NUCLEI

(CHAPTER-13)

* Nucleus is made up of neutron and proton.

$\boxed{\text{NEUTRON + PROTON = NUCLEONS}}$

PROTON :- CHARGE- $+e = 1.6 \times 10^{-19} C$

MASS- $1.67 \times 10^{-27} kg = 1$ amu

SYMBOL- ${}^{1}_{1}H$

NEUTRON :- CHARGE :- 0

MASS :- $1.674 \times 10^{-27} kg \approx 1$ amu

SYMBOL :- ${}^{1}_{0}n$

ATOMIC NUMBER :- (Z)

It is the number of protons present inside nucleus.

Z = Number of protons
= Number of electrons (in neutral atom)

MASS NUMBER :- (A)

It is the total number of protons and neutrons inside the atomic nucleus of the element.

A = Number of protons + Number of neutrons

Eg :- ${}^{16}_{8}O$ = 8p, 8n

ISOTOPES :- Atoms of the same element whose nuclei have same number of protons but different number of neutrons.

Atomic number → same
Mass number → Different

Eg:- Protium , Deuterium , Tritium
^1_1H ^2_1H ^3_1H

ISOBAR:-

Atoms of different element whose nuclei have same number of nucleons but different number of protons and neutrons.

$$\boxed{\begin{array}{l}\text{Atomic number — Different}\\ \text{Mass number — Same}\end{array}}$$

Eg:- $^{14}_{6}C$ $^{14}_{7}N$
(6p, 8n) (7p, 7n)

ISOTONES:-

Atoms of different element whose nuclei have same number of neutrons but different number of protons.

$$\boxed{\begin{array}{l}\text{Atomic number — Different}\\ \text{Mass number — Different}\end{array}}$$

Eg:- $^{3}_{1}H$ $^{4}_{2}He$
(1p, 2n) (2p, 2n)

SIZE OF NUCLEUS:-

Volume of nucleus $\propto$ Mass number

$$\Rightarrow \frac{4}{3}\pi R^3 \propto A$$

$$\Rightarrow R^3 \propto A$$

$$\Rightarrow R \propto (A)^{1/3}$$

$$\Rightarrow \boxed{R = R_0(A)^{1/3}}$$ It is also known as nuclear unit radius

where, R = radius of nucleus.
A = mass number
$R_0 = 1.2 \times 10^{-15} m = 1.2\ fm$

NUCLEAR DENSITY:- (ρ)

$$\rho = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} = \frac{A \times amu}{V} = \frac{A \times 1 amu}{\frac{4}{3}\pi R^3}$$

$$= \frac{A \times 1 amu}{\frac{4}{3}\pi [R_0 (A)^{1/3}]^3} = \frac{A \times 1 amu}{\frac{4}{3}\pi R_0^3 A} = \frac{3 \times 1 amu}{4\pi R_0^3}$$

$$\boxed{\rho = \frac{3 \times 1 amu}{4\pi R_0^3}}$$

$$\rho = \frac{3 \times 1.6 \times 10^{-27}}{4 \times 3.14 \times (1.25 \times 10^{-15})^3} \approx 2 \times 10^{17} \, kg/m^3$$

MASS-ENERGY RELATION:-

According to Einstein relation of mass and energy

$$\boxed{E = mc^2 \text{ —— (I)}}$$

$$E = E_0 + K \cdot E$$

$$\Rightarrow E = m_0 c^2 + \frac{1}{2} m v^2 \text{ —— (II)}$$

where, E = total energy
E_0 = rest mass energy
m_0 = normal mass (rest)
m = mass (light speed)

From equation (I) and (II),

$$mc^2 = m_0 c^2 + K \cdot E$$

$$\Rightarrow K \cdot E = mc^2 - m_0 c^2$$

$$\Rightarrow K \cdot E = c^2 (m - m_0)$$

$$\Rightarrow \boxed{K \cdot E = \Delta m c^2}$$

MASS DEFECT:- (Δm)

The mass of nucleus is less than the sum of mass of all nucleons making it. The mass that dissapered is termed as 'Mass Defect'.

$$\boxed{\Delta m = [Z m_p + (A - Z) m_n] - M} \text{ , } M = \text{mass of nucleus.}$$

- Mass defect is taken in amu.

NUCLEAR BINDING ENERGY :-

Binding energy :- (ΔE_b)

It is the amount of energy required to separate all nucleons from the nucleus.

$$\boxed{\Delta E_b = [Zm_p + (A-Z)m_n - M]\,C^2}$$

$$\boxed{\text{Energy in 1 amu} = 931\ \text{MeV}}$$

$$\Delta E_b = \Delta m c^2 \quad \text{—— (I)}$$

If mass defect is taken in amu,

$$\Delta m = [Zm_p + (A-Z)m_n - M]\ \text{amu} \quad \text{—(II)}$$

Substituting eqn (II) in eqn (I)

$$\Delta E_b = [Zm_p + (A-Z)m_n - M]\, 931\ \text{MeV}$$

$$\Rightarrow \boxed{\Delta E_b = \Delta m \times 931\ \text{MeV}}$$

- More binding energy means more stable nucleus.

Nuclear binding energy per nucleons :-

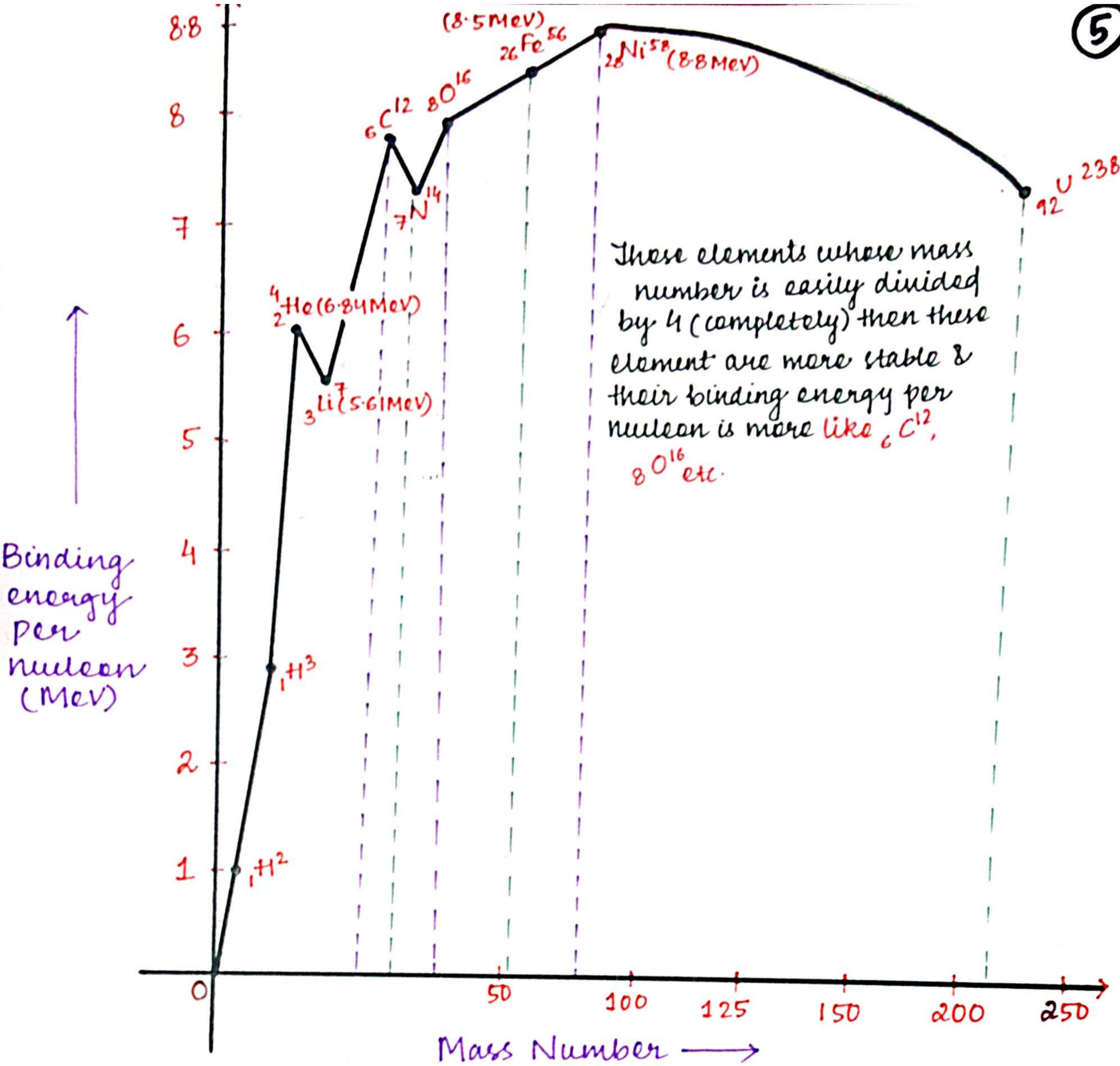
The ratio of total binding energy of nucleus to total number of nucleons is defined as nuclear binding energy per nucleons.

$$\bar{E}_b = \frac{\text{Total binding energy}}{\text{Number of nucleons}(A)}$$

$$\Rightarrow \bar{E}_b = \frac{E_b}{A}$$

- The average energy required to release nucleons from a nucleus is called binding energy per nucleons.

BINDING ENERGY CURVE :-



The following are the features of the plot:-

1. Average binding energy per nucleon for mass number less than 3 is very small. (hydrogen).

2. Some nuclei with mass number (3 to 20) have large binding energy per nucleon than their neighbouring nuclei. For eg:- $_2He^4$, $_4Be^8$, $_6C^{12}$, $_8O^{16}$ and $_{10}Ne^{20}$.

3. For (30-56) binding energy per nucleons increases gradually till it attains a max. value 8.8 MeV. Thus, Iron, nickel are stable element.

(4) For nuclei whose mass number is greater than 56, their binding energy per nucleon decreases. For uranium, one of the heaviest natural element, the binding energy per nucleon drops to 7.5 MeV

CONCLUSION:-

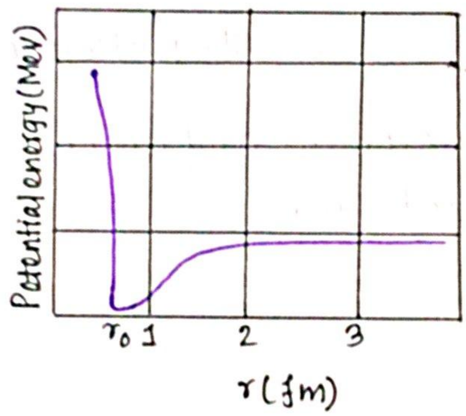
(1) NUCLEAR FISSION:-

When a heavy nucleus spitts up into lighter nuclei (eg. uranium) then binding energy per nucleons of lighter nuclei is more than that of the original heavy nucleus. This process is called nuclear fission.

(2) NUCLEAR FUSION:-

When two very light nuclei (eg hydrogen) combine to form a heavy nucleus then binding energy per nucleons of heavy nucleus becomes more than the lighter nuclei. In other words, the nucleons of the fused heavy nucleus are tightly bound. i.e energy is released. This process is called as nuclear fusion

NUCLEAR FORCE:-



Some of the important characterstics are:-

1. Nuclear forces are independent of charge.
2. Nuclear forces are very short range forces.
3. They are (dependent) on spin or angular momentum of nuclei
4. The nuclear force is much stronger than the coulomb force acting between charges or gravitational forces between masses.

RADIOACTIVITY:-

The unstable nuclei gains stability by emitting α-particles or β-particles and γ- EM waves. This phenomenon is called radioactivity

LAW OF RADIOACTIVE DECAY:-

- It is also known as Rutherford and soddy law)
- Radioactivity is a random process.

STATISTICAL LAW:-

When there is a large number of nuclei, rate of decay or disintegratio is directly proportional to the number of nuclei in the sample.

$$\text{Rate of decay} = \frac{\text{no. of nuclei decays}}{\text{time}}$$

$$= \frac{-dN}{dt}$$

$$\Rightarrow \frac{-dN}{dt} \propto N$$

$$\Rightarrow \boxed{\frac{-dN}{dt} = \lambda N}$$

λ = decay constant or disintegration constant.

$\lambda \rightarrow$ depends on choice of element and isotope.

$N \rightarrow$ number of undecayed nuclei

$$\int_{N_0}^{N} \frac{dN}{N} = \int_0^t -\lambda dt$$

$$\Rightarrow [\log N]_{N_0}^{N} = -\lambda(t)_0^t$$

$$\Rightarrow \log N - \log N_0 = -\lambda(t-0)$$

$$\Rightarrow \boxed{\log \frac{N}{N_0} = -\lambda t}$$ Logarithmic form

$$\frac{N}{N_0} = e^{-\lambda t}$$

$N_0 \rightarrow$ initial number of nuclei

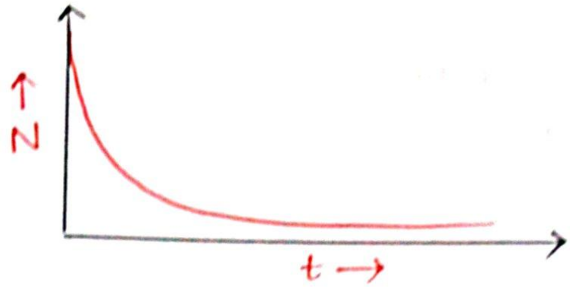
$$\Rightarrow \boxed{N = N_0 e^{-\lambda t}}$$ Exponential form.

DECAY CONSTANT (λ):-

when $t = 1/\lambda$, $N = N_0 e^{-1} = N_0(1/e) = 0.368 N_0$

Decay constant is the reciprocal of time in which no. of nuclei left undecayed at 36.8% of initial number of nuclei.

GRAPH OF N VS t :-



HALF LIFE OF DECAY ($t_{1/2}$) :-

when $t = t_{1/2}$, $N = N_0/2$

$$N = N_0 e^{-\lambda t}$$

$$\log\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\Rightarrow \log\left(\frac{N_0}{2N_0}\right) = -\lambda t_{1/2}$$

$$\Rightarrow \log(2)^{-1} = -\lambda t_{1/2}$$

$$\Rightarrow -\log 2 = -\lambda t_{1/2}$$

$$\Rightarrow t_{1/2} = \frac{\log 2}{\lambda}$$

$$\Rightarrow \boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

NUMBER OF NUCLEI LEFT AFTER 'n' HALF LIVES:-

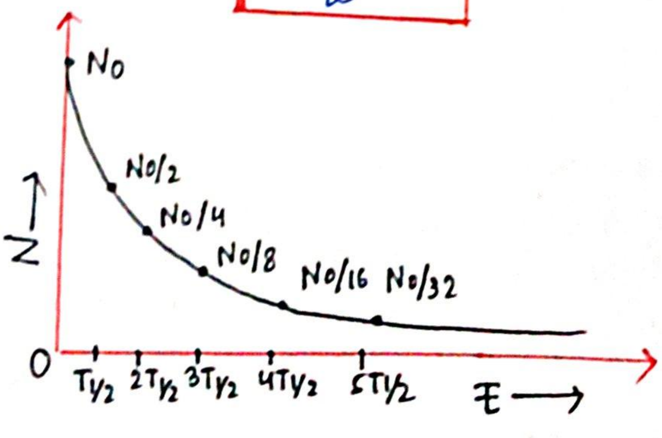
After 1 half life, $N = \frac{N_0}{2} = \frac{N_0}{2}$

After 2 half life, $N = \frac{N_0}{4} = \frac{N_0}{2^2}$

After 3 half life, $N = \frac{N_0}{8} = \frac{N_0}{2^3}$

⋮

After n half life, $\boxed{N = \frac{N_0}{2^n}}$.



ACTIVITY OF A RADIOACTIVE SAMPLE:-

$$\text{Activity} = \frac{\text{Rate of decay}}{\text{Disintegration}}$$

$$R = -\frac{dN}{dt}$$

$$\Rightarrow R = \lambda N$$

$$\Rightarrow R = \lambda N_0 e^{-\lambda t}$$

$$\boxed{R = R_0 e^{-\lambda t}}$$

S.I units of activity:-

① Becquerel (Bq)

② Curie (Ci)

③ Rutherford (rd)

α-DECAY:-

① α-particle is helium nucleus
(2p, 2n)

② Mass = 4 amu
Charge = +2e

③ $${}^{A}_{Z}X \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$$

Parent nucleus — Daughter nucleus

eg: $${}^{238}_{92}U \rightarrow {}^{4}_{2}He + {}^{234}_{90}Th + Q$$

④ $$Q = [m_X - m_Y - m_{He}]c^2$$

β-decay:-

ⓐ $\beta^{\ominus}$ decay (${}^{0}_{-1}e$ or ${}^{0}_{-1}\beta$)

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$$

(Antineutrino)

eg:- $^{14}_{6}C \rightarrow ^{0}_{-1}e + ^{14}_{7}N + \bar{\nu}$

* Actually $\beta^{\ominus}$ decay is conversion of a neutron to a proton inside nucleus.

(b) $\beta^{\oplus}$ decay:-

(1) $^{A}_{Z}X \longrightarrow ^{A}_{Z-1}Y + ^{0}_{+1}e + \nu$

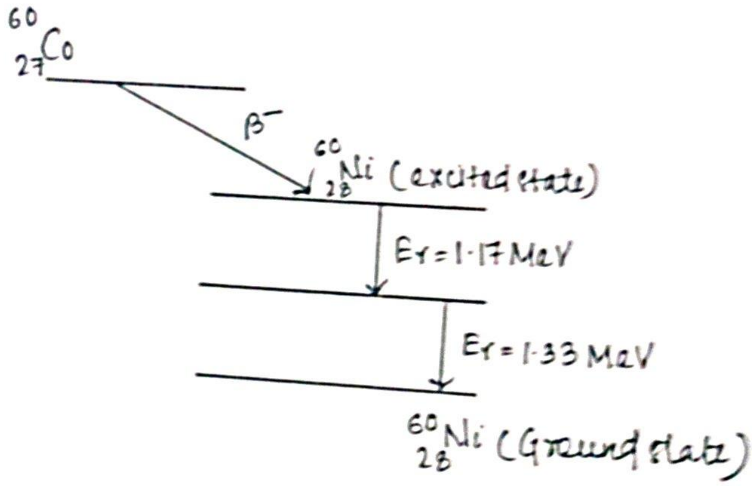
eg:- $^{22}_{11}Na \rightarrow ^{22}_{10}Ne + ^{0}_{+1}e + \nu$

GAMMA DECAY:-

The process of emission of γ-ray photon during the radioactive disintegration of nucleus is called gamma decay.

$$\underset{\text{(Excited state)}}{^{A}_{Z}X} \longrightarrow \underset{\text{(Ground state)}}{^{A}_{Z}X + \gamma}$$

eg:-



NUCLEAR FISSION:-

In this nuclear reaction, a heavy nucleus splits into lighter nuclei and large amount of energy is produced.

eg:- $^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{141}_{56}Ba + ^{92}_{36}Kr + 3\,^{1}_{0}n + Q$

When a slow moving neutron strikes with uranium splits into barium and krypton.

NUCLEAR FUSION :-

The process in which two very light nuclei combine to form a nucleus with a large mass number along with release of large amount of energy is called fusion.

Eg:-

$$_{1}^{3}H + {}_{1}H^{2} \rightarrow {}_{2}He^{4} + {}_{0}n^{1} + Q\ (17.6\,MeV)$$

$$_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{1}H^{3} + {}_{1}H^{1} + Q\ (4.0\,MeV)$$

- Nuclear fusion is known as thermo nuclear reaction because it cannot take place so easily.
- A temperature of the order of 10^{8} kelvin is required to start nuclear fusion.