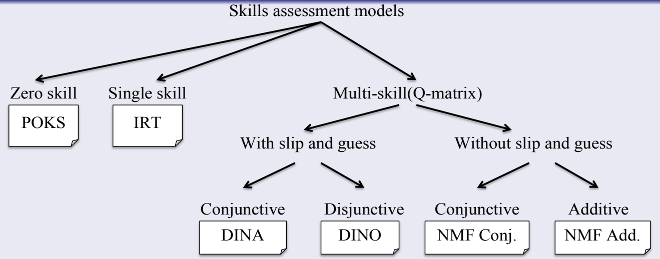
- A general answer : best performer
- Our contribution
  - To make a comprehensive comparison of educational data model performances
  - To propose a new approach to assessing model fit
- The proposed approach :
  - Assessing the fit of the model to the underlying ground truth using a methodology based on **synthetic data**

Student skills assessment models



- Number of Skills

Student skills assessment models



- Number of Skills
- Q-matrix

s_1 : fraction multiplication

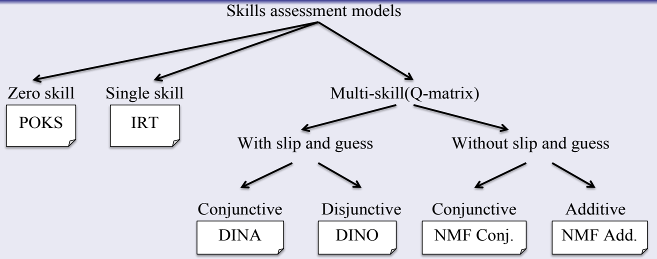
s_2 : fraction addition

s_3 : fraction reduction

i_1	$\frac{4}{\frac{12}{3}} + \frac{3}{5} = \frac{8}{5}$
i_2	$\frac{4}{\frac{12}{3}} = \frac{4\times 3}{12} = \frac{12}{12} = 1$
i_3	$1 + \frac{3}{5} = \frac{8}{5}$
i_4	$2\times\frac{1}{2} = 1$

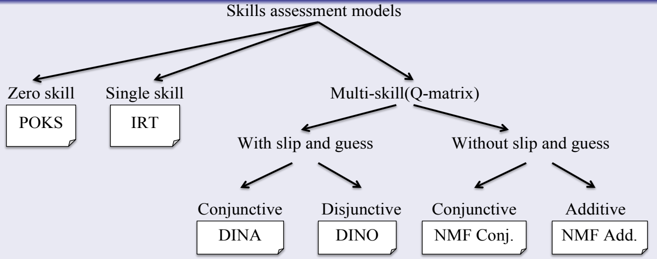
$$
\text{Items}\quad
\begin{array}{c}
 \\ i_1 \\ i_2 \\ i_3 \\ i_4
\end{array}
\begin{array}{c}
\text{Skills} \\
\begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\
\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}
\end{array}
$$

Student skills assessment models



- Number of Skills
- Q-matrix
- Slip and Guess

Student skills assessment models



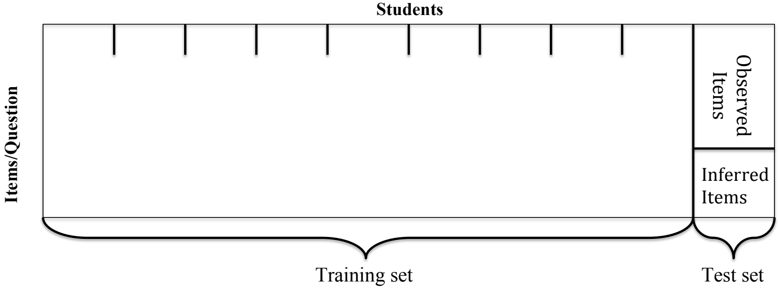
- Number of Skills
- **Q-matrices (types)**
- Slip and Guess

1. Conjunctive
2. Additive
3. Disjunctive

Skills

$$\text{Items}\quad \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} \begin{array}{c} \begin{array}{ccc} s_1 & s_2 & s_3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

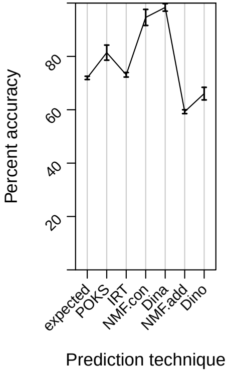
- **Performance of a model** over a data set

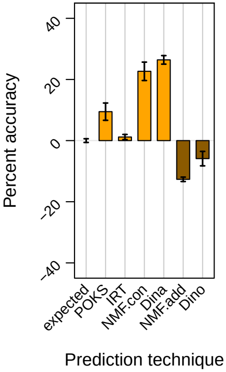


- **Performance of a model** over a data set
- Model parameters

Contributed skills		Skills Model	Parameters estimated from: Training set		Parameters estimated from: Observed items
Multiple		NMF Conj.		• Q-matrix	• Students skills mastery matrix
		NMF Add.		• Q-matrix	• Students skills mastery matrix
		DINA	• Slip • Guess	• Q-matrix	• Students skills mastery matrix
		DINO	• Slip • Guess	• Q-matrix	• Students skills mastery matrix
Single		IRT	• Item difficulty • Item discrimination		• Student Ability
		Expected	• Item Odds		• Student Odds
Zero		POKS	• Initial Odds • Odds ratio • Partial order structure		

- **Performance of a model** over a data set
- Model parameters
- Performance vector
- Performance signature





Model	*Performace*
Expected	0.72%
POKS	0.80%
IRT	0.74%
NMF.Conj	0.94%
Dina	0.99%
NMF.Add	0.60%
Dino	0.65%

- **Performance of a model** over a data set
- Model parameters
- Performance vector
- Performance signature
- Performance prototype

The *performance vector* associated with the synthetic data of a model class.

- **Performance of a model** over a data set
- Model parameters
- Performance vector
- Performance signature
- Performance prototype
- Target performance vector

The *performance vector* of the real data set to classify.

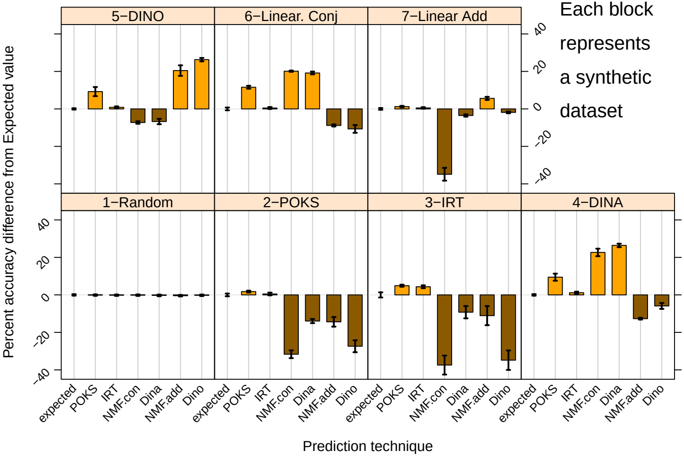
Research questions

1. ✓What is the *performance vector* of student skills assessment models over real and over synthetic data created using the same models ?
   - Experiment 1 : Predictive performance of models over real and synthetic data sets

Datasets

Data set	Number of Skills	Number of Items	Number of Students	Mean Score	Q-matrix
Synthetic					
1.Random	7	30	700	0.75	$\mathbf{Q}_{01}$
2.POKS	7	20	500	0.50	$\mathbf{Q}_{02}$
3.IRT-2PL	5	20	600	0.50	$\mathbf{Q}_{03}$
4.DINA	7	28	500	0.31	$\mathbf{Q}_5$
5.DINO	7	28	500	0.69	$\mathbf{Q}_6$
Linear (Matrix factorization)					
6. Conj.	8	20	500	0.24	$\mathbf{Q}_1$
7. Comp.	8	20	500	0.57	$\mathbf{Q}_1$
Real					
8.Fraction	8	20	536	0.53	$\mathbf{Q}_1$
9.Vomlel	6	20	149	0.61	$\mathbf{Q}_4$
10.ECPE	3	28	2922	0.71	$\mathbf{Q}_3$
Fraction subsets and variants of $\mathbf{Q}_1$					
11. 1	5	15	536	0.53	$\mathbf{Q}_{10}$
12. 2/1	3	11	536	0.51	$\mathbf{Q}_{11}$
13. 2/2	5	11	536	0.51	$\mathbf{Q}_{12}$
14. 2/3	3	11	536	0.51	$\mathbf{Q}_{13}$

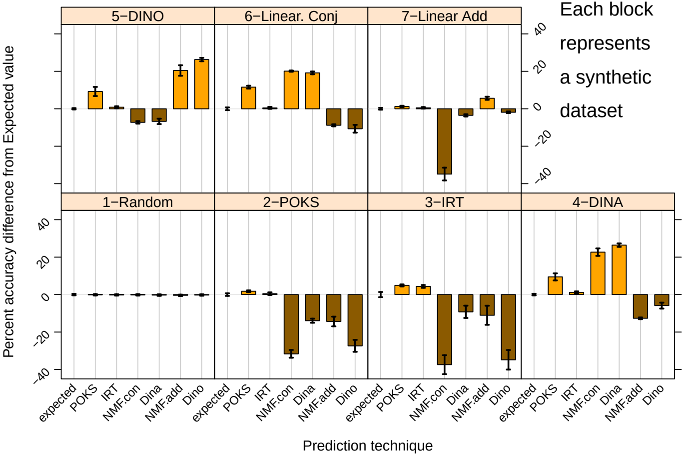
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

The highest performance is for the generative model behind the dataset

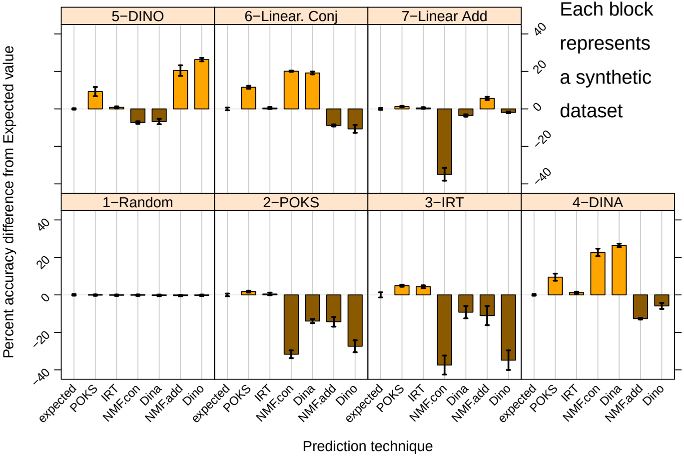
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

Data sets have unique pattern of performance vector across models

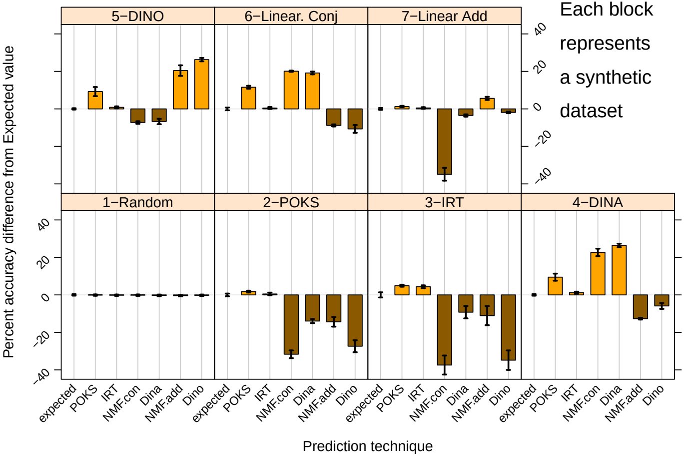
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

The random data set has a flat performance across techniques

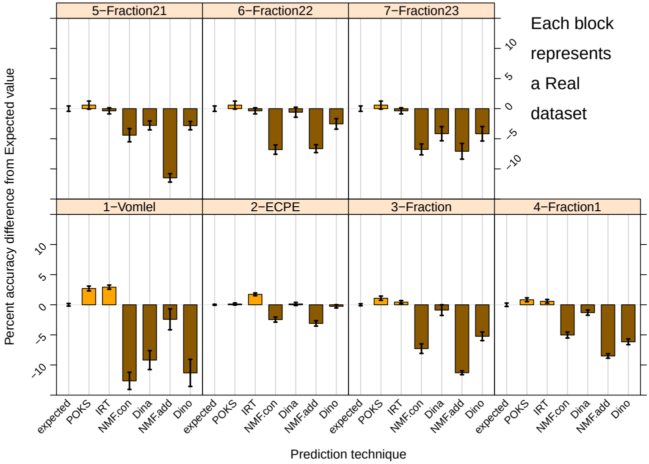
Predictive performance of models over synthetic datasets



Each block represents a synthetic dataset

The capacity of recognizing a data set's true model relies on this uniqueness characteristic

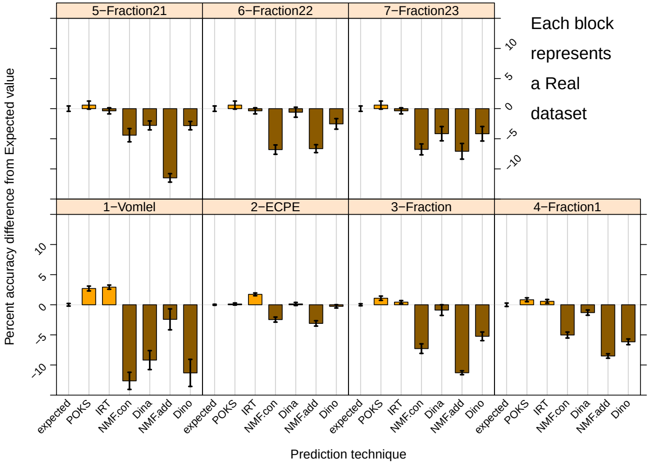
Predictive performance of models over real datasets



Each block represents a Real dataset

In most cases, the best performer is close to the baseline

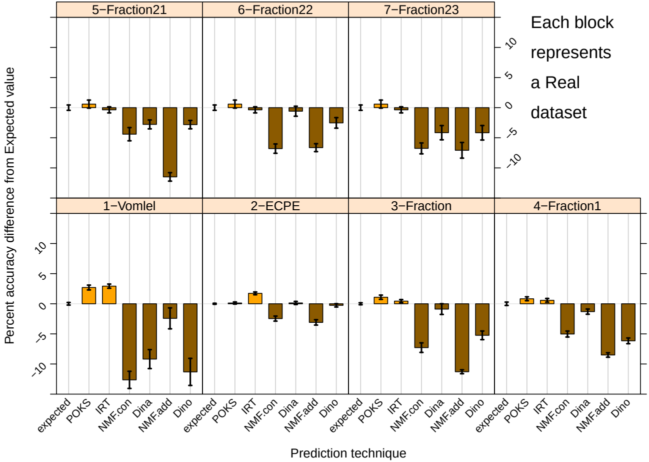
Predictive performance of models over real datasets



Each block represents a Real dataset

The pattern of the Fraction performance data set repeats over its subsets

Predictive performance of models over real datasets



Each block represents a Real dataset

None of the real data sets show the large the amplitude and the differences found in the synthetic data sets models

Vector space of accuracy performances

- Performance vectors of datasets in columns(Data points in the performance space)

Model	**Synthetic data set**						
	Random	POKS	IRT	DINA	DINO	Linear .Conj	Linear .Comp
Expected	**0.75**	0.91	0.90	0.72	0.72	0.78	0.93
POKS	0.75	**0.94**	0.94	0.81	0.81	0.90	0.94
IRT	0.75	0.91	**0.95**	0.73	0.73	0.79	0.89
DINA	0.75	0.77	0.81	**1.00**	0.65	**0.98**	0.89
DINO	0.75	0.63	0.56	0.66	**1.00**	0.68	0.91
NMF.Conj	0.75	0.59	0.53	0.95	0.65	0.97	0.58
NMF.Comp	0.75	0.76	0.79	0.59	0.93	0.70	**0.98**

The diagonal generally displays the best performance

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1. ✓What is the *performance vector* of student skills assessment models over real and over synthetic data created using the same models ?
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2. **Is the performance vector unique to each synthetic data type (data from the same ground truth model) ?**
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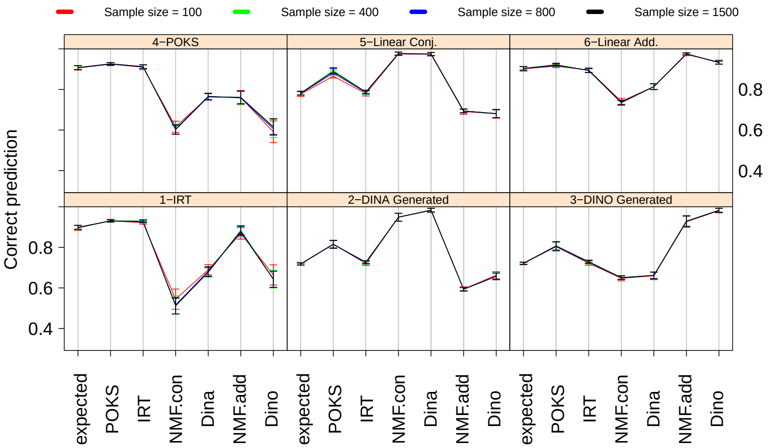
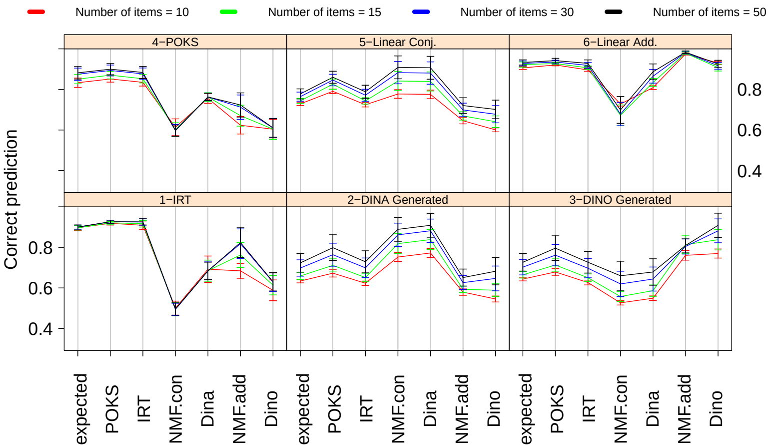
Are they stable in addition to be unique.

Variation of sample size over synthetic data sets



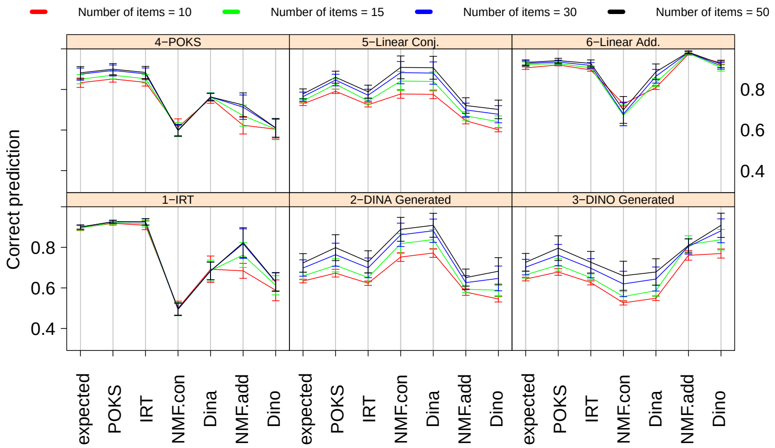
Obviously, the signature pattern did not change significantly for some parameters such as **Sample size**.

Variation of number of items over synthetic data sets



Even for synthetic data the ground truth should not necessarily be close to 100

Variation of number of items over synthetic data sets



The Performance signature shifts down once the number of items degrades.

Data specific parameters

1. Sample size (Number of students)
2. Number of items
3. Number of latent skills
4. Item score variance
5. Student score variance
6. Average success rate

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Conclusion :

- Contextual factors can potentially influence the performance of a model
- For better comparison of the results, we can also consider **data specific parameters** of the real data in the generation process

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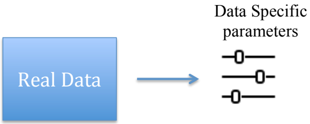
Signature framework

This approach relies on generating synthetic datasets



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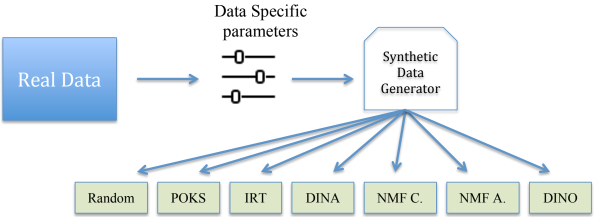

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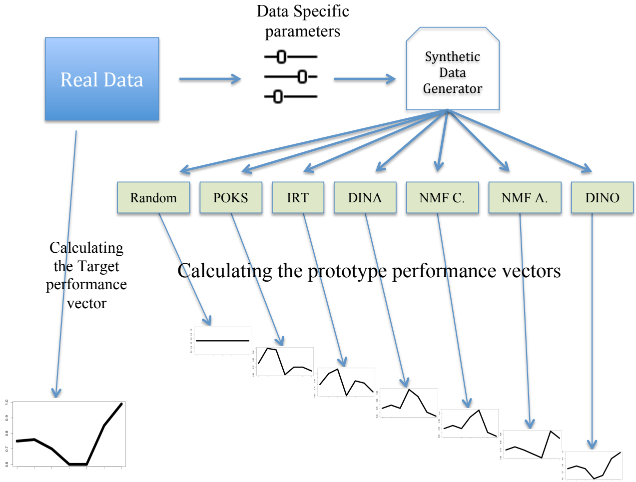
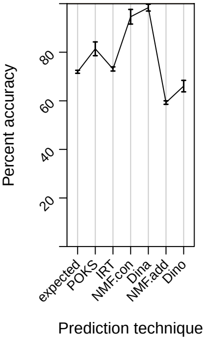


Recall



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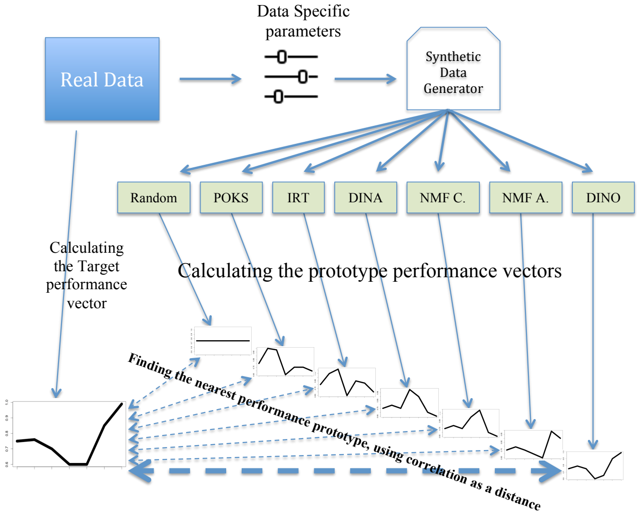
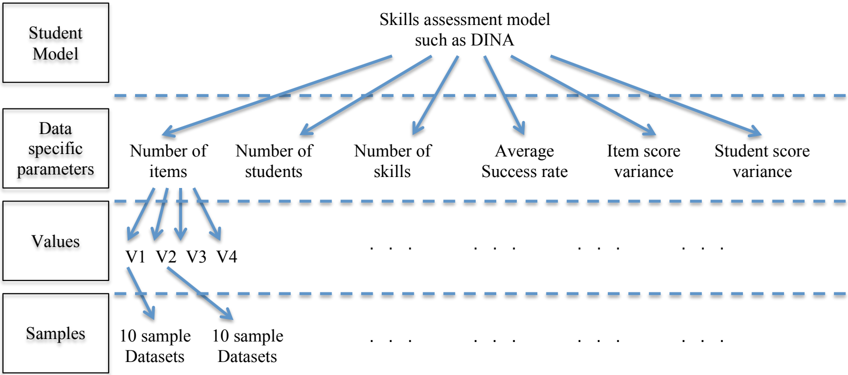
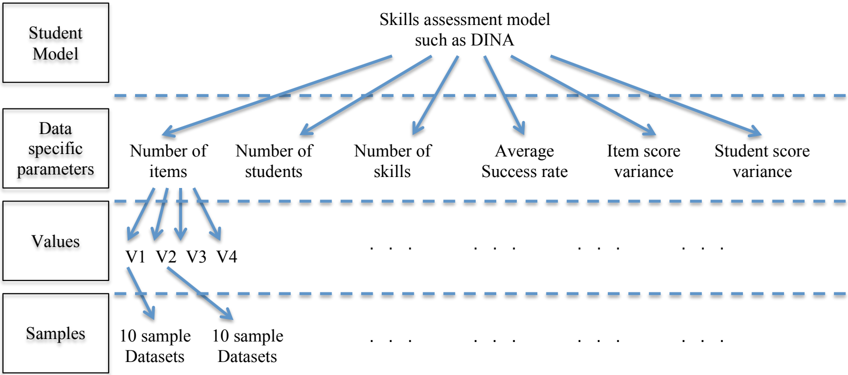


Pool of synthetic datasets



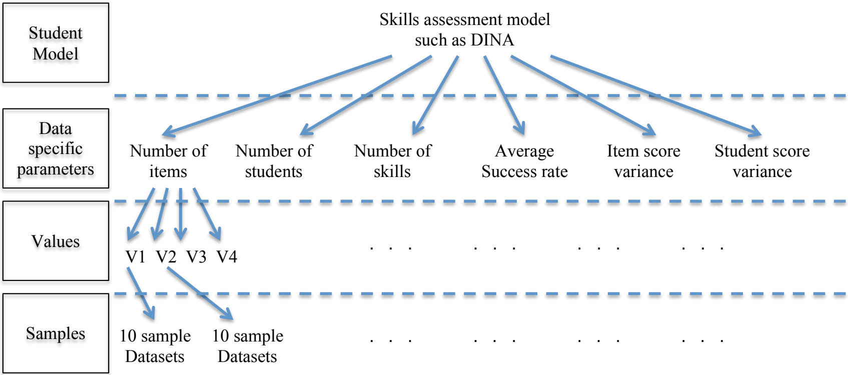
There exists 6 skills assessment models

Pool of synthetic datasets



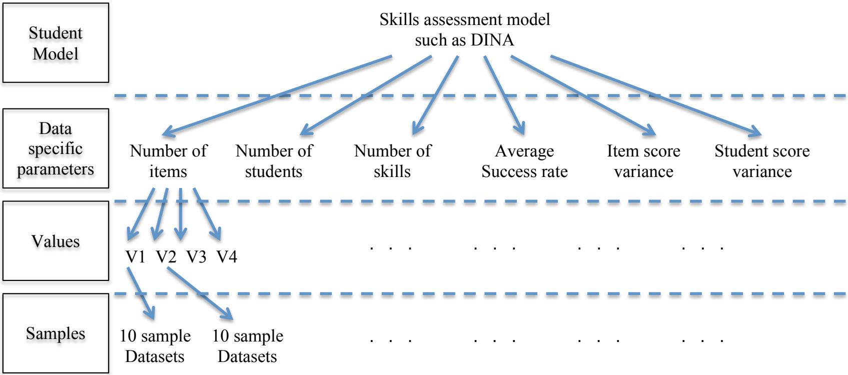
There exists 6 skills assessment models X 6 data specific parameters

Pool of synthetic datasets



There exists 6 skills assessment models X 6 data specific parameters X 4 values

Pool of synthetic datasets



There exists 6 skills assessment models X 6 data specific parameters X 4 values X 10 samples = 1440 samples in the pool

Degree of similarity between six synthetic datasets based on the correlation

Synthetic Datasets

Synthetic Datasets

	POKS	IRT	NMF Conj.	DINA	NMF Add.	DINO
POKS	**0.96**					
IRT	0.86	**0.96**				
NMF Conj.	0.22	-0.20	**0.96**			
DINA	0.02	-0.40	0.94	**0.96**		
NMF Add.	0.44	0.75	-0.62	-0.73	**0.93**	
DINO	-0.15	0.20	-0.70	-0.69	0.63	**0.95**

Item1 Item1

Degree of similarity between six synthetic datasets and the ground truth based on the correlation

Real Datasets

Synthetic Datasets	Vomlel	ECPE	Fraction	Fraction subsets 1	21	22	23
Random	0.58	**0.73**	0.61	0.43	0.24	0.61	0.57
IRT	**0.90**	0.42	0.72	0.88	0.60	0.77	0.61
DINA	-0.38	-0.09	0.23	0.30	0.56	0.06	0.38
DINO	0.34	0.15	-0.18	-0.31	0.10	-0.08	0.38
POKS	0.75	0.40	**0.83**	**0.95**	**0.70**	**0.83**	**0.80**
NMF Conj.	-0.05	0.54	0.51	0.55	0.66	0.33	0.57
NMF Add.	0.39	0.06	-0.04	-0.19	-0.03	0.13	0.28

Item1 Item1

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4. **How does the method compare with the standard practice of using the model with the best performance ?**
   - Experiment 4 : Signature vs. best performer classification

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Thank you