

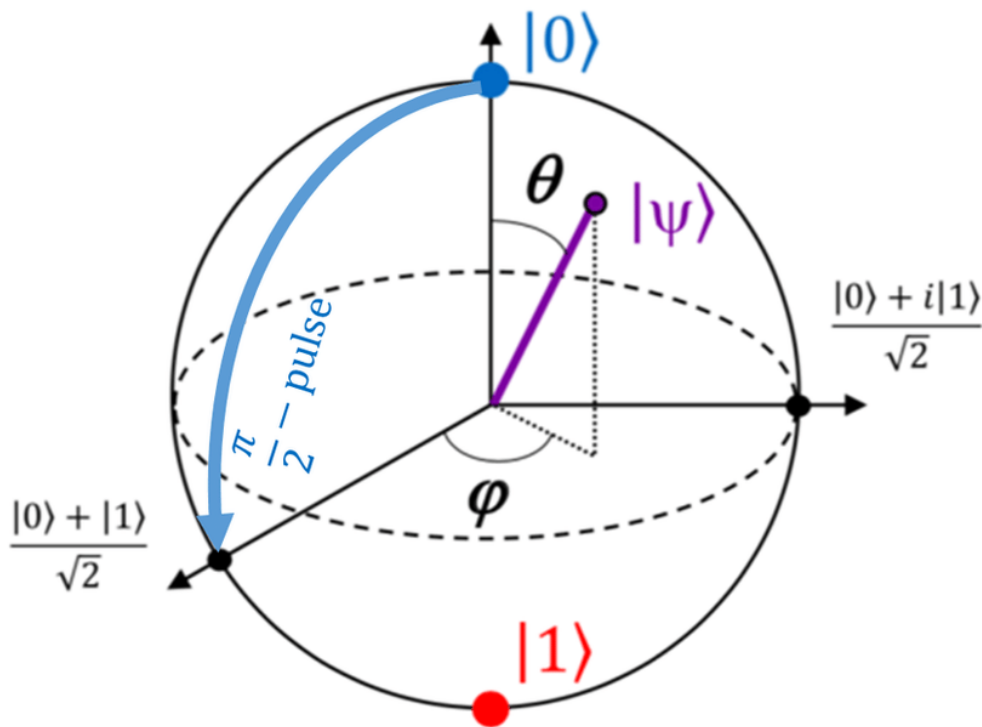
Quantum Key Distribution

Based on Random Grouping

Bell State Measurement

transmission information
secret magic measurement
privacy resend system
distance calculate receiver
configuration legitimate
probability paradox
loss mutual technique
supremacy protocol
key Eve attack spin Bob error
Bell sender
QKD gets state method secure
distribution grouping channel quantum binary
eavesdropper intercept system
information
cryptography
threshold efficiency position mechanism message
detection combination classical photon sequence entropy
Alice Trojan communication
no information

Qubits



- **Complex 2D-Hilbert space**
- **No phase**
- **Normalization**
- **Two independent parameters**

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$

Bell States

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle)$$
$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle)$$

- **Basis in four-dimensional state space**
- **Maximum entangled state of 2 qubits**
- **Easy to combine and measure**
(in comparison with peers)

QKD Timeline

BB84

first QKD protocol
1984

E91

EPR paradox
1991

Gao

first BSM usage
2008

Yuan

2 BSM combination
2008

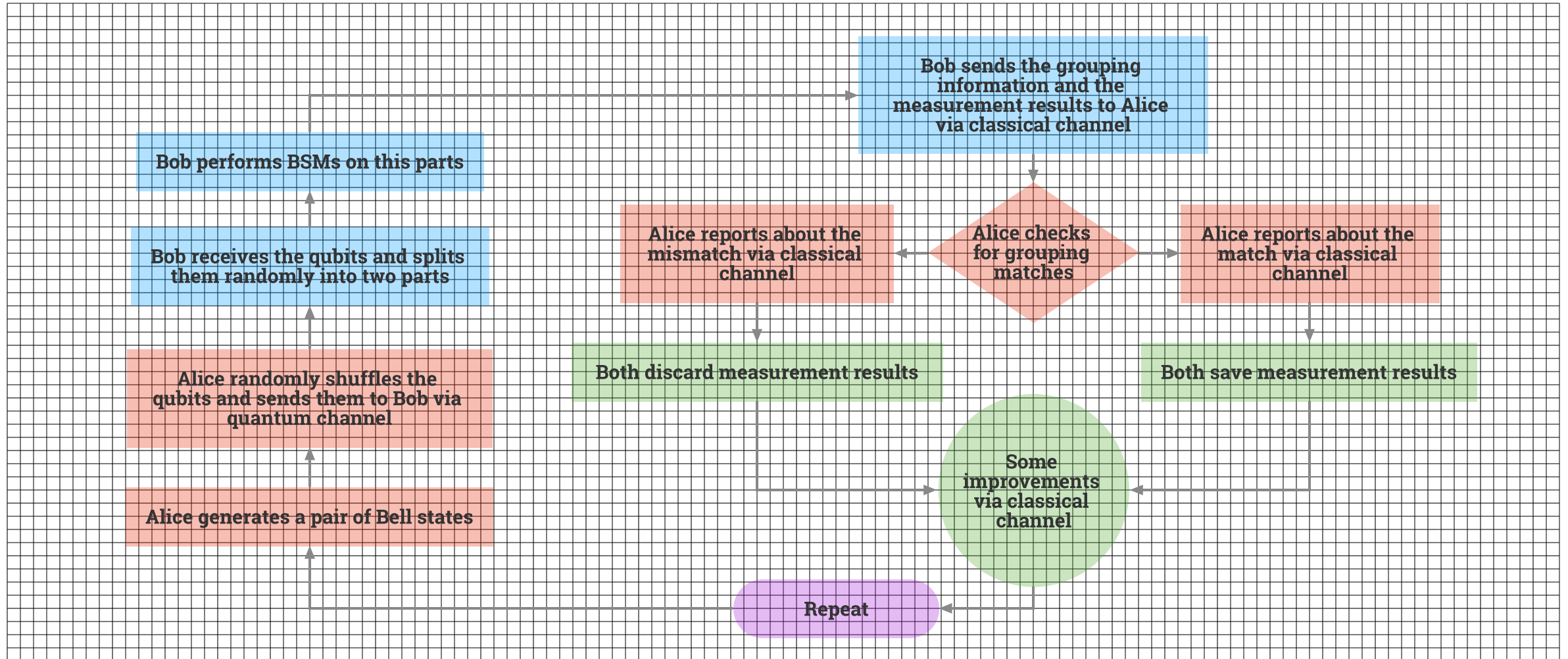
Protocol*

Random Grouping
2020

Groupings & Spooky Scary Formulas

$$\begin{aligned} |\mathcal{C}\rangle_{1234} &= |\phi^-\rangle_{12} \otimes |\phi^+\rangle_{34} = \frac{1}{2} (|00\rangle_{12} - |11\rangle_{12}) \otimes (|00\rangle_{34} + |11\rangle_{34}) = \\ &= \frac{1}{2} (|0000\rangle + |0011\rangle - |1100\rangle - |1111\rangle)_{1234} = \\ &= \frac{1}{4} (2|\textcolor{red}{0}\textcolor{blue}{0}\textcolor{red}{0}\textcolor{blue}{0}\rangle + 2|\textcolor{red}{0}\textcolor{blue}{0}\textcolor{red}{1}\textcolor{blue}{1}\rangle - 2|\textcolor{red}{1}\textcolor{blue}{1}\textcolor{red}{0}\textcolor{blue}{0}\rangle - 2|\textcolor{red}{1}\textcolor{blue}{1}\textcolor{red}{1}\textcolor{blue}{1}\rangle)_{\textcolor{red}{1}\textcolor{blue}{2}\textcolor{red}{3}\textcolor{blue}{4}} = \\ &= \frac{1}{4} ([|\textcolor{red}{0}\textcolor{blue}{0}\textcolor{red}{0}\textcolor{blue}{0}\rangle - |\textcolor{red}{0}\textcolor{blue}{1}\textcolor{red}{0}\textcolor{blue}{1}\rangle + |\textcolor{red}{1}\textcolor{blue}{0}\textcolor{red}{1}\textcolor{blue}{0}\rangle - |\textcolor{red}{1}\textcolor{blue}{1}\textcolor{red}{1}\textcolor{blue}{1}\rangle] + [|\textcolor{red}{0}\textcolor{blue}{0}\textcolor{red}{0}\textcolor{blue}{0}\rangle + |\textcolor{red}{0}\textcolor{blue}{1}\textcolor{red}{0}\textcolor{blue}{1}\rangle - |\textcolor{red}{1}\textcolor{blue}{0}\textcolor{red}{1}\textcolor{blue}{0}\rangle - |\textcolor{red}{1}\textcolor{blue}{1}\textcolor{red}{1}\textcolor{blue}{1}\rangle] + \\ &+ [|\textcolor{red}{0}\textcolor{blue}{0}\textcolor{red}{1}\textcolor{blue}{1}\rangle - |\textcolor{red}{0}\textcolor{blue}{1}\textcolor{red}{1}\textcolor{blue}{0}\rangle + |\textcolor{red}{1}\textcolor{blue}{0}\textcolor{red}{0}\textcolor{blue}{1}\rangle - |\textcolor{red}{1}\textcolor{blue}{1}\textcolor{red}{0}\textcolor{blue}{0}\rangle] + [|\textcolor{red}{0}\textcolor{blue}{0}\textcolor{red}{1}\textcolor{blue}{1}\rangle + |\textcolor{red}{0}\textcolor{blue}{1}\textcolor{red}{1}\textcolor{blue}{0}\rangle - |\textcolor{red}{1}\textcolor{blue}{0}\textcolor{red}{0}\textcolor{blue}{1}\rangle - |\textcolor{red}{1}\textcolor{blue}{1}\textcolor{red}{0}\textcolor{blue}{0}\rangle])_{\textcolor{red}{1}\textcolor{blue}{2}\textcolor{red}{3}\textcolor{blue}{4}} = \\ &= \frac{1}{4} ([|\textcolor{red}{0}\textcolor{blue}{0}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{0}\rangle_{24} - |\textcolor{red}{0}\textcolor{blue}{0}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{1}\rangle_{24} + |\textcolor{red}{1}\textcolor{blue}{1}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{0}\rangle_{24} - |\textcolor{red}{1}\textcolor{blue}{1}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{1}\rangle_{24}] + \\ &+ [|\textcolor{red}{0}\textcolor{blue}{0}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{0}\rangle_{24} + |\textcolor{red}{0}\textcolor{blue}{0}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{1}\rangle_{24} - |\textcolor{red}{1}\textcolor{blue}{1}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{0}\rangle_{24} - |\textcolor{red}{1}\textcolor{blue}{1}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{1}\rangle_{24}] + \\ &+ [|\textcolor{red}{0}\textcolor{blue}{1}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{1}\rangle_{24} - |\textcolor{red}{0}\textcolor{blue}{1}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{0}\rangle_{24} + |\textcolor{red}{1}\textcolor{blue}{0}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{1}\rangle_{24} - |\textcolor{red}{1}\textcolor{blue}{0}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{0}\rangle_{24}] + \\ &+ [|\textcolor{red}{0}\textcolor{blue}{1}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{1}\rangle_{24} + |\textcolor{red}{0}\textcolor{blue}{1}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{0}\rangle_{24} - |\textcolor{red}{1}\textcolor{blue}{0}\rangle_{13}|\textcolor{blue}{0}\textcolor{red}{1}\rangle_{24} - |\textcolor{red}{1}\textcolor{blue}{0}\rangle_{13}|\textcolor{red}{1}\textcolor{blue}{0}\rangle_{24}]) = \\ &= \frac{1}{2} (|\textcolor{red}{\phi}^+\rangle_{13}|\textcolor{blue}{\phi}^-\rangle_{24} + |\textcolor{red}{\phi}^-\rangle_{13}|\textcolor{blue}{\phi}^+\rangle_{24} + |\textcolor{red}{\psi}^+\rangle_{13}|\textcolor{blue}{\psi}^-\rangle_{24} + |\textcolor{red}{\psi}^-\rangle_{13}|\textcolor{blue}{\psi}^+\rangle_{24}) \end{aligned}$$

Key Distribution Algorithm



Qubit Error Ratio

50%

15%

4%



BB84

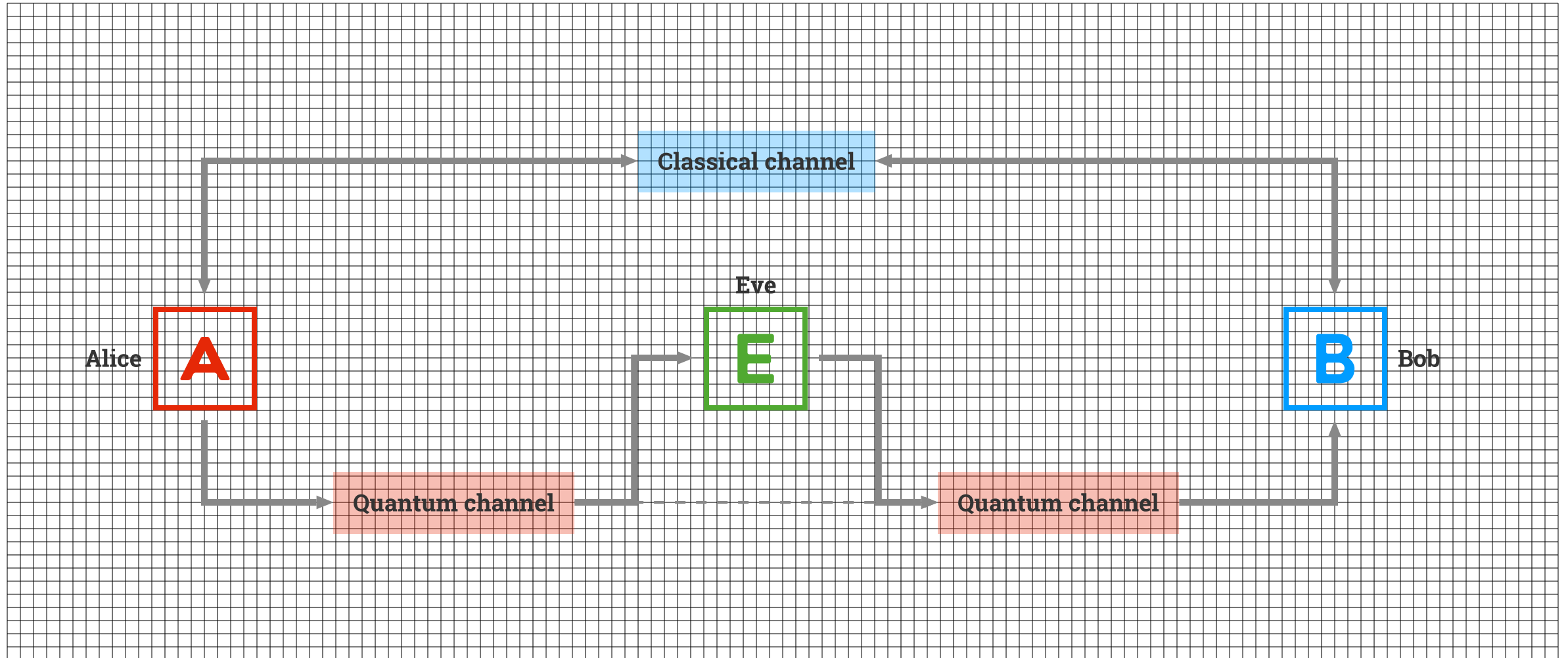


E91

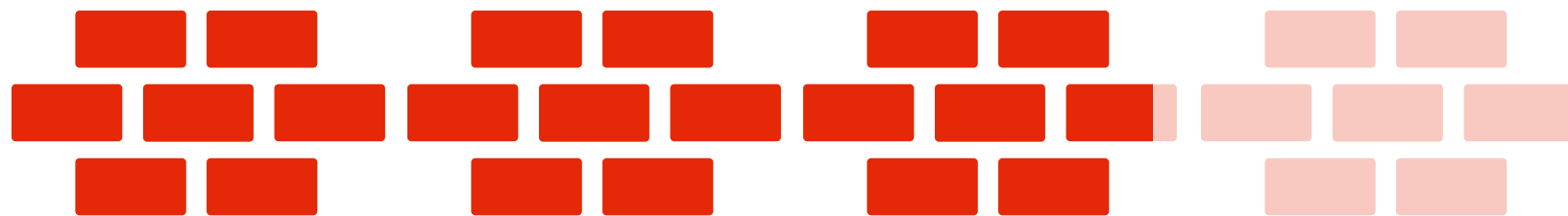


Protocol*

Intercept-Resend Attack

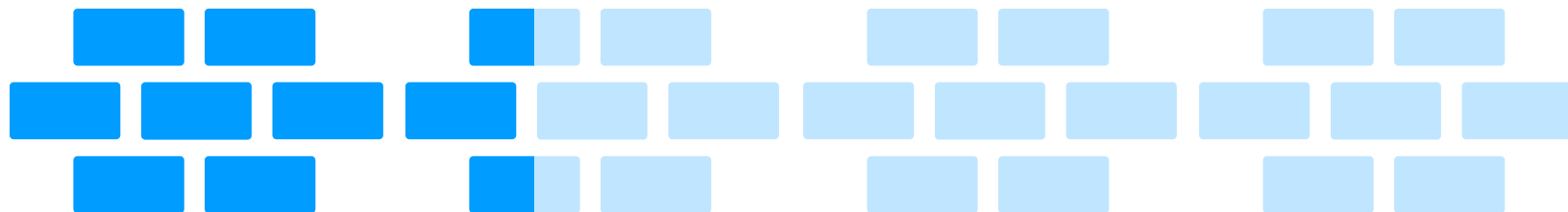


72



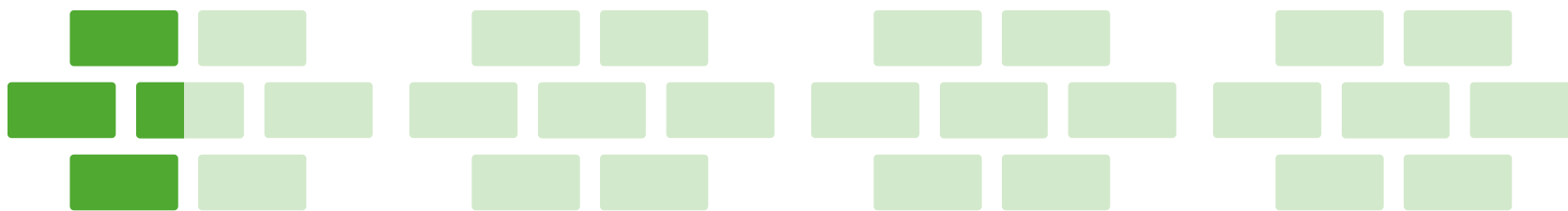
bits needed to detect Eve in **BB84**

33



bits needed to detect Eve in **E91**

11



bits needed to detect Eve in **Protocol***

Current Problems

- **QKD usually relies on having authenticated classical channel**
- **Quantum gates noise makes probabilistic estimates worse**
- **QKD systems are still quite expensive**

Questions?

