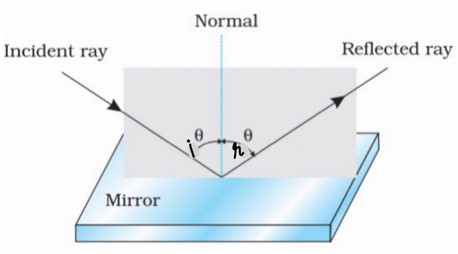
Ray optics and Optical instruments

Some basic properties of light

- Light travels with a constant speed in vacuum i.e. c= 3 × 10^8 m/s
- **Rectilinear propagation-** Light travels in a straight line
- **Ray-** The path of light is called a ray
- **Beam-** Bundle of rays is called beam



Reflection of light (PYQ 2020, 2018, 2016, 2014, 2011)

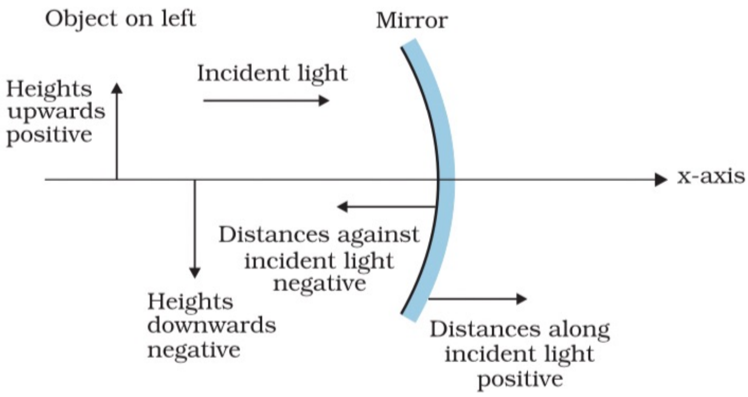
Laws of reflection

1st law- The angle of incidence (i) is equal to the angle of reflection (r)
2nd law- The incident ray, the reflected ray and the normal, at the point Of incidence, all lie in the same plane.

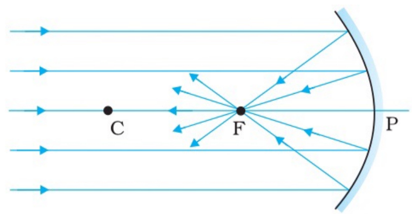
Sign convention

We follow the system of cartesian sign conventions-

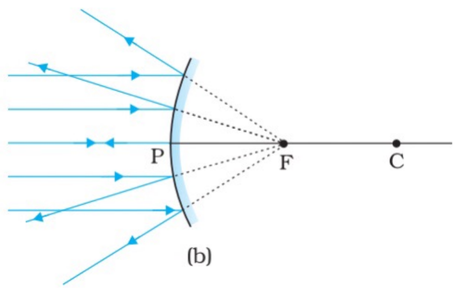
- All distances are measured from the pole or the mirror or optical center of the lens
- The direction of incident light is taken as positive and the opposite direction is taken as negative
- The distances measured upwards w.r.t. the x-axis and perpendicular to the principal axis are positive and the heights measured downwards are negative.



Focal length of spherical mirrors



Principal focus- A paraxial ray of light after reflection converge or appear to diverge from a point after reflection form a concave or convex mirror respectively. This point F is called the principal focus of the mirror.

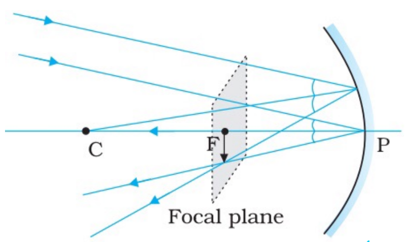


(b)

Focal plane- If the paraxial rays of light were incident making some angle with the principal axis, the reflected rays converge or appear to diverge from a point in a plane through F normal to the principal axis. This is called the focal plane of the mirror.

Focal length- The distance between the focus F and pole P of a mirror is called the focal length of the mirror *(f)*.

Radius of curvature- The radius of the sphere of which the mirror is a part Of is called the radius of curvature of the mirror. (R)



Note: We will make all calculations and formulas for paraxial rays i.e. rays which are incident at points close to the pole and make small angles with the principal axis.

Apni Kaksha

Relation between focal length and radius of curvature

Let the center of curvature of the mirror be C. Consider a ray parallel to the principal axis, incident at M. let angle of incidence be θ.

In ΔMCF

$\angle MFP = 2\theta$ (External angle property)

Also,

$$Tan\theta = \frac{MD}{CD}$$

$$Tan2\theta = \frac{MD}{FD}$$

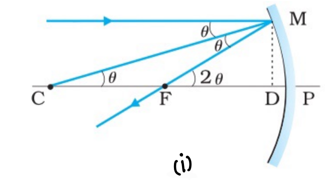
Also, if $\theta \approx 0, Tan\theta \approx \theta$

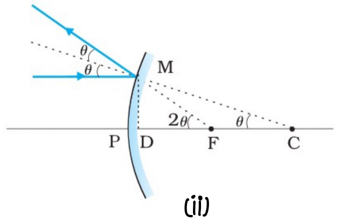
$$\Rightarrow \frac{MD}{FD} = 2 \times \frac{MD}{CD}$$

$$\Rightarrow FD = \frac{CD}{2}$$

For $\theta \approx 0, FD \approx f, CD \approx R$

$$\boxed{\Rightarrow f = \frac{R}{2}}$$





The Mirror equation (PYQ 2020, 2018, 2016, 2011)

If rays emanating from a point actually meet at a point after reflection and/or refraction, that point is called the image of the point. The image is real if the rays actually converge at a point and it is virtual if the do not actually meet but appear to diverge from a point when produced backwards.

Rules for drawing Ray diagrams

1. Rays of light parallel to the principal axis pass through the principal focus after reflection.
2. The rays of light passing through the center of curvature of a concave mirror or appearing to pass through it for a convex mirror retrace their path after reflection.
3. Rays passing through or appearing to pass through the principal focus of a concave and convex mirror respectively, become parallel to the principal axis after reflection.
4. Rays incident at any angle at the pole, follow laws of reflection.

The mirror equation is the relation between the object distance (u), image distance (v) and focal length (f). Consider the following ray diagram for object AB-

$$\Delta A'B'F \sim \Delta MPF$$

$$\frac{B'A'}{PM} = \frac{B'F}{FB}$$

$$\Rightarrow \frac{B'A'}{BA} = \frac{B'F}{FB}, (\because PM = AB)$$

Also,

$$\Delta A'B'P \sim \triangle ABP$$

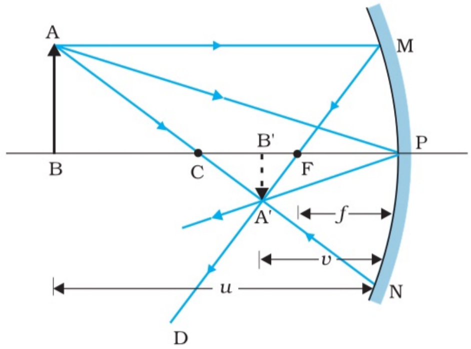
$$\frac{B'A'}{BA} = \frac{B'P}{BP}$$

Substituting values from figure considering sign conventions

$$\frac{-v+f}{-f} = \frac{-v}{u}$$

$$\frac{v-f}{f} = \frac{v}{u}$$

$$\boxed{\frac{1}{v} + \frac{1}{u} = \frac{1}{f}}$$



Linear Magnification (m) (PYQ 2018, 2016, 2014)

It is defined as the ratio of the height of the image(h') to that of the object (h) (h and h' are taken positive or negative according to the sign convention)

$$\boxed{m = \frac{h'}{h}}$$

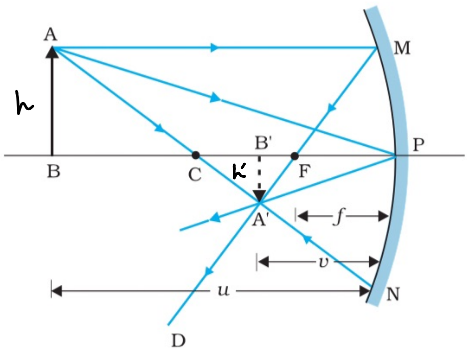
$$\triangle A'B'P \sim \triangle ABP$$

$$\frac{B'A'}{BA} = \frac{BP}{BP}$$

Substituting values from figure considering sign conventions

$$\frac{-h'}{h} = -\frac{v}{u}$$

$$\boxed{m = -\frac{v}{u}}$$



Magnification	Image type
Negative	inverted
Positive	Virtual
>1	Magnified
<1	Diminished

Image formation for concave mirror

Object Position	Image Position	Nature
1. At Infinity	At Focus	Real, inverted and diminished
2. Between infinity and C	Between F and C	Real, inverted and diminished
3. At C	At C	Real, inverted and same sized
4. Between F and C	Between c and infinity	Real, inverted and magnified
5. At F	At infinity	Real, inverted and magnified
6. Between F and P	Behind the mirror	Virtual, erect and magnified

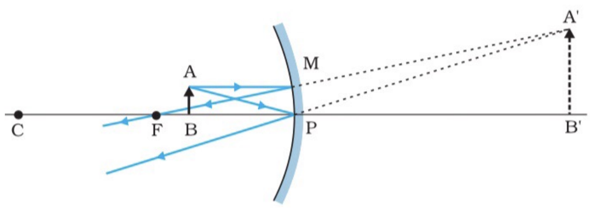
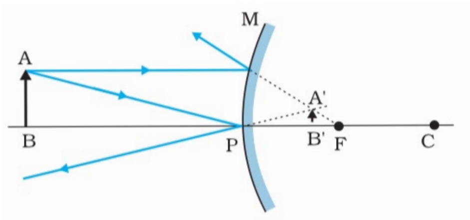


Image formation for convex mirror

Object	Image	Nature
1. At infinity	At F behind the mirror	Virtual, erect and diminished
2. Anywhere else	Behind the mirror	Virtual, erect and diminished



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Important PYQs

Ques: (a) Calculate the distance of an object of height h from a concave mirror of radius of curvature of 20cm, so as to obtain a real image of magnification 2. Find the location of the image as well.
(b) Using mirror formula, explain why a convex mirror always produces a virtual image. **(PYQ 2016) [3M]**

Ans: a) ATQ,

$$-\frac{v}{u} = -2$$

$$\Rightarrow v = 2u \quad - 1$$

Using mirror formula-

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \frac{1}{-10} = \frac{1}{2u} + \frac{1}{u} \quad \text{(From 1)}$$

$$\Rightarrow -\frac{1}{10} = \frac{3}{2u}$$

$$\Rightarrow u = -15\text{cm}$$

Also,

$$v = 2u$$

$$v = -2 \times 15 = -30\text{cm}$$

b) using mirror equation for a convex mirror-

$$\frac{1}{f} = \frac{1}{-u} + \frac{1}{v}$$

$$v = \frac{fu}{f+u}$$

$$v > 0 \quad - 2$$

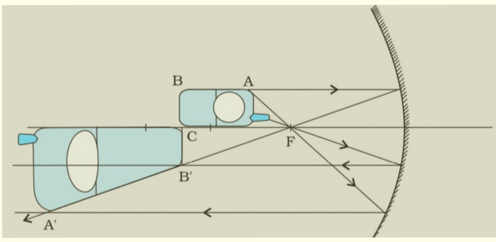
$$m = -\frac{v}{u} = -\frac{v}{-u} = \frac{v}{u} \quad \text{(From 2)}$$

$$m > 0$$

Therefore, it always produces a virtual image

Ques: (a) A mobile phone lies along the principal axis of a concave mirror. Show with the help of a suitable diagram, the formation of its image. Explain why magnification is not uniform
(b) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect will this have on the Image of the object. Explain **(PYQ 2014) [3M]**

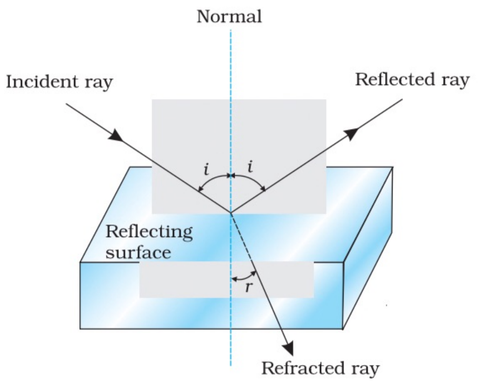
Ans: (a)The magnification of the phone is not uniform because different parts of the phone are at different distances from the pole. So, the magnification of the different parts will be different and the image formed will be distorted.



(b) If the lower half of the mirror is covered with an opaque material, still then complete of the object is formed. But, the intensity of the image will be reduced (halved in this case). This is because the laws of reflection are true for every part of the mirror.

Refraction (PYQ 2019, 2017, 2013)

The direction of propagation of an obliquely incident ray of light that enters the other medium, changes at the interface of the two media. This phenomenon is called refraction of light.



Laws of refraction

1. The incident ray, the refracted ray, the normal to the interface at the point of incidence, all lie in the same plane.
2. **Snell's Law-** The ratio of the sine of angle of incidence(i) to the sine Of angle of refraction(r) is constant for a given color of light and a given pair of media.

$$\boxed{\frac{\sin i}{\sin r} = n_{21}}$$

Refractive index (PYQ 2019)

The constant n_{21} or μ_{21} is called the refractive index of medium 2 w.r.t medium 1. The refractive index of medium 2 w.r.t. 1 can also be defined as the ratio of speed of light in medium 1 to that in medium 2 i.e.

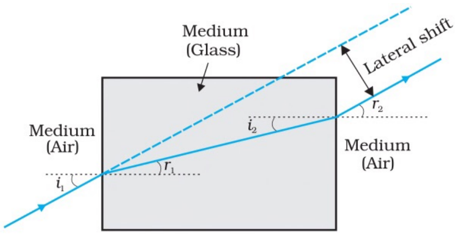
$$\boxed{\mu_{21} = \frac{V_1}{V_2}}$$

Where v_1 and v_2 are speed of light in medium 1 and 2 respectively.

Note: The refractive index of a medium depends on the wavelength of light in the medium but is independent of the angle of incidence. μ decreases with increase in wavelength i.e. $\mu_{red} < \mu_{violet}$

- If n_{21}>1, light travels slower in medium 2 and the refracted ray bends towards the normal. Such a medium is called an optically denser medium than medium 1.
- If n_{21}<1, light travels faster in medium 2 and the refracted ray bends away from the normal. Such a medium is called an optically rarer medium than medium 1.
- Optical density and mass density (d=mass/volume) should not be confused. An optically denser medium may have a lower mass density than the other medium. For e.g. mass density of turpentine is less than that of water but it is more optically dense.
- $n_{21} = 1/n_{12}$

Refraction in a rectangular glass slab

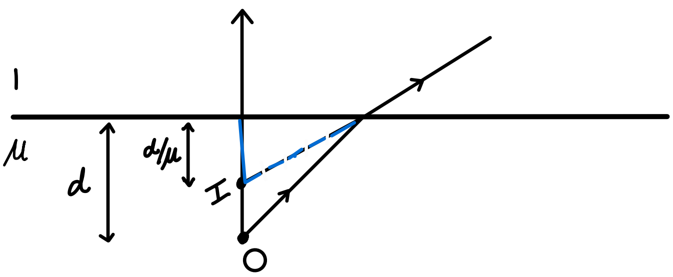


Consider a rectangular glass slab as shown in the figure. For a rectangular glass slab-

- refraction takes place at two interfaces (air-glass and glass-air)
- in fig., r_2=i_1 i.e. the emergent ray is parallel to the incident ray i.e. there is no deviation
- the emergent ray is laterally shifted/displaced w.r.t. the incident ray

Apparent Depth

When an object in a denser medium is viewed from a rarer medium, for e.g. a fish being viewed by a bird –



Object distance = d
Image distance = d/μ

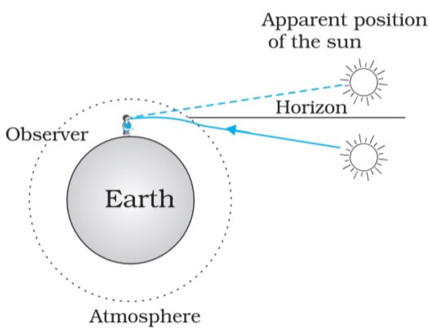
Real depth (R.D) = d
Apparent Depth (A.D) = d/μ

So, we see-
R.D > A.D
i.e. object appears closer than it is.

Shift-
R.D- A.D = d - d/μ
= d(1-1/μ)

Phenomenon due to refraction

1. **Early sunrise/ Delayed sunset-** As light coming from the sun passes through the atmosphere, it gets refracted. As a result of which, the sun is visible little before the actual sunrise and little after the actual sunset. The time difference between actual sunset and apparent sunset is about two minutes.
2. **Apparent flattening (oval shape) of the sun at sunset and sunrise.**



Important PYQs

Ques: When monochromatic light travels from a rarer to denser medium, explain the following, giving reasons-
Is the frequency of the reflected and refracted light same as that of the incident light? **(PYQ 2013) [1M]**

Ans: Yes, since the frequency of light depends only on the source which produces it, the frequency of reflected and refracted light is the same as that of the incident light.

Ques: Monochromatic light of wavelength 589nm is incident from air on a water surface. If μ for water surface is 1.33, find the wavelength, frequency, and speed of refracted light. **(PYQ 2017) [2M]**

Ans: 1) $\mu = \frac{\lambda_1}{\lambda_2}$

$\Rightarrow \lambda_2 = 443nm$

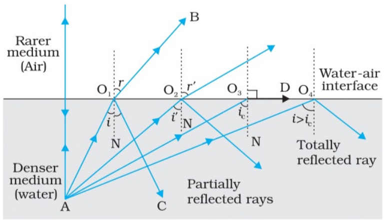
2) $f_1 = f_2 = \frac{c}{\lambda_1} = 0 \cdot 51\text{MHz}$

3) $\mu = \frac{C}{V_2}$

$V_2 = \frac{c}{\mu} = 2.25 \times 10^8 \times m / s$

Total internal reflection (TIR) (PYQ 2019, 2016, 2013, 2010)

When a ray of light travels from an optically denser medium to an optically rarer medium, if angle of incidence (i) is greater than the critical angle, then the ray of light is completely reflected back into the same medium. This is called Total Internal Reflection (TIR).



Critical angle (θ_c/i_c) - It is the angle of incidence at which angle of refraction is 90°

So, using Snell's Law-

$$\frac{\sin\theta_c}{\sin 90} = n_{21}$$

$$\sin\theta_c = n_{21}$$

So refractive index of denser medium-

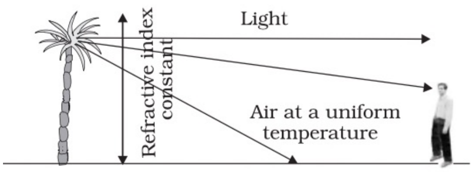
$$\boxed{n_{12} = \frac{1}{\sin\theta_C}}$$

For values of i greater than i_c, Snell's Law cannot be satisfied, therefore refraction is not possible.

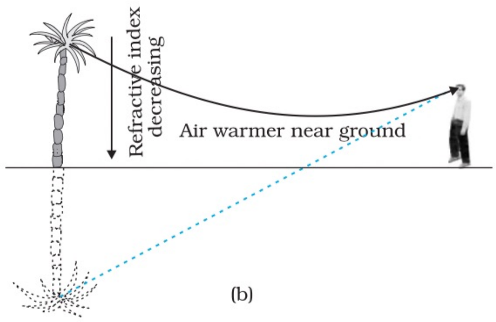
Note: When light travels from an optically denser medium to an optically rarer medium at the interface, it is partially reflected and partially refracted to the second medium. This reflection is called internal reflection.

TIR in Nature and Its Technological Applications-

1. **Mirage-** On hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with increasing density. Hot air is less danse than cold air therefore, the air near the ground is optically rarer than the air at higher levels. So, light from a tall object e.g. a tree, passes from a denser medium to a rarer medium and continuously bends towards the normal. If the angle of incidence exceeds the critical angle, the light suffers TIR. To a distant observer, the light appears to be coming from below the ground and the observer naturally assumes that the light is being reflected from the ground by a pool of water near the tall object. This phenomenon is called a mirage. It is very common in hot deserts.

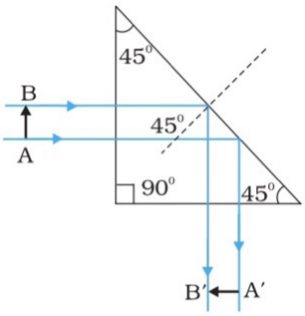


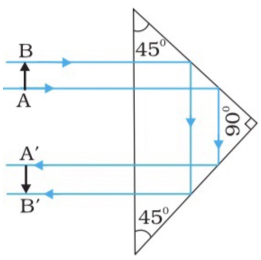
(a)



(b)

2. **Diamonds-** Diamonds are known for their spectacular brilliance their brilliance is mainly due to the total internal reflection. The critical angle for the diamond air interface is very small and light is likely to undergo TIR when it enters a diamond.
3. **Prism-** Prisms are used to bend light by 90° or 180° by making use of TIR. Such a prism is also used to invert images without changing their size.





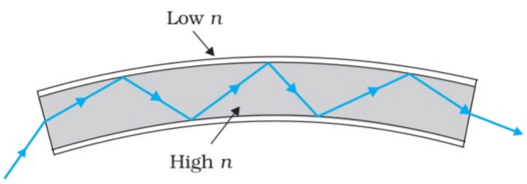
4. **Optical fibers:**
 Construction- They are made of composite glass/quartz fibers. Each fiber has a core and cladding. The refractive index of the material of the core is higher than of the cladding

Principle- Total internal reflection

Working- When a light signal is sent through one end of the fiber at a suitable angle, it undergoes repeated total internal reflections along the length of the fiber and comes out of the other end. Optical fibers are fabricated in such a way that light reflected at one side of inner surface strikes the other at angle larger than the critical angle. Even if the fiber is bent, light can easily travel along its length.

Uses- 1. Used for transmitting audio signals through long distances
2. Used for transmitting and receiving electrical signals which can be converted to light by transducers
3. Used as 'light pipe' to examine internal organs like esophagus, stomach, intestines.
4. used in decorative lamps

Note: The main requirement of optical fibers is that there should be very little absorption of light as it travels over long distances.
With certain materials like silica glass, it is possible to transmit more than 95% of light over a distance of 1km



Important PYQs

Ques: A ray of light incident normally on one surface of a right isosceles prism is total reflected as shown in the fig. What must be the minimum refractive index of the prism? **(PYQ 2016) [1M]**

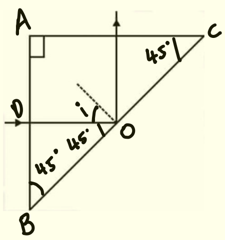
Ans: ATQ and using properties of triangle we see that-

$$\angle ABO = \angle DOB = \angle i = 45^\circ$$

We know for TIR-

$$n_{12} = \frac{1}{\sin\theta_C}$$

$$n_{12} = \sqrt{2}$$

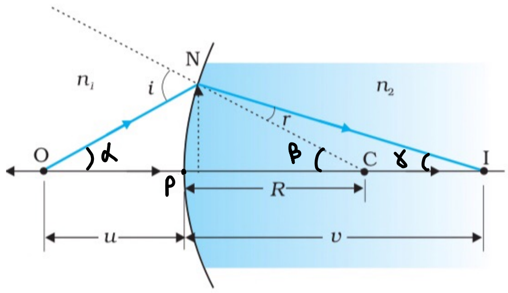


Refraction at Spherical surfaces and by Lenses (PYQ 2020, 2019, 2015, 2014)

- An infinitesimal part of a spherical surface can be regarded as planar and the same laws of refraction can be applied at every point on the surface.
- **Lens-** A thin lens is a transparent optical medium bound by two thin surfaces; at least one of which should be spherical

Refraction at spherical surface-

Consider the following spherical surface with center of curvature C and radius of curvature R. The rays are incident from a medium of refractive index n_1 to another of refractive index n_2. Let O be the object and I image. For small angles,



Using Snell's Law we have-

$$n_1 \sin i = n_2 \sin r$$

$$\Rightarrow n_1 i = n_2 \cdot r \quad \text{(Since they are paraxial rays)}$$

$$\Rightarrow n_1(\alpha + \beta) = n_2(\beta - \gamma) \quad \text{(External angle property)}$$

$$\Rightarrow n_1\left(\frac{NP}{P0} + \frac{NP}{PC}\right) = n_2\left(\frac{NP}{PC} - \frac{NP}{PI}\right)$$

$$\Rightarrow n_1\left(\frac{1}{-u} + \frac{1}{R}\right) = n_2\left(\frac{1}{R} - \frac{1}{v}\right)$$

$$\Rightarrow -\frac{n_1}{u} + \frac{n_1}{R} = \frac{n_2}{R} - \frac{n_2}{v}$$

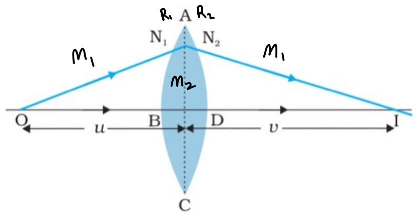
$$\boxed{\Rightarrow \frac{n_2 - n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}}$$

Lens Maker's Formula (PYQ 2020, 2017, 2015, 2012, 2011, 2010)

Assumptions- 1. Lens is thin 2. Media on both sides are same

Consider the following double convex lens of radius of curvature R_1 and R_2 and refractive index n_2 kept in a medium of refractive index n_1.

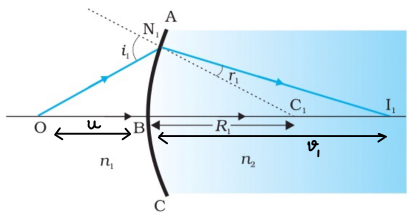
The image formation will take place in two steps. i) Th refracting surface 1 creates image I_1 of the object O. ii) The image I_1 acts as a virtual object for the refracting surface 2 which forms the final image I. Applying the formula derived to the first interface-



$$\frac{n_2}{v_1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1}$$

And for the second interface-

$$\frac{n_1}{v} - \frac{n_2}{v_1} = \frac{n_1 - n_2}{R_2}$$



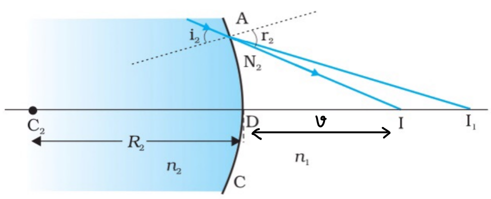
Adding both equations-

$$\frac{n_1}{v} - \frac{n_1}{u} = (n_2 - n_1) \times \left(\frac{1}{R_1} \ \frac{-1}{R_2}\right)$$

Dividing by n_1-

$$\boxed{\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right) \times \left(\frac{1}{R_1} \ \frac{-1}{R_2}\right)}$$

Suppose the object is at infinity, therefore v= f



$$\boxed{\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right) \times \left(\frac{1}{R_1} \ \frac{-1}{R_2}\right)}$$

From 3,4 we get-

$$\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

This is called the Thin Lens formula

Note: The two foci of double concave and convex lens are equidistant from the optical center. The focus on the side of the original source of light is called the first focal point and the other is called the second focal point.

Important PYQs

Ques: A biconvex lens of refractive index 1.47 is immersed in a liquid, it becomes invisible and behaves as a plane glass plate. The refractive index of the liquid is? **(PYQ 2020) [1M]**

Ans: Since the lens behaves as a plane glass plate i.e. its power becomes zero. Therefore, from lens makers formula we see-

$$P = \left(\frac{\mu_2}{\mu_1} - 1\right) \times \left(\frac{2}{R}\right) = 0$$

$$\Rightarrow \frac{\mu_2}{\mu_1} = 1$$

$$\Rightarrow \mu_2 = \mu_1$$

Therefore, the refractive index of the liquid is 1.47

Ques: An object is placed 30cm Infront of a plano-convex lens with its spherical surface of radius of curvature 20cm. if the refractive index of the material of the lens is 1.5. Find the position and nature of the image formed. **(PYQ 2020) [3M]**

Ans: $\frac{1}{v} - \frac{1}{u} = \left(\frac{n_2}{n_1} - 1\right) \times \left(\frac{1}{R_1} - \frac{-1}{R_2}\right)$

$$\frac{1}{v} - \frac{1}{-30} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{\infty}\right)$$

$$v = -120\text{cm}$$

$$m = \frac{v}{u} = \frac{-120}{-30} = 4$$

Virtual, erect and magnified

Ques: A double convex lens is made of glass of refractive index 1.55 with both faces of the same radius of curvature. Find the radius of curvature required if focal length is 20cm **(PYQ 2017) [2M]**

Ans: $\frac{1}{f} = (\mu - 1) \times \left(\frac{2}{R}\right)$

$$\frac{1}{20} = (1 \cdot 55 - 1) \times \frac{2}{R}$$

$$R = 22\text{cm}$$

Ques: A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? **(PYQ 2015) [1M]**

Ans: $\frac{1}{f} = (\mu - 1) \times \left(\frac{-2}{R}\right)$

$$\frac{1}{f} = \left(\frac{1 \cdot 5}{1.65} - 1\right) \times \left(\frac{-2}{R}\right)$$

$$\frac{1}{f} = \left(-\frac{0 \cdot 15}{1 \cdot 65}\right) \times \left(\frac{-2}{R}\right) > 0$$

Since f is positive, lens is diverging

Ques: A convex lens is placed in contact with a plane mirror. A point object at a distance of 20 cm on the axis of this combination has its image coinciding with itself. What is the focal length of the lens? **(PYQ 2014) [1M]**

Ans: First, an image is formed behind the plane mirror by the convex lens at a distance v_1. This acts as an object for the plane mirror which forms the image at a distance v_1 in front of the mirror-lens combination. This image acts as an object for the lens and the final image is formed at u

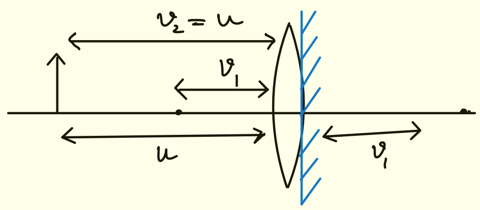
$$\frac{1}{f} = \frac{1}{v_1} - \frac{1}{-u}$$

$$v_1 = \frac{fu}{u - f}$$

$$\frac{1}{f} = \frac{1}{u} - \frac{1}{v_1}$$

$$\frac{1}{f} = \frac{1}{u} - \frac{(u - f)}{fu}$$

$$\Rightarrow u = f = 20\text{cm}$$



Ques: Under what condition does a biconvex lens of a certain refractive index acts as a plane glass sheet when immersed in a liquid? **(PYQ 2012) [1M]**

Ans: When the refractive index of the liquid is same as that of the lens, it will behave as a plane glass sheet.

Ques: A converging lens of focal length of 20cm in air is made of a material of refractive index 1.6. It is immersed in a liquid of refractive index 1.3. calculate its new focal length **(PYQ 2011) [3M]**

Ans: $\frac{1}{f} = (\mu - 1) \times \left(\frac{2}{R}\right)$

$\frac{1}{20} = (1 \cdot 6 - 1) \times \left(\frac{2}{R}\right)$ — (1)

$\frac{1}{f} = (1 \cdot 3 - 1) \times \left(\frac{2}{R}\right)$ — (2)

Dividing 1 and 2

$\frac{f}{20} = \frac{0 \cdot 6}{0.3}$

$f = 40\text{cm}$

Ques: A glass lens of refractive index 1.45 disappears when immersed in a liquid. What is the value of refractive index of the liquid? **(PYQ 2010) [1M]**

Ans: 1.45

Ques: The radii of curvature of the faces of a double convex lens are 10cm and 15cm. if focal length of the lens is 12cm, find the refractive index of the material of the lens. **(PYQ 2010) [2M]**

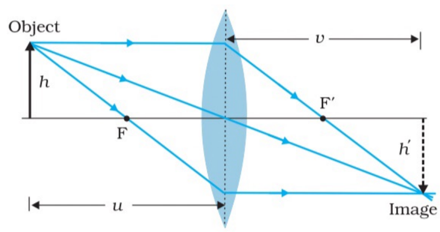
Ans: $\frac{1}{f} = (\mu - 1) \times \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

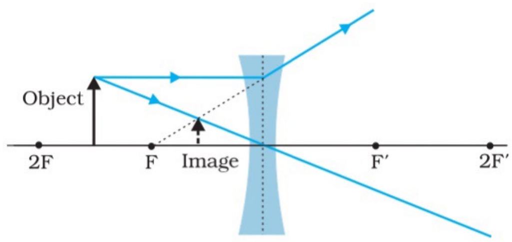
$\frac{1}{12} = (\mu - 1) \times \left(\frac{1}{10} - \frac{1}{-15}\right)$

$\mu = 1.5$

Image formation by a lens

1. Rays parallel to the principal axis after refraction pass through the second principal focus in a convex lens and appear to diverge from the first principal focus in a concave lens.
2. Ray of light passing through the optical center passes through undeviated.
3. A ray of light passing through the first principal focus (for a convex) or appearing the meet at it (for a concave lens) emerges parallel to the principal axis after refraction.





Magnification produced by a lens (m)

It is defined as the ratio of the size of the image to the size of the object.

$$\boxed{m = \frac{h'}{h} = \frac{v}{u}}$$

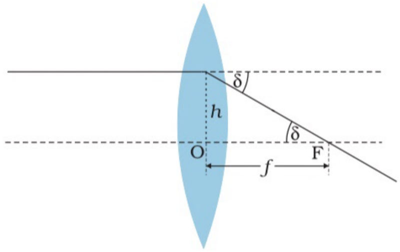
Power of a lens (PYQ 2019, 2018)

Power of a lens is a measure of the convergence or divergence which a lens introduces in the light falling on it. It can be defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical center. SI unit- m^{-1}/ D (diopter)

$$Tan\delta = \frac{h}{f};\ \tan\delta = \frac{1}{f} \quad \text{(h=1)}$$

$$\Rightarrow \delta = \frac{1}{f} \qquad \text{(for small angles)}$$

$$\boxed{\Rightarrow P = \frac{1}{f}}$$



Important PYQs

Ques: Calculate the radius of curvature of an equi-concave lens of refractive index 1.5, when kept in a medium of refractive index 1.4, to have a power of -5D **(PYQ 2019) [2M]**

Ans: $\frac{1}{f} = P = (n_{21} - 1) \times \left(\frac{-2}{R}\right)$ (Using lens makers formula)

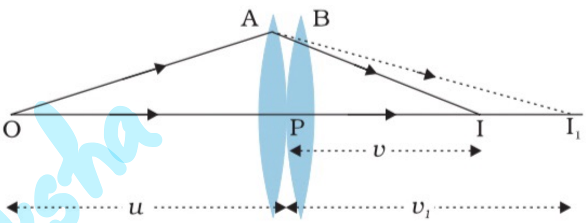
$$-5 = \left(\frac{1\cdot 5}{1\cdot 4} - 1\right) \times \left(\frac{-2}{R}\right)$$

$$R = \frac{1}{35}m$$

Combination of lenses in contact (PYQ 2017)

Consider two lenses A and B of focal lengths f_1 and f_2 placed in contact with each other. Let the object O be placed beyond the focus of the first lens. The first lens produces image I_1 which acts as a virtual object for the second lens which in turn produces image I. since the lenses are thin, we assume that the optical centers of the two lenses coincide at P. For the first lens-

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$



For the second lens-

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$$

Adding 1,2

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}$$

If the two-lens system is regarded as equivalent to a single lens of focal length f we have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

So, we get-

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

This derivation is valid for any number of thin lenses in contact. For e.g., for n number of thin lenses in contact with focal length f_1, f_2, f_3, ……., f_n-

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \dots}$$

In terms of power-

$$\boxed{P = P_1 + P_2 + P_3 + \dots}$$

Note: The net magnification of the combination of lenses will be written as-

$$\boxed{m = m_1\, m_2\, m_3\ \dots}$$

Such systems are used to design lenses for cameras, microscopes, telescopes and other optical instruments.

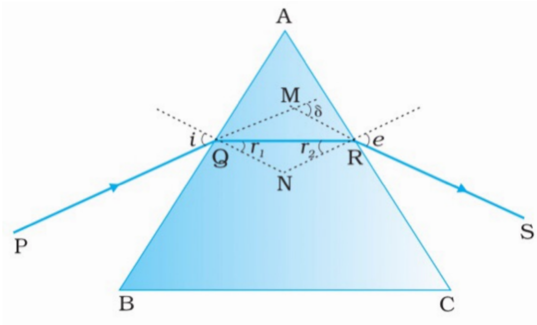
Refraction through a Prism (PYQ 2020, 2019, 2017, 2016, 2013, 2012)

Prism- A prism is a combination of 2 refracting surfaces where the incident and the emergent surfaces are not parallel

Angle of prism (A) – It is defined as the angle between the incident and the emergent face

Angle of deviation (δ) – It is defined as the angle between the incident ray produced forward and the emergent ray produced backwards.

Consider the following prism ABC. The angle of incidence and refraction on the face AB are i and r_1 respectively while on the other face the angle of refraction and emergence are r_2 and e respectively.



Consider the quadrilateral AQNR-

$$\angle A + \angle QNR = 180^\circ \quad - ①$$

From triangle QNR-

$$r_1 + r_2 + \angle QNR = 180^\circ \quad - ②$$

From 1, 2-

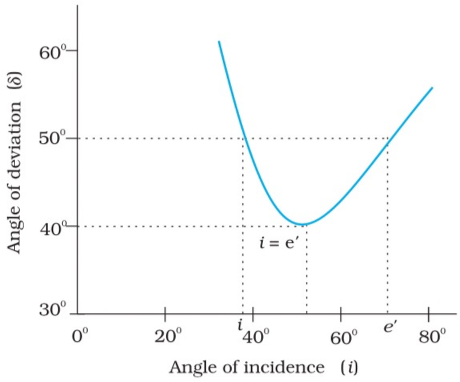
$$\boxed{r_1 + r_2 = A}$$

The total deviation is the sum of deviations on the two faces. Therefore,

$$\delta = (i - r_1) + (e - r_2)$$

$$\boxed{\delta = i + e - A} \quad - ③$$

We see that the deviation depends on the angle of incidence hence plotting the graph between i and δ, we get the following curve-



When i = e, the deviation is minimum i.e. $\delta = \delta_m$. At minimum deviation, the refracted ray becomes parallel to the base of the prism which implies $r_1 = r_2$. Therefore-

$$\boxed{2r = A; r = \frac{A}{2}}$$

Substituting in 3

$$\delta_m = 2i - A$$

$$\boxed{\Rightarrow i = \left(\frac{\delta_m + A}{2}\right)}$$

The refractive index of the prism-

$$\boxed{\mu_{21} = \frac{\mu_2}{\mu_1} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin(A/2)}}$$

For small angled prisms i.e. thin prisms, δ_m is also very small and we get-

$$\mu_{21} = \frac{\mu_2}{\mu_1} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin(A/2)} \sim \frac{(A + \delta_m)/2}{(A/2)}$$

$$\boxed{\Rightarrow \delta_m = (\mu_{21} - 1) \cdot A}$$

This implies that prisms do not deviate light much.

Important PYQs

Ques: For a glass prism, the angle of minimum deviation will be smallest for the light of-

A) Red color C) yellow color

B) Blue color D) green color (**PYQ 2020) [1M]**

Ans: We know that $\delta_m = (n_{21} - 1)$ A. therefore, min deviation will be smallest for the color with smallest refractive index i.e. A) Red color.

Ques: A ray of light on passing through an equilateral glass prism, suffers a minimum deviation equal to the angle of the prism. The value of refractive index of the material of prism is? **(PYQ 2020) [1M]**

Ans: $\delta_m = A = 60^\circ$

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin(A/2)}$$

$$\mu = \frac{\sin 60}{\sin 30}$$

$$\mu = \sqrt{3}$$

Ques: An equilateral glass prism has a refractive index 1.6 in air. Calculate the angle of minimum deviation of the prism when kept in a medium of refractive index 4 √2/5 **(PYQ 2019) [2M]**

Ans: $$\frac{\mu_2}{\mu_1} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin(A/2)}$$

$$\frac{1\cdot 6}{\frac{4\times\sqrt{2}}{5}} = \frac{\sin\left(\frac{\delta_m + 60}{2}\right)}{\sin 30^\circ}$$

$$\sin\left(\frac{\delta_m + 60}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\frac{\delta_m + 60}{2} = 45^\circ$$

$$\delta_m = 30^\circ$$

Ques: How does the angle of minimum deviation of the prism change if incident violet light is replaced by red light? **(PYQ 2017) [1M]**

Ans: The smaller is the refractive index of light, the smaller is the angle of minimum deviation and since $\mu_{red} < \mu_{violet}$, the angle of minimum deviation will become smaller.

Ques: A ray of light passing from air through an equilateral glass prism undergoes minimum deviation when angle of incidence is 3/4th of the angle of prism. Calculate the speed of light in the prism. **(PYQ 2017) [2M]**

Ans: ATQ,

$$r = \frac{A}{2} = \frac{60}{2} = 30^\circ$$

$$i = \frac{3}{4}A = 45^\circ$$

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 45^\circ}{\sin 30^\circ}$$

$$\mu = \sqrt{2}$$

$$\mu = \frac{c}{v}$$

$$v = \frac{c}{\mu} = 2\cdot 12\times 10^8 \times m/s$$

Ques: A ray of light incident on an equilateral glass prism of refractive index √3 moves parallel to the base line of prism. Find angle of incidence of this ray. **(PYQ 2012) [2M]**

Ans: Since ray undergoes min deviation -

$$r = \frac{A}{2} = \frac{60}{2} = 30^\circ$$

$$\frac{\sin i}{\sin r} = \mu \quad \text{(Snell's law)}$$

$$\sin i = \sqrt{3}\times\frac{1}{2}$$

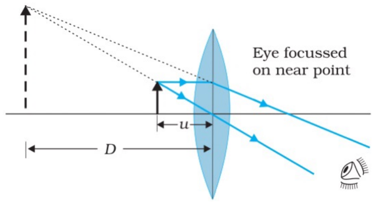
$$i = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$i = 60^\circ$$

Optical instruments (PYQ 2020, 2019, 2017, 2014, 2013, 2012, 2011, 2010)

1. The Simple Microscope-

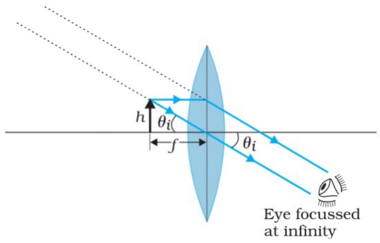
A simple microscope is a converging lens of a small focal length. The idea is to get an erect, magnified and virtual image of the object at a distance of about 25cm or more. The linear magnification m for the image formed at near point D by a simple microscope can be obtained by-



$$m = \frac{v}{u} = v\left(\frac{1}{v} - \frac{1}{f}\right) = \left(1 - \frac{v}{f}\right)$$

According to sign convention, v is negative and is equal in magnitude to D.
Thus,

$$\boxed{m = \left(1 + \frac{D}{f}\right)}$$

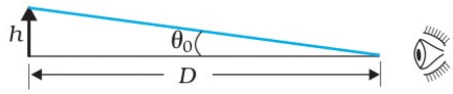


The magnification m = h'/h where h and h' are the size of the object and the image respectively. This is also the ratio of angle subtended by the image to that suspended by the object.
Although the closest comfortable distance for viewing an image is when it is at the near point (D=25cm) but it causes strain in the eyes. Therefore, the image formed at infinity is considered more comfortable for viewing. So, let's find magnification (**Angular magnification**) when the image is at infinity. This setting of the microscope is called the **Normal adjustment.**
Let height of object be of height h. the maximum angle it can subtend to be visible without a lens is when it is at a distance D from the observer. The angle subtended will be-

$$\tan\,\theta_o = \left(\frac{h}{D}\right) \approx \theta_o$$

The angle subtended by the image (θ_i) at the eye when the object is at a distance u-

$$\tan\theta_i = \frac{h'}{-v} = \frac{h}{-v}\cdot\frac{v}{u} = \frac{h}{-u} \approx \theta.$$



For image to be at infinity, u= *-f.* Therefore-

$$\theta_i = \left(\frac{h}{f}\right)$$

Angular magnification (A.M)

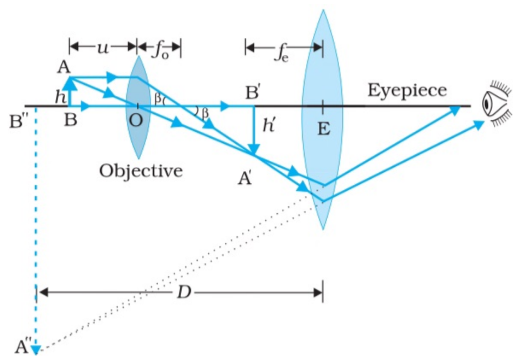
$$\boxed{= \left(\frac{\theta_i}{\theta_o}\right) = \frac{D}{f}}$$

Note: This is one less than the magnification when the object is at the near point

Limitation of Simple microscope-
A simple microscope has limited maximum magnification for realistic focal lengths.

2. Compound Microscope (PYQ 2014, 2013, 2012, 2011, 2010)

For much larger magnification one uses two lenses, one compounding the effect of the other. This is called a compound microscope.
Objective lens- The lens nearest to the object is called the objective and produces a real inverted and magnified image which serves as an object for the second lens.
Eyepiece- The eyepiece functions like a simple microscope and produces the final image which virtual erect and magnified.
Therefore, the first inverted image will be near the focal plane of the eyepiece. The final image will be inverted w.r.t to the object.



The linear magnification due to the objective equals-

$$\boxed{m_O = \frac{h'}{h} = \frac{L}{f_o}}$$

where we have used the resul

$$\tan\beta = \left(\frac{h}{f_o}\right) = \left(\frac{h'}{L}\right)$$

Where L is approximately the length of the microscope tube or the distance between the second focus of the objective and the first focus of the eyepiece (focal length f_e), h is height of object, h' height of first image and f_o is the focal length of objective. the first image is formed near the focus of the eyepiece.
From the discussion about the simple microscope, the magnification due to the eye piece when the object is at f_e and image is to be formed at D-

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

And when the final image is at infinity-

$$m_e = \frac{D}{f_e}$$

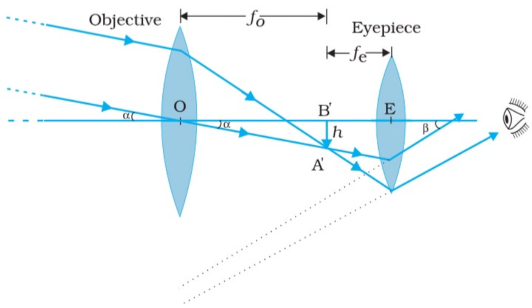
We learnt that net magnification of two lenses in combination is the product of their individual magnifications. Therefore, the net magnification when the final image is at infinity-

$$m_0 m_e = \left(\frac{L}{f_0}\right) \times \left(\frac{D}{f_e}\right)$$

Note: To get large magnification for small objects, the objective and eyepiece should have small focal lengths. The objective has a smaller aperture than they eyepiece in a compound microscope.

3. Telescope (PYQ 2020, 2019, 2017, 2012)

The telescope is used to provide angular magnification of distant objects.



Construction-
It has an objective and an eye piece but here the objective has a large focal length and a much larger aperture than the eyepiece (unlike in a microscope)
Working-
Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. the eyepiece magnifies this image producing a final inverted image
Magnification-
The magnification power m is the ratio of the angle subtended by the final image to that subtended by the object at the eye or lens-

$$m \approx \frac{\beta}{\alpha} \approx \frac{h}{f_e} \cdot \frac{f_o}{h} = \frac{f_o}{f_e}$$

Note: 1. The length of the telescope tube = $f_e + f_o$
2. Refracting telescopes have an additional pair of inverting lenses to make the final image erect.
3. For an Astronomical Telescope, resolution and resolving power depend on the area of the objective. With larger diameters, fainter objects can be observed. So, in telescopes, the diameter of the objective is large.

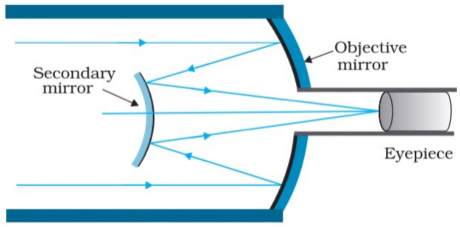
Limitations of Refracting telescopes-
1. They require large lenses and such big lenses tend to be heavy and they are difficult to make and support by their edges.
2. It is very expensive to make such large lenses
3. The images formed are not free from chromatic aberrations and distortions.

Reflecting telescopes
The telescopes with mirror objectives are called reflective telescopes
Advantages-
1. There are no chromatic aberrations in a mirror
2. If parabolic reflecting surfaces are used, spherical aberrations are also removed
3. More stable as mirrors weigh less than lenses of equal optical quality

Limitation-
The objective focuses the light inside the telescope and the eyepiece and observer obstruct some light

Cassegrain Telescope-

In this, the light being focused by another mirror is deflected. A convex secondary mirror focuses the incident light which passes through a hole in the objective primary mirror. It is advantageous as we can get a large focal length in a short telescope.



Important PYQs

Ques: Larger aperture of the objective lens in an astronomical telescope

A) Increases the resolving power
B) Decreases the brightness of the image
C) Increases the size of the image
D) Decreases the length of the telescope

Ans: A) Increases the resolving power of the telescope.

Ques: An astronomical telescope has an objective lens of focal length 20m and eyepiece of focal length 1cm.

1. Find angular magnification
2. If the telescope is used to view the moon, find the diameter of the image formed by the objective lens. Given the diameter of the moon is 3.5 ×10^6 m and radius of lunar orbit is 3.8×10^8 m **(PYQ 2020, 2019, 2011) [2M]**

Ans: $A \cdot M = \dfrac{f_o}{f_e}$

$A \cdot M = \dfrac{20}{1 \times 10^{-2}}$

$A \cdot M = 2000$

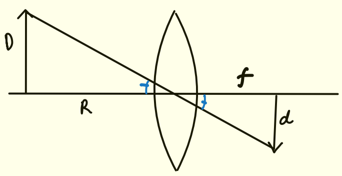
2) using the principle that angle sub tended by the image and object is same-

$\dfrac{D}{R} = \dfrac{d}{f}$

$\dfrac{3 \cdot 5 \times 10^6}{3 \cdot 8 \times 10^8} = \dfrac{d}{20}$

$d = 0.18m$

(D- diameter of moon
d- diameter of image
R- radius of orbit
f- focal length of objective)



Ques: You are given the following three lenses. Which two lenses will use as an eyepiece and objective to construct an astronomical telescope? **(PYQ 2017) [2M]**

Lenses	Power (D)	Aperture (cm)
L_1	3	8
L_2	6	1
L_3	10	1

Ans: The objective should have a large focal length and a large aperture and the eyepiece should have a small focal length and small aperture. So, I would use L_1 as an objective and L_2 as an eyepiece (we did not use L_3 as focal length will be too small)

Ques: The total magnification produced by a compound microscope is 20. The magnification produced by the eye piece is 5. The distance between the objective and eyepiece is observed to be 14cm and if least distance of distinct vision is 20cm, calculate the focal length of both the lenses **(PYQ 2014) [2M]**

Ans: 1) $m=\left(\frac{L}{f_0}\right)\times\left(\frac{D}{f_e}\right)$

$20=\frac{14}{f_0}\times 5$

$f_0=3\cdot 5\text{cm}$

2) $m_e=\left(\frac{D}{f_e}\right)$

$5=\frac{20}{f_e}$

$f_e=4\text{cm}$

Derivations and Definitions asked as PYQs

Ques: An object is placed in front of a concave mirror. It is observed that a virtual image is formed. Draw a ray diagram to show the image formation and hence derive the mirror equation **(PYQ 2020) [3M]**

Ques: (a) With the help of a ray diagram show how a concave mirror is used to obtain an erect and magnified image of an object
(b) Using the above ray diagram, obtain the mirror formula and the expression for linear magnification **(PYQ 2018) [3M]**

Ques: Define refractive index (**PYQ 2019) [1M]**

Ques: Under what conditions is the phenomenon of total internal reflection of light observed? Obtain the relation between the critical angle of incidence and the refractive index of the medium. **(PYQ 2019, 2013, 2010) [2M]**

Ques: a) A point object O is kept in a medium of refractive index n_1 in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index n_2 from the first one as shown in the figure. Draw the ray diagram showing the image formation and deduce the relationship between the object distance and image distance in terms of n_1, n_2, and R

b) When the image formed above acts as a virtual object for the concave spherical surface separating the medium n_2 from n_1 ($n_2>n_1$) draw the ray diagram similar to n_1. Hence obtain the expression for lens maker's formula **(PYQ 2015) [5M]**

Ques: Using the lens makers formula, derive the thin lens formula. **(PYQ 2020) [2M]**

Ques: Define power of lens and write its SI unit. **(PYQ 2018) [1M]**

Ques: Draw a ray diagram to show the image formation by a combination of two thin convex lenses in contact. Obtain the expression for the power of this combination in terms of the focal lengths of the lenses. **(PYQ 2017) [2M]**

Ques: What is the relationship between the angle of incidence 'i' angle of prism A and angle of minimum deviation for a triangular prism? **(PYQ 2013) [1M]**

Ques: Draw the ray diagram of an astronomical telescope when the final image is formed at infinity. Write the expression for the power of telescope **(PYQ 2020) [3M]**

Ques: Draw labelled diagram of an astronomical telescope in the near point adjustment **(PYQ 2019) [1M]**

Ques: Draw ray diagram of a telescope in the normal adjustment **(PYQ 2017) [1M]**

Ques: Draw a labelled diagram of a compound microscope for near point adjustment **(PYQ 2014) [1M]**

Ques: Draw a ray diagram showing image formation by a compound microscope. Hence obtain total magnification when image is formed at infinity. **(PYQ 2013) [3M]**