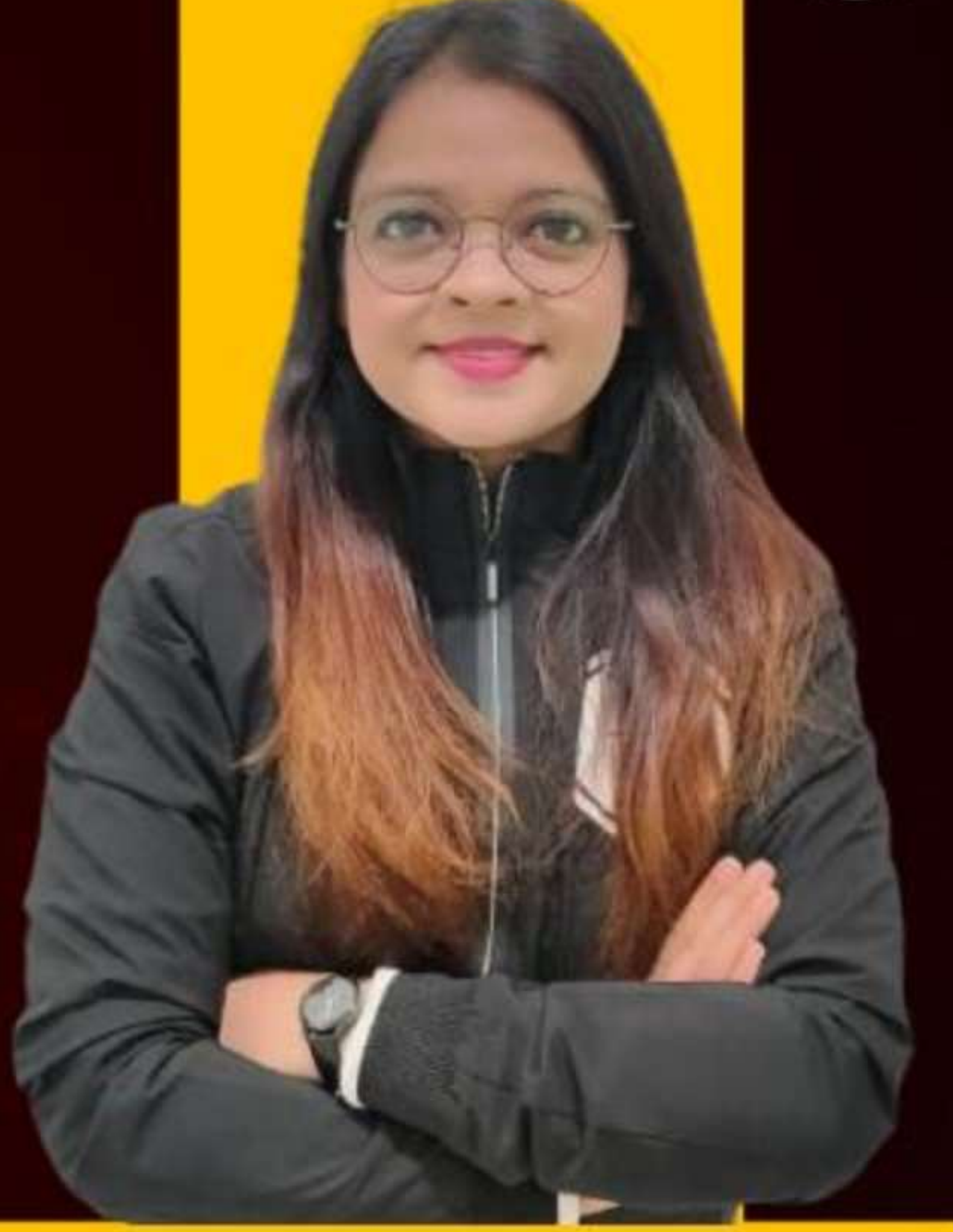
NCERT MIND MAP FOR 12TH BOARDS

- Subject - Mathematics
- Chapter –Determinants

(One Shot)



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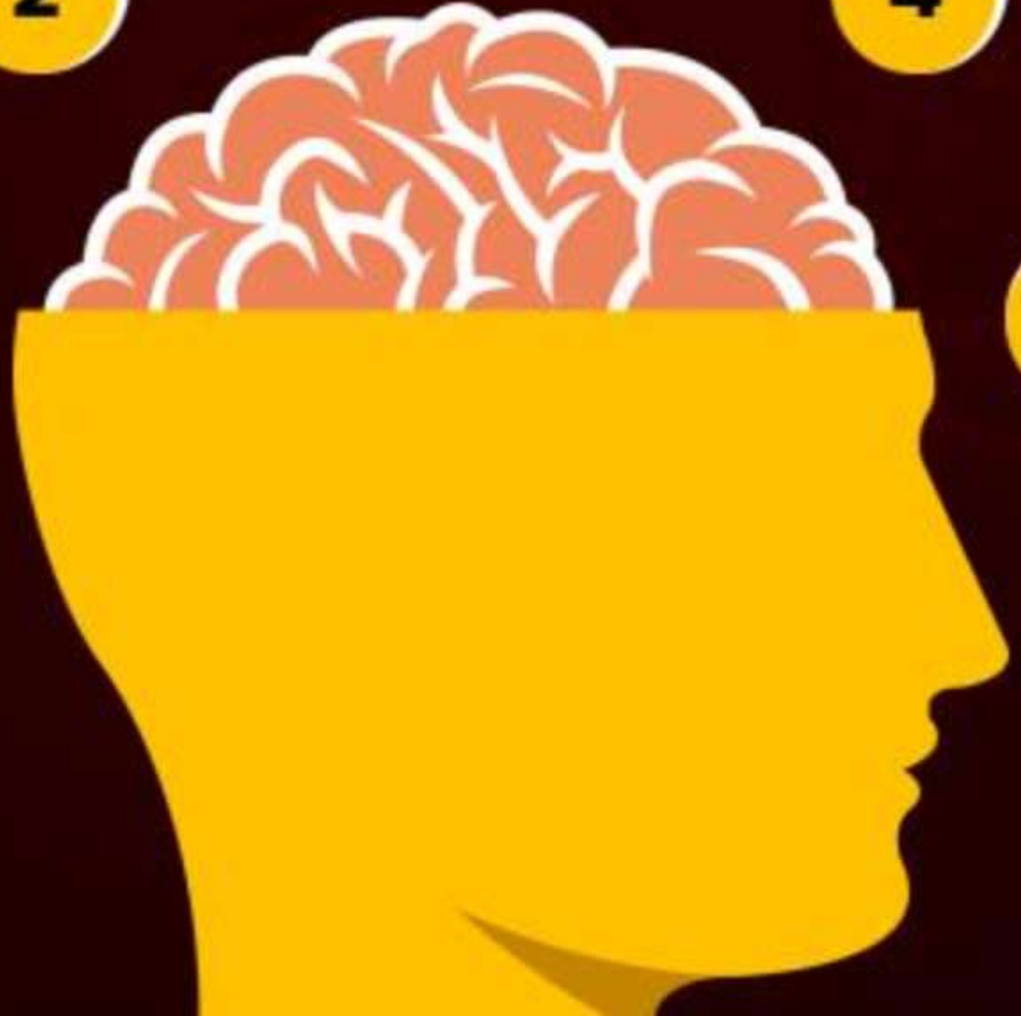
Topics to be covered

1. Value of determinant
2. Properties of determinant
3. Minors and cofactors
4. Adjoint and inverse
5. Application of determinant



Topic : value of determinant

(1) Value of det. of 1^{st} order Matrix : $|a_{11}| = a_{11}$

$A = [-3]$ $B = [2]$ $C = [0]$ $D = [1]$

$|A| = -3$ $|B| = 2$ $|C| = 0$ $|D| = 1.$

(2) 2^{nd} order, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = |A| = a_{11}a_{22} - a_{21}a_{12}.$$

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $|A| = \underline{1} - 0$

$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |B| = 4 - 6 = -2.$

(3) Value of det of 3rd order matrix :

$$A = \begin{vmatrix} \overset{+}{a_{11}} & \overset{-}{a_{12}} & \overset{+}{a_{13}} \\ \overset{-}{a_{21}} & \overset{+}{a_{22}} & \overset{-}{a_{23}} \\ \overset{+}{a_{31}} & \overset{-}{a_{32}} & \overset{+}{a_{33}} \end{vmatrix}$$

Selecting R_1

$$+a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\Delta = a_{11}(a_{22}a_{33} - a_{32}a_{23})$$
$$- a_{12}(a_{21}a_{33} - a_{31}a_{23})$$
$$+ a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$A = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & 3 \end{vmatrix} \end{array}$$

Expanding Using R_1

$$1\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}$$

$$1(3-(-2)) - 1(0) - 1(0)$$

$$5$$

Expand using C_1

$$+1\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} -$$

$$= 5.$$

For Expanding always prefer Row/column with max. zeroes.

Topic : Properties of determinants

1) Value of det. does not change when its rows and columns are interchange.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \Delta.$$

$$\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} = \Delta.$$

$$\boxed{\begin{aligned} |A| &= \Delta \\ |A^T| &= \Delta. \end{aligned}}$$

Skew symmetric Matrix

$$A^T = -A.$$

$$|A|^T = |-A| = -1^n |A|$$

$$|A^T| = (-1)^n |A|$$

$$|A| = (-1)^n (|A|)$$

n - Even

$$|A| = |A|$$

n - odd

$$|A| = -|A|$$

$$|A| = 0$$

Determinant value of odd order skew symmetric matrix is zero.

2) $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\Delta.$$

when two Rows/columns are interchange then value of det become negative.

P.3: If a Row/column of det has all its elements as 0 then value of det is 0.

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 4 & 5 & 6 \end{vmatrix} = 0.$$

P.4

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$KA = \begin{vmatrix} 1 & 2 & 3 \\ 4K & 5K & 6K \\ 7 & 8 & 9 \end{vmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad |A| = ad - bc.$$

$$2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} \quad |2A| = 4ad - 4bc$$

$$= 4(ad - bc)$$

$$= 2 \times 2 \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Generalise: $|A| = \Delta$

$|KA| = K^n \Delta.$

where n is order of matrix.

P.5 :

$$\begin{vmatrix} x+a & x+b & x+c \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} = \begin{vmatrix} x & x & x \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix} + \begin{vmatrix} a & b & c \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{vmatrix}$$

Ex :

$$\begin{vmatrix} x_1 & x_2 \\ x_3 & x_4 \end{vmatrix} + \begin{vmatrix} y_1 & y_2 \\ y_3 & y_4 \end{vmatrix}$$

$$x_1x_4 - x_2x_3 + y_1y_1 - y_2y_3$$

P.6 The value of Δ does not change by adding to its elements of any row/column, multiple of elements of any other row/column.

$R_1 \to R_1 + \lambda R_2$

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11}+\lambda a_{21} & a_{12}+\lambda a_{22} & a_{13}+\lambda a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Ex: $\Rightarrow \Delta = \begin{vmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \\ 3 & 5 & 7 \end{vmatrix}$

$R_2 \to R_2 - 2R_1$ ✓

$$\Delta = \begin{vmatrix} 2 & 4 & 6 \\ 0 & 0 & 0 \\ 3 & 5 & 7 \end{vmatrix} = 0.$$

Topic :Minors and cofactors

Minors:

In a det minor of an elements is the det obtained by removing Row and column containing that element.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Minor of a_{11}

$$\Rightarrow \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

Cofactors:

$$C_{a_{ij}} = (-1)^{i+j} M_{a_{ij}}.$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{3} & a_{32} & a_{33} \end{vmatrix}$$

$$M_{a_{12}} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

$$C_{a_{12}} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

Ex: $|A| = \begin{vmatrix} \overset{+}{a} & \overset{-}{b} \\ \overset{-}{c} & \overset{+}{d} \end{vmatrix}$

$M_a = d$ $\qquad C_a = d$

$M_b = c$ $\qquad C_b = -c$

$M_c = b$ $\qquad C_c = -b$

$M_d = a$ $\qquad C_d = a$

$\Delta = a_{11} \begin{vmatrix} \quad \end{vmatrix} - a_{12} \begin{vmatrix} \quad \end{vmatrix} + a_{13} \begin{vmatrix} \quad \end{vmatrix}$

$\Delta = a_{11} M_{a_{11}} - a_{12} M_{a_{12}} + a_{13} M_{a_{13}}$

$$\boxed{\Delta = a_{11} C_{a_{11}} + a_{12} C_{a_{12}} + a_{13} C_{a_{13}}}$$

Topic : Adjoint and inverse

Adjoint of A → Adj A.

It is transpose of matrix obtained by replacing each element with their cofactors.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{adj}\,A = \begin{bmatrix} Ca_{11} & Ca_{21} & Ca_{13} \\ Ca_{12} & Ca_{22} & Ca_{32} \\ Ca_{13} & Ca_{23} & Ca_{33} \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$$

$$\text{adj} A = \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}.$$

$A \text{ adj} A = (\text{adj} A) A = |A| I.$

$$A \left(\frac{(\text{adj } A)}{|A|} \right) = I.$$

$$\boxed{A^{-1} = \frac{\text{adj } A}{|A|.}}$$

Ex: $A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj} A}{|A|} = \frac{\begin{bmatrix} 4 & +3 \\ -2 & 1 \end{bmatrix}}{10.} = \begin{bmatrix} 4/10 & 3/10 \\ -2/10 & 1/10 \end{bmatrix}$$

$$|kA| = k^n |A|.$$

$$A \operatorname{adj} A = |A| I$$

$$|A| \, |\operatorname{adj} A| = \big| |A| I \big|$$

$$|A| \, |\operatorname{adj} A| = |A|^n \underbrace{|I|}_{1}$$

$$|\operatorname{adj} A| = |A|^{n-1}$$

n - order of matrix.

Ex: If $|A| = 9$ and A is 3^{rd} order matrix then $|\operatorname{adj} A| = ?$

$$|A|^{3-1} = 9^{3-1} = 81.$$

Topic : Application based questions

Area.

If vertices of Δ are

$A(x_1, y_1)$ $B(x_2, y_2)$ $C(x_3, y_3)$.

Area of ΔABC

$$= \left| \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

$\Rightarrow$ mod

If $\Delta = 0$ then points are collinear

If $\Delta = a$.

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm a.$$

System of linear Eqns:

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3.$$

A = coefficient Matrix.

$$= \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

X = Varible Matrix

$$= \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

B = constant matrix.

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

$$AX = B.$$

$$X = A^{-1}B$$

$$X = \frac{(\text{adj}A)\,B}{|A|}$$

$|A| \neq 0$ Unique solⁿ.

$|A| = 0$

$(\text{adj } A)B = O$ Infinite solⁿ.

$(\text{adj } A)B \neq O$ no solution.

Homework

Try to complete ncert

THANK YOU