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Q1 - a - $x^T y = (1\times4) + (-2\times0) + (3\times-1) = 1$

b - $\|x\|_2 = \sqrt{1^2 + -2^2 + 3^2} = \sqrt{14}$, $\|y\|_2 = \sqrt{17}$

c = $\cos\theta = \dfrac{1}{\sqrt{14}\sqrt{17}} = 0.0648$

Q2 - a - No

b - $V_3 = C_1 V_1 + C_2 V_2$ $\quad C_1 = 1, C_2 = 1$

$\begin{pmatrix}1\\0\\2\end{pmatrix} + \begin{pmatrix}2\\1\\2\end{pmatrix} = \begin{pmatrix}3\\1\\4\end{pmatrix} \simeq V_3 = V_1 + V_2$

c = A is 2

Q3 - a - $Y = AX$, $1 = (1\times2) + (2\times-1) = 0$

$2 - (0\times2) + (1\times-1) = -1$, $3 = (1\times2) + (-1\times-1) = 3$

$Y = \begin{pmatrix}0\\-1\\3\end{pmatrix}$

b - $Y = 2\begin{pmatrix}1\\0\\2\end{pmatrix} - 1\begin{pmatrix}2\\1\\2\end{pmatrix}$

Q4- a = $AB = (1\times0) + (2\times1) = 2, (1\times1) + (2\times0) = 1$

$(0)(0) + (1\times1) = 1, (0\times1) + (1\times0) = 0$

$AB = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$, $BA = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$

b = no $AB \neq BA$

Q5- $\det(A) = (?)(4) - (-1)(2) = 14$

Q6-

Q7- $\det(A) = (2\times3) - (5\times1) = 1$

$A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = \frac{1}{1} \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}$

Q8 - a - the eigen values are: 1, 3, 5

Q9- $(x_1 \ x_2) \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

$\begin{pmatrix} x_1 + 2x_2 \\ 2x_1 + 3x_2 \end{pmatrix} (x_1 \ x_2) = x_1(x_1 + 2x_2) + x_2$

$(2x_1 + 3x_2)$

$x^T A x = x_1^2 + 4x_1x_2 + 3x_2^2$

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$Q1$ - A - $x^T A x (x_1 \ x_2) \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2x_1^2 + x_2^2$

b - $x, x^T A x = 2x_1^2 + x_2^2 \geq 0$

if $x \neq 0$ then either x_1 or x_2 is non zero

eigen values for A is 2 and 1 both positive

$Q11$ - a - Rank$(B) = 1$

$B = C_1 \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \ C_2 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$C_2 = -\frac{1}{2} C_1 \quad = \left\{ \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\}$

1B - $\text{Proj}_a(x) = \frac{a^T x}{a^T a} a$

$a^T x = (1 \times 3) + (2 \times 1) = 5$

$a^T a = (1^2) + (2)^2 = 5$

$\text{Proj}_a(x) = \frac{5}{5} a = a = \frac{1}{2}$

$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix}$

$AB = \begin{pmatrix} 6 & 9 \\ 3 & 4 \end{pmatrix}$

$tr(AB) = 6 + 4 = 10$

$BA = \begin{pmatrix} 0 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 3 & 10 \end{pmatrix}$

$tr(BA) = 0 + 10 = 10$

$\boxed{tr(AB) = Tr(BA) = 10}$

Calculus

21- $9x^2 - 10x + 2$

22- $u = 2x^2 + 1$, $f(x) = e^u$

$f'(x) = e^u \cdot \frac{du}{dx}$, $\frac{du}{dx} = 4x$

$f'(x) = e^{2x^2+1} \cdot 4x$

23- $u = x^2 + 1$, $\ln(u) = f(x)$

$\frac{dy}{du} = 1/u$, $\frac{du}{dx} = 2x$

$f'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$

b- $U = 2x$ $u' = 2$, $v = x^2 + 1$, $v' = 2x$

$\frac{U}{V}$, $g' = \frac{u'v - uv'}{v^2}$

$f''(x) = \frac{2(x^2+1) - 4x^2}{(x^2+1)^2} = \frac{2(1-x^2)}{(x^2+2)^2}$

24. $\begin{cases} 0 & x < 0 \\ x & x > 1 \end{cases}$

ReLU' $\begin{cases} 0 & x < 0 \\ 1 & x > 1 \end{cases}$

b. bcs left hand is 0 and right hand is 1 so $0 \neq 1$

25. $\frac{\partial f}{\partial x_1} = 2x_1 x_2 \quad \frac{\partial f}{\partial x_2} = x_1 + 4$

26- $f(x_1, x_2, x_3) = (1, 4x_2, -9x_3^2)$

when $x_2 = -1$ and $x_3 = 1$

$f(1, -1, 1) = (1, -4, -9)$

27 = $\nabla f(x_1, x_2) = (2x_1 + 4x_2, 4x_1 + 6x_2)$

b - $H_f \begin{pmatrix} f''(x_1) & \cancel{f''_{x_1 x_2}} \\ \frac{\partial^2 f}{\partial x_1 \partial x_2} & f''(x_2) \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 4 & 6 \end{pmatrix}$

C - indefinite bcs $a_{11} = 2 > 0$ and $\det(H) = -4 < 0$

28- $J(x_1, x_2) = \begin{pmatrix} 2x_1 & 0 \\ x_2 & x_1 \\ 0 & \cos x_2 \end{pmatrix}$

$J(1,0) = \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$

29- $\nabla_x f(x) = w$

30- 1 - $\nabla f(x) = 2Ax$

31- $\frac{dL}{d\theta} = \frac{d}{d\theta}\left[(\theta - 3)^2\right] = 2(\theta - 3) \cdot 1$

$= 2\theta - 6$

b- $2(0) - 6 = -6$

$\theta_1 = 0 - (0.1 \times -6) = 0.6$

32. $\frac{dz}{dw} = 6(w^T x)(2 - 6(w^T x))\, x$

33- a $= f'(x) = 4x^3 - 4x$, $f''(x) = 12x^2 - 4$

b- $4x^3 - 4x = 0$, $(x = 0, x = 1)$

34- $\frac{dy}{du} = 2u$, $\nabla_x u = a$, $\nabla_x y = (2u) \cdot a$

$\nabla_x y = 2(a^T x) a$

36 - a - $|S| = 6 \times 6 = 36$

b - $(4,6), (5,5), (6,4)$ - $P(sum=10) = \frac{3}{36} = \boxed{\frac{1}{12}}$

37 - $9 = 4, \quad 10 = 3, \quad 11 = 2, \quad 12 = 1$

$P(sum \geq 9) = \frac{10}{36} = \frac{5}{18}$

38 - $P(A) = 0.3 \quad , \quad P(A') = 1 - P(A) = 0.7$

b - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.5 + 0 = 0.8$

37 - a - $P(B) = \frac{1}{10} \quad , P(F) = \frac{9}{10} \quad , P(T|B) = 1$

$P(T|F) = \frac{1}{2}$

$P(T) = P(T|B)P(B) + P(T|F)P(F)$

$= \frac{11}{20}$

b - $P(B|T) = \frac{P(T|B)P(B)}{P(T)} = \frac{\frac{1}{10}}{\frac{11}{20}}$

$= \frac{2}{11}$

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4o - $P(D) = 0.01$, $P(D') = 0.99$

$P(Pos|D) = 0.99$, $P(Pos|D') = 0.05$

a - $P(Pos) = P(Pos|D)P(D) + P(Pos|D')P(D')$

$= 0.0594$

b - $P(D|Pos) = \frac{P(Pos|D)P(D)}{P(Pos)}$

$= \frac{1}{6}$

41 - $P(Pass) = P(Pass|CS)P(CS) + P(Pass|EE) + P(EE) = 0.82$

b - $\frac{27}{41}$

42 - $P(X = x) = \frac{1}{6}$

$E[X] = \frac{1}{6}(1+2+3+4+5+6) = 3.5$

$Var(x) = E[x^2] - E(x)^2 = \frac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2)$

$= \frac{91}{6} - 3.5^2 = \frac{35}{12}$

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4) - $X\begin{cases} 0 & 1, 2 \\ 1 & 3, 4, 5, 6 \end{cases}$

a- $P(X=0) = \frac{2}{6} = \frac{1}{3}$,

$P(X=1) = \frac{4}{6} = \frac{2}{3}$,

b- $E[X] = 0 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3} = \frac{2}{3}$

$Var(X) = E[X^2] - E[X]^2 = p(1-p) = \frac{2}{9}$

44 - $\{1, 2, 4\}$,

$P(X=1)\ 0.2$, $P(X=2) = 0.5$, $P(X=4) = 0.3$

$E[X] = 1(0.2) + 2(0.5) + 4(0.3) = 2.4$

$Var(X) = 7 - (2.4)^2 = 1.24$