

ELECTROSTATIC POTENTIAL AND CAPACITANCE

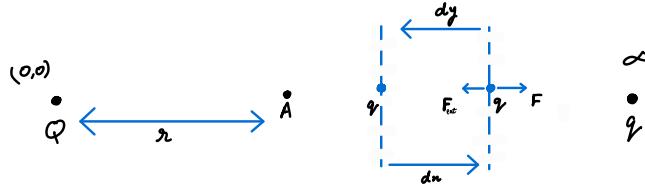


⚡ Electrostatic Potential Energy (U) (PYQ 2020, 2018, 2014, 2010)

Electrostatic potential energy of a body at a point in an electric field can be defined as the work done by external force in moving a charge (slowly, without acceleration) from infinity to the given point, against the field. SI Unit is Joule.

Derivation (PYQ 2020)

Consider a source charge Q at origin and another charge q at infinity. An external force F_{ext} acts on q such that it is equal in magnitude but opposite in direction the Coulomb force between Q and q . So the work done by F_{ext} on q on moving it by a distance dx is-



$$dW = \vec{F}_{ext} \cdot d\vec{y} = F_{ext} dy \cos 0^\circ$$

$$dW = -F_{ext} dx = -\frac{1}{4 \times \pi \epsilon_0} \times \frac{Qq}{x^2} dx \quad (\because dx = -dy)$$

$$\int dW = \frac{-Qq}{4 \times \pi \epsilon_0} \int_{\infty}^r \frac{dx}{x^2}$$

$$W = \frac{Qq}{4 \times \pi \epsilon_0} \left[\frac{1}{x} \right]_{\infty}^r$$

$$W = \frac{1}{4 \times \pi \epsilon_0} \times \frac{Qq}{r}$$

$$W = U_f - U_i = U_r - U_{\infty}$$

$$\Rightarrow U_r = U_{\infty} + \frac{1}{4 \times \pi \epsilon_0} \times \frac{Qq}{r}$$

$$U_{\infty} = 0$$

$$\Rightarrow U_r = \frac{1}{4 \times \pi \epsilon_0} \times \frac{Qq}{r}$$

Note: 1) We cannot calculate absolute potential energy of a body; we can only calculate the difference in potential energy of the body between two points.

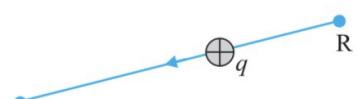
2) Electrostatic force is a conservative force so work done and hence potential energy difference only depends on the initial and final positions of the object

⚡ ELECTROSTATIC POTENTIAL (V) (PYQ 2018, 2016, 2012, 2011)

Electrostatic potential at a point in an electric field is defined as the work done by external force to bring a unit positive charge (slowly, without acceleration) from infinity to the given point. (SI Unit- joule/coulomb or Volt)

By definition,

$$V_p - V_R = \frac{U_p - U_R}{q}$$

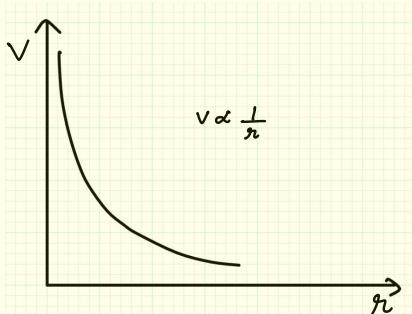


Note: We cannot calculate absolute potential at a point; we can only calculate the potential difference between two points.

*To derive expression for potential, just substitute Q_2 for 1 in the derivation for U .

Ques: Draw a graph showing variation of potential (V) and distance (r) (PYQ 2012)

Ans:



Similar PYQs



Two point charges q and $-q$ are located at points $(0, 0, -a)$ and $(0, 0, a)$ respectively.

- Find the electrostatic potential at $(0, 0, z)$ and $(x, y, 0)$
- How much work is done in moving a small test charge from the point $(5, 0, 0)$ to $(-7, 0, 0)$ along the x-axis?

- How would your answer change if the path of the test charge between the same points is not along the x-axis but along any other random path? (Hint: electrostatic force is conservative)

(PYQ 2018)

⚡ POTENTIAL DUE TO AN ELECTRIC DIPOLE (PYQ 2019)

Consider a dipole of dipole moment \mathbf{p} and length $2a$. Let us calculate the potential at a point O at a distance r from the center of the dipole.

$$r_2 = r + a \cos \theta$$

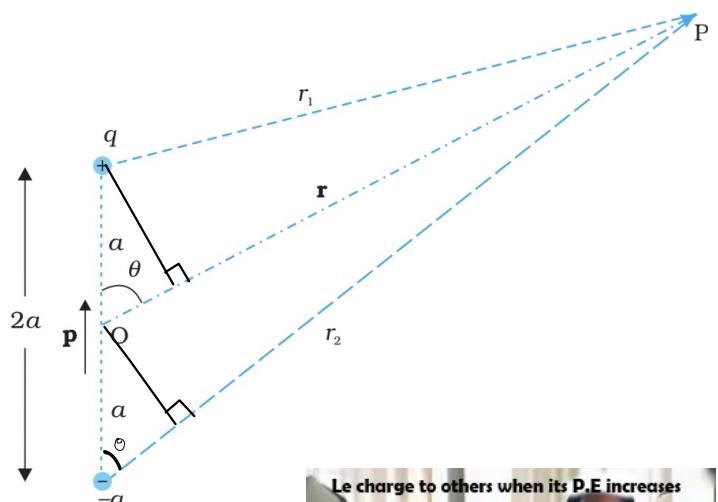
$$r_1 = r - a \cos \theta$$

$$V_p = V_q + V_{-q}$$

$$\begin{aligned} V_p &= \frac{1}{4 \times \pi \epsilon_0} \times \frac{q}{r_1} - \frac{1}{4 \times \pi \epsilon_0} \times \frac{q}{r_2} \\ &= \frac{q}{4 \times \pi \epsilon_0} \times \left[\frac{1}{r - a \cos \theta} - \frac{1}{r + a \cos \theta} \right] \\ &= \frac{q}{4 \times \pi \epsilon_0} \times \left[\frac{r + a \cos \theta - r - a \cos \theta}{r^2 - a^2 \cos^2 \theta} \right] \\ &= \frac{q}{4 \times \pi \epsilon_0} \times \left[\frac{2 \times a \cos \theta}{r^2 - a^2 \cos^2 \theta} \right] \\ &= \frac{1}{4 \times \pi \epsilon_0} \times \left[\frac{(2qa) \times \cos \theta}{r^2 - a^2 \cos^2 \theta} \right] \end{aligned}$$

$$r \gg a$$

$$V_p = \frac{1}{4 \times \pi \epsilon_0} \times \frac{p \cos \theta}{r^2}$$



1) At axial position

$$\theta = 0^\circ$$

$$\cos \theta = 1$$

$$V_A = \frac{1}{4 \times \pi \epsilon_0} \times \frac{q}{r^2}$$

2) At equatorial position

$$\theta = 90^\circ$$

$$\cos \theta = 0$$

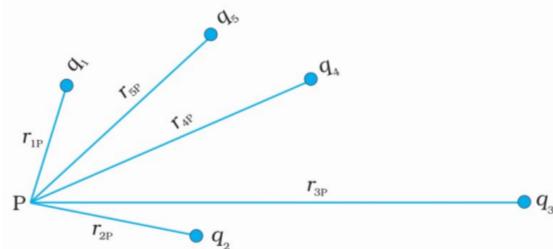
$$V_B = 0$$

Important PYQs 

Ques: Derive an expression for the electrostatic potential at any point along the axial line of a dipole (PYQ 2019)

⚡ POTENTIAL DUE TO A SYSTEM OF CHARGES

Total potential at a point due to a system of charges is equal to the algebraic sum of potentials due to all the individual charges at that point.



Note: In the previous chapter we saw that electric field inside a charged hollow sphere is 0. So potential inside the shell is constant and is equal to its value at the surface as no work is done on moving a charge inside the shell.

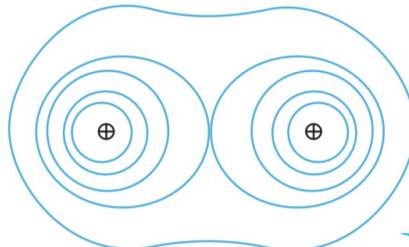
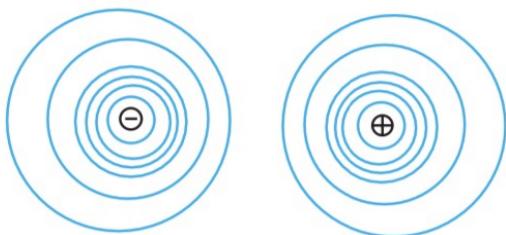
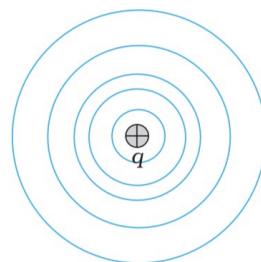
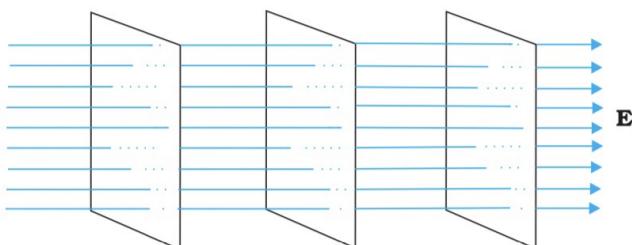
Important PYQs 

Ques: The potential at the surface of a spherical hollow metal shell with radius 6cm is 12V. What will be the potential at its center? (PYQ 2011)

Ans: Since the electric field inside a shell is 0, no work is done to move a charge inside the shell therefore potential at the center will be the same as the potential at its surface i.e. 12V

⚡ EQUIPOTENTIAL SURFACES (PYQ 2020, 2019, 2014, 2013, 2010)

Equipotential surfaces are surfaces which have a constant value of potential at all points.



*Equipotential surfaces are always perpendicular to the electric field. (PYQ 2014)

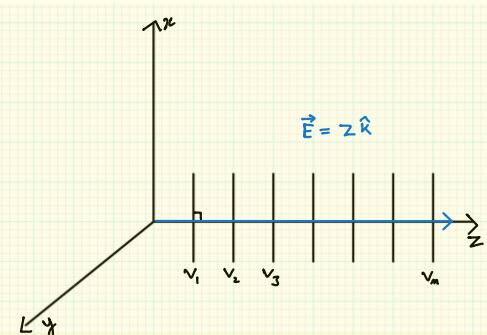
Proof: To prove this, let us assume that electric field is not perpendicular to the equipotential surfaces. This implies that the electric field can be resolved into two components, one of which will be perpendicular to the equipotential surface and one will be parallel or along the surface.

Now, consider a charge at a point A on the surface. Since there is a component of field along the surface, when we move the charge to a point B on the surface some work will be done. This means that potential at points A and B will not be the same which is not possible as it is an equipotential surface so potential at all points on it must be the same. Therefore, by contradiction we can conclude that electric field is always perpendicular to the equipotential surface.

Important PYQ

Ques: Draw the equipotential surfaces due to a uniform electric field along z direction (PYQ 2019)

Ans:



⚡ Relation between potential (V) and field (E)

$$V_A - V_B = dV$$

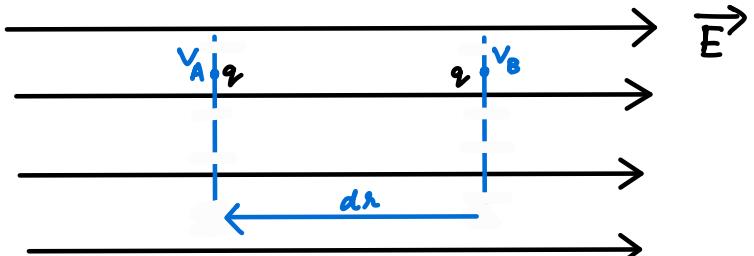
Work done in moving q from B to A-

$$dW = qdV$$

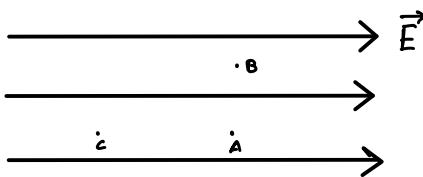
$$dW = q\vec{E} \cdot d\vec{r} = -qEdr$$

$$qdV = -qEdr$$

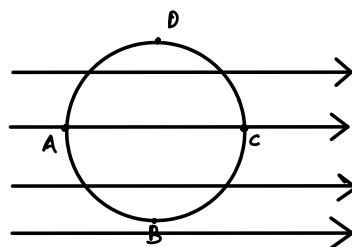
$$E = -\frac{dV}{dr}$$



- 1) Electric field is in the direction where potential decreases the steepest.
- 2) Magnitude of electric field is given by the change in magnitude of potential per unit displacement normal to the equipotential surface at the point.



$$V_C > V_A = V_B$$



$$V_A > V_B = V_D > V_C$$

Ques: The following diagram depicts some equipotential surfaces. Calculate the magnitude and direction of electric field.

Ans: We know electric field is perpendicular to the equipotential Surfaces so let us draw a perpendicular to these surfaces.

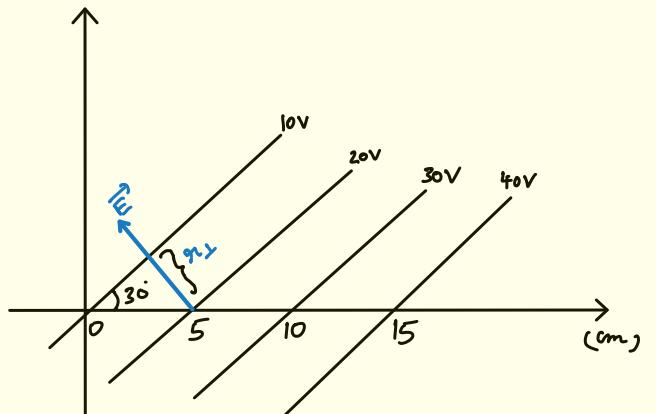
$$r_1 = 5 \times \sin 30^\circ$$

$$r_1 = 2.5 \text{ cm}$$

$$|E| = \frac{\Delta V}{r_1} = \frac{10}{2.5 \times 10^{-2}} = 400 \text{ NC}^{-1}$$

$$\vec{E} = -400 \times \cos 60^\circ \hat{i} + 400 \times \sin 60^\circ \hat{j}$$

$$\vec{E} = -200 \hat{i} + 200 \times \sqrt{3} \hat{j} \text{ NC}^{-1}$$



⚡ Potential energy due to a system of charges (PYQ 2020, 2010)

Consider a charge q_1 which is brought from infinity to a point A. Since there is no electric field no work is done in this process. Now, another charge q_2 is brought from infinity to a point B at a distance r from q_1 . Since, the potential at B is $V = kq_1/r$, work is done in bringing the charge q_2 to B which will be equal to Vq_2 . Therefore, the potential energy of the system will be

$$U = \frac{1}{4 \times \pi \epsilon_0} \times \frac{q_A q_B}{r}$$

Ques: Three point charges $1\mu\text{C}$, $-1\mu\text{C}$, $2\mu\text{C}$ were initially infinite distance apart. Calculate the work done in assembling these charges at the vertices of an equilateral triangle of side 10cm (PYQ 2020)

Ans: To calculate the work done, we calculate the difference in the final and Initial potential energy of the system.

The initial P.E of the system is zero as all charges were infinitely separated

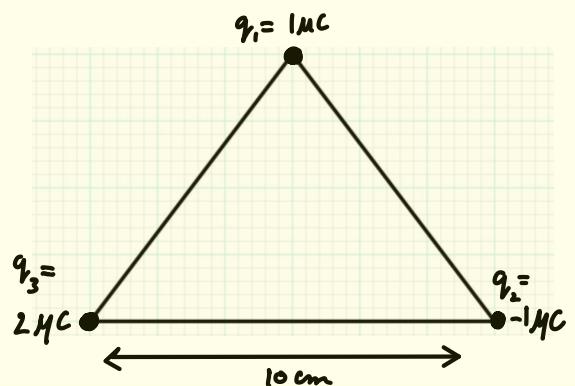
The final P.E of system can be calculated as-

$$U_T = U_{12} + U_{13} + U_{23} \\ = \frac{1}{4 \times \pi \epsilon_0 r} \times (q_1 q_2 + q_2 q_3 + q_1 q_3) = -9 \times 10^{-2} \text{ J}$$

$$W = U_f - U_i$$

$$W = -9 \times 10^{-2} - 0$$

$$W = -9 \times 10^{-2} \text{ J}$$



Note: Work done by system (W_{sys}) = -Work done by ext. force (W_{ext})

⚡ Potential energy in an external field

1. Potential energy of a single charge

If a charge q is brought from infinity to a given point in an external electric field $\mathbf{E}(r)$, its potential energy will be equal to $q \cdot V(r)$ where $V(r)$ is the potential due to the external field at that point.

For e.g. if an electron is accelerated through a potential difference of V it will gain energy equal to eV. This unit of energy is defined as electron volt (eV) and $1\text{eV} = 1.6 \times 10^{-19} \text{ J}$.

2. Potential energy of a system of two charges in an external field (PYQ 2018)

To calculate the potential energy of a system of two charges (q_1, q_2) in

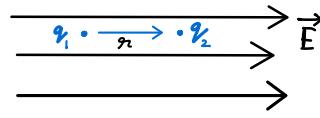
An external field, first we calculate the work done in bringing charge q_1

From infinity which will be $q_1 \cdot V(r_1)$. Then we calculate the work done in

Bringing q_2 from infinity, but this time the work done is not just against the external electric field but also against the field due to q_1 .

Work done on q_2 due to external field = $q_2 \cdot V(r_2)$

Work done on q_2 due to q_1 = kq_1q_2/r_{12}



Therefore, total potential energy of the system = $q_1 \cdot V(r_1) + q_2 \cdot V(r_2) + kq_1q_2/r_{12}$

Ques: Two charges $7\mu\text{C}$ and $-2\mu\text{C}$ are placed at $(-9\text{cm}, 0, 0)$ and $(9\text{cm}, 0, 0)$ resp. in an external electric field $E = A/r^2$ where $A = 9 \times 10^5 \text{ Cm}^{-2}$. What is the potential energy of the system?

Ans: $U = q_1 \cdot V(r_1) + q_2 \cdot V(r_2) + kq_1q_2/r_{12}$

First we need to calculate the expression for potential due to the field –

$$E = -\frac{dV}{dt}$$

$$\int dV = \int -Edr$$

$$V = -A \int \frac{dr}{r^2}$$

$$V = \frac{A}{r}$$

$$= \frac{A}{r_1}q_1 + \frac{A}{r_2}q_2 + \frac{1}{4 \times \pi \epsilon_0} \times \frac{q_1q_2}{r_{12}}$$

$$= 70 - 20 - 0 \cdot 7$$

$$= 49.3 \text{ J}$$

⚡ Potential energy of dipole in external field

We know that when a dipole is kept in an external electric field, it Experiences a torque (τ) = $\mathbf{p} \times \mathbf{E}$ which causes it to rotate.

Let the dipole be at an angle θ_0 with the field initially. An external Torque (τ_{ext}) is applied to it which is equal and opposite to the torque Due to E . τ_{ext} rotates the dipole (very slowly and without acceleration) Such that it now makes an angle θ_1 with the field.

Let us calculate the work done in the process-

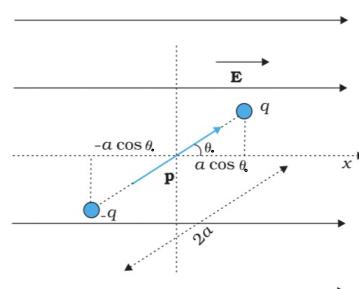
Let the dipole rotate in the plane of the paper by an infinitesimal amount $d\theta$. Let the work done in this process be dW

$$dW = \tau \cdot d\theta$$

$$dW = pE \sin \theta d\theta$$

$$W = pE \int_{\theta_0}^{\theta_1} \sin \theta d\theta$$

$$W = -pE(\cos \theta_1 - \cos \theta_0)$$



Dipoles, when they meet an electric field at the perfect angle:



$\tau = p \times E$, max at 90°

Electrostatics Dance, anybody?

Apni Kaksha

This work done by the ext torque is stored in the form of potential energy of the dipole. Let $U = 0$ when $\theta = \Pi/2$

$$U = -pE\cos\theta$$

$$U = -\vec{p} \cdot \vec{E}$$

Note: We take P.E of dipole 0 at $\theta = \Pi/2$ because the work done in bringing the two charges from infinity will be equal and opposite and will cancel out in this configuration.

Important PQYs



Ques: calculate P.E of a dipole with charges $q, -q$ And length L and in stable equilibrium with field E (PYQ 2020)

Ans: stable equilibrium means $\theta = 0^\circ$

$$U = -p \cdot E = -qL E \cos 0^\circ = -qLE$$



Ques: A dipole of length 4cm makes an angle of 60° with the electric field and experiences a torque of $4\sqrt{3}$ Nm. Calculate P.E of dipole if it has a charge of $\pm 8nC$ (PYQ 2014)

$$\text{Ans: } p = 8 \times 10^{-9} \times 4 \times 10^{-2} = 0.32 nCm$$

$$\tau = PE\sin\theta$$

$$4\sqrt{3} = 0.32 \times 10^{-9} \times E \times \frac{\sqrt{3}}{2}$$

$$E = 2.5 \times 10^{10} \times N/c$$

$$U = -PE\cos\theta$$

$$U = -0.32 \times 10^{-9} \times 2.5 \times 10^{10} \times \frac{1}{2}$$

$$U = -4J$$

⚡ Electrostatics of conductors

Metallic conductors have mobile charge carriers in the form of free electrons.

1. Inside a conductor, electrostatic field is zero (PYQ 2012)

In electrostatic situation, when there is no current inside or on the surface of the conductor, the electric field is zero inside the conductor. This is because, in static situation the free charges inside the conductor are distributed in such a way that the net electrostatic field inside a conductor is zero everywhere.

2. At the surface of a charged conductor electrostatic field must be normal to the surface at every point

If E was not normal to the surface, there would be a component of E parallel to the surface which would cause the charges on the surface to move. But since it is a static situation, that should not be the case. Therefore, electrostatic field should be normal to the surface of a conductor at all points.

3. The interior of a conductor can have no excess charge in the static situation

When a conductor is charged, all the excess charge must reside at the surface in static situation. This follows from Gauss's law. Consider any arbitrary Gaussian surface inside a conductor, since electric field is 0 inside the conductor, net flux through the surface is also 0. This implies that the net charge enclosed inside the surface is also zero.

4. Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on the surface

This follows from results 1 and 2. Since $E=0$ everywhere inside the conductor and has no tangential component, no work is done in moving a charged particle inside or on the surface of the conductor hence there is no potential difference between any two points inside or on the surface of a conductor.

5. Electric field at the surface of a charged conductor

$$E = \frac{\sigma}{\epsilon_0} \hat{n}$$

Where σ is the surface charge density and \hat{m} is a unit vector normal to the surface of the conductor in the Outward direction.

Proof: consider a short cylindrical Gaussian surface that is half inside and half outside the conductor.

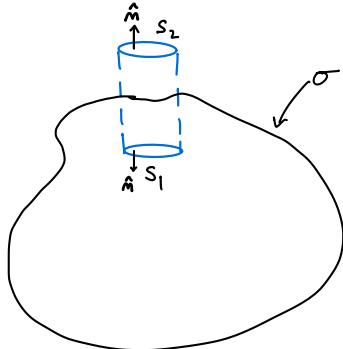
$$\phi = \oint \vec{E} \cdot d\vec{s} = \int E \cdot ds_1 + \int E \cdot \cancel{ds_2^0} \quad (\text{Field inside conductor is } 0)$$

$$\phi = E \cdot S_1$$

$$\phi = \frac{q}{\epsilon_0} = \frac{\sigma \cdot S_1}{\epsilon_0}$$

$$\frac{\sigma S_1}{\epsilon_0} = ES_1$$

$$E = \frac{\sigma}{\epsilon_0}$$



6. Electrostatic shielding

Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence i.e. the field inside the cavity is always zero.

Use- to protect sensitive instruments from outside electrical influence.

Note: when a conductor is kept in an external electric field, the charge carriers (free e⁻s) move and are redistributed in such a way that the electric field produced due to the induced charges opposes the external field such that the two fields cancel each other out completely and net electrostatic field inside the conductor is zero.

⚡ Dielectrics and polarization

Dielectrics are non-conducting substances.

Dielectric in an external field

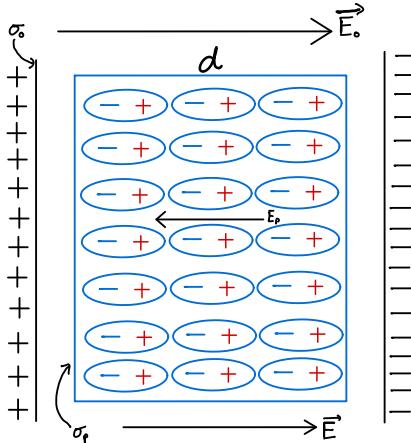
When a dielectric is kept in an external electric field, the field causes re orientation of the molecules of the dielectric such that each molecule becomes a dipole and the net result of this is the appearance of charges on the surface of the dielectric which produces a field which opposes the external field. This is called polarization of a dielectric.

Unlike a conductor, this induced field doesn't cancel out the external field completely but can only reduce it.

The extent of polarization depends on two factors-

- i) The potential energy of the dipole in the external field trying to align the dipole along the field
- ii) The thermal energy tending to disrupt the alignment

Let E_o be the external field and E_p be the field due to polarization of the dielectric and let E be the net field inside the dielectric



$$E = E_0 - E_p \quad - \textcircled{1}$$

$$\frac{\sigma_0}{k\epsilon_0} = \frac{\sigma_0}{\epsilon_0} - \frac{\sigma_p}{\epsilon_0}$$

$$\sigma_p = \sigma_0 \left(1 - \frac{1}{k} \right)$$

Note:

- **linear isotropic dielectrics**- dielectrics for which the induced dipole moment is the direction of the field and is proportional to the field strength
 - **polar molecules**- the +ve and –ve centers coincide | **non-polar molecules**- the +ve and –ve centers don't coincide
 - e.g. O₂, H₂
 - e.g. HCl, H₂O

Polarization (P)

It is a vector | Polarization is defined as the induced dipole moment per unit volume. Its direction is given by the direction of the induced dipole moment.

$$p = Q_p d$$

$$p = \sigma_p A d$$

$$\frac{p}{Ad} = \sigma_p$$

$$\vec{p} = \sigma_p \cdot \hat{n}$$

i.e. Polarization vector is numerically equal to the induced surface charge density

Electrical displacement (D)

It is a vector | Electrical displacement is directly related to free charge density in a dielectric medium (i.e. it replaces \mathbf{E} which is directly related to the free charge density in free space)

$$\sigma_p = \vec{p} \cdot \hat{n}$$

$$\vec{E} \cdot \hat{n} = \frac{\sigma_0 - \vec{p} \cdot \hat{n}}{\epsilon_0}$$

$$\Rightarrow (\epsilon_0 \vec{E} + \vec{p}) \cdot \hat{n} = \sigma_0$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{p}$$

Since \mathbf{P} is in the same direction as \mathbf{E} , all three quantities \mathbf{D} , \mathbf{P} , \mathbf{E} are parallel.

The ratio of magnitudes of **D** and **E** –

$$\frac{D}{E} = \frac{\sigma_0 \epsilon_0}{\sigma_0 - \sigma_p} = \epsilon_0 K$$

$$\vec{D} = \epsilon_0 k \vec{E}$$

$$\vec{p} = \vec{D} - \epsilon_0 \vec{E}$$

$$\vec{p} = (k - 1)\epsilon_0 \vec{E}$$

$$\vec{p} = \chi_e \vec{E}$$

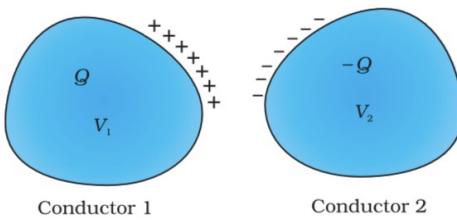
$$\chi_e = (k - 1)\epsilon_0$$

Which is called electric susceptibility

⚡Capacitors and Capacitance (PYQ 2020, 2019, 2018, 2017, 2016, 2015, 2014, 2013, 2011, 2010)

Capacitor: A system of two conductors separated by an insulator which is used to store electrical energy/ charge.

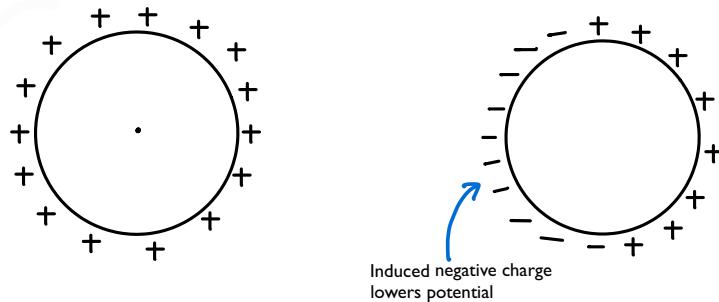
- The two conductors have charges Q and $-Q$. Q is called the charge of the capacitor.
- The total charge on capacitor = 0
- The electric field in the region between the conductors is proportional to the charge.
- The pot. Difference (V) b/w the conductors is the work done in taking a unit +ve charge from one conductor to another



Capacitance: The ratio Q/V for a capacitor is constant and is called the capacitance (C) of the capacitor. | SI unit: Farad (F)

Note: C is independent of both Q and V and only depends on the geometrical configuration (shape, size, separation) of the system of the conductors

Principle of capacitors: When a charged conductor is kept near an uncharged conductor, its capacitance increases.



Types of capacitors

1. Spherical capacitor (NCERT back exercise Q 2.29)

Isolated spherical capacitor- it consists of a single isolated hollow metal sphere. It stores energy in the region from its surface to infinity.

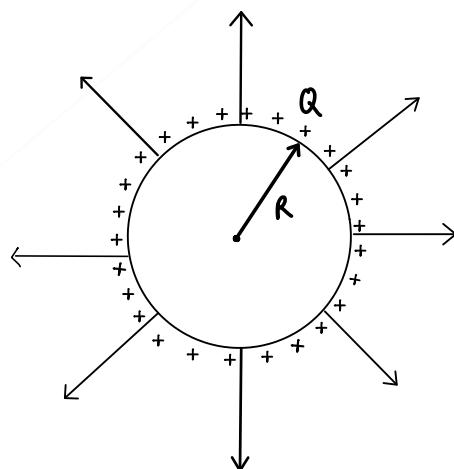
Consider a hollow metal sphere with charge Q and radius R . The potential at its surface can be written as-

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{Q}{R}$$

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Q}{4\pi\epsilon_0 R}} = 4\pi\epsilon_0 R$$

$$C = 4\pi\epsilon_0 R$$



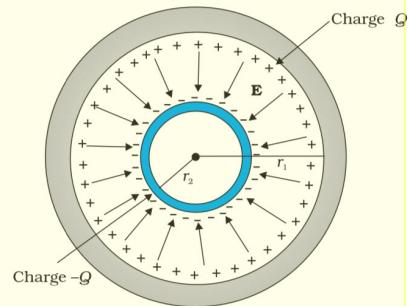
 **Ques:** 2.29

A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36). Show that the capacitance of a spherical capacitor is given by



$$C = \frac{4\pi\epsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.



Ans: Concentric spherical capacitor- it consists of two charged concentric spheres. Energy is stored inside the cavity b/w the spheres.

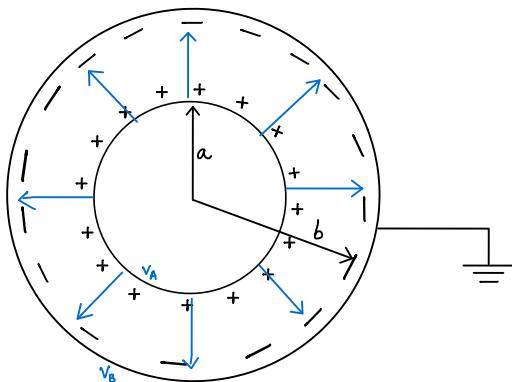
Consider two concentric spheres with radius a and b ($b > a$). Let charge on a be $+Q$ and on b be $-Q$. The potential difference between sphere a and b -

$$\Delta V = V_A - V_B = V_A - 0 = V_A$$

$$V_A = \frac{KQ}{a} - \frac{KQ}{b} = KQ\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{1}{4\pi\epsilon_0} \times \frac{(b-a)}{ab}$$

$$C = \frac{Q}{V}$$

$$C = \frac{4\pi\epsilon_0(ab)}{b-a}$$



2. Cylindrical capacitor (NCERT back exercise Q 2.32)

 **Ques:** 2.32



A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC . Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

It consists of two concentric cylinders of radius a and b ($b > a$). Let charge density of the cylinder be λ . To calculate the potential difference b/w cylinder a and b , let us consider the field E at a point P at a distance r from the axis of a . We know that,

$$dV = -\vec{E} \cdot d\vec{r} = -Edr \cos 0^\circ$$

$$dV = \frac{-\lambda}{2\pi\epsilon_0} \times \frac{dr}{r}$$

$$\int_v^v dV = \frac{-\lambda}{2\pi\epsilon_0} \int_b^a \frac{dr}{r}$$

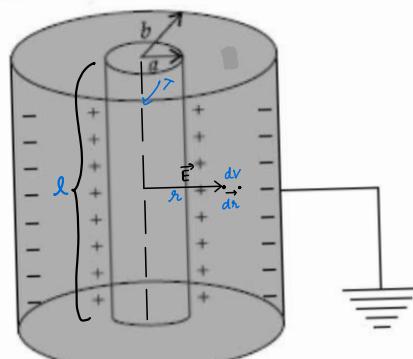
$$V_v - V_b = \frac{-\lambda}{2\pi\epsilon_0} [\ln r]_b^a$$

$$V = \frac{\lambda}{2\pi\epsilon_0} \times \ln\left(\frac{b}{a}\right)$$

$$C = \frac{Q}{V} = \frac{\lambda l}{\lambda \ln(b/a)} \times 2\pi\epsilon_0$$

$$C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$

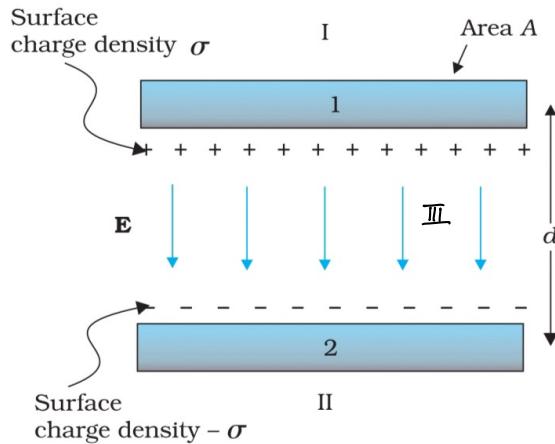
Cylindrical Capacitor



3. Parallel plate capacitor (PYQ 2014)

A parallel plate capacitor consists of two large plane parallel conducting plates separated by a small distance.

Consider the following setup; two conducting plates of area A with charge Q and $-Q$ respectively and surface charge density $\sigma = Q/A$ are separated by a distance d.



Let us calculate the Electric field in the three regions-

$$E_x = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

$$E_x = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

$$E_x = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$$

We know,

$$|E| = \frac{\Delta V}{r}$$

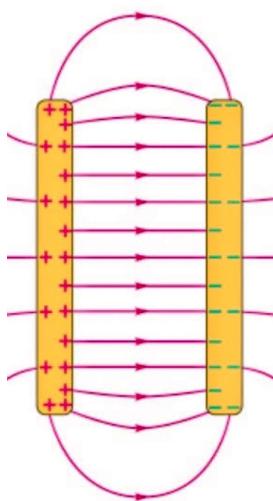
$$\frac{\sigma}{\epsilon_0} = \frac{\Delta V}{d}$$

$$\Delta V = \frac{\sigma d}{\epsilon_0}$$

$$C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{\sigma A \epsilon_0}{\sigma d}$$

$$C = \frac{A \epsilon_0}{d}$$

Note: Fringing effect/ End effect of field- for plates of finite area, the field lines bend outwards at the edges.



Important PYQ

Ques:

A capacitor of unknown capacitance is connected across a battery of V volts. The charge stored in it is $360 \mu\text{C}$. When potential across the capacitor is reduced by 120 V , the charge stored in it becomes $120 \mu\text{C}$.

Calculate :

- The potential V and the unknown capacitance C.
- What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V ? (PYQ 2013)

Ans:1) $C = Q/V$

$$C = 360 \mu\text{C}/V; C = 120 \mu\text{C}/V - 120$$

Equating both we get,

$$V = 180 \text{ V}, C = 2 \mu\text{F}$$

$$2) Q = CV$$

$$= 2 \mu\text{F} \times 400 \text{ V}$$

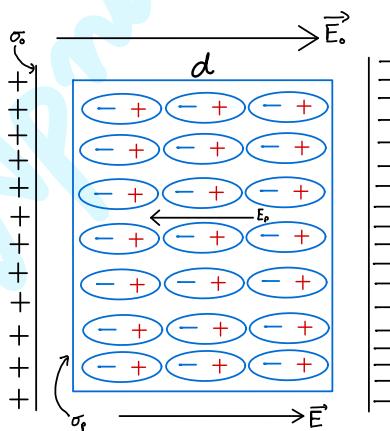
$$= 800 \mu\text{C}$$

⚡ Effect of dielectric on capacitance (PYQ 2020, 2016, 2010)

A dielectric medium of dielectric constant K is inserted between the plates of a parallel plate capacitor

Earlier the field b/w the plates was $E_0 = \sigma/\epsilon_0$. But now since there is a dielectric between the plates, the electric field will be modified (as described in polarisation of dielectrics)

Case 1: when space b/w the plates are completely filled with dielectric



Let the new field be E -

$$E = E_0 - E_p = \frac{\sigma_0 - \sigma_p}{\epsilon_0}$$

$$V = Ed = \frac{\sigma_0 - \sigma_p}{\epsilon_0} d$$

$$\sigma_0 - \sigma_p = \frac{\sigma_0}{k}$$

$$V = \frac{\sigma d}{\epsilon_0 k}$$

$$C = \frac{Q}{V} = \frac{\sigma A \epsilon_0 K}{\sigma \cdot d}$$

$$C = \frac{k A \epsilon_0}{d}$$

$$C = K C_0$$

Case 2: When space b/w the plates is partially filled with dielectric .

Consider a case where a dielectric medium of dielectric constant K and thickness t is placed b/w the plates of a capacitor of plate separation d ($d > t$)

Let us calculate the electric field at the 3 points-

$$E_{p_1} = \frac{\sigma}{\epsilon_0}$$

$$E_{p_2} = \frac{\sigma}{\epsilon_0}$$

$$E_{p_3} = \frac{\sigma}{k\epsilon_0}$$

$$\Delta V = E p_1 \Delta r_1 + E p_2 \Delta r_2 + E p_3 \Delta r_3$$

$$= E p_1 (\Delta r_1 + \Delta r_2) + E p_3 \Delta r_3$$

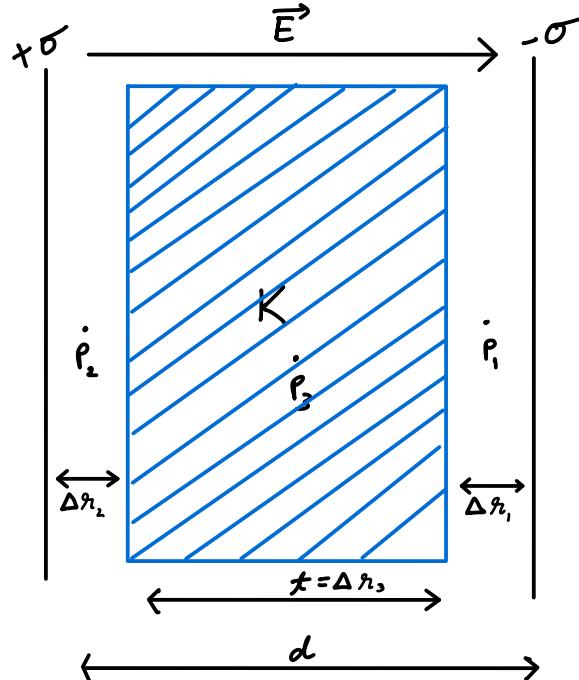
$$= \frac{\sigma}{\epsilon_0} \times (d - t) + \frac{\sigma}{k\epsilon_0} t$$

$$\Delta V = \frac{\sigma}{\epsilon_0} \times \left(d - t + \frac{t}{K} \right)$$

$$C = \frac{Q}{V}$$

$$C = \frac{\sigma A \epsilon_0}{\sigma(d - t + t / K)}$$

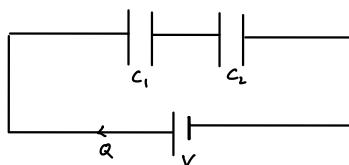
$$C = \frac{A \epsilon_0}{(d - t + t / K)}$$



⚡ Combination of Capacitors

1. Series combination

Consider the two capacitors C_1 & C_2 , connected with each other and a battery of emf V (as shown in the figure), have charge Q each. Let the potential drop across C_1 & C_2 be V_1 and V_2 resp. Then we can say that the total potential drop across the combination is equal to the sum of the potential drops across C_1 & C_2 i.e. –



$$V = V_1 + V_2$$

$$V = \frac{Q}{C_1} + \frac{Q}{C_2}$$

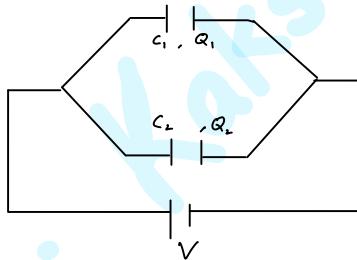
$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

We can see the combination as a capacitor with an effective capacitance C, charge Q and potential drop V-

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

2. Parallel combination

Consider the two capacitors C_1 & C_2 , connected with each other and a battery of emf V (as shown in the figure). In this case, both capacitors will have the same potential difference. Let the charge on C_1 & C_2 be Q_1 & Q_2 resp. We can imagine the combination as a capacitor (C) with charge Q, which is the sum of the charges Q_1 & Q_2 -



$$Q = Q_1 + Q_2$$

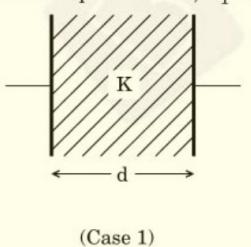
$$CV = C_1V + C_2V$$

$$C = C_1 + C_2$$

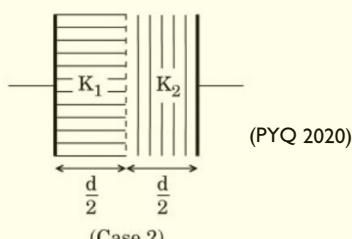
Note: Both of these results are true for any arbitrary number (n) of capacitors.

Important PYQ

The space between the plates of a parallel plate capacitor is completely filled in two ways. In the first case, it is filled with a slab of dielectric constant K. In the second case, it is filled with two slabs of equal thickness and dielectric constants K_1 and K_2 respectively as shown in the figure. The capacitance of the capacitor is same in the two cases. Obtain the relationship between K, K_1 and K_2 .



(Case 1)



(Case 2)

Case 1

$$C = \frac{KA\epsilon_0}{d}$$

Case 2

In the second case, we can imagine the capacitor as a combination of two capacitors connected in series.

$$C_1 = \frac{2k_1 A \epsilon_0}{d}$$

$$C_2 = \frac{2k_2 A \epsilon_0}{d}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{d}{2A\epsilon_0} \times \left(\frac{1}{k_1} + \frac{1}{k_2} \right)$$

$$C = \frac{2A\epsilon_0 K_1 K_2}{d(K_1 + K_2)}$$

Equating both

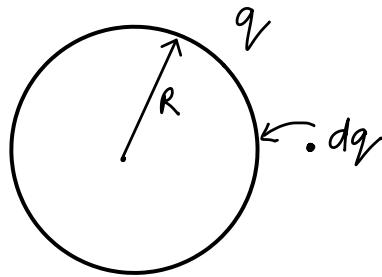
$$\frac{2A\epsilon_0 k_1 k_2}{d(k_1 + k_2)} = \frac{KA\epsilon_0}{d}$$

$$K = \frac{2K_1 K_2}{K_1 + K_2}$$

⚡ Energy stored in capacitor (U) (PYQ 2019, 2018, 2017, 2016, 2015, 2014, 2011, 2010)

Consider an uncharged isolated hollow metal sphere. To calculate the energy stored when it will be fully charged to a charge Q, we need to find the work done in charging this sphere to a total charge Q. To calculate the total work done, we will first calculate the work done in transferring an infinitesimal charge dq (from infinity) when the sphere has acquired a total charge q.

Work done (dw) = change in potential energy of charge dq = V · dq (where V is potential of sphere with total charge q)



$$dW = V \cdot dq$$

$$dW = \frac{1}{4 \times \pi \epsilon_0} \times \frac{q}{R} \cdot dq$$

$$W = \frac{1}{4 \times \pi \epsilon_0 R} \int_0^q q dq$$

$$W = \frac{1}{4 \times \pi \epsilon_0 R} \left[\frac{q^2}{2} \right]_0^q$$

$$W = \frac{1}{4 \times \pi \epsilon_0 R} \times \frac{Q^2}{2}$$

$$W = \frac{1}{2} \times \frac{Q^2}{(4 \times \pi \epsilon_0 R)} = \frac{Q^2}{2C}$$

This work done is stored as P.E
in the capacitor

$$U = \frac{Q^2}{2C}$$

Also,

$$Q = CV$$

$$U = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} QV$$

⚡ Energy stored per unit volume/ energy density (u) (PYQ 2014)

consider a parallel plate capacitor with plate area A, distance b/w plates d and surface charge density σ. The energy stored in it will be-

$$U = \frac{Q^2}{2C}$$

$$U = \frac{(\sigma A)^2}{2 \times \frac{A \epsilon_0}{d}} = \frac{\sigma^2 Ad}{2 \epsilon_0}$$

$$U = \frac{\sigma^2}{2 \epsilon_0} \times \frac{\epsilon_0}{\epsilon_0} \times Ad = \frac{1}{2} \times \left(\frac{\sigma}{\epsilon_0} \right)^2 \epsilon_0 (Ad)$$

$$U = \frac{1}{2} E^2 \epsilon_0 (Ad)$$

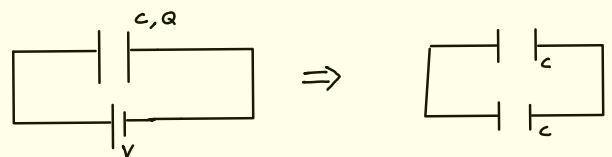
$$\frac{U}{Ad} = u$$

$$u = \frac{1}{2} \epsilon_0 E^2$$

Important PYQs



- Q(b) A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another uncharged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor. (PYQ 2019)



Ans:

1) Initial energy

$$U_i = \frac{1}{2}CV^2$$

$$Q = CV$$

2) Final energy

Total charge will remain conserved,

$$Q = Q_1 + Q_2$$

$$CV = CV' + CV'$$

$$V' = \frac{V}{2}$$

$$U_f = 2 \times \frac{1}{2}C\left(\frac{V}{2}\right)^2 = \frac{1}{4}CV^2$$

$$\frac{U_f}{U_i} = \frac{1}{4}$$

Similar PYQ

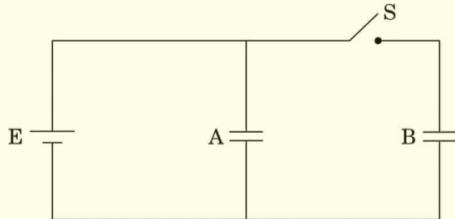
◎ A capacitor of capacitance C_1 is charged to a potential V_1 while another capacitor of capacitance C_2 is charged to a potential difference V_2 . The capacitors are now disconnected from their respective charging batteries and connected in parallel to each other.

- Find the total energy stored in the two capacitors before they are connected.
- Find the total energy stored in the parallel combination of the two capacitors.
- Explain the reason for the difference of energy in parallel combination in comparison to the total energy before they are connected.

(PYQ 2018)



Ques:



(PYQ 2017)

Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.

Ans: 1) Initial energy

$$U_i = 2 \times \frac{1}{2}CV^2 \\ = CV^2$$

2) After dielectric is added

$$C_A = C_B = kC$$

For A, V will remain const. As it remained connected to the battery

$$U_A = \frac{1}{2}C_A V^2 \\ = \frac{1}{2}kCV^2$$

For B, Q will remain constant (conservation of charge)

$$Q_B = CV$$

$$U_B = \frac{Q_B^2}{2C_B} = \frac{(CV)^2}{2kC} = \frac{1}{2} \times \frac{CV^2}{k}$$

$$U_T' = U_A + U_B = \frac{1}{2}CV^2 \left(k + \frac{1}{k} \right)$$

$$\frac{U_T'}{U_T} = \frac{2CV^2K}{CV^2(k^2 + 1)}$$

$$\frac{U_T'}{U_T} = \frac{2k}{k^2 + 1}$$

Ques: Two parallel plate capacitors X and Y have the same area of plates and same separation between them. X has air between the plates while Y contains a dielectric medium of $\epsilon_r = 4$.

- (i) Calculate capacitance of each capacitor if equivalent capacitance of the combination is $4 \mu\text{F}$.

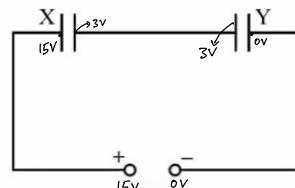
- (ii) Calculate the potential difference between the plates of X and Y.

Ans: i) $C_x = C; C_y = 4C$

$$ATQ, \frac{1}{4\mu} = \frac{1}{C} + \frac{1}{4C} \quad (\text{Series comb.})$$

$$\Rightarrow C_x = 5\mu\text{F}, C_y = 20\mu\text{F}$$

ii)



$$Q = CV$$

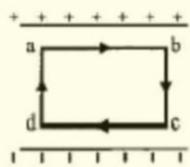
$$Q = 4\mu \times 15 = 60\mu\text{C}$$

$$V_x = \frac{Q}{C_x} = \frac{60\mu}{5\mu} = 12V$$

$$V_y = 15 - V_x = 3V$$

Similar PYQ

Ques: The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop a b c d a.



(PYQ 2014)



(Hint=- electric field is conservative)

Ques: Net capacitance of three identical capacitors in series is $3 \mu\text{F}$. What will be their net capacitance if connected in parallel?

Find the ratio of energy stored in the two configurations if they are both connected to the same source. (PYQ 2011)



Ques: A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will (i) the capacitance of the capacitor, (ii) potential difference between the plates and (iii) the energy stored in the capacitor be affected?

Justify your answer in each case.

(PYQ 2010)

Ans:

$$1) \quad C' = kC$$

2) Charge will remain conserved ,

$$Q = CV$$

$$V = \frac{Q}{C}$$

$$V = \frac{C}{Q} = \frac{KC}{Q} = KV'$$

$$3) \quad E = \frac{Q^2}{2C}$$

$$E' = \frac{Q^2}{2C'} = \frac{Q^2}{2kC}$$

$$E' = \frac{E}{k}$$

SOLUTIONS FOR SIMILAR PYQS



1) PYQ 2018 soln

a. Potential at (0,0,z)

$$V = V_q + V_{-q}$$

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{z+a} - \frac{1}{4\pi\epsilon_0} \times \frac{q}{|z-a|}$$

$$V = \frac{q}{4\pi\epsilon_0} \times \left(\frac{1}{z+a} - \frac{1}{|z-a|} \right) \quad (\text{mod because } z \text{ can be less than or greater than } a)$$

Potential at (x,y,0)

Since this point lies in the XY plane, it will be equidistant from both the charges lying on the z axis. Therefore,

$$V = V_q + V_{-q}$$

$$V = \frac{1}{4\pi\epsilon_0} \times \frac{q}{d} - \frac{1}{4\pi\epsilon_0} \times \frac{q}{d}$$

$$V = 0V$$

b. Similarly, both these points lie on the x axis and hence will be equidistant from the two charges and potential at both points will be zero. So,

$$\Delta V = 0$$

$$W = q\Delta V$$

$$W = 0$$

c. Since electrostatic force is a conservative force, work done by it depends only on the initial and final position of the charge and not the path followed

2) PYQ 2018 soln

a. let P.E. Of capacitors be U_1 and U_2

$$U_1 = \frac{1}{2}C_1V_1^2$$

$$U_2 = \frac{1}{2}C_2V_2^2$$

$$U_T = U_1 + U_2$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

b. Let charge on capacitors be Q_1 and Q_2

$$Q_1 = C_1V_1$$

$$Q_2 = C_2V_2$$

$$Q = Q_1 + Q_2 \quad (\text{Total charge remains conserved})$$

$$(C_1 + C_2)V = C_1V_1 + C_2V_2 \quad (Q = CV)$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$U = \frac{1}{2}CV^2$$

$$U = \frac{1}{2} \times \frac{(c_1v_1 + c_2v_2)^2}{c_1 + c_2}$$

c. When the two capacitors are connected in parallel, redistribution of charge takes place between them and some energy is lost as heat during this process.

3. PYQ 2014 soln

Electrostatic field is conservative in nature i.e. the work done by it depends only on the initial and final position of the charge and not the path followed. Since the charge is moved in a closed loop abcd i.e. initial and final position are same, the net work done in the process is zero.

4. PYQ 2011 soln

- a. Let capacitance of each be C

We know that net capacitance in series is-

$$\frac{1}{C_0} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$C_0 = \frac{C}{3} = 3\mu F$$

$$C = 9\mu F$$

Therefore, net capacitance in parallel-

$$C_0 = C + C + C$$

$$C_0 = 3C$$

$$C_0 = 27\mu F$$

- b. Let energy in series and parallel be U_1 and U_2 resp

$$U_1 = \frac{1}{2} \times (3\mu F) V^2$$

$$U_2 = \frac{1}{2} \times (27\mu F) V^2$$

$$\frac{U_1}{U_2} = \frac{3}{27} = \frac{1}{9}$$

$$U_1 : U_2 = 1 : 9$$

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