AIM

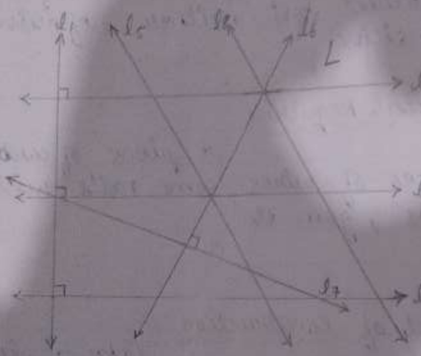
To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \perp m\}$ is symmetric but neither reflexive nor transitive.

Material Required

A piece of cardboard, 8 pieces of wires, some nails, white paper, gum etc.

Method of Construction

Take a piece of cardboard and paste a white paper on it. Fix all the 8 wires randomly on the cardboard with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in the given figure on next page.



Figure

Demonstuation

1. Let the wires represent the line $l_1, l_2, \ldots l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4 (as in figure).
3. l_6 is perpendicular to l_7.
4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.
5. $(l_1, l_2), (l_1, l_3), (l_1, l_4), (l_6, l_7) \in R$

Observation

1. In Figure, no line is perpendicular to itself, so relation $R = \{(l, m) : l \perp m\}$ ______ reflexive.

2. In Figure, $l_1 \perp l_2$. Is $l_2 \perp l_1$? ______ (Yes/No)
$\therefore (l_1, l_2) \in R \Rightarrow (l_2, l_1)$ ______ R ($\notin$ / $\in$)
Similarly, $(l_3 \perp l_1)$. Is $l_1 \perp l_3$? (Yes/No)
$\therefore (l_3, l_1) \in R \Rightarrow (l_1, l_3)$ ______ R ($\notin$ / $\in$)
Also, $l_6 \perp l_7$. Is $l_7 \perp l_6$? (Yes/No)
$\therefore (l_6, l_7) \in R \Rightarrow (l_7, l_6)$ R ($\notin$ / $\in$)
$\therefore$ The relation R ______ symmetric (is/is not)

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3. In Figure, $l_2 \perp l_1$ and $l_1 \perp l_3$.
Is $l_2 \perp l_3$? (Yes/No)
i.e. $(l_2, l_1) \in R$ and $(l_1, l_3) \in R$
$\Rightarrow (l_2, l_3)$. R ($\cancel{\notin}$ / $\in$)
$\therefore$ The relation R . transitive (is/is not)

Result

The given relation is symmetric but neither reflexive nor transitive.

Teacher Signature

Aim

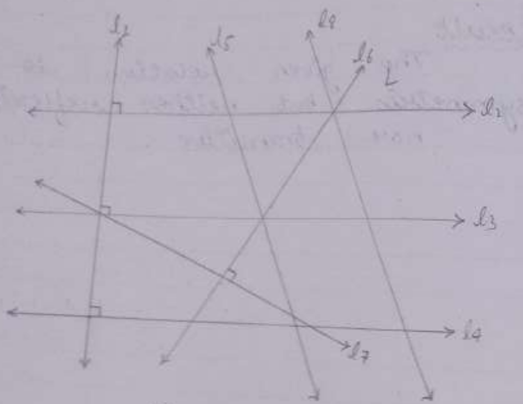
To verify that the relation R in the set L of all lines in a plane, defined by $R = \{(l, m) : l \parallel m\}$ is an equivalence relation.

Material Required

A piece of cardboard, 8 pieces of wire, some nails, white paper, gum, etc.

Method of construction

Take a piece of cardboard and paste a white paper on it. Fix all the 8 wires randomly on the cardboard with the help of nails such that some of them are parallel, some are perpendicular to each other and some are inclined as shown in the figure.



Figure

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Demonstration

1. Let the wires represent the lines $l_1, l_2, \ldots\ldots, l_8$.
2. l_1 is perpendicular to each of the lines l_2, l_3, l_4.
3. l_6 is perpendicular to l_7.
4. l_2 is parallel to l_3, l_3 is parallel to l_4 and l_5 is parallel to l_8.
5. $(l_2, l_3), (l_3, l_4), (l_5, l_8) \in R$

Observation

1. In Figure, every line is parallel to itself. So, the relation $R = \{(l, m) : l \parallel m\}$ reflexive relation (is / is not).

2. In Figure, observe that $l_2 \parallel l_3$. Is $l_3 \parallel l_2$? (Yes/No)
So, $(l_2, l_3) \in R \Rightarrow (l_3, l_2)$ R ($\notin$/$\in$)
Similarly, $l_3 \parallel l_4$. Is $l_4 \parallel l_3$? (Yes/No)
So, $(l_3, l_4) \in R \Rightarrow (l_4, l_3)$ R ($\notin$/$\in$)

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and $(l_5, l_8) \in R \Rightarrow (l_8, l_5)$ R $(\notin/\in)$
$\therefore$ The relation R symmetric relation.

3. In Figure, observe that $l_2 \parallel l_3$ and $l_3 \parallel l_4$. Is $l_2 \parallel l_4$? (Yes/No)
So, $(l_2, l_3) \in R$ and $(l_3, l_4) \in R \Rightarrow (l_2, l_4)$ R.
Similarly $l_3 \parallel l_4$ and $l_4 \parallel l_2$. Is $l_3 \parallel l_2$? (Yes/No)
So, $(l_3, l_4) \in R$, $(l_4, l_2) \in R \Rightarrow (l_3, l_2)$ R
Thus, the relation R transitive relation (is/is not).
Hence, the relation R is reflexive, symmetric and transitive. So, R is an equivalence relation.

Result

The given relation is an equivalence relation

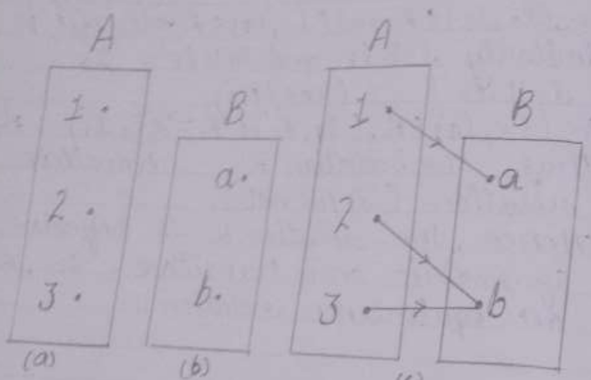

Figure 3.1

Aim

To demonstrate a function which is not one-one but is onto.

Material Required

A piece of cardboard, some nails, some strings of 5-6cm length, gum and some plastic strips.

Method of Construction

1. Paste a plastic strip on the left hand side of the cardboard and fix three nails on it as shown in the figure 3.1 (a) Name them as 1, 2, and 3.

2. Paste another strip on the right hand side of the cardboard and fix two nails in the plastic strip as shown in Figure 3.1 (b). Name them as a and b.

3. Join the nails on the left strip to the nails on the right strip as shown in Figure 3.1(c).

Demonstration

1. Take the set $A = \{1, 2, 3\}$
2. Take the set $B = \{a, b\}$
3. Join elements of set A to the elements of set B as shown in Figure 3.1(c).

Observation

1. The image of the element 1 of set A in set B is ______.
The image of the element 2 of set A in set B is ______.
The image of the element 3 of set A in set B is ______.
So, Figure 3.1 (c) represents a

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2. Every element in set A has a image in set B.
So, Function is _______.

3. The pre-image of each element of set B in set A _______ is _______. So, the function _______.

Result
The function is not one-one but is onto.

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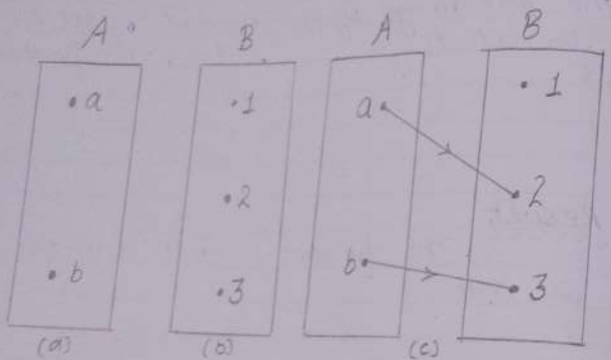

Figure 4.1

Aim

To demonstrate a function which is one-one but not onto.

Material Required

A piece of cardboard, some nails, some strings of 5-6 cm length, gum and some plastic strips.

Method of Construction

1. Paste a plastic strip on the left hand side of the cardboard and fix two nails on it as shown in Figure 4.1(b). Name them as a, ~~1~~ and b.
2. Paste another strip on the right hand side of the cardboard and fix three nails on it as shown in Figure 4.1(b). Name them as 1, 2 and 3.
3. Join the nails on the left strip to the nails on the right strip as shown

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in Figure 4.1(c).

Demonstration

1. Take set $A = \{a, b\}$
2. Take set $B = \{1, 2, 3\}$
3. Join elements of set A to the element of set B as shown in Figure 4.1(c).

Observation

1. The image of the element a of set A in set B is .
   The image of the element b of set A in set B is .
   So, the Figure 4.1(c) represents a
2. Every element in set A has image in set B. So, the function is
3. The preimage of the element 1 of set

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B in set A
function is . So, the
Thus, Figure 4.1(c) represents a function
which is but not onto.

Result
The function is one-one but not onto.

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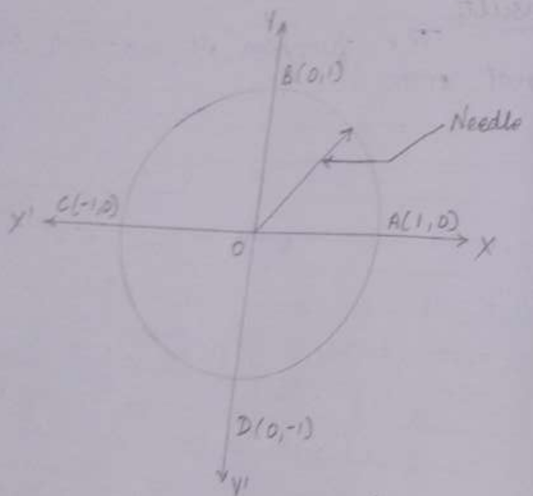

Figure 6.1

Aim

To explore the principal value of the function $\sin^{-1}x$ using a unit circle.

Materials Required

1. Cardboard, white chart sheet, string, ruler, glue, steel wire and needle.

Method of construction

1. Take a cardboard of a convenient size and paste a white paper on it.

2. Draw a unit circle with centre O on it.

3. Through the centre of the circle, draw two perpendicular lines X'OX and YOY' representing the x-axis and y-axis, respectively as shown in Figure 6.1.

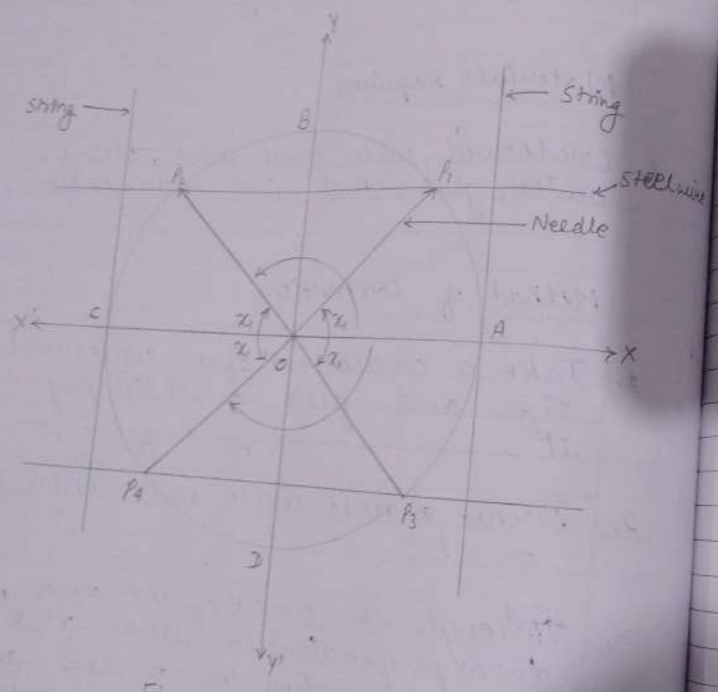

Figure 6.2

4. Mark the points A, C, B and D, where the circle cuts the x-axis and y-axis, respectively as shown in Fig 6.1.

5. Fix two nails on opposite side of cardboard which are parallel to y-axis. Fix one steel wire between the nails such that the wire can be moved parallel to x-axis as shown in Fig. 6.2.

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle Fig 6.2.

Demonstration

1. Keep the needle at an arbitrary angle, say x_1 with the positive direction of x-axis. Measure of angle in radian is equal to the length of intercepted arc of the unit circle.

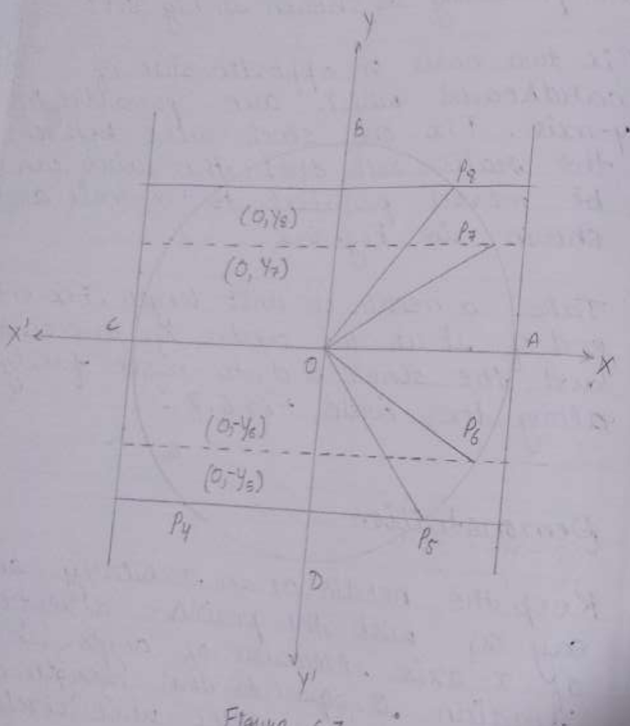

Figure 6.3

2. Slide the steel wire between the rails, parallel to x-axis such that the wire meets with free end of the needle (say P_1) (Fig 6.2)

3. Denote the y-coordinate of the point P_1 as y_1, where y_1 is perpendicular distance of steel wire from the x-axis of the unit circle giving $y_1 = \sin x_1$

4. Rotate the needle further anticlockwise and keep it at the angle $\pi - x_1$. Find the value of y-coordinate of intersecting point P_3 with the help of sliding steel wire. Value of y-coordinate for the points P_1 and P_2 are same for the different value of angles, $y_1 = \sin x_1$ and $y_1 = \sin(\pi - x_1)$. This demonstrates that sine function is not one-to-one for angles considered in first and second quadrants.

5. Keep the needle at angle $-x_1$ and $(-\pi + x_1)$, respectively. By sliding down

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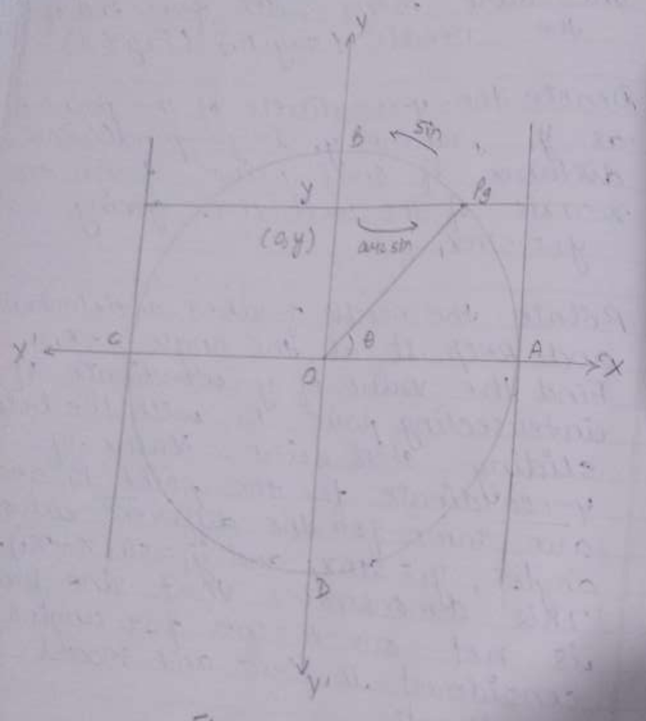

Figure 6.4

the steel wire parallel to x-axis, demonstrate that y-coordinate for the points P_3 and P_4 are the same and thus sine function is not one-to-one for points considered in 3rd and 4th quadrants as shown in Fig. 6.2.

6. However, the y-coordinate of the points P_3 and P_1 are different. Move the needle in anticlockwise direction starting from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ and look at the behaviour of y-coordinates of points P_5, P_6, P_7 and P_8 by sliding the steel wire parallel to x-axis accordingly, y-coordinate of points P_5, P_6, P_7 and P_8 are different (see fig 6.3). Hence, sine function is one-to-one in the domain $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and its range lies b/w -1 and 1.

7. Keep the needle at any arbitary angle say θ lying in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and denote the y-coordinate of the intersecting point P_9 as

y (see Fig 6.4). Then $Y = \sin\theta$ or θ = arc $\sin^{-1}y$) as sine function is one-one and onto in the domain $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and range $[-1, 1]$. So, its inverse arc sine function exist. The domain of arc sine function is $[-1, 1]$ and range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. This range is called principal value of arc sine function (or $\sin^{-1}$ function).

Observation

1. Sine function is non-negative in and quadrants.
2. For the quadrants 3rd and 4th, sine function is
3. $\theta = \sin^{-1}y \Rightarrow y = \quad \theta$, where $-\frac{\pi}{2} \leq \theta \leq$
4. The other domains of sine function on which it is one-one and onto provides for $\sin^{-1}x$ function.

Result

The principal value of $\sin^{-1}x$ is

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Aim

To find analytically the limit of a function $f(x)$ at $x = c$ and also check the continuity of the function at that point.

Material Required

Paper, pencil and calculator.

Method of construction

1. Consider the function given by $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 7, & x = 3 \end{cases}$

2. Take some points on the left side and some points on the right side of $c (=3)$, which are very near to c.

3. Calculate the corresponding values of $f(x)$ for each of the points considered in above step.

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4. Record the values of points on the left side and right side of c as x and the corresponding values of f(x) in a form of a table.

Demonstration

The values of x and f(x) are recorded as follws

1. Table 1 For points on left of $c(=3)$

x	2.9	2.99	2.999	2.9999	2.99999	2.999999	2.9999999
$f(x)$	5.9	5.99	5.999	5.9999	5.99999	5.999999	5.9999999

2. Table 2 For points on right of $c(=3)$

x	3.1	3.01	3.001	3.0001	3.00001	3.000001	3.0000001
$f(x)$	6.1	6.01	6.001	6.0001	6.00001	6.000001	6.0000001

Observation

1. The value of f(x) is approaching to

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, as $x \to 3$ from the left.

2. The value of $f(x)$ is approaching to , as $x \to 3$ from the right.

3. So, $\lim_{x \to 3^-} f(x) =$ and $\lim_{x \to 3^+} f(x) =$

4. Therefore, $\lim_{x \to 3} f(x) =$, $f(3) =$

5. Is $\lim_{x \to 3} f(x) = f(3)$?

6. Since $f(c) \neq \lim_{x \to c} f(x)$, so the function is at $x = 3$.

Result

This activity is very helpful in understanding the existence of limit of a function at a point and check its continuity.

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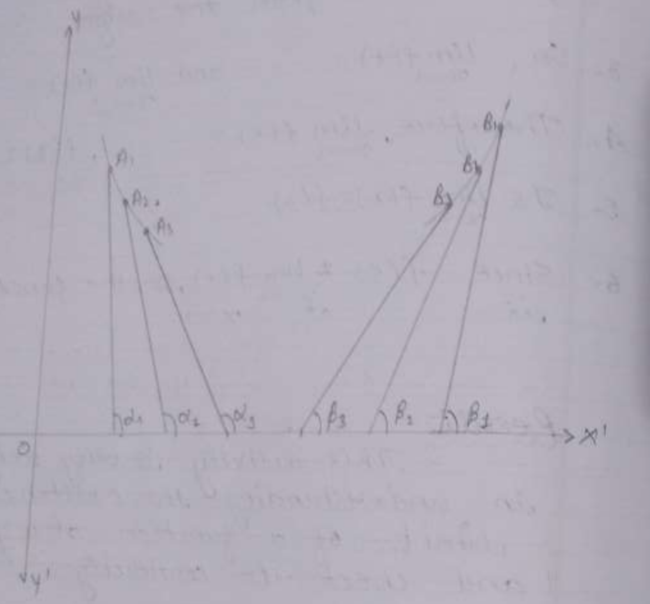

Figure

Aim

To understand the concepts of decreasing and increasing functions.

Materials Required

Some pieces of wire of different lengths, piece of cardboard, white paper, gum, geometry box, trigonometric tables.

Method of construction

1. Take a piece of cardboard of a convenient size and using gum paste a white paper on it.

2. Take two pieces of wire of length say 25 cm each and fix them on the white paper to represent the X-axis and Y-axis.

3. Take two more pieces of wire each of suitable length and bend them in the shape of curve representing two functions, fix them on the paper as

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shown in Figure.

4. Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.

Demonstration :

1. Take one straight wire and place it on the left curve such that it is tangent to the curve at the point say A_1 and making an angle α_1 with the positive direction of X-axis.

2. α_1 is an obtuse angle, so $\tan \alpha_1$ is negative, i.e the slope of the tangent at A_1 is negative.

3. Take another two points say A_2 and A_3 on the same curve, and make tangents using the same wire, at A_2 and A_3 making angles α_2 and α_3 respectively with positive direction of X-axis.

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4. Here again α_2 and α_3 are obtuse angles and therefore slope of the tangents $\tan\alpha_2$ and $\tan\alpha_3$ are both negative, i.e. derivatives of functions at A_2 and A_3 are negative.

5. The function given by the left curve is a decreasing function.

Observation

1. $\alpha_1 =$ >90
$\alpha_2 =$ $>$
$\alpha_3 =$ $>$
$\tan\alpha_1 =$, negative
$\tan\alpha_2 =$, ( )
$\tan\alpha_3 =$, ( )
Thus the function is

2. $\beta_1 =$, <90
$\beta_2 =$, $<$
$\beta_3 =$, $<$
$\tan\beta_1 =$, positive
$\tan\beta_2 =$
$\tan\beta_3 =$

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Thus, function is

Result

We have found points on the curve where the function is decreasing and increasing.

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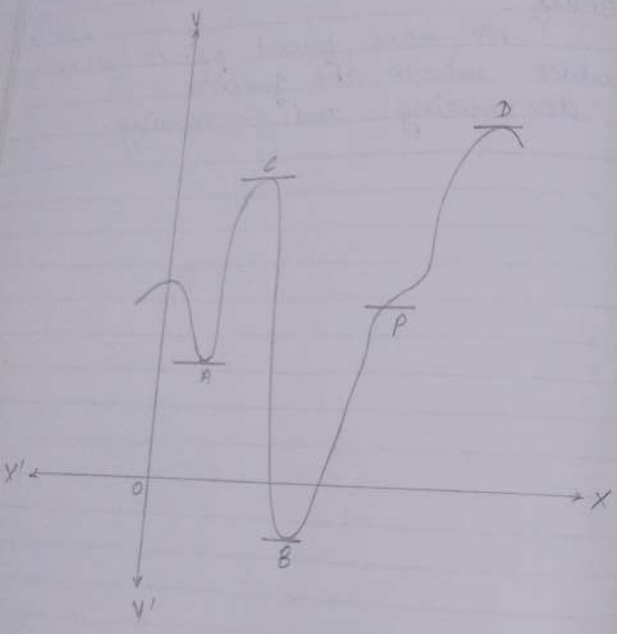

Figure

Aim

To understand the concept of local maxima, local minima and point of inflection.

Material Required

Drawing board, some pieces of wire, glue, white sheet.

Method of Construction

1. Take a drawing board of suitable size and paste a white sheet on it.

2. Take two pieces of wires and fix them on the sheet on the drawing board in the form of X-axis and Y-axis.

3. Take another wire of suitable length and bend it in the shape of a curve. Fix this curved wire on the white sheet pasted on the drawing board, as shown in Figure.

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4. Take five more wires each of length say 1.5 cm and fix them at the points A, C, B, P and D as shown in Figure.

Demonstration

1. In the figure, wires at points A, C, B, P and D represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e. the value of first derivative at these points is zero. The tangent at P intersect the curve.

2. The sign of first derivative changes from negative to positive at the points A and B. So, they are the points of local minima.

3. The sign of first derivative changes from positive to negative at the points C and D. So, they are the points of local maxima.

4. The sign of first derivative does not

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change at the point P. So, it ~~is~~ is the point of inflection.

Observation

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is

2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is

3. Sign of the first derivative at a point on the curve to immediate left of B is .

4. Sign of the first derivative at a point on the curve to immediate right of B is

5. Sign of the first derivative at a point on the curve to immediate left of C is

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6. Sign of the first derivative at a point on the curve to immediate right of C is ______.

7. Sign of the first derivative at a point on the curve to immediate left of D is ______.

8. Sign of the first derivative at a point on the curve to immediate right of D is ______.

9. Sign of the first derivative at a point immediate left of P is ______ and immediate right of P is ______.

10. A and B are points of local ______

11. C and D are points of local ______

12. P is a point of ______.

Result

We have found point of local maxima, local minima and point of inflection

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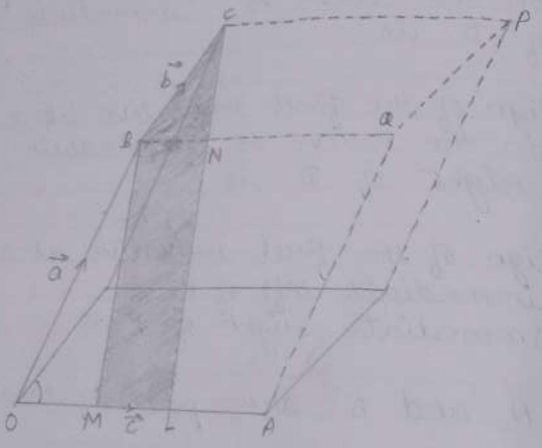

Figure

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Aim

To verify geometrically $[\vec{c}\times(\vec{a}+\vec{b}) = \vec{c}\times\vec{a} + \vec{c}\times\vec{b}]$

Material Required

Geometry box, cardboard, white paper, cutter, sketch pen, cellotape.

Method of construction

1. Fix a white paper on the cardboard of suitable size.
2. Draw a line segment OA = (8 cm, say) and let it represent $\vec{c}$.
3. Draw another line segment OB (= 6cm, say) at an angle 60° with OA. Let $\vec{OB} = \vec{a}$
4. Draw BC = (5cm, say) making an angle (say 30°) with $\vec{OA}$. Let $\vec{BC} = \vec{b}$.
5. Draw perpendiculars BM, CL and BN.

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6. Complete parallelograms OAPC, OAQB and BQPC.

Demonstration

1. $\vec{OC} = \vec{OB} + \vec{BC} = \vec{a} + \vec{b}$

2. Let $\angle COA = \alpha$

3. We know that, $|\vec{c} \times (\vec{a}+\vec{b})| = |\vec{c}|\,|\vec{a}+\vec{b}|\sin\alpha$
= Area of parallelogram OAPC.

4. Again, $|\vec{c} \times \vec{a}|$ = Area of ||gm OAQB.

5. Also, $|\vec{c} \times \vec{b}|$ = Area of ||gm BQPC.

6. Therefore, area of ||gm OAPC = (OA)(CL)
= (OA)(BM+NC) = (OA)(BM) + (OA)(NC)
= Area of ||gm OAQB + area of ||gm BQPC
$= |\vec{c} \times \vec{a}| + |\vec{c} \times \vec{b}|$
So, $|\vec{c} \times (\vec{a}+\vec{b})| = |\vec{c} \times \vec{a}| + |\vec{c} \times \vec{b}|$

Direction of each of these vectors, $\vec{c} \times (\vec{a}+\vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ is ⊥ to same plane.
So, $\vec{c} \times (\vec{a}+\vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

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Observation

$|\vec{c}| = |\vec{OA}| = OA =$

$|\vec{a}+\vec{b}| = |\vec{OC}| = OC =$

$CL =$

$|\vec{c}\times(\vec{a}+\vec{b})| =$ Ar. of ||gm OAPC — (i)

$= (OA)(CL) =$ sq units

$|\vec{c}\times\vec{a}| =$ Ar. of ||gm OAQB — (ii)

$= (OA)(BM) = \quad \times \quad =$

$|\vec{c}\times\vec{b}| =$ Ar. of ||gm BQPC — (iii)

$= (OA)(CN) = \quad \times \quad =$

From eq (i), (ii) & (iii), we get

ar. of ||gm OAPC = ar. of ||gm OAQB + area of ||gm

Thus $|\vec{c}\times(\vec{a}+\vec{b})| = |\vec{c}\times\vec{a}| + |\vec{c}\times\vec{b}|$

$\vec{c}\times\vec{a}$, $\vec{c}\times\vec{b}$ and $\vec{c}\times(\vec{a}+\vec{b})$ are all in the direction of to the plane of paper.

Therefore, $\vec{c}\times(\vec{a}+\vec{b}) = \vec{c}\times\vec{a} +$

Result

We have verified that $\vec{c}\times(\vec{a}+\vec{b}) = \vec{c}\times\vec{a}+\vec{c}\times\vec{b}$.

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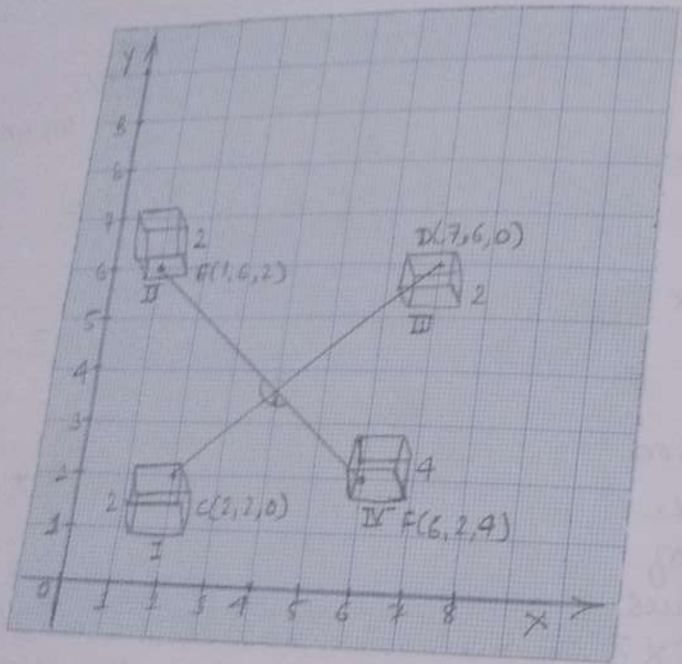

Aim

To measure the shortest distance between two skew-lines and verify it analytically.

Material Required

A piece of cardboard of size 40cm X 30cm, graph paper, three wooden block of size 2cm X 2cm X 2cm each and one wooden blocks of size 2cm X 2cm X 4cm, some wire of different lengths, set squares, gum, pen/pencil, etc.

Method of Construction

1. Paste a graph paper on a piece of cardboard
2. On the graph paper, draw two lines OA and OB to represent X-axis and Y-axis, respectively.
3. Name the three blocks of size 2cm X 2cm X 2cm as I, II and III. Name the other

wooden block of size 2cm × 2cm × 4cm as IV.

4. Place blocks I, II, III such that their base centres are at points (2,2), (1,6) and (7,6), respectively, and block IV with its base centre at (6,2).

5. Place a wire joining the points C and D, the centres of the top of the blocks I, and III and another wire joining the centres E and F of the bases of blocks II and IV as shown in Figure.

6. The lines represented by these two wire are skew-lines.

7. Take a wire and join it perpendicularly with the skew-lines and calculate the actual distance.

Demonstration

1. A set-square is placed in such a way that its one perpendicular side is along the wire CD.

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2. Move the set-square along CD till its other perpendicular side touches the other line.

3. Using set-squares, measure the distance between the two lines in this position. This is known as the shortest distance between two skew-lines.

4. Analytically, find the equation of the line joining C(2,2,0) and D(7,6,8) and other line joining E(1,6,2) and F(6,2,9) and find SD using $\left|\frac{(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2})}{|\vec{b_1}\times\vec{b_2}|}\right|$

The two distances obtained will be equal.

Observation

1. Coordinates of point C are

2. Coordinates of point D are

3. Coordinates of point E are

Teacher Signature ______

4. Coordinates of point F are

5. Equation of line CD is

6. Equation of line EF is

Shortest distance between CD and EF analytically =

Shortest distance by actual measurement =

The results so obtained are

Result

We have measured the shortest distance between two skew-lines.