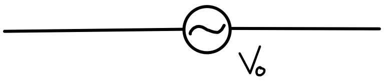
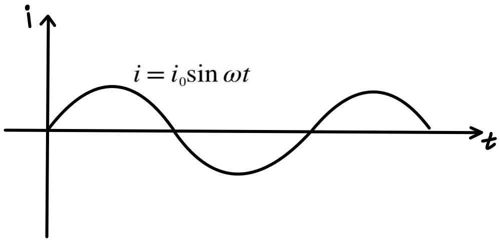
Alternating Current

The current which changes in direction and magnitude with time is called alternating current. For e.g. the electric mains supply in our homes which varies sinusoidally with time. AC voltage is represented as-





Average value of a function

The average value of any function y = f(x) over an interval from x_1 to x_2 can be calculated as-

$$\bar{y} = \frac{\int_{x_1}^{x_2} f(x)\cdot dx}{\int_{x_1}^{x_2} dx}$$

The following are some results for average value of trigonometric functions which should be memorized-

1) $\int_0^T \sin(\omega t)\cdot dt = \int_0^T \cos(\omega t)\cdot dt = 0$

2) $\int_0^T \sin^2(\omega t)\cdot dt = \int_0^T \cos^2(\omega t)\cdot dt = \frac{T}{2}$

For a sinusoidal wave, the average value in one time period will be 0 as equal current flows in forward and backward direction. Therefore, the average value is calculated over half a time period-

$$i_{avg} = \frac{\int_0^{T/2} i_0 \sin(\omega t) dt}{\int_0^{T/2} dt}$$

$$I_{avg} = \frac{2i_0}{\pi} = 0.637 i_0$$

Root mean squared (RMS) value/ Effective value

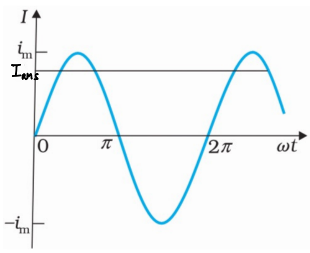
RMS value for an ac current is that value of dc current which when applied to the same resistor for the same amount of time, produces the same amount of heat as ac.

Mathematically,

$$I_{RMS} = \left[\frac{\int_0^T i^2 dt}{\int_0^T dt}\right]^{\frac{1}{2}}$$

Note: - 'An ac current of 4A' implies that the RMS value is 4A

Let current be I = i_0 Sin(ωt), where i_0 is the peak value. The RMS value is related to peak value as-



$$i_{rms} = \frac{i_0}{\sqrt{2}} = 0.707 i_0$$

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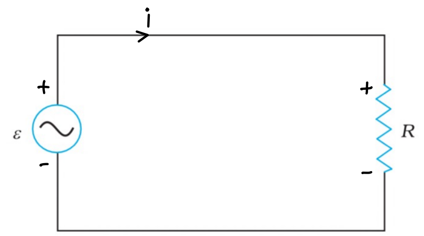
Similarly, the rms voltage or effective voltage-

$$\boxed{V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707V_0}$$

AC voltage applied to a Resistor

Consider a resistor of resistance R is connected to an alternating voltage source v= v_oSinωt, where v_o is the amplitude of the oscillating potential difference and ω is the angular frequency. Using Kirchhoff's loop law-



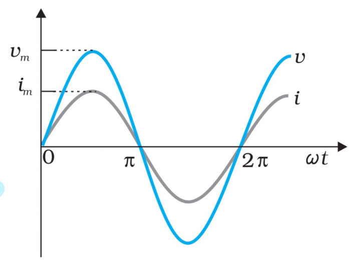
$$V_0 \sin \omega t - iR = 0$$

$$i = \frac{V_0}{R} \times \sin \omega t$$

$$\boxed{i = i_0 \sin(\omega t)}$$

Where-

$$\boxed{i_0 = \frac{V_0}{R}} \quad - (1)$$



In the graph, both v and i reach minimum and maximum values Simultaneously. Therefore, for a resistor voltage and current are In phase with each other i.e. phase difference = 0.

Power consumed

In complete cycle, since equal current flows in both directions, the avg current over a complete cycle is 0 but the average power consumed is not zero. This is because the power consumed is given by i^2R (according to Joule's law) which depends on i^2 and not i and i^2 is always positive whether i is positive or negative. Thus, there is Joule heating and dissipation of electrical energy when ac current passes through a resistor.

Instantaneous power dissipated in the circuit-

$$\boxed{p_i = i^2R = i_0^2R\sin^2(\omega t)}$$

Therefore, average power dissipated-

$$\overline{p} = \langle i^2R \rangle = i_0^2R \langle \sin^2(\omega t) \rangle$$

$$\overline{p} = i_0^2R \times \frac{1}{2}$$

$$\boxed{\overline{p} = \frac{1}{2}i_0^2R}$$

Also-

$$\overline{p} = \left(\frac{i_0}{\sqrt{2}}\right)^2 R$$

$$\boxed{\overline{p} = i_{RMS}^2R}$$

From 1

$$V_0 = I_0R$$

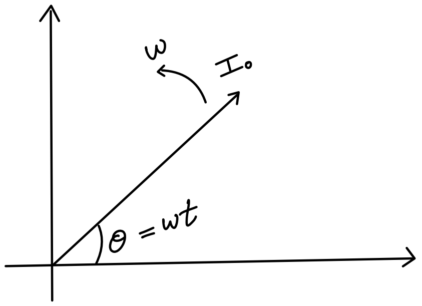
$$\frac{V_0}{\sqrt{2}} = \frac{I_0R}{\sqrt{2}}$$

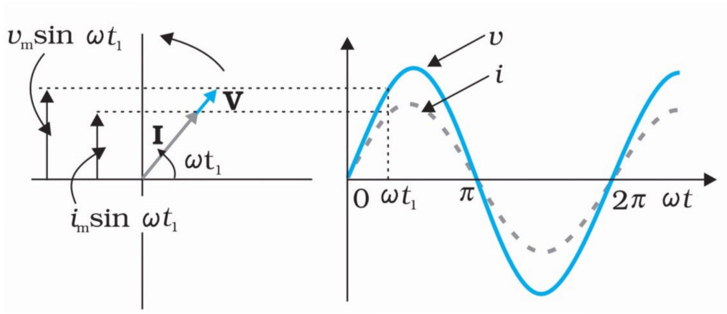
$$\boxed{V_{RMS} = I_{RMS} \cdot R}$$

Note: The instantaneous values of ac current/ voltage obey dc laws like Kirchhoff's loop and junction law.

Representation of AC voltage and current by Rotating vectors- Phasors

A phasor is a vector which rotates about the origin with angular speed ω. The orthogonal components of phasors **v** and **I** give the instantaneous values v and i and their magnitude represents the amplitude v_o and i_o



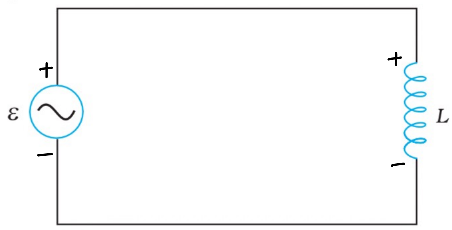


Note: Though the voltage and current in an ac circuit are represented by vectors they are not vectors themselves. They are scalar quantities.

AC voltage applied to an inductor (PYQ 2011)

Consider an inductor of self inductor L and negligible resistance in its windings. Thus, the circuit is purely inductive. Let the voltage be v= v_o Sinωt. Using Kirchhoff's loop law-

$$V - L\frac{di}{dt} = 0$$



The negative sign is in accordance with Lenz' law

$$V = L\frac{di}{dt}$$

$$V_0 \sin(\omega t) = L\frac{di}{dt}$$

$$\int di = \frac{V_0}{L} \int \sin(\omega t) dt$$

$$i = -\frac{V_0}{\omega L} \times \cos(\omega t) + c$$

$$i = \frac{V_0}{\omega L} \times \sin\left(\omega t - \frac{\pi}{2}\right) + c$$

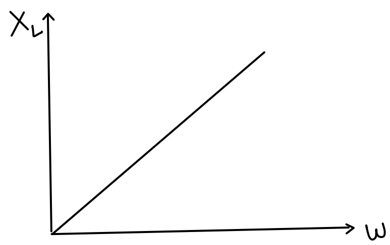
The integration constant has dimensions of current and is time independent. Since the source emf and the current oscillate symmetrically about zero, the integration constant will be 0

$$\boxed{I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \qquad I_0 = \frac{V_0}{\omega L}}$$

Inductive reactance (X_L)

The quantity ωL is analogous to resistance and is called Inductive reactance.

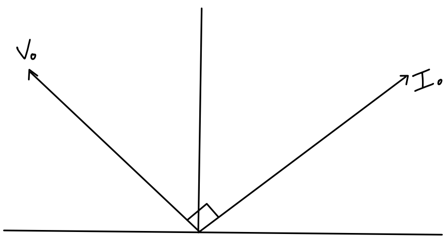
$$\boxed{X_L = \omega L \qquad I_0 = \frac{V_0}{X_L}}$$

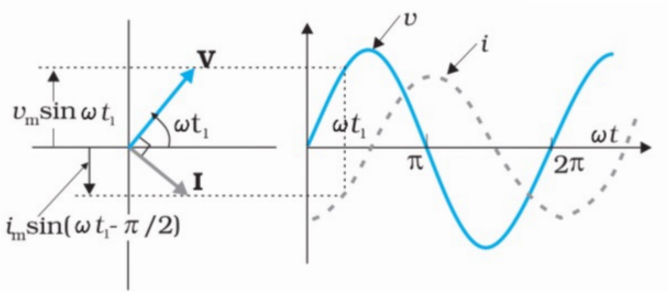


- Dimensions are same as Resistance | SI Unit- Ohm (Ω)

Phasor

Comparing the expression for current and voltage in a purely inductive circuit, we see that the current lags behind voltage by π/2 or 1/4 th of the cycle.





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Power consumption

The instantaneous value of power supplied to the inductor is-

$$P_i = iV = I_0\sin\left(\omega t - \frac{\pi}{2}\right) \times V_0\sin(\omega t)$$

$$P_i = -I_0V_0\cos(\omega t) \times \sin(\omega t)$$

$$\boxed{P_i = -\frac{I_0V_0}{2} \times \sin(2\omega t)}$$

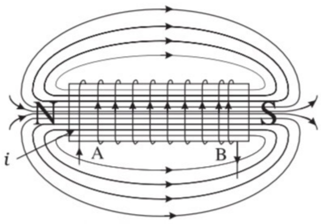
The average power over a complete cycle-

$$\overline{P} = \left\langle -\frac{i_0V_0}{2} \times \sin(2\omega t)\right\rangle$$

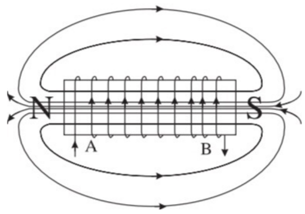
$$\boxed{\overline{P} = -\frac{i_0V_0}{2} < \sin(2\omega t) > \quad = 0}$$

Thus, the average power supplied to an inductor in one complete cycle is zero

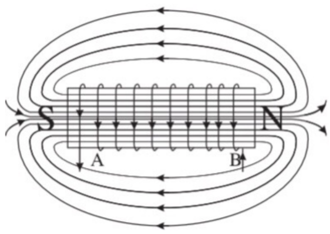
Magnetization and demagnetization of an inductor



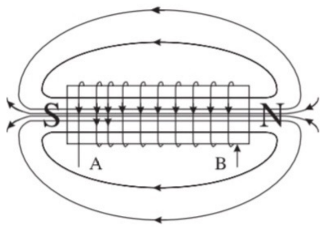
0-1 Current i flows through the coil entering at A and Increases from 0 to maxm value and the core gets Magnetized. Current and voltage are both positive So, P=vi is positive. Hence, energy is absorbed from source



1-2 current in the coil is positive but decreasing the core gets demagnetized. v Is negative as di/dt is negative and hence vi is also negative i.e. energy is returned to the source



2-3 current I becomes negative entering at B. The polarity Of the magnet is revered, v and i are negative hence vi is Positive. Hence, energy is being absorbed.

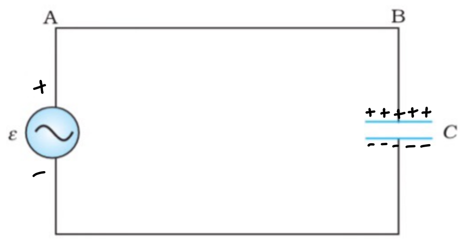


3-4 current decreases and reaches 0. The voltage is positive but current is negative hence vi is Negative. Hence power is returned to the source.

Hence, over the entire cycle, net power absorbed or lost is zero.

AC voltage applied to a capacitor (PYQ 2015, 2017)

Consider a capacitor of capacitance C connect to an ac voltage v= v_0 Sinωt. When a capacitor is connected to an ac source, it limits or regulates the The current but does not completely limit the flow of charged. The capacitor Is repeatedly charged and discharged as the current reverses direction every Half cycle. Let q be the charge on a capacitor at a time t. the instantaneous Voltage v across the capacitor will be-



$$V = \frac{q}{c}$$

Using Kirchhoff's law

$$V_0\sin\omega t = \frac{q}{c}$$

$$i = \frac{V_0}{(1/\omega C)} \times \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$I_0 = \frac{V_0}{(1/\omega C)}$$

$$i = \frac{V_0}{(1/\omega C)} \times \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\boxed{i = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \qquad I_0 = \frac{V_0}{(1/\omega C)}}$$

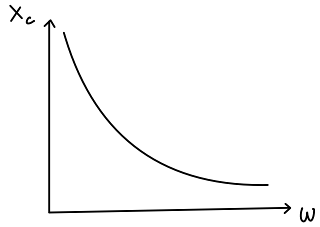
Capacitive reactance (X_R) (PYQ 2015)

Comparing $i_0 = v_0/R$ for a purely resistive circuit, we find that $1/\omega C$ plays the role of resistance. It limits the magnitude of current in a purely capacitive circuit just like resistance limits the value of current in a purely resistive circuit

- SI unit- Ohm (Ω)

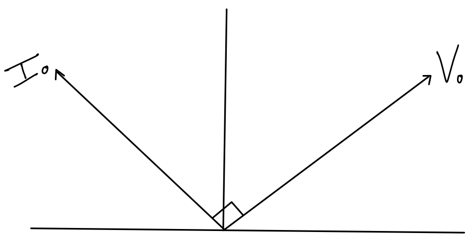
$$\boxed{X_c = \frac{1}{\omega C}}$$

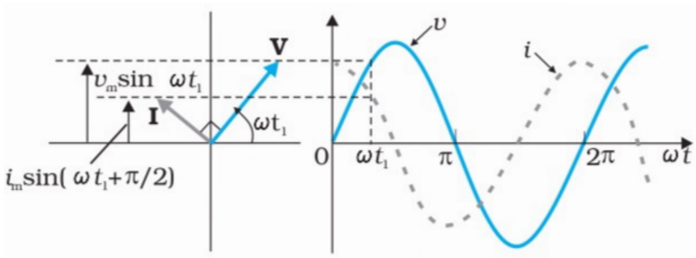
$$I_0 = \frac{V_0}{X_c}$$



Phasor

Comparing the expression for voltage and current for a purely capacitive circuit we see that current leads voltage by π/2 or by ¼ th of a period





Power consumed

The instantaneous value of power supplied to a capacitor is-

$$P_i = iv = I_0 \cos(\omega t) V_0 \sin(\omega t)$$

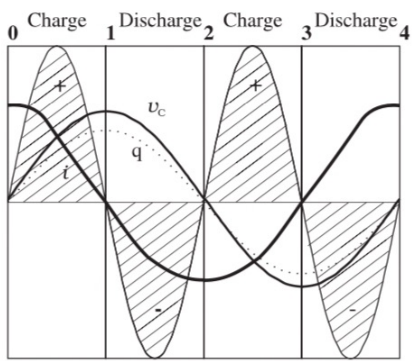
$$\boxed{p_i = \frac{I_0 V_0}{2} \times \sin(2\omega t)}$$

The average power over a complete cycle-

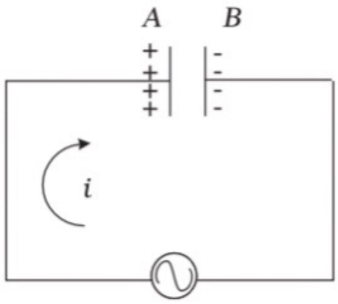
$$\overline{P} = \langle \frac{I_0 V_0}{2} \times \sin(2\omega t) \rangle$$

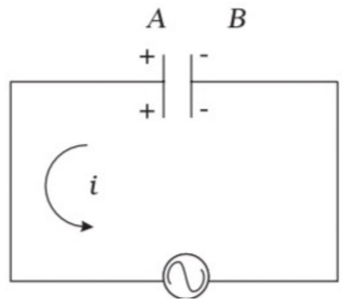
$$\boxed{\overline{p} = \frac{I_0 V_0}{2} \langle \sin(2\omega t) \rangle = 0}$$

Thus, the average power supplied to a capacitor in complete cycle is zero.



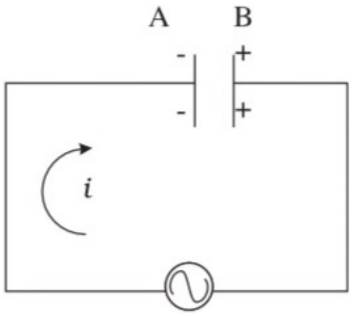
Charging and discharging of a capacitor

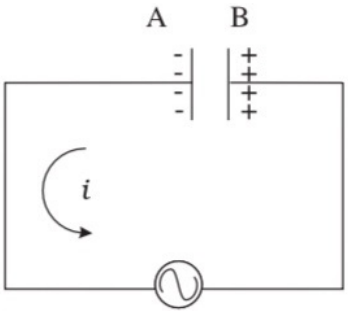




0-1 the current flows starting from its maximum value. Plate A is charged +vely and B is charged -vely. The charge on plate becomes maxm and Current becomes 0. Let charge on capacitor at Time t be q, then, voltage v = q/C. Since P=vi is +ve, energy is absorbed from the source.

1-2 the current I reverses direction so the charge is depleted and capacitor is discharged. The voltage is reduced but remains +ve and I is -ve therefore p=vi is negative i.e. energy is returned to the source.





2-3 as current continues to flow from A to B, the capacitor is charged to reverse polarity. Both current and voltage are -ve so P=vi is +ve. Hence, energy is absorbed from the source

3-4 the current again reverses its direction. The charge is again depleted and the voltage v is reduced till it becomes 0 and capacitor is discharged. P=vi is -ve hence, energy is returned to the source

important PYQs 

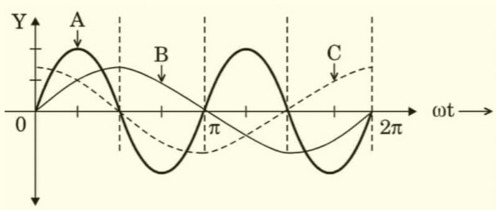
Ques: A device X is connected to an ac source $V=V_0 \sin\omega t$. The variation and voltage, current and power in one cycle is shown in the following graph

a. identify X

b. which of the curves A, B, C represent the voltage, current, and the power consumed in the circuit? Justify

c. how does its impedance vary with frequency of ac source? Show graphically

d. obtain an expression for the current in the circuit and its phase relation with ac voltage **(PYQ 2017) [5M]**



Ans: b) From the information in the question we can conclude that-

B- V | C- I | A- power. This is because the current will have a phase difference of $\pi/2$ With the voltage and the power will have twice the frequency of the voltage/ current.

$$i = I_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$$

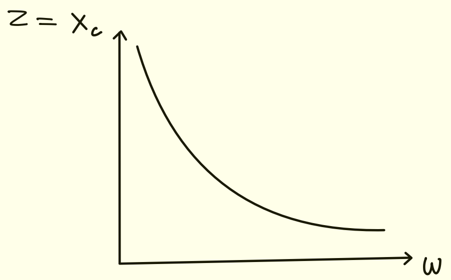
$$P = \pm \frac{I_0 V_0}{2} \times \sin(2\omega t)$$

a) from the diagram we can see that the current leads the voltage by a difference of $\pi/2$. Therefore the device is a capacitor

d) already done above in notes. Refer to capacitor in an ac circuit

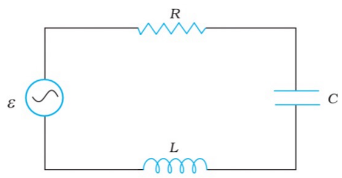
c) Since it is a purely capacitive circuit, the impedance will be equal to the capacitive reactance hence-

$$Z = X_c$$



AC voltage applied to series LCR circuit (PYQ 2020, 2018, 2016, 2015,2014, 2013, 2012, 2010)

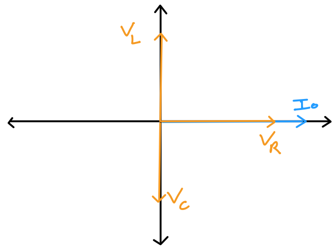
Consider a resistance R, capacitor C and inductor L, connected to an ac source v= v_0Sinωt. Let charge on C be q and current in the circuit be i= i_0Sinωt then using Kirchhoff's loop law-



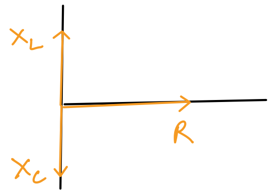
$$L\frac{dI}{dt}+iR+\frac{q}{C}=V$$

Phasor solution

To calculate the current i in the circuit let us take help of the phasors. Since all 3 components L, C, R are in series the current through all of them will be same. In the phasor diagram let this current be represented by $\mathbf{I_0}$. let voltage across R, C and L be $\mathbf{V_R}$, $\mathbf{V_C}$, $\mathbf{V_L}$ respectively and net voltage be $\mathbf{V_0}$. We know that for a resistor current is in phase with voltage, for a capacitor current leads by π/2 and for an inductor, current lags by π/2. This can be shown as-



Dividing the voltages by $\mathbf{I_0}$ we can draw the same diagram as-



(since, $\mathbf{V_R/I_0 = R, V_C/I_0 = X_C, V_L/I_0 = X_L}$)

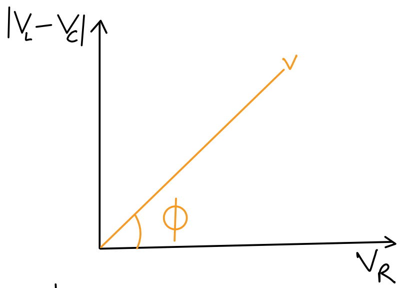
Now, since $\mathbf{V_L}$ and $\mathbf{V_C}$ are in the same line their net result can be written as- $|\mathbf{V_L - V_C}|$ and then the net voltage $\mathbf{V_0}$ can be written as-

$$V_0^2 = V_R^2 + (V_L - V_c)^2$$

$$V = IR$$

$$V_0^2 = I_0^2R^2 + I_0^2(X_L - X_c)^2$$

$$\boxed{I_0 = \frac{V_0}{\sqrt{R^2+(X_L - X_c)^2}}} \quad \text{— (1)}$$



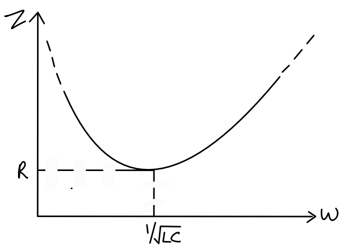
(Where ϕ Is phase diff between voltage and current)

Impedance (Z)

We define a quantity called impedance in an ac circuit which is analogous to resistance in a dc circuit| SI unit- Ohm

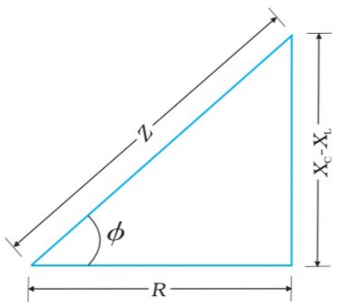
$$\boxed{I_0 = \frac{V_0}{Z} \qquad Z = \sqrt{R^2+(X_L - X_c)^2}}$$

$$\boxed{Z = \sqrt{R^2+\left(\omega L \frac{-1}{\omega C}\right)^2}}$$



Impedance diagram

Impedance diagram is a right-angled triangle with Z as its hypotenuse.



(where ϕ is the phase difference b/w current i and Voltage v in the circuit)

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From the diagram, we can see-

$$\boxed{\begin{aligned} Tan\phi &= \frac{X_c - X_L}{R} \\ \cos\phi &= \frac{R}{Z} \end{aligned}} \quad — ②$$

Note: 1. Equation (1) gives the amplitude of the current and equation (2) gives its phase angle.
2. If $X_C > X_L$, ϕ is +ve, the circuit is predominantly capacitive and the current leads the voltage.
3. If $X_L > X_C$, ϕ is -ve, the circuit predominantly inductive and the current lags behind the voltage.

Disadvantages of using Phasors-

1. The phasor diagram gives nothing about the initial condition
2. The solution obtained is called the steady state solution and it is not a general solution
3. We have a transient solution which exists for v= 0. The general solution is the sum of the transient solution and the steady state solution. After sufficiently long time, the effects of the transient solution dies out and only the steady state solution defines the circuit.

Analytical solution

Using Kirchhoff's law we can write-

$$L\frac{di}{dt} + Ri + \frac{q}{c} = V_0 \sin(\omega t)$$

$$i = \frac{dq}{dt}, \frac{di}{dt} = \frac{d^2q}{dt^2}$$

$$\Rightarrow L \cdot \frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = V_0 \sin\omega t \quad — ③$$

This is like the equation for a forced, damped oscillator. So let us assume-

$$q = q_0 \sin(\omega t + \theta)$$

$$\Rightarrow \frac{dq}{dt} = q_0\omega\cos(\omega t + \theta)$$

$$\Rightarrow \frac{d^2q}{dt^2} = -q_0\omega^2 \sin(\omega t + \theta)$$

Substituting these in 3

$$-Lq_0\omega^2 \sin(\omega t + \theta) + Rq_0\omega\cos(\omega t + \theta) + \frac{q_0}{c} \times \sin(\omega t + \theta) = v_0 \sin\omega t$$

$$q_0\omega\left[\left(\frac{1}{\omega C} - \omega L\right) \times \sin(\omega t + \theta) + R\cos(\omega t + \theta)\right] = V_0 \sin\omega t$$

We know- $\frac{1}{\omega C} = X_C, \omega L = X_L$

$$\Rightarrow q_0\omega Z\left[\left(\frac{(X_c - X_L)}{Z} \times \sin(\omega t + \theta) + \frac{R}{Z} \times \cos(\omega t + \theta)\right)\right] = V_0 \sin\omega t$$

From impedance diagram $\frac{R}{Z} = \cos\phi, \frac{X_c - X_L}{Z} = \sin\phi$

$$\Rightarrow q_0\omega Z[\sin\phi \times \sin(\omega t + \theta) + \cos\phi \times \cos(\omega t + \theta)] = V_0 \sin\omega t$$

$$q_0\omega Z\cos(\omega t + \theta - \phi) = v_0 \sin\omega t$$

Comparing both sides we get-

$$q_0\omega Z = V_0 = I_0 Z$$

$$\boxed{I_0 = q_0\omega}$$

$$\theta - \phi = -\frac{\pi}{2}$$

$$\theta = -\frac{\pi}{2} + \phi$$

Therefore, current in the circuit-

$$i = \frac{dq}{dt} = q_0\omega\cos(\omega t + \theta)$$

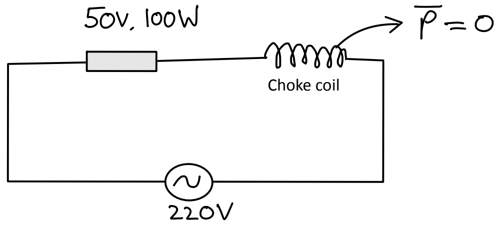
$$\boxed{\Rightarrow i = I_0 \sin(\omega t + \phi)}$$

Where-

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_c - X_L)^2}}, \ \phi = \tan^{-1}\left(\frac{X_c - X_L}{R}\right)$$

Choke coil (PYQ 2014)

A choke coil (inductor) is a device which is kept in series with a fluorescent tube light so that it provides the required potential across the coil without consuming any energy.



Important PYQs

Ques: A resistance R and a capacitor C are connected in series to a source v= v_0Sinωt. Find:

a. The peak value of the voltage across the i. resistance and the ii. Capacitor

b. The phase difference between the applied voltage and current. Which one of them is ahead? **(PYQ 2020) [3M]**

Ans: We know-

$$Z = \sqrt{R^2 + (X_c - X_L)^2}$$

$$Z = \sqrt{R^2 + (1/\omega^2 c^2)}$$

Also, current in the circuit-

$$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + 1/(\omega^2 c^2)}}$$

a) $V_R = i_0 R = \dfrac{V_0 R}{\sqrt{R^2 + 1/(\omega^2 \cdot c^2)}}$

$$V_c = i_0 X_c = \frac{V_0 \cdot 1/(\omega c)}{\sqrt{R^2 + 1/(\omega^2 c^2)}}$$

b) $\phi = Tan^{-1}\left(\dfrac{X_c - X_L}{R}\right) = Tan^{-1}\left(\dfrac{1}{\omega CR}\right)$ Since there is no inductive element, current will lead the voltage

Ques: a. Draw graphs showing variation of inductive reactance and capacitive reactance with the frequency of the applied ac source

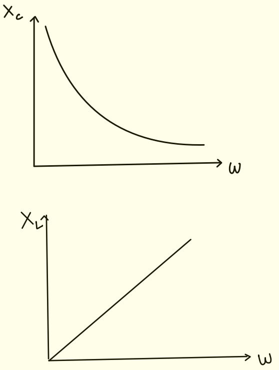
b. Draw the phasor diagram for a series RC circuit connected to an ac source

c. An alternating voltage of 220V is applied across a device X, a current of 0.25 A flows, which lags behind the voltage by a phase difference of π/2. If the same voltage is applied across another device Y, same current flows but now it is in phase with the applied voltage

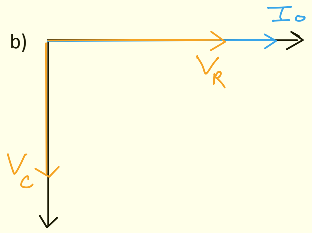
i. name devices X and Y

ii. Calculate the current in the circuit when the same voltage is applied across series combination of X and Y **(PYQ 2018) [5M]**

Ans: a)



b)



c) i) X- inductor ; Y- Resistor

ii) ATQ-

$$I_{RMS} = \frac{V_{Rms}}{X_L}$$

$$X_L = \frac{220}{0.25} = 880\Omega$$

Illy. $R = 880\Omega$

Therefore-

$$Z = \sqrt{R^2 + X_L^2}$$

$$Z = 880.\sqrt{2}\Omega$$

So, current in the circuit-

$$I_{RMS} = \frac{V_{RMS}}{Z}$$

$$I_{RMS} = \frac{220}{880 \times \sqrt{2}} = \frac{1}{4 \times \sqrt{2}} A$$

Ques: An inductor L of inductive reactance X_L is connected in series with a bulb B and an ac source. How would the brightness of the bulb change when i. number of turns in the inductor are reduced ii. An iron rod is inserted in the inductor iii. A capacitor of capacitive reactance $X_c = X_L$ is inserted in series. Justify **(PYQ 2015) [3M]**

Ans: i) As number of turns are reduced, the inductance and hence the inductive reactance decreases, therefore the impedance decreases and hence current in the circuit increases. So, brightness of bulb increases

ii) as an iron rod is inserted, the inductance and hence the reactance increases. Therefore current decreases so bulb glows less brightly

$$X_L = \omega L \qquad I_0 = \frac{V_0}{Z}$$

iii) as a capacitor of same reactance is introduced, the inductance becomes minimum and hence current increases and bulb glows more brightly

$$Z = \sqrt{R^2 + (X_c - X_L)^2}$$

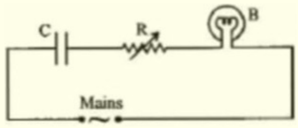
$$Z = R$$

Circuit will be in resonance and amplitude of current will be maximum

Ques: A capacitor C, a variable resistor R and a bulb B are connected in series to the ac mains in the circuit as shown. The bulb glows with some brightness. How will the glow of the bulb change if i. a dielectric is introduced between the plates of the capacitor, keeping the resistance R to be the same, ii. the resistance R is increased keeping the capacitance same? **(PYQ 2014) [2M]**

Ans: i) the dielectric increases the capacitance and hence reduces the reactance. So, the impedance decreases and hence the bulb glows more brightly $X_c = \dfrac{1}{\omega C}$

ii) as resistance is increased the impedance increases and current decreases so, bulb glows less brightly



Ques: An electric lamp having coil of negligible inductance connected in series with a capacitor and an AC source is glowing with certain brightness. How does the brightness of the lamp change on reducing the i. capacitance and ii. The frequency. Justify **(PYQ 2010) [2M]**

Ans: We know that- $X_c = \dfrac{1}{\omega C}$

So on reducing he capacitance and the frequency, the capacitive reactance increases. This increases the impedance and hence reduces the current in the circuit. Therefore, the bulb glows less brightly.

Resonance (PYQ 2016, 2013, 2012, 2010)

The phenomenon of resonance is a characteristic of systems which have a tendency to oscillate at a particular frequency. This frequency is called the system's natural frequency. If such a system is driven by an external energy source at a frequency almost equal to the system's natural frequency, then the amplitude of oscillation becomes very large.

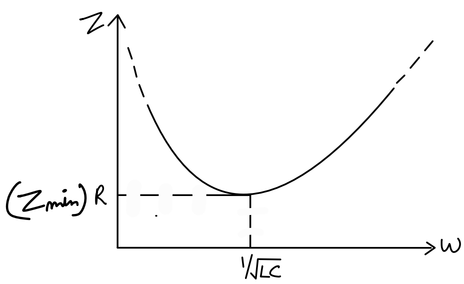
Consider an RLC circuit with amplitude of voltage= v_o and frequency ω. The current in the circuit can be written as-

$$I_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + (X_c - X_L)^2}}$$

$$X_c = \frac{1}{\omega C}, X_L = \omega L$$

So, if ω is varied, then at a particular frequency (ω_o), $X_C = X_L$, impedance will be minimum-

$$\boxed{Z = \sqrt{R^2 + (0)^2} = R}$$



This frequency is called the resonant frequency

$$X_c = X_L; \omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{\Rightarrow \omega_0 = \sqrt{\frac{1}{LC}}}$$

At resonant frequency, the current amplitude is maximum-

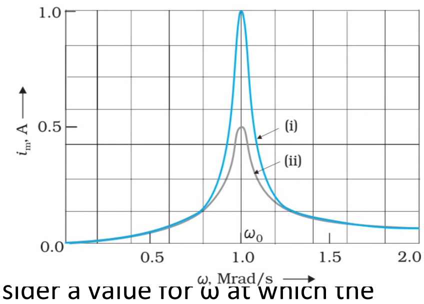
$$\boxed{I_{max} = \frac{V_0}{R}}$$

Uses of resonance in LCR circuit

Tuning of radio set or TV- to hear the signal from particular station, we need to tune the radio for which we vary the capacitance of a capacitor such that the resonant frequency of the circuit becomes almost equal to the frequency of the radio signal received. When this happens, the amplitude of the current with the frequency of the signal becomes maximum.

In which circuit does resonance take place?
Resonance is only exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltages across L and C cancel each other as they are out of phase and current amplitude is maximum i.e. the total source voltage appears across R. This means we cannot have resonance in a RL and RC circuit.

Sharpness of resonance



The amplitude of current in the series LCR circuit is given by-

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2}}$$

We know that it is maximum for $\omega = \omega_0 = \sqrt{\frac{1}{LC}}$ and the maximum value is- $I_{max} = \frac{V_0}{R}$

For values other than ω_0, the amplitude of current is less than the maxm value. Consider a value for ω at which the amplitude becomes $1/\sqrt{2}$ times the maxm value. At this value, the power dissipated becomes half ($P \propto I^2$)
From the graph above, we see that there are two such values for ω. Let them be ω_1 and ω_2 –

$$\omega_1 = \omega_0 + \Delta\omega$$

$$\omega_2 = \omega_0 - \Delta\omega$$

Bandwidth of the circuit
The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is called the bandwidth of the circuit.

The quantity $\omega_0/2\Delta\omega$ is called the sharpness. The smaller the value of Δω, the sharper or narrower is the resonance. We know that the relation b/w I_0 for ω_1 and I_0 for ω_0 is as follows-

$$I_{\omega_1} = \frac{1}{\sqrt{2}} i_{max}$$

$$I_{\omega_1} = \frac{V_0}{\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2}} = \frac{I_{max}}{\sqrt{2}}$$

$$I_{\omega_1} = \frac{V_0}{R\sqrt{2}}$$

or $$\sqrt{R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2} = R\sqrt{2}$$

or $$R^2 + \left(\omega_1 L - \frac{1}{\omega_1 C}\right)^2 = 2R^2$$

$$\omega_1 L - \frac{1}{\omega_1 C} = R$$

which may be written as,

$$(\omega_0 + \Delta\omega)L - \frac{1}{(\omega_0 + \Delta\omega)C} = R$$

$$\omega_0 L\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \frac{1}{\omega_0 C\left(1 + \frac{\Delta\omega}{\omega_0}\right)} = R$$

We can approximate $\left(1 + \frac{\Delta\omega}{\omega_0}\right)^{-1}$ as $\left(1 - \frac{\Delta\omega}{\omega_0}\right)$ since $\frac{\Delta\omega}{\omega_0} << 1$. Therefore,

$$\omega_0 L\left(1 + \frac{\Delta\omega}{\omega_0}\right) - \omega_0 L\left(1 - \frac{\Delta\omega}{\omega_0}\right) = R$$

$$= \omega_0 L \frac{2\Delta\omega}{\omega_0} = R$$

$$\boxed{= \Delta\omega = \frac{R}{2L}}$$

Therefore, sharpness of resonance is given by-

$$\boxed{\frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R}}$$

Quality factor (Q) (PYQ 2013)

The ratio $\omega_0 L/R$ is called the quality factor of the circuit

$$\boxed{Q = \frac{\omega_0 L}{R}}$$

We see that-

$$\boxed{2\Delta\omega = \frac{\omega_0}{Q}}$$

So, larger the value of Q, the smaller the value of $2\Delta\omega$ or the bandwidth of the circuit and sharper is the resonance.

Using $\omega_0^2 = 1/LC$ we get-

$$\boxed{Q = \frac{1}{\omega_0 \cdot CR}}$$

Selectivity of the circuit

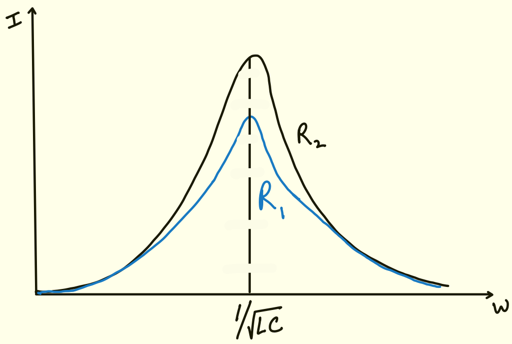
If the resonance is less sharp, not only is the maxm current less, the circuit is close to resonance for a larger range ($\Delta\omega$) of frequencies and tuning of the circuit is not good. So, less sharp is the resonance, less is the selectivity of the circuit or vice versa.

Also, if Q is large i.e. R is low or L is large, selectivity is also large.

Important PYQs 

Ques: In a series LCR circuit connected to an ac source of variable frequency and voltage $V = V_m \sin\omega t$, draw a plot showing the variation of current I with angular frequency ω for two different resistances R_1 and R_2 ($R_1 > R_2$). Write the condition under which resonance takes place. For which out of the two curves, a sharper value of resonance is produced? Define Q factor of the circuit and give its significance **(PYQ 2013) [5M]**

Ans: a)



C) Greater is the value of Q, sharper is the resonance, therefore, resonance is sharper for R_2

$$Q = \frac{1}{\omega_0 \cdot CR}$$

(b and d already shown in notes)

Ques: the figure shows a series LCR circuit with L= 5.0 H, C= 80 μF, R= 40 Ω connected to a variable frequency 240 V source. Calculate-

a. The angular frequency of the source which drives the circuit At resonance
b. The current at resonating frequency
c. The rms potential drop across the capacitor at resonance

(PYQ 2012) [3M]

Ans: a) $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80\mu}} = 50 rad/s$

b) $I_0 = \frac{V_0}{R} = \frac{240}{40} = 6A$

$$V_{RMS} = I_{RMS} \cdot X_c = \frac{I_0}{\sqrt{2}} \cdot X_c = \frac{240}{\sqrt{2}} \times \frac{10^6}{50 \times 80}$$

$$V_{RMS} = 4.2 \times 10^4 V$$

Ques: Define quality factor and write its SI unit **(PYQ 2016) [1M]**

Ans: It is defined as the ratio $\omega_0 L/R$. it is dimensionless and unitless quantity

Power in an AC circuit: Power factor (PYQ 2017, 2010)

(not in CBSE syllabus for 2020 Boards)

Consider a series RLC circuit with a voltage source $v = v_0 \sin\omega t$. Let current in the circuit be $i = i_0 \sin(\omega t + \phi)$. We know-

$$i_0 = \frac{V_0}{Z}; \phi = Tan^{-1}\left(\frac{X_c - X_L}{R}\right)$$

Therefore, the instantaneous power supplied by the source is-

$$p = vi = (V_0 \sin\omega t) \times [I_0 \sin(\omega t + \phi)]$$

$$\boxed{p = \frac{V_0 I_0}{2} \times [\cos\phi - \cos(2\omega t + \phi)]}$$

Average power-

The average of the time dependent term in one complete cycle will be 0 hence the average power-

$$\boxed{\overline{P} = \frac{V_0 I_0}{2} \times \cos\phi} = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \times \cos\phi$$

$$\boxed{\overline{P} = V_{RMS} \cdot I_{Rms} \cos\phi}$$

$$V = IZ$$

$$\boxed{\overline{P} = I^2 Z \cos\phi}$$

Where Cosϕ is the **power factor** of the circuit

case(i): Resistive circuit- If the circuit contains only pure R, it is called resistive. For such a circuit-

$$\boxed{\phi = 0, \cos\phi = 1}$$

therefore, there is maximum power dissipation.

Case (ii) Purely inductive or capacitive circuit- If the circuit contains only a capacitor or an inductor, the phase difference is π/2. Therefore, cosϕ=0 i.e. no power is dissipated even though a current flows. This is called Wattless current.

$$\boxed{\phi = \frac{\pi}{2}; \cos\phi = 0}$$

Case(iii)- LCR series circuit- In series LCR circuit power is given by –

$$\boxed{\overline{P} = V_{RMS} \cdot I_{Rms} \cos\phi}$$

So ϕ may be non 0 is RLC, RL or RC circuits. Even here, power is dissipated only across R.

Case (iv): Power dissipated at resonance- maximum power is dissipated (through R) at resonance

$$\boxed{\phi = 0, \cos\phi = 1 \qquad P = I^2 Z = I^2 R}$$

Important PYQs

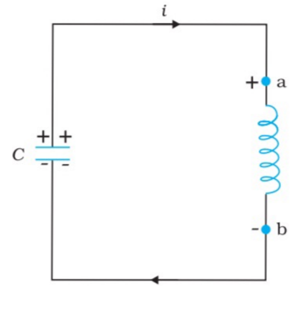
Ques: In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When a capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. Calculate the ratio P_1/P_2 **(PYQ 2017) [1M]**

Ans: $P_1 = \cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + R^2}} = \frac{1}{\sqrt{2}}$

$$P_2 = \cos\phi = \frac{R}{\sqrt{R^2 + (X_L - X_c)^2}} = 1$$

$$\frac{P_1}{P_2} = \frac{1}{\sqrt{2}}$$

LC oscillations



We know that a capacitor and inductor can store electrical and magnetic energy respectively. Consider a capacitor C, initially charged(q_0), connected to an inductor L. As the circuit is completed, current begins to grow in the circuit and the charge on the capacitor decreases. Let q and i be the charge on the capacitor and the current in the circuit resp. Using Lenz law and Kirchhoff's loop law-

$$\frac{q}{c} - L\frac{di}{dt} = 0$$

We can write i= -dq/dt (as q decreases, i increases) therefore,

$$\frac{d^2q}{dt^2} + \frac{1}{LC}\cdot q = 0$$

Comparing this equation with that of a simple harmonic motion-

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

We see that the charge is oscillating and the natural frequency of oscillation is-

$$\boxed{\omega^2 = \frac{1}{LC};\ \omega = \frac{1}{\sqrt{LC}}}$$

Further, we can also deduce that charge varies with time as-

$$q = q_0\cos(\omega t + \phi)$$

Where q_0 is the amplitude/ maxm value of charge and ϕ is phase constant. For the case given above, q=q_0 @ t=0 , ϕ=0 so-

$$\boxed{q = q_0\cos(\omega t)}$$

For current-

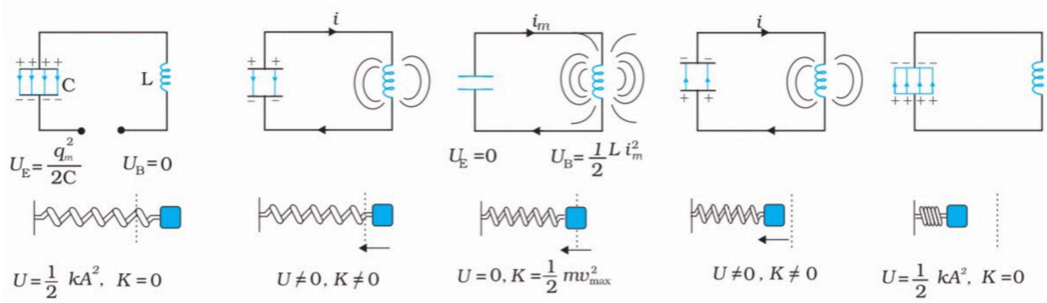
$$i = -\frac{dq}{dt}$$

$$\boxed{\begin{aligned} i &= i_0\sin(\omega t) \\ i_0 &= q_0\omega \end{aligned}}$$

Note: Since circuit has no dissipative element, total energy remains constant

Visualization of LC oscillations

1. At t=0, the charge on capacitor is q_0 and the current in the circuit is 0. Hence, there is no energy stored in the inductor and the total energy of the circuit is the electrical energy stored in the capacitor-
2. When the switch is closed, current starts growing in the circuit, the charge on the capacitor and hence the electrical energy starts decreasing and then some energy gets stored as magnetic energy in the inductor. If at time t current is I then the magnetic energy-
3. At time t= T/4, current reaches its maximum value i_0, all the energy is stored as magnetic energy and the capacitor has no charge and no energy.
4. As the current continues to flow, it again starts charging the capacitor. This process continues till the capacitor is again fully charged at t=T/2 but in the opposite polarity.
5. This whole process repeats itself till the system is reverted to its original state. Thus, the energy oscillates between the capacitor and inductor



This oscillation can be compared to that of a block connected to a spring-

$$\frac{d^2x}{dt^2} + \omega_\circ^2 x = 0 \qquad \varepsilon = -L\frac{d^2q}{dt^2} \qquad F = m\frac{d^2x}{dt^2}$$

Here $\omega_\circ = \sqrt{k/m}$ where k is the spring constant. Comparing the two equations, we see that L is analogous to mass 'm'. L is a measure of resistance to change in current in the circuit.

For an LC circuit-

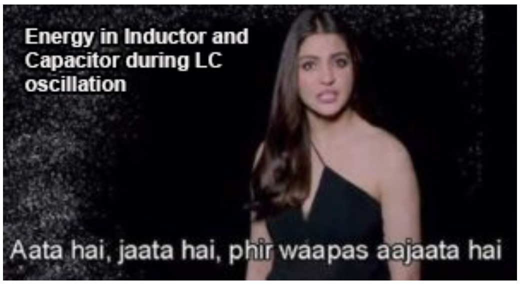
$$\omega = \frac{1}{\sqrt{LC}}$$

And for a spring-

$$\omega_0 = \sqrt{\frac{k}{m}}$$

So, 1/C is analogous to k. the constant k= F/x tells us the force required to produce unit displacement similarly, 1/C= v/q tells us the potential difference required to store unit charge

Mechanical system	Slectrical system
Mass m	Inductance L
Force constant k	Reciprocal capacitance $1/C$
Displacement x	Charge q
Velocity $v = dx/dt$	Current $i = dq/dt$
Mechanical energy	Electromagnetic energy
$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$	$U = \frac{1}{2}\frac{q^2}{C} + \frac{1}{2}Li^2$



The above discussion is not realistic because-

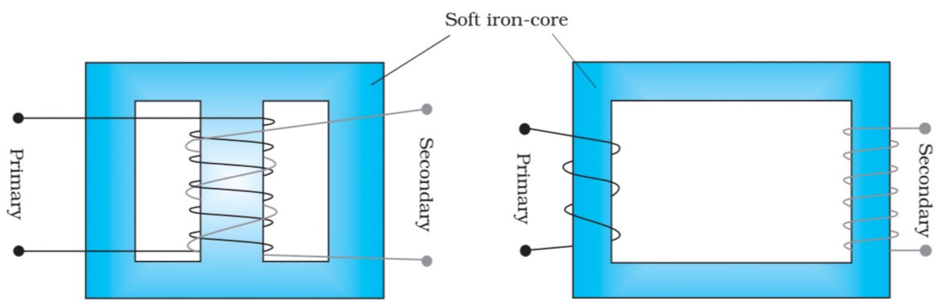
1. Every inductor has some resistance. The resistance causes damping of charge and current which causes the oscillations to die away
2. Even if the total resistance is 0, the total energy will not remain constant, it is radiated away in the form of electromagnetic waves.

Transformer (PYQ 2020, 2019, 2018, 2017, 2015, 2011)

It is a device used to step up or step-down alternating voltage

Principle- mutual induction

Construction- A transformer consists of two sets of coils insulated from each other. They are wound over a soft iron core either on top of each other or on separate limbs of the core. One of the coils, known as the primary coil has N_1 turns and the other coil, called secondary coil has N_2 turns. Usually, primary coil is for input and secondary coil is for output.



Working- When an alternating current is passed through the primary coil, an alternating magnetic flux is induced in the coil. Through mutual induction, the alternating emf in the primary coil sets up an alternating emf and hence alternating current in the secondary coil. We assume that the coils have no resistance and entire flux of the primary coil is linked with the secondary coil i.e. there is no flux leakage. According to Faraday's laws, the emf induced in N_1 turns of the primary coil-

$$\varepsilon_1 = -N_1\frac{d\phi}{dt}$$

Similarly, the emf induced in N_2 turns of the secondary coil- $\varepsilon_2 = -N_2\frac{d\phi}{dt}$

Assuming $\varepsilon_1=V_1$ and $\varepsilon_2=V_2$ where V_1 and V_2 are the potential across primary and secondary coil respectively

$$V_1 = -N_1\frac{d\phi}{dt}$$

$$V_2 = -N_2\frac{d\phi}{dt}$$

Dividing both we get-

$$\boxed{\frac{V_1}{V_2} = \frac{N_1}{N_2}}$$

Now assuming that there is no power loss-
Power at primary = power at secondary

$$V_1 I_1 = V_2 I_2$$

$$\frac{I_1}{I_2} = \frac{V_2}{V_1}$$

Where I_1 and I_2 are currents in primary and secondary coils respectively. Therefore,

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1}}$$

Assumptions made-
1. The primary resistance the current is small
2. Entire flux of primary coil is linked with the secondary coil i.e. there is no flux leakage
3. The secondary current is small

Types of transformers-
1. **Step-up transformer**
   if the number of turns of secondary coil is more than that of the primary coil i.e. $N_1<N_2$, we can see that $V_2>V_1$. Such a transformer is called a step-up transformer

2. **Step-down transformer**
   If number of turns of primary coil is more than that of the secondary coil i.e. $N_1>N_2$, we can see that $V_2<V_1$. Such a transformer is called step down transformer

Energy losses in transformers/ factors affecting efficiency of a transformer (PYQ 2020, 2019, 2018, 2017, 2011)
1. **Flux leakage-** The complete flux of primary and secondary coil cannot be linked. There are always some leakages. It can be reduced by winding the coils over one another.
2. **Resistance of the windings-** The windings have some resistance which causes loss of energy in the form of heat. They are minimized by using thick wires
3. **Eddy currents-** The alternating emf induces eddy currents and causes loss of energy as heat
4. **Hysteresis-** The magnetization of core is continuously reversed by alternating magnetic field which causes loss of energy due to hysteresis. It can be reduced by using materials of low hysteresis loss.

Use of transformers-
The large-scale transmission of electrical energy is done with the help of transformers. The source voltage is stepped up to reduce current and hence minimize I^2R losses. At the point of consumption, the voltage is stepped down to about 240 V which reaches our home.

Efficiency of a transformer (η) (PYQ 2018)
It is defined as the ratio of useful output power to the input power

Important PYQS

Ques: Laminated iron sheets are used to minimize___________ currents in the core of a transformer **(PYQ 2020) [1M]**

Ans: Eddy currents

Ques: A small town with a demand of 1200 kW of electric power at 220V is situated 20 km away from an electric plant generating power at 440V. The resistance of the two-wire line carrying power is 0.5 Ω per km. the town gets power from the line through a 4000-220 V step down transformer at a sub-station in the town. Estimate the line power loss in the form of heat. **(PYQ 2019) [2M]**

Ans: Total resistance-

$$R = 2 \times 0.5 \times 20$$

$$R = 20\Omega$$

Current in the wires-

$$I = \frac{P}{V} = \frac{1200 \times 10^3}{4000} = 300A$$

Heat loss-

$$P_H = I^2 R$$

$$P_H = \left(300^2 \times 20\right)$$

$$P_H = 1800kW$$

Ques: Calculate the current drawn by the primary of a 90% efficient transformer which steps down 220V to 22V, if the output resistance is 440Ω **(PYQ 2018) [2M]**

Ans:

$$P_2 = \frac{V_2^2}{R}$$

$$P_2 = \frac{(22)^2}{440} = 1 \cdot 1W$$

$$\frac{P_2}{P_1} \times 100 = 90$$

$$P_1 = \frac{P_2 \times 100}{90} = V_1 I_1$$

$$I_1 = \frac{1 \cdot 1 \times 100}{90 \times 220}$$

$$I_1 = 5 \cdot 5mA$$

Ques: The primary coil of a step-up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are respectively 220V and 1100W. calculate

a. Number of turns in secondary
b. Current in primary
c. Voltage across secondary
d. Current in secondary
e. Power in secondary **(PYQ 2017) [3M]**

Ans: Since it is a step up transformer-

a) $\frac{N_2}{N_1} = 100$

$$N_2 = 100 \times 100 = 10{,}000$$

b) $P = VI$

$$I = \frac{P}{V} = \frac{1100}{220} = 5A$$

c) $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$$V_2 = \frac{V_1 N_2}{N_1} = 220 \times 100 = 22kV$$

d) $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

$$I_2 = \frac{1}{100} \times 5 = 50mA$$

e) $P_1 = P_2 = 1100W$

Ques: Is it possible to use a transformer to bring down high dc voltage? Explain **(PYQ 2015) [1M]**

Ans: No, a transformer cannot be used to step down dc voltage because the working principle of a transformer is mutual induction which requires a time varying magnetic flux and hence a time varying current. Since, dc is not time varying, it can't produce a variable flux and hence cannot be stepped down.

Derivations and definitions asked as PYQs

Ques: Define capacitive reactance and write its SI unit **(PYQ 2015) [1M]**

Ques: show that in an ac circuit containing a pure inductor, the voltage is ahead of current by $\pi/2$ in phase **(PYQ 2011) [2M]**

Ques: A series LCR circuit is connected to an ac source having voltage $v = v_m \sin\omega t$. Derive the expression for the instantaneous current I and its phase relationship to the applied voltage
Obtain the condition for resonance to occur. Define power factor and state the conditions where it is i. maximum ii. Minimum **(PYQ 2010) [5M]**

Ques: An ac voltage source $V = V_0 \sin\omega t$ is connected to a series combination of L, C and R. use the phasor diagram to obtain an expression for the impedance of the circuit and the phase angle between voltage and current. Find the condition where the current will be in phase with the voltage. What is this circuit condition called? **(PYQ 2016) [4M]**

Ques: Why is a choke coil needed in the use of fluorescent tubes with ac mains **(PYQ 2014) [1M]**

Ques: Define Q factor of a circuit and give its significance **(PYQ 2013) [1M]**

Ques: With the help of a labelled diagram, explain the working of a step-up transformer. Give reason to explain the following-

1. The core of a transformer is laminated
2. Thick copper windings are used in windings **(PYQ 2020) [3M]**

Ques: Draw the diagram of the device used to decrease high ac voltage into a low ac voltage and state its working principle. Write 4 sources of energy loss of this device **(PYQ 2019) [3M]**

Ques: 1. state working principle of transformers
2. define efficiency of a transformer
3. state two factors which affect the efficiency of a transformer **(PYQ 2018) [3M]**

Ques: write the function of a transformer. State its working principle with the help of a labelled diagram. Mention various energy losses in the device **(PYQ 2017) [3M]**

Ques: What device is used to bring high ac voltage down to low voltage ac. What is the principle of working? **(PYQ 2015) [1M]**

Ques: Mention the various energy losses of a transformer **(PYQ 2011) [2M]**