1 History and Overview

Fracture is a problem that society has faced for as long as there have been man-made structures. The problem may actually be worse today than in previous centuries, because more can go wrong in our complex technological society. Major airline crashes, for instance, would not be possible without modern aerospace technology.

Fortunately, advances in the field of fracture mechanics have helped to offset some of the potential dangers posed by increasing technological complexity. Our understanding of how materials fail and our ability to prevent such failures have increased considerably since World War II. Much remains to be learned, however, and existing knowledge of fracture mechanics is not always applied when appropriate.

While catastrophic failures provide income for attorneys and consulting engineers, such events are detrimental to the economy as a whole. An economic study [1] estimated the annual cost of fracture in the U.S. in 1978 at $119 billion (in 1982 dollars), about 4% of the gross national product. Furthermore, this study estimated that the annual cost could be reduced by $35 billion if current technology were applied, and that further fracture mechanics research could reduce this figure by an additional $28 billion.

1.1 WHY STRUCTURES FAIL

The cause of most structural failures generally falls into one of the following categories:

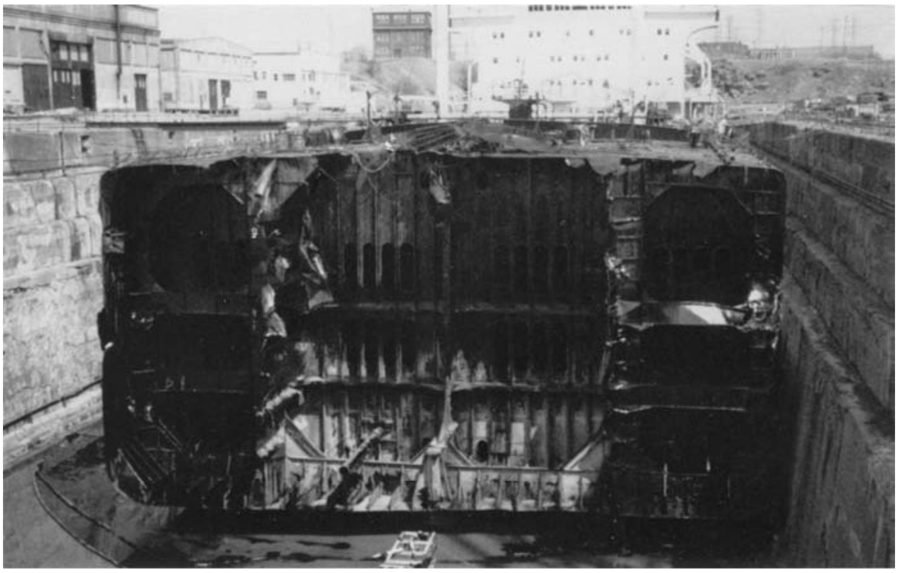
1. Negligence during design, construction, or operation of the structure.
2. Application of a new design or material, which produces an unexpected (and undesirable) result.

In the first instance, existing procedures are sufficient to avoid failure, but are not followed by one or more of the parties involved, due to human error, ignorance, or willful misconduct. Poor workmanship, inappropriate or substandard materials, errors in stress analysis, and operator error are examples of where the appropriate technology and experience are available, but not applied.

The second type of failure is much more difficult to prevent. When an "improved" design is introduced, invariably, there are factors that the designer does not anticipate. New materials can offer tremendous advantages, but also potential problems. Consequently, a new design or material should be placed into service only after extensive testing and analysis. Such an approach will reduce the frequency of failures, but not eliminate them entirely; there may be important factors that are overlooked during testing and analysis.

One of the most famous Type 2 failures is the brittle fracture of World War II Liberty ships (see Section 1.2.2). These ships, which were the first to have all-welded hulls, could be fabricated much faster and cheaper than earlier riveted designs, but a significant number of these vessels sustained serious fractures as a result of the design change. Today, virtually all steel ships are welded, but sufficient knowledge was gained from the Liberty ship failures to avoid similar problems in present structures.

Knowledge must be applied in order to be useful, however. Figure 1.1 shows an example of a Type 1 failure, where poor workmanship in a seemingly inconsequential structural detail caused a more recent fracture in a welded ship. In 1979, the Kurdistan oil tanker broke completely in two



(a)

(b)

FIGURE 1.1 The MSV Kurdistan oil tanker, which sustained a brittle fracture while sailing in the North Atlantic in 1979: (a) fractured vessel in dry dock and (b) bilge keel from which the fracture initiated. (Photographs provided by S.J. Garwood.)

while sailing in the North Atlantic [2]. The combination of warm oil in the tanker with cold water in contact with the outer hull produced substantial thermal stresses. The fracture initiated from a bilge keel that was improperly welded. The weld failed to penetrate the structural detail, resulting in a severe stress concentration. Although the hull steel had adequate toughness to prevent fracture initiation, it failed to stop the propagating crack.

Polymers, which are becoming more common in structural applications, provide a number of advantages over metals, but also have the potential for causing Type 2 failures. For example, polyethylene (PE) is currently the material of choice in natural gas transportation systems in the U.S. One advantage of PE piping is that maintenance can be performed on a small branch of the line without shutting down the entire system; a local area is shut down by applying a clamping tool to the PE pipe and stopping the flow of gas. The practice of pinch clamping has undoubtedly saved vast sums of money, but has also led to an unexpected problem.

In 1983, a section of a 4-in. diameter PE pipe developed a major leak. The gas collected beneath a residence where it ignited, resulting in severe damage to the house. Maintenance records and a visual inspection of the pipe indicated that it had been pinch clamped 6 years earlier in the region where the leak developed. A failure investigation [3] concluded that the pinch clamping operation was responsible for the failure. Microscopic examination of the pipe revealed that a small flaw apparently initiated on the inner surface of the pipe and grew through the wall. Figure 1.2 shows a low-magnification photograph of the fracture surface. Laboratory tests simulated the pinch clamping operation on sections of PE pipes; small thumbnail-shaped flaws (Figure 1.3) formed on the inner wall of the pipes, as a result of the severe strains that were applied. Fracture mechanics tests and analyses [3, 4] indicated that stresses in the pressurized pipe were sufficient to cause the observed time-dependent crack growth, i.e., growth from a small thumbnail flaw to a through-thickness crack over a period of 6 years.

The introduction of flaws in PE pipe by pinch clamping represents a Type 2 failure. The pinch clamping process was presumably tested thoroughly before it was applied in service, but no one anticipated that the procedure would introduce damage in the material that could lead to failure after several years in service. Although specific data are not available, pinch clamping has undoubtedly led to a significant number of gas leaks. The practice of pinch clamping is still widespread in the natural gas industry, but many companies and some states now require that a sleeve be fitted to the affected region in order to relieve the stresses locally. In addition, newer grades of PE pipe material have lower density and are less susceptible to damage by pinch clamping.

Some catastrophic events include elements both of Type 1 and Type 2 failures. On January 28, 1986, the Challenger Space Shuttle exploded because an O-ring seal in one of the main boosters did not respond well to cold weather. The shuttle represents relatively new technology, where

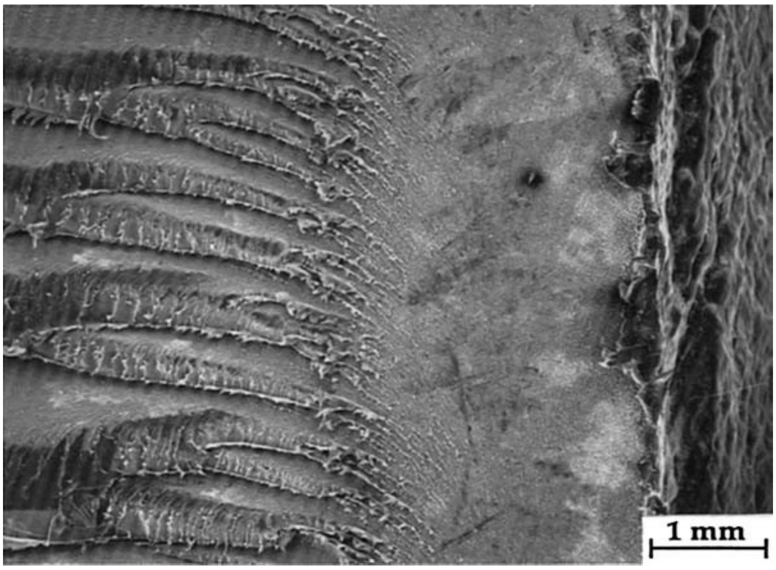


FIGURE 1.2 Fracture surface of a PE pipe that sustained time-dependent crack growth as a result of pinch clamping. (Taken from Jones, R.E. and Bradley, W.L., *Forensic Engineering,* Vol. I, 1987.) (Photograph provided by R.E. Jones, Jr.)

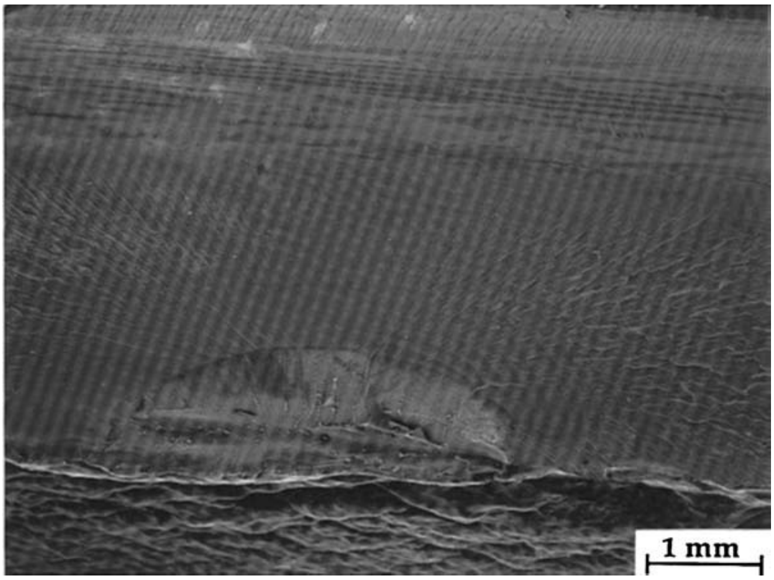


FIGURE 1.3 Thumbnail crack produced in a PE pipe after pinch clamping for 72 h. (Photograph provided by R.E. Jones, Jr.)

service experience is limited (Type 2), but engineers from the booster manufacturer suspected a potential problem with the O-ring seals and recommended that the launch be delayed (Type 1). Unfortunately, these engineers had little or no data to support their position and were unable to convince their managers or NASA officials. The tragic results of the decision to launch are well known.

On February 1, 2003, almost exactly 17 years after the Challenger accident, the Space Shuttle Columbia was destroyed during reentry. The apparent cause of the incident was foam insulation from the external tank striking the left wing during launch. This debris damaged insulation tiles on the underside of the wing, making the orbiter vulnerable to reentry temperatures that can reach 3000°F. The Columbia Accident Investigation Board (CAIB) was highly critical of NASA management for cultural traits and organizational practices that, according to the board, were detrimental to safety.

Over the past few decades, the field of fracture mechanics has undoubtedly prevented a substantial number of structural failures. We will never know how many lives have been saved or how much property damage has been avoided by applying this technology, because it is impossible to quantify disasters that *don't* happen. When applied correctly, fracture mechanics not only helps to prevent Type 1 failures but also reduces the frequency of Type 2 failures, because designers can rely on rational analysis rather than trial and error.

1.2 HISTORICAL PERSPECTIVE

Designing structures to avoid fracture is not a new idea. The fact that many structures commissioned by the Pharaohs of ancient Egypt and the Caesars of Rome are still standing is a testimony to the ability of early architects and engineers. In Europe, numerous buildings and bridges constructed during the Renaissance Period are still used for their intended purpose.

The ancient structures that are still standing today obviously represent successful designs. There were undoubtedly many more unsuccessful designs with much shorter life spans. Because knowledge of mechanics was limited prior to the time of Isaac Newton, workable designs were probably achieved largely by trial and error. The Romans supposedly tested each new bridge by requiring the design engineer to stand underneath while chariots drove over it. Such a practice would not only provide an incentive for developing good designs, but would also result in the social equivalent of Darwinian natural selection, where the worst engineers were removed from the profession.

The durability of ancient structures is particularly amazing when one considers that the choice of building materials prior to the Industrial Revolution was rather limited. Metals could not be produced in sufficient quantity to be formed into load-bearing members for buildings and bridges. The primary construction materials prior to the 19th century were timber, brick, and mortar; only the latter two materials were usually practical for large structures such as cathedrals, because trees of sufficient size for support beams were rare.

Brick and mortar are relatively brittle and are unreliable for carrying tensile loads. Consequently, pre-Industrial Revolution structures were usually designed to be loaded in compression. Figure 1.4 schematically illustrates a Roman bridge design. The arch shape causes compressive rather than tensile stresses to be transmitted through the structure.

The arch is the predominant shape in pre-Industrial Revolution architecture. Windows and roof spans were arched in order to maintain compressive loading. For example, Figure 1.5 shows two windows and a portion of the ceiling in King's College Chapel in Cambridge, England. Although these shapes are aesthetically pleasing, their primary purpose is more pragmatic.

Compressively loaded structures are obviously stable, since some have lasted for many centuries; the pyramids in Egypt are the epitome of a stable design.

With the Industrial Revolution came mass production of iron and steel. (Or, conversely, one might argue that mass production of iron and steel fueled the Industrial Revolution.) The availability of relatively ductile construction materials removed the earlier restrictions on design. It was finally feasible to build structures that carried tensile stresses. Note the difference between the design of the Tower Bridge in London (Figure 1.6) and the earlier bridge design (Figure 1.4).

The change from brick and mortar structures loaded in compression to steel structures in tension brought problems, however. Occasionally, a steel structure would fail unexpectedly at stresses well below the anticipated tensile strength. One of the most famous of these failures was the rupture of a molasses tank in Boston in January 1919 [5]. Over 2 million gallons of molasses were spilled, resulting in 12 deaths, 40 injuries, massive property damage, and several drowned horses.

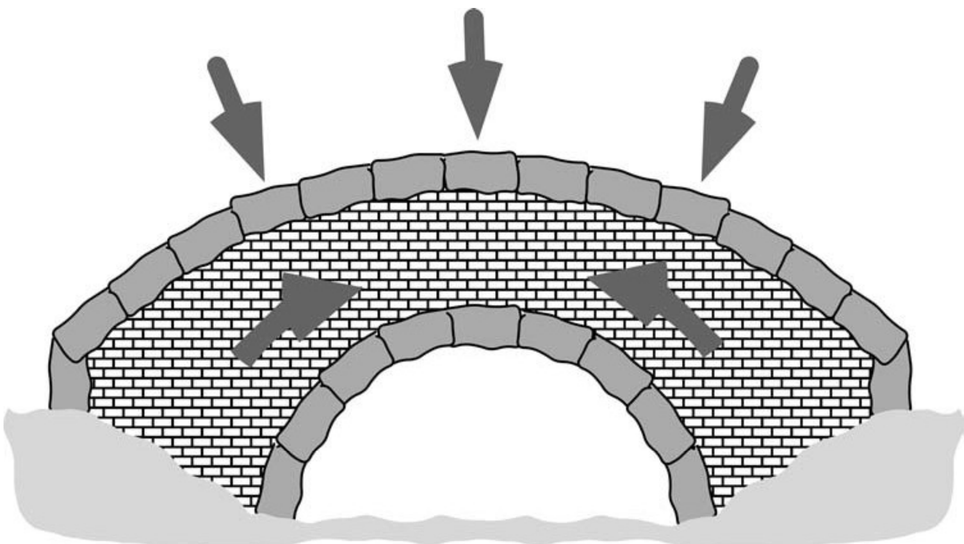


FIGURE 1.4 Schematic Roman bridge design. The arch shape of the bridge causes loads to be transmitted through the structure as compressive stresses.

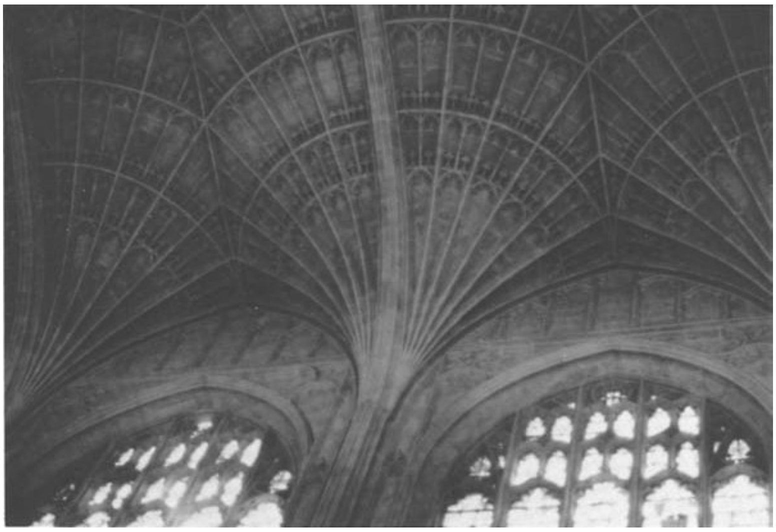


FIGURE 1.5 Kings College Chapel in Cambridge, England. This structure was completed in 1515.

The cause of the failure of the molasses tank was largely a mystery at the time. In the first edition of his elasticity text published in 1892, Love [6] remarked that "the conditions of rupture are but vaguely understood." Designers typically applied safety factors of 10 or more (based on the tensile strength) in an effort to avoid these seemingly random failures.

1.2.1 Early Fracture Research

Experiments performed by Leonardo da Vinci several centuries earlier provided some clues as to the root cause of fracture. He measured the strength of iron wires and found that the strength varied inversely with wire length. These results implied that flaws in the material controlled the strength;

FIGURE 1.6 The Tower Bridge in London, completed in 1894. Note the modern beam design, made possible by the availability of steel support girders.

a longer wire corresponded to a larger sample volume, and a higher probability of sampling a region containing a flaw. These results were only qualitative, however.

A quantitative connection between fracture stress and flaw size came from the work of Griffith, which was published in 1920 [7]. He applied a stress analysis of an elliptical hole (performed by Inglis [8] seven years earlier) to the unstable propagation of a crack. Griffith invoked the first law of thermodynamics to formulate a fracture theory based on a simple energy balance. According to this theory, a flaw becomes unstable, and thus fracture occurs, when the strain-energy change that results from an increment of crack growth is sufficient to overcome the surface energy of the material (see Section 2.3). Griffith's model correctly predicted the relationship between strength and flaw size in glass specimens. Subsequent efforts to apply the Griffith model to metals were unsuccessful. Since this model assumes that the work of fracture comes exclusively from the surface energy of the material, the Griffith approach applies only to ideally brittle solids. A modification to Griffith's model, that made it applicable to metals, did not come until 1948.

1.2.2 The Liberty Ships

The mechanics of fracture progressed from being a scientific curiosity to an engineering discipline, primarily because of what happened to the Liberty ships during World War II [9].

In the early days of World War II, the U.S. was supplying ships and planes to Great Britain under the Lend-Lease Act. Britain's greatest need at the time was for cargo ships to carry supplies. The German navy was sinking cargo ships at three times the rate at which they could be replaced with existing ship-building procedures.

Under the guidance of Henry Kaiser, a famous construction engineer whose previous projects included the Hoover Dam, the U.S. developed a revolutionary procedure for fabricating ships quickly. These new vessels, which became known as the Liberty ships, had an all-welded hull, as opposed to the riveted construction of traditional ship designs.

The Liberty ship program was a resounding success, until one day in 1943, when one of the vessels broke completely in two while sailing between Siberia and Alaska. Subsequent fractures occurred in other Liberty ships. Of the roughly 2700 Liberty ships built during World War II, approximately 400 sustained fractures, of which 90 were considered serious. In 20 ships the failure was essentially total, and about half of these broke completely in two.

Investigations revealed that the Liberty ship failures were caused by a combination of three factors:

- The welds, which were produced by a semi-skilled work force, contained crack-like flaws.
- Most of the fractures initiated on the deck at square hatch corners, where there was a local stress concentration.
- The steel from which the Liberty ships were made had poor toughness, as measured by Charpy impact tests.

The steel in question had always been adequate for riveted ships because fracture could not propagate across panels that were joined by rivets. A welded structure, however, is essentially a single piece of metal; propagating cracks in the Liberty ships encountered no significant barriers, and were sometimes able to traverse the entire hull.

Once the causes of failure were identified, the remaining Liberty ships were retrofitted with rounded reinforcements at the hatch corners. In addition, high toughness steel crack-arrester plates were riveted to the deck at strategic locations. These corrections prevented further serious fractures.

In the longer term, structural steels were developed with vastly improved toughness, and weld quality control standards were developed. Also, a group of researchers at the Naval Research Laboratory in Washington, DC. studied the fracture problem in detail. The field we now know as fracture mechanics was born in this lab during the decade following the war.

1.2.3 Post-War Fracture Mechanics Research[1]

The fracture mechanics research group at the Naval Research Laboratory was led by Dr. G.R. Irwin. After studying the early work of Inglis, Griffith, and others, Irwin concluded that the basic tools needed to analyze fracture were already available. Irwin's first major contribution was to extend the Griffith approach to metals by including the energy dissipated by local plastic flow [10]. Orowan independently proposed a similar modification to the Griffith theory [11]. During this same period, Mott [12] extended the Griffith theory to a rapidly propagating crack.

In 1956, Irwin [13] developed the energy release rate concept, which was derived from the Griffith theory but in a form that was more useful for solving engineering problems. Shortly afterward, several of Irwin's colleagues brought to his attention a paper by Westergaard [14] that was published in 1938. Westergaard had developed a semi-inverse technique for analyzing stresses and displacements ahead of a sharp crack. Irwin [15] used the Westergaard approach to show that the stresses and displacements near the crack-tip could be described by a single constant that was related to the energy release rate. This crack-tip characterizing parameter later became known as the "stress-intensity factor." During this same period of time, Williams [16] applied a somewhat different technique to derive crack tip solutions that were essentially identical to Irwin's results.

A number of successful early applications of fracture mechanics bolstered the standing of this new field in the engineering community. In 1956, Wells [17] used fracture mechanics to show that the fuselage failures in several Comet jet aircraft resulted from fatigue cracks reaching a critical size. These cracks initiated at windows and were caused by insufficient reinforcement locally, combined with square corners that produced a severe stress concentration. (Recall the unfortunate hatch design in the Liberty ships.) A second early application of fracture mechanics occurred at General Electric in 1957. Winne and Wundt [18] applied Irwin's energy release rate approach to the failure of large rotors from steam turbines. They were able to predict the bursting behavior of large disks extracted from rotor forgings, and applied this knowledge to the prevention of fracture in actual rotors.

It seems that all great ideas encounter stiff opposition initially, and fracture mechanics is no exception. Although the U.S. military and the electric power generating industry were very supportive of the early work in this field, such was not the case in all provinces of government and industry. Several government agencies openly discouraged research in this area.

In 1960, Paris and his coworkers [19] failed to find a receptive audience for their ideas on applying fracture mechanics principles to fatigue crack growth. Although Paris et al. provided convincing experimental and theoretical arguments for their approach, it seems that design engineers were not yet ready to abandon their S-N curves in favor of a more rigorous approach to fatigue design. The resistance to this work was so intense that Paris and his colleagues were unable to find a peer-reviewed technical journal that was willing to publish their manuscript. They finally opted to publish their work in a University of Washington periodical entitled *The Trend in Engineering*.

1.2.4 Fracture Mechanics from 1960 to 1980

The Second World War obviously separates two distinct eras in the history of fracture mechanics. There is, however, some ambiguity as to how the period between the end of the war and the present should be divided. One possible historical boundary occursed around 1960, when the fundamentals of linear elastic fracture mechanics were fairly well established, and researchers turned their attention to crack-tip plasticity.

[1] For an excellent summary of early fracture mechanics research, refer to *Fracture Mechanics Retrospective: Early Classic Papers* (*1913–1965*), John M. Barsom, ed., American Society of Testing and Materials (RPS 1), Philadelphia, PA, 1987. This volume contains reprints of 17 classic papers, as well as a complete bibliography of fracture mechanics papers published up to 1965.

Linear elastic fracture mechanics (LEFM) ceases to be valid when significant plastic deformation precedes failure. During a relatively short time period (1960–1961) several researchers developed analyses to correct for yielding at the crack tip, including Irwin [20], Dugdale [21], Barenblatt [22], and Wells [23]. The Irwin plastic zone correction [20] was a relatively simple extension of LEFM, while Dugdale [21] and Barenblatt [22] each developed somewhat more elaborate models based on a narrow strip of yielded material at the crack tip.

Wells [23] proposed the displacement of the crack faces as an alternative fracture criterion when significant plasticity precedes failure. Previously, Wells had worked with Irwin while on a sabbatical at the Naval Research Laboratory. When Wells returned to his post at the British Welding Research Association, he attempted to apply LEFM to low- and medium-strength structural steels. These materials were too ductile for LEFM to apply, but Wells noticed that the crack faces moved apart with plastic deformation. This observation led to the development of the parameter now known as the crack-tip-opening displacement (CTOD).

In 1968, Rice [24] developed another parameter to characterize nonlinear material behavior ahead of a crack. By idealizing plastic deformation as nonlinear elastic, Rice was able to generalize the energy release rate to nonlinear materials. He showed that this nonlinear energy release rate can be expressed as a line integral, which he called the J integral, evaluated along an arbitrary contour around the crack. At the time his work was being published, Rice discovered that Eshelby [25] had previously published several so-called conservation integrals, one of which was equivalent to Rice's J integral. Eshelby, however, did not apply his integrals to crack problems.

That same year, Hutchinson [26] and Rice and Rosengren [27] related the J integral to crack-tip stress fields in nonlinear materials. These analyses showed that J can be viewed as a nonlinear, stress-intensity parameter as well as an energy release rate.

Rice's work might have been relegated to obscurity had it not been for the active research effort by the nuclear power industry in the U.S. in the early 1970s. Because of legitimate concerns for safety, as well as political and public relations considerations, the nuclear power industry endeavored to apply state-of-the-art technology, including fracture mechanics, to the design and construction of nuclear power plants. The difficulty with applying fracture mechanics in this instance was that most nuclear pressure vessel steels were too tough to be characterized with LEFM without resorting to enormous laboratory specimens. In 1971, Begley and Landes [28], who were research engineers at Westinghouse, came across Rice's article and decided, despite skepticism from their co-workers, to characterize the fracture toughness of these steels with the J integral. Their experiments were very successful and led to the publication of a standard procedure for J testing of metals 10 years later [29].

Material toughness characterization is only one aspect of fracture mechanics. In order to apply fracture mechanics concepts to design, one must have a mathematical relationship between toughness, stress, and flaw size. Although these relationships were well established for linear elastic problems, a fracture design analysis based on the J integral was not available until Shih and Hutchinson [30] provided the theoretical framework for such an approach in 1976. A few years later, the Electric Power Research Institute (EPRI) published a fracture design handbook [31] based on the Shih and Hutchinson methodology.

In the United Kingdom, Well's CTOD parameter was applied extensively to fracture analysis of welded structures, beginning in the late 1960s. While fracture research in the U.S. was driven primarily by the nuclear power industry during the 1970s, fracture research in the U.K. was motivated largely by the development of oil resources in the North Sea. In 1971, Burdekin and Dawes [32] applied several ideas proposed by Wells [33] several years earlier and developed the CTOD design curve, a semiempirical fracture mechanics methodology for welded steel structures. The nuclear power industry in the UK developed their own fracture design analysis [34], based on the strip yield model of Dugdale [21] and Barenblatt [22].

Shih [35] demonstrated a relationship between the J integral and CTOD, implying that both parameters are equally valid for characterizing fracture. The J-based material testing and structural design approaches developed in the U.S. and the British CTOD methodology have begun to merge in recent years, with positive aspects of each approach combined to yield improved analyses. Both parameters are currently applied throughout the world to a range of materials.

Much of the theoretical foundation of dynamic fracture mechanics was developed in the period between 1960 and 1980. Significant contributions were made by a number of researchers, as discussed in Chapter 4.

1.2.5 Fracture Mechanics from 1980 to the Present

The field of fracture mechanics matured in the last two decades of the 20th century. Current research tends to result in incremental advances rather than major gains. The application of this technology to practical problems is so pervasive that fracture mechanics is now considered an established engineering discipline.

More sophisticated models for material behavior are being incorporated into fracture mechanics analyses. While plasticity was the important concern in 1960, more recent work has gone a step further, incorporating time-dependent nonlinear material behavior such as viscoplasticity and viscoelasticity. The former is motivated by the need for tough, creep-resistant high temperature materials, while the latter reflects the increasing proportion of plastics in structural applications. Fracture mechanics has also been used (and sometimes abused) in the characterization of composite materials.

Another trend in recent research is the development of microstuctural models for fracture and models to relate local and global fracture behavior of materials. A related topic is the efforts to characterize and predict geometry dependence of fracture toughness. Such approaches are necessary when traditional, so-called single-parameter fracture mechanics break down.

The continuing explosion in computer technology has aided both the development and application of fracture mechanics technology. For example, an ordinary desktop computer is capable of performing complex three-dimensional finite element analyses of structural components that contain cracks.

Computer technology has also spawned entirely new areas of fracture mechanics research. Problems encountered in the microelectronics industry have led to active research in interface fracture and nanoscale fracture.

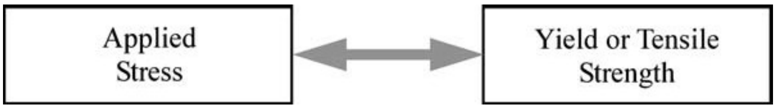
1.3 THE FRACTURE MECHANICS APPROACH TO DESIGN

Figure 1.7 contrasts the fracture mechanics approach with the traditional approach to structural design and material selection. In the latter case, the anticipated design stress is compared to the flow properties of candidate materials; a material is assumed to be adequate if its strength is greater than the expected applied stress. Such an approach may attempt to guard against brittle fracture by imposing a safety factor on stress, combined with minimum tensile elongation requirements on the material. The fracture mechanics approach (Figure 1.7(b)) has three important variables, rather than two as in Figure 1.7(a). The additional structural variable is flaw size, and fracture toughness replaces strength as the relevant material property. Fracture mechanics quantifies the critical combinations of these three variables.

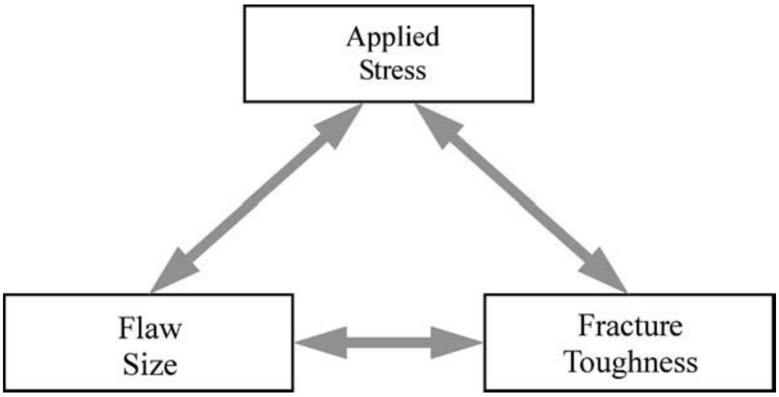
There are two alternative approaches to fracture analysis: the energy criterion and the stress-intensity approach. These two approaches are equivalent in certain circumstances. Both are discussed briefly below.

1.3.1 The Energy Criterion

The energy approach states that crack extension (i.e., fracture) occurs when the energy available for crack growth is sufficient to overcome the resistance of the material. The material resistance may include the surface energy, plastic work, or other types of energy dissipation associated with a propagating crack.



(a)



(b)

FIGURE 1.7 Comparison of the fracture mechanics approach to design with the traditional strength of materials approach: (a) the strength of materials approach and (b) the fracture mechanics approach.

Griffith [7] was the first to propose the energy criterion for fracture, but Irwin [13] is primarily responsible for developing the present version of this approach: the energy release rate $\mathcal{G}$ which is defined as the rate of change in potential energy with the crack area for a linear elastic material. At the moment of fracture $\mathcal{G} = \mathcal{G}_c$, the critical energy release rate, which is a measure of fracture toughness.

For a crack of length $2a$ in an infinite plate subject to a remote tensile stress (Figure 1.8), the energy release rate is given by

$$\mathcal{G} = \frac{\pi \sigma^2 a}{E} \tag{1.1}$$

where E is Young's modulus, σ is the remotely applied stress, and a is the half-crack length. At fracture $\mathcal{G} = \mathcal{G}_c$, and Equation (1.1) describes the critical combinations of stress and crack size for failure:

$$\mathcal{G}_c = \frac{\pi \sigma_f^2 a_c}{E} \tag{1.2}$$

Note that for a constant $\mathcal{G}_c$ value, failure stress σ_f varies with $1/\sqrt{a}$. The energy release rate $\mathcal{G}$ is the driving force for fracture, while $\mathcal{G}_c$ is the material's resistance to fracture. To draw an analogy to the strength of materials approach of Figure 1.7(a), the applied stress can be viewed as the driving force for plastic deformation, while the yield strength is a measure of the material's resistance to deformation.

The tensile stress analogy is also useful for illustrating the concept of similitude. A yield strength value measured with a laboratory specimen should be applicable to a large structure; yield strength does not depend on specimen size, provided the material is reasonably homogeneous. One of the fundamental assumptions of fracture mechanics is that fracture toughness ($\mathcal{G}_c$ in this case) is

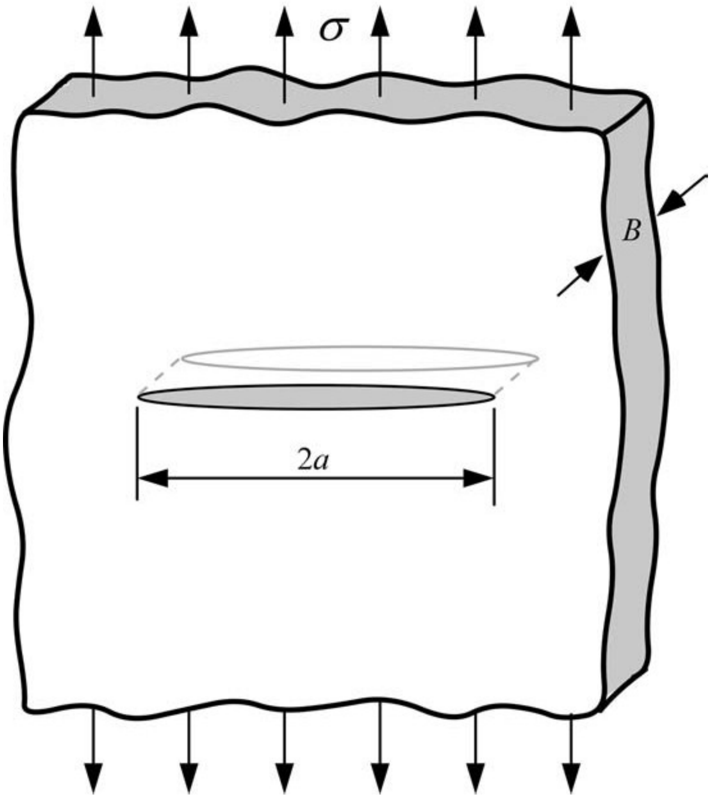


FIGURE 1.8 Through-thickness crack in an infinite plate subject to a remote tensile stress. In practical terms, "infinite" means that the width of the plate is >> $2a$.

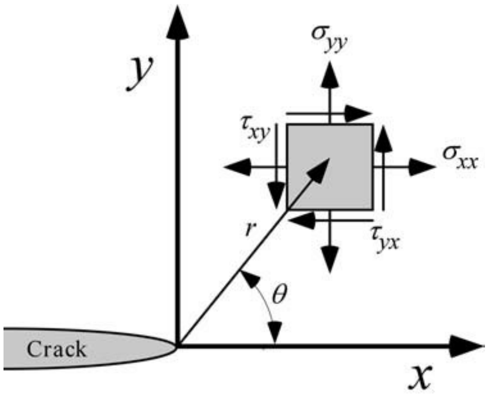
independent of the size and geometry of the cracked body; a fracture toughness measurement on a laboratory specimen should be applicable to a structure. As long as this assumption is valid, all configuration effects are taken into account by the driving force $\mathcal{G}$. The similitude assumption is valid as long as the material behavior is predominantly linear elastic.

1.3.2 The Stress-Intensity Approach

Figure 1.9 schematically shows an element near the tip of a crack in an elastic material, together with the in-plane stresses on this element. Note that each stress component is proportional to a single constant K_I. If this constant is known, the entire stress distribution at the crack tip can be computed with the equations in Figure 1.9. This constant, which is called the stress-intensity factor, completely characterizes the crack-tip conditions in a linear elastic material. (The meaning of the subscript on K is explained in Chapter 2.) If one assumes that the material fails locally at some critical combination of stress and strain, then it follows that fracture must occur at a critical stress intensity K_{Ic}. Thus, K_{Ic} is an alternate measure of fracture toughness.

For the plate illustrated in Figure 1.8, the stress-intensity factor is given by

$$K_I = \sigma\sqrt{\pi a} \tag{1.3}$$



$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 - \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\left[1 + \sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{3\theta}{2}\right)$$

FIGURE 1.9 Stresses near the tip of a crack in an elastic material.

Failure occurs when $K_I = K_{Ic}$. In this case, K_I is the driving force for fracture and K_{Ic} is a measure of material resistance. As with $\mathcal{G}_c$, the property of similitude should apply to K_{Ic}. That is, K_{Ic} is assumed to be a size-independent material property.

Comparing Equation (1.1) and Equation (1.3) results in a relationship between K_I and $\mathcal{G}$:

$$\mathcal{G} = \frac{K_I^2}{E} \tag{1.4}$$

This same relationship obviously holds for $\mathcal{G}_c$ and K_{Ic}. Thus, the energy and stress-intensity approaches to fracture mechanics are essentially equivalent for linear elastic materials.

1.3.3 Time-Dependent Crack Growth and Damage Tolerance

Fracture mechanics often plays a role in life prediction of components that are subject to time-dependent crack growth mechanisms such as fatigue or stress corrosion cracking. The *rate* of cracking can be correlated with fracture mechanics parameters such as the stress-intensity factor, and the critical crack size for failure can be computed if the fracture toughness is known. For example, the fatigue crack growth rate in metals can usually be described by the following empirical relationship:

$$\frac{da}{dN} = C(\Delta K)^m \tag{1.5}$$

where da/dN is the crack growth per cycle, ΔK is the stress-intensity range, and C and m are material constants.

Damage tolerance, as its name suggests, entails allowing subcritical flaws to remain in a structure. Repairing flawed material or scrapping a flawed structure is expensive and is often unnecessary. Fracture mechanics provides a rational basis for establishing flaw tolerance limits.

Consider a flaw in a structure that grows with time (e.g., a fatigue crack or a stress corrosion crack) as illustrated schematically in Figure 1.10. The *initial* crack size is inferred from nondestructive examination (NDE), and the *critical* crack size is computed from the applied stress and fracture toughness. Normally, an *allowable* flaw size would be defined by dividing the critical size by a safety factor. The predicted service life of the structure can then be inferred by calculating the time required for the flaw to grow from its initial size to the maximum allowable size.

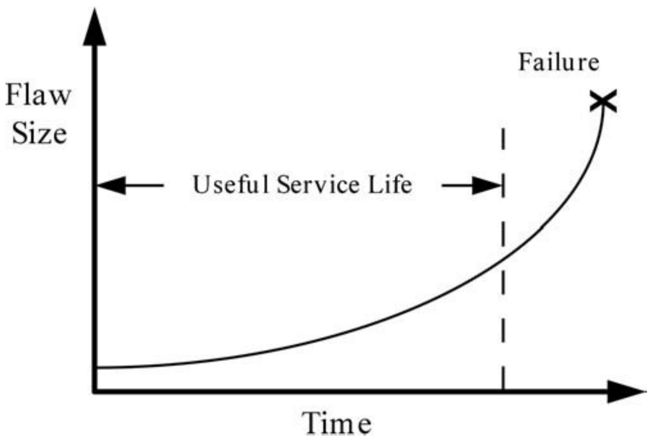


FIGURE 1.10 The damage tolerance approach to design.

1.4 EFFECT OF MATERIAL PROPERTIES ON FRACTURE

Figure 1.11 shows a simplified family tree for the field of fracture mechanics. Most of the early work was applicable only to linear elastic materials under quasistatic conditions, while subsequent advances in fracture research incorporated other types of material behavior. Elastic-plastic fracture mechanics considers plastic deformation under quasistatic conditions, while dynamic, viscoelastic, and viscoplastic fracture mechanics include time as a variable. A dashed line is drawn between linear elastic and dynamic fracture mechanics because some early research considered dynamic linear elastic behavior. The chapters that describe the various types of fracture behavior are shown in Figure 1.11. Elastic-plastic, viscoelastic, and viscoplastic fracture behavior are sometimes included in the more general heading of *nonlinear fracture mechanics*. The branch of fracture mechanics one should apply to a particular problem obviously depends on material behavior.

Consider a cracked plate (Figure 1.8) that is loaded to failure. Figure 1.12 is a schematic plot of failure stress vs. fracture toughness K_{Ic}. For low toughness materials, brittle fracture is the governing

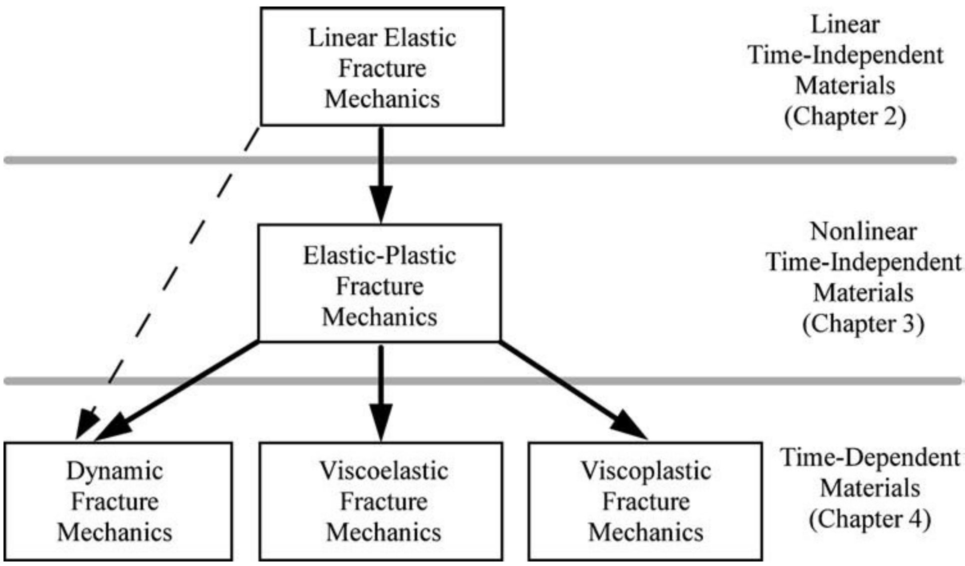


FIGURE 1.11 Simplified family tree of fracture mechanics.

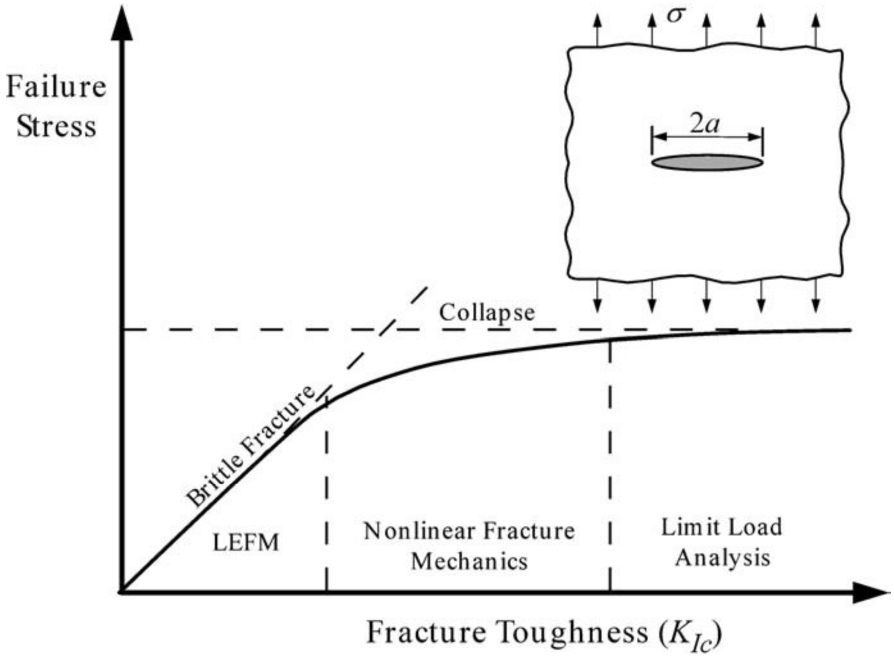


FIGURE 1.12 Effect of fracture toughness on the governing failure mechanism.

failure mechanism, and critical stress varies linearly with K_{Ic}, as predicted by Equation (1.3). At very high toughness values, LEFM is no longer valid, and failure is governed by the flow properties of the material. At intermediate toughness levels, there is a transition between brittle fracture under linear elastic conditions and ductile overload. Nonlinear fracture mechanics bridges the gap between LEFM and collapse. If toughness is low, LEFM is applicable to the problem, but if toughness is sufficiently high, fracture mechanics ceases to be relevant to the problem because failure stress is insensitive to toughness; a simple limit load analysis is all that is required to predict failure stress in a material with very high fracture toughness.

Table 1.1 lists various materials, together with the typical fracture regime for each material.

TABLE 1.1
Typical Fracture Behavior of Selected Materials[a]

Material	Typical Fracture Behavior
High strength steel	Linear elastic
Low- and medium-strength steel	Elastic-plastic/fully plastic
Austenitic stainless steel	Fully plastic
Precipitation-hardened aluminum	Linear elastic
Metals at high temperatures	Viscoplastic
Metals at high strain rates	Dynamic/viscoplastic
Polymers (below T_g)[b]	Linear elastic/viscoelastic
Polymers (above T_g)[b]	Viscoelastic
Monolithic ceramics	Linear elastic
Ceramic composites	Linear elastic
Ceramics at high temperatures	Viscoplastic

[a] Temperature is ambient unless otherwise specified.
[b] T_g—Glass transition temperature.

1.5 A BRIEF REVIEW OF DIMENSIONAL ANALYSIS

At first glance, a section on dimensional analysis may seem out of place in the introductory chapter of a book on fracture mechanics. However, dimensional analysis is an important tool for developing mathematical models of physical phenomena, and it can help us understand existing models. Many difficult concepts in fracture mechanics become relatively transparent when one considers the relevant dimensions of the problem. For example, dimensional analysis gives us a clue as to when a particular model, such as linear elastic fracture mechanics, is no longer valid.

Let us review the fundamental theorem of dimensional analysis and then look at a few simple applications to fracture mechanics.

1.5.1 The Buckingham Π-Theorem

The first step in building a mathematical model of a physical phenomenon is to identify all of the parameters that may influence the phenomenon. Assume that a problem, or at least an idealized version of it, can be described by the following set of scalar quantities: $\{u, W_1, W_2, \ldots, W_n\}$. The dimensions of all quantities in this set is denoted by $\{[u], [W_1], [W_2], \ldots, [W_n]\}$. Now suppose that we wish to express the first variable u as a function of the remaining parameters:

$$u = f(W_1, W_2, \ldots, W_n) \tag{1.6}$$

Thus, the process of modeling the problem is reduced to finding a mathematical relationship that represents f as best as possible. We might accomplish this by performing a set of experiments in which we measure u while varying each W_i independently. The number of experiments can be greatly reduced, and the modeling processes simplified, through dimensional analysis. The first step is to identify all of the *fundamental dimensional units* (fdu's) in the problem: $\{L_1, L_2, \ldots, L_m\}$. For example, a typical mechanics problem may have $\{L_1 = \text{length}, L_2 = \text{mass}, L_3 = \text{time}\}$. We can express the dimensions of each quantity in our problem as the product of the powers of the fdu's; i.e., for any quantity X, we have

$$[X] = L_1^{a_1} L_2^{a_2}, \ldots, L_m^{a_m} \tag{1.7}$$

The quantity X is dimensionless if $[X] = 1$.

In the set of Ws, we can identify *m primary quantities* that contain all of the fdu's in the problem. The remaining variables are secondary quantities, and their dimensions can be expressed in terms of the primary quantities:

$$[W_{m+j}] = [W_1]^{a_{m+j(1)}}, \ldots, [W_m]^{a_{m+j(m)}} \quad (j = 1, 2, \ldots, n - m \tag{1.8}$$

Thus, we can define a set of new quantities π_i that are dimensionless:

$$\pi_i = \frac{W_{m+j}}{W_1^{a_{m+j(1)}}, \ldots, W_m^{a_{m+j(m)}}} \tag{1.9}$$

Similarly, the dimensions of u can be expressed in terms of the dimensions of the primary quantities:

$$[u] = [W_1]^{a_1}, \ldots, [W_m]^{a_n} \tag{1.10}$$