Motion Planner for K-Legged Robots Trajectory Optimization

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1 Centroidal System Dynamics

$$m(\ddot{c} + g) = F_e + \sum_i F_i \tag{1}$$

$$mc \times (\ddot{c} + g) + \dot{L} = c \times F_e + \sum_i (S_i \times F_i) \tag{2}$$

$$\dot{x} = g(x, u) \tag{3}$$

2 Variables

- c   CoM *x,y,z*
- l   Linear momentum *x,y,z*
- k   Angular momentum *x,y,z*
- F_λ Contact force of λ leg *x,y,z*
- S_λ Contact foot step position of λ leg *x,y,z*
- w_k, w_F, w_S Parameter weights for k, F_λ, S_λ in objective function.

3 Objective function and constraints

3.1 Objective Function

$$\min_{l,k} \frac{1}{2} \sum_{i=1}^{N} \left(||l - l_d||^2 + w_k ||k - k_d||^2 + \sum_{\lambda=1}^{k\_legs} w_F ||F_\lambda||^2 + \sum_{\lambda=1}^{k\_legs} w_S ||S_\lambda - S_{d\lambda}||^2 \right) \tag{4}$$

3.2 Constraints

$$|F^x| \leq \mu F^z$$

$$|F^y| \leq \mu F^z$$

$$\begin{aligned} |F^x_\lambda| &\leq \mu F^z_\lambda \\ |F^y_\lambda| &\leq \mu F^z_\lambda \end{aligned} \quad \lambda \in \{1, \ldots, k\_legs\}$$

Equation label 1 - 4 λ

$$\begin{aligned} 0 &\leq \mu F^z_\lambda - F^x_\lambda \\ 0 &\leq \mu F^z_\lambda + F^x_\lambda \\ 0 &\leq \mu F^z_\lambda - F^y_\lambda \\ 0 &\leq \mu F^z_\lambda + F^y_\lambda \end{aligned} \quad \lambda \in \{1, \ldots, k\_legs\}$$

Equation label 5 *x,y,z*

$$l_{i+1} - \left(\sum_{\lambda=1}^{k\_legs} F_{\lambda i} + mg \right) \cdot \Delta t - l_i = 0$$

Equation label 6 *x,y,z*

$$k_{i+1} - \left(\sum_{\lambda=1}^{k\_legs} (c_i - S_{\lambda i} \times F_{\lambda i}) \right) \cdot \Delta t - k_i = 0$$

Equation label 7 *x,y,z*

$$c_{i+1} - c_i - \frac{1}{m} l_i \cdot \Delta t = 0$$

3.3 Jacobian gradient of objective function

$$J_{obj.} = \left[\frac{\partial f_{obj}}{\partial c}, \quad \frac{\partial f_{obj}}{\partial l}, \quad \frac{\partial f_{obj}}{\partial k}, \quad \frac{\partial f_{obj}}{\partial F_0}, \ldots, \quad \frac{\partial f_{obj}}{\partial F_i}, \quad \frac{\partial f_{obj}}{\partial S_0}, \ldots, \quad \frac{\partial f_{obj}}{\partial S_i} \right]$$

3.4 Jacobian gradient of constraints

$$J_{g.} = \left[\frac{\partial g}{\partial c}, \quad \frac{\partial g}{\partial l}, \quad \frac{\partial g}{\partial k}, \quad \frac{\partial g}{\partial F_0}, \ldots, \quad \frac{\partial g}{\partial F_i}, \quad \frac{\partial g}{\partial S_0}, \ldots, \quad \frac{\partial g}{\partial S_i} \right]$$

Correct Assumption Consider only foots with contact to the ground. Objective function and Constraint equations have to adapt for this assumption. Re-write them, where $k\_legs \longrightarrow contact\_legs$ 5. For this, we need to keep which legs has contact to the ground and consider only them.

$$\lambda \in \{1, \ldots, k\_legs\}$$

$$\min_{l,k} \frac{1}{2} \sum_{i=1}^{N} \Bigg(||l - l_d||^2 + w_k ||k - k_d||^2 + \underbrace{\sum_{\lambda=1}^{con\_legs} w_F ||F_\lambda||^2}_{\text{legs in contact}} + \underbrace{\sum_{\lambda=1}^{con\_legs} w_S ||S_\lambda - S_{d\lambda}||^2}_{\text{legs in contact}} \Bigg) \tag{5}$$

Important The last affects the gradient of Objective function and the gradient of Constraints equation. Repetitively cyclic *i_leg* parameter plays an important role in evaluation of functions (f), constraints (g) and much more in their gradient.

4 Future work

Add phases based on number of legs. Add phase for each *i_leg* in motion. Then, problem description will have k phases, that can be developed parametric consider the i leg to be in motion. Then, call this problem-phase repetitively for every $i \in k$. Now we have moved all the legs. Swing phases and k-supports needs also to be considered and developed as different phases. Get them in row, and you have described a centroidal k-legged motion planner totally correctly!

5 Results

Ipopt Info we take from ipopt when it tries to solve the problem with initial incorrect assumption.
Number of Iterations....: 4000
Number of objective function evaluations = 6809
Number of objective gradient evaluations = 6
Number of equality constraint evaluations = 6889
Number of inequality constraint evaluations = 6889
Number of equality constraint Jacobian evaluations = 4082
Number of inequality constraint Jacobian evaluations = 4082
Number of Lagrangian Hessian evaluations = 0
Total seconds in IPOPT = 121.177

EXIT: Maximum Number of Iterations Exceeded.
*** The problem FAILED!

Plots With initial assumption we get the following plots.

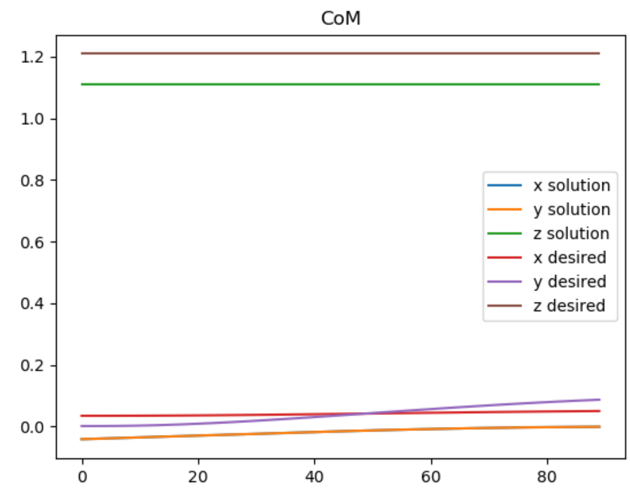

Figure 1: CoM solution

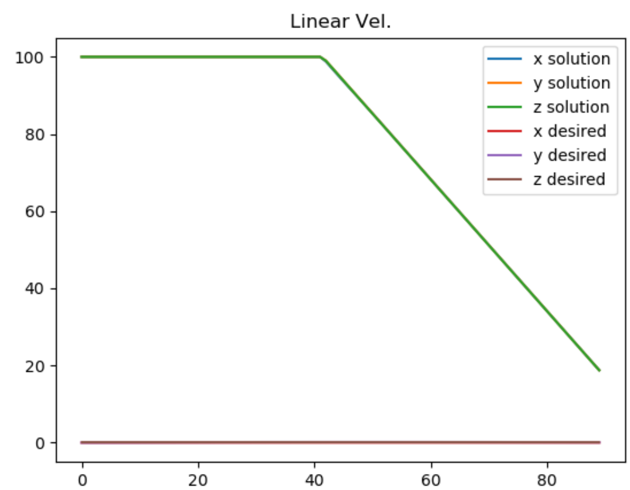

Figure 2: Linear momentum solution

6 Appendix

More figures to describe the problem.

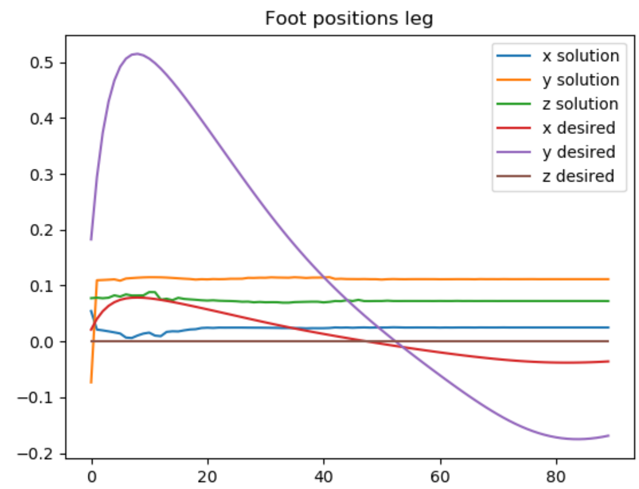

Figure 3: Foot position 0-leg solution

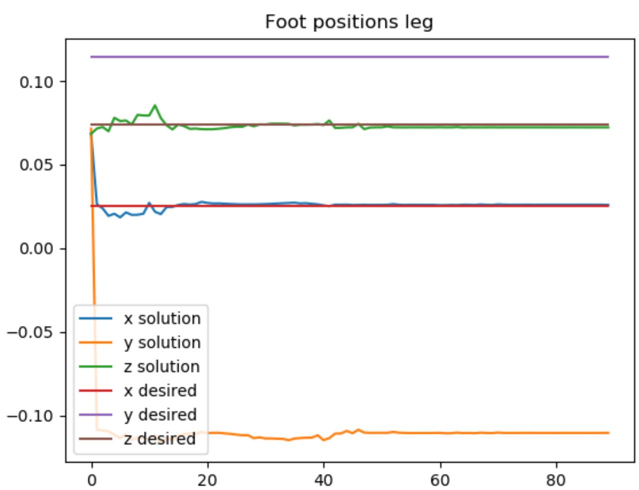

Figure 4: Foot position 1-leg solution

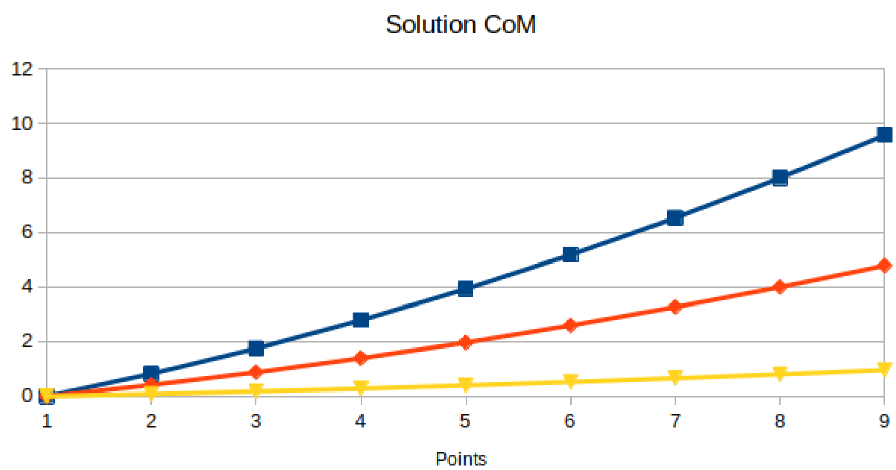

Figure 5: Simplified dynamic solution

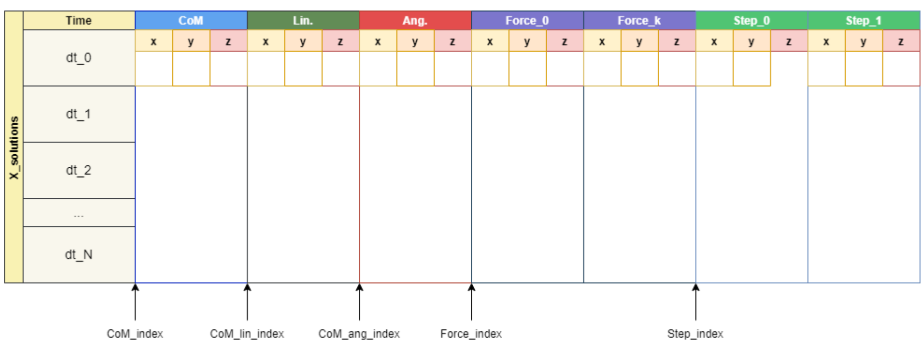

Figure 6: Solution variables mapping in ipopt of centroidal dynamic

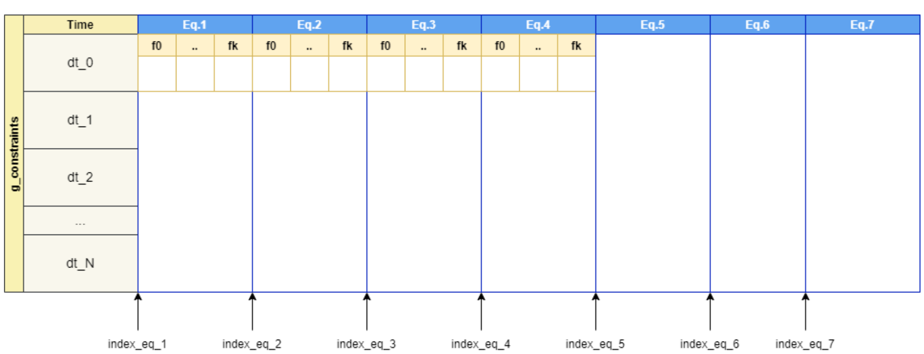

Figure 7: Constraints mapping in ipopt of centroidal dynamic

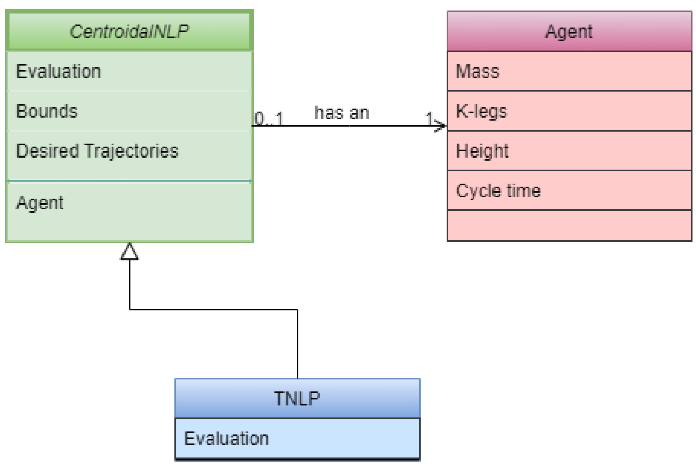

Figure 8: Class diagram