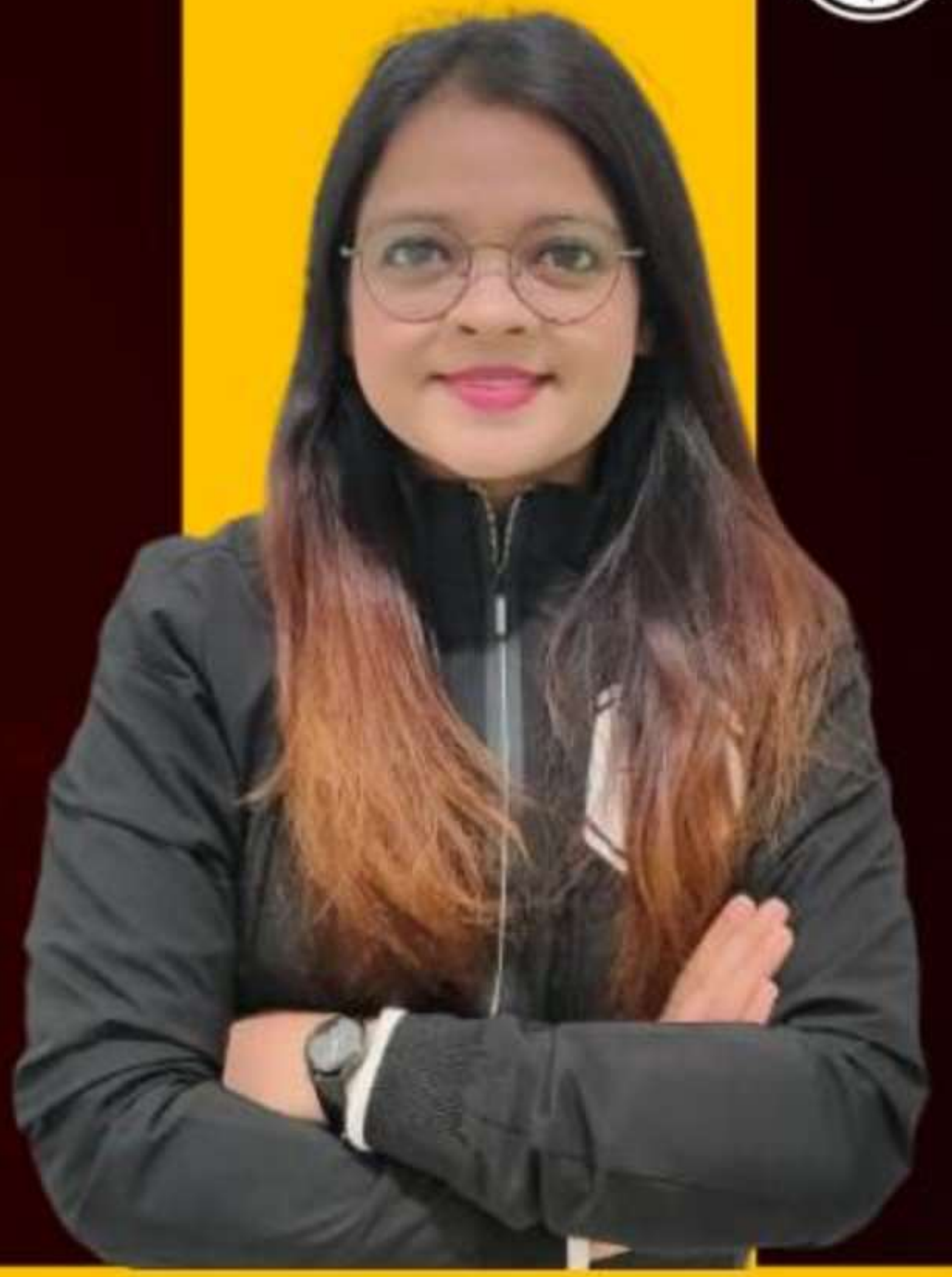
NCERT MIND MAP FOR 12TH BOARDS

- Subject - Mathematics
- Chapter –Matrices

(One Shot)



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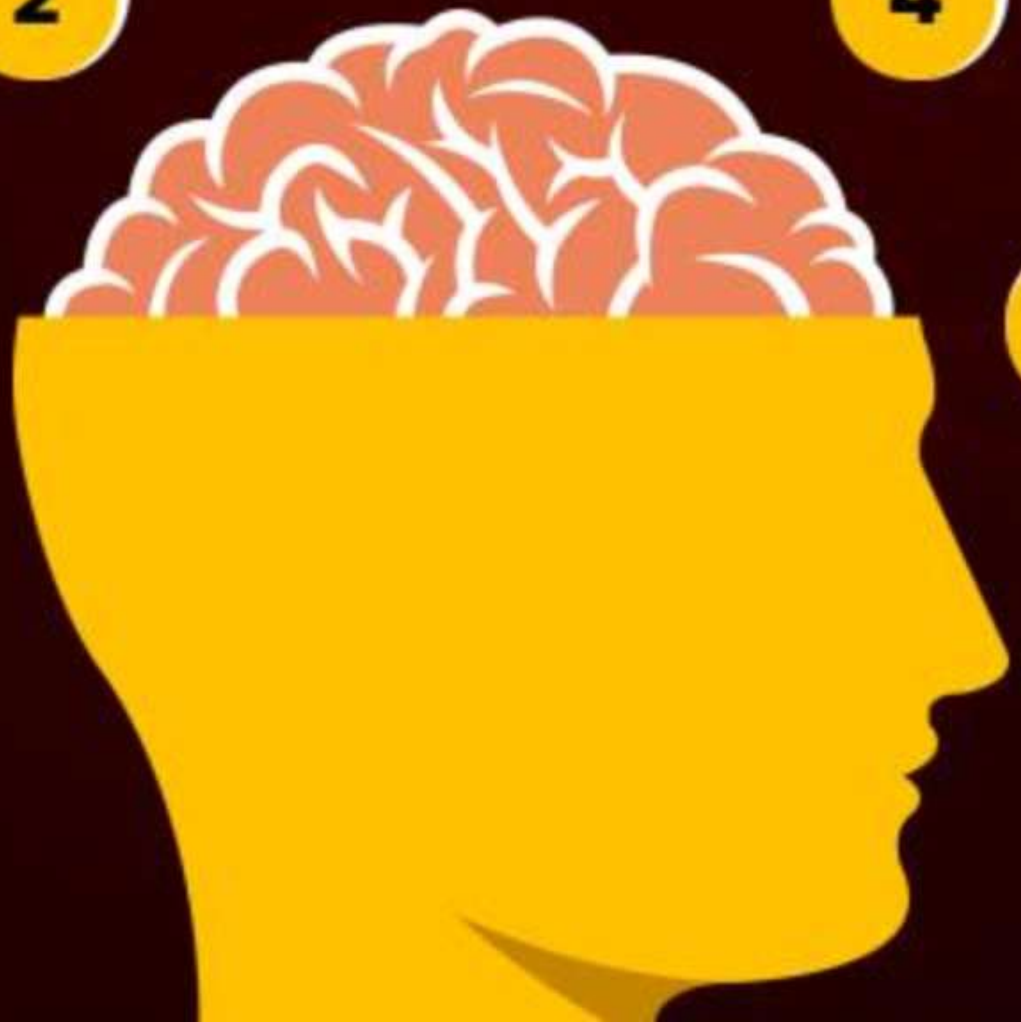
Topics to be covered

1. Matrices
2. Types of matrices
3. Transpose of matrices
4. Operations on matrices
5. Invertible matrices



Topic : Matrices

Matrix: An array of elements.

$$\begin{bmatrix} \checkmark & \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark & \checkmark \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

Order: no. of rows × no. of colums.

$m \times n$ matrix will have m rows and n column.

No. of elements $= mn$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \end{bmatrix}$$

$$A = [a_{ij}]_{m \times n}$$

$$1 \leq i \leq m$$
$$1 \leq j \leq n$$

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 3 & 4 \end{bmatrix}_{3 \times 2}$$

$a_{32} = 4$ $a_{21} = 1$

$A = [a_{ij}]_{2 \times 2}$

where $a_{ij} = i + j$

$a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{21} = 2 + 1 = 3$

$a_{22} = 2 + 2 = 4$

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$$

Ex: If a Matrix have 12 elements find its possible order?

clearly,

$mn = 12$

$$\begin{cases} 1 \times 12 & 12 \times 1 \\ 2 \times 6 & 6 \times 2 \\ 3 \times 4 & 4 \times 3 \end{cases}$$

Topic : Types of matrices

Row Matrix

↓

Order : $1 \times n$

$$\begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \end{bmatrix}_{1 \times n}$$

Column Matrix :

Order : $m \times n$.

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ \vdots \\ a_{m1} \end{bmatrix}$$

Square Matrix :

Equal no. of rows and columns.

→

Diagonal Matrix :

Square matrix have elements other than a_{ii} = 0.

↓

Scalar Matrix :

Diagonal Matrix having all diagonal elements equal

←

Identity Matrix :

Scalar matrix having diagonal elements = 1.

Square

$$= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Diagonal:

$$= \begin{bmatrix} \underline{1} & 0 & 0 \\ 0 & \underline{2} & 0 \\ 0 & 0 & \underline{3} \end{bmatrix}$$

Scalar:

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Identity:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Topic : Transpose of Matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2\times 3}$$

Then $A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3\times 2}$.

A^T: Matrix obtained by Interchanging rows and columns of a given matrix.

$\boxed{\text{If order of } A \text{ is } m\times n \text{ then order of } A^T = n\times m}$

Symmetric: $A^T = A$.

$$\begin{bmatrix} a & x & y \\ x & b & z \\ y & z & c \end{bmatrix}$$

- Square

Skew symmetric: $A^T = -A$

$$\begin{bmatrix} 0 & x & -y \\ -x & 0 & -z \\ y & z & 0 \end{bmatrix}$$

Properties of Transpose:

1. $(A+B)^T = A^T + B^T$
2. $(A-B)^T = A^T - B^T$.
3. $(AB)^T = B^T A^T$
4. $(KA)^T = K(A^T)$
5. $(A^T)^T = A$.

Ex: If A and B are symmetric Matrix then $A+B$, $A-B$, $AB+BA$ are symmetric as well.

Soln: Given: $A^T = A$.
$B^T = B$.

$$(A+B)^T = A^T + B^T$$
$$= A+B. \quad \therefore A+B \text{ is symmetric}$$

$$(AB+BA)^T = (AB)^T + (BA)^T.$$
$$= B^T A^T + A^T B^T.$$
$$= BA + AB. \text{ — symmetric}$$

$A+A^T$ is always symmetric.

& $A-A^T$ is always skew symmetric.

$$A = \underbrace{\frac{A+A^T}{2}}_{\text{Symmetric}} + \underbrace{\frac{A-A^T}{2}}_{\text{skew symmetric.}}$$

Topic : Operations on matrices

Equality : $A = B$ if order of A = order of B.

corresponding elements are equal.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$$

$a = 1 \quad b = 4 \quad c = -1 \quad d = 0.$

Addition: $A+B$ is only possible when order of A = order of B.

Just add corresponding elements.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} a+x & b+y \\ c+z & d+w \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} a-x & b-y \\ c-z & d-w \end{bmatrix}$$

Property :

(1) $A+B = B+A$.

(2) $A+(B+C) = (A+B)+C$

(3) Null Matrix : Having all its elements as zero. O

$A+O = A$.

Multiplication :

(1) By scalar : $A = \begin{bmatrix} a_{ij} \end{bmatrix}_{m\times n}$

$kA = \begin{bmatrix} ka_{ij} \end{bmatrix}_{m\times n}$.

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad -A = \begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$

$2A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ $\quad \frac{A}{2} = \begin{bmatrix} \frac{1}{2} & 1 \\ 3/2 & 2 \end{bmatrix}$

(2) Multiplication By Matrix : AB.

(i) If no. of columns in A (first matrix)
= No. of rows in B (second matrix).

$A \quad B = (AB)$? Order of AB

$2\times 2 \quad 2\times 3 = \checkmark \qquad 2\times 3$

$2\times 1 \quad 2\times 1 = \times \qquad \_\ \_\_\_$

$3\times 2 \quad 2\times 4 = \checkmark \qquad 3\times 4.$

Ex:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Row by column.

$R_1C_1 \quad R_1C_2$

$R_2C_1 \quad R_2C_2$

$$= \begin{bmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \end{bmatrix}$$

Properties of Matrix Multiplication:

(1) AB does not necessarily equal to BA.

(2) $A(BC) = (AB)C$

(3) $C(A+B) = CA + CB.$

$(A+B)C = AC + BC$

(4) $A^2 = A \cdot A$ (If A square).

$A^3 = A^2 \cdot A = A \cdot A \cdot A.$

$A^2 \cdot A^3 = A^5$

Topic : Invertible matrices

B is called Inverse of A if

$AB = BA = I$.

denoted by $B = A^{-1}$

→ Inverse of A is possible when $|A| \neq 0$.

In this case A is called invertible matrix.

$I^2 = I^3 = I^n$
$= I$.

$AI = A$

$BI = B$.

$\rightarrow \quad AA^{-1} = I.$

$\rightarrow \quad A^2 \cdot A^{-1} = A.$

$\rightarrow \quad A^3 \cdot A^{-1} = A^2.$

$\rightarrow \quad (AB)^{-1} = B^{-1}A^{-1}$

$\rightarrow \quad (A^2)^{-1} = (A^{-1})^2$

$\rightarrow \quad (A^T)^{-1} = (A^{-1})^T$

$\rightarrow$ Inverse of A is unique.

Proof: let B and C be two inverse of A.

$$AB = BA = I.$$

$$AC = CA = I.$$

$$AB = I$$

pre-multiply by C

$$CAB = CI$$

$$IB = CI.$$

$$\boxed{B = C}$$

Homework

Ncert → chapter · 3.

THANK YOU