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SN-BHM: A hierarchical Bayesian model for Supernova Cosmology

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ABSTRACT

Abstract Abstract

Key words: keyword1 – keyword2 – keyword3

1 INTRODUCTION

Almost two decades have passed since the discovery of the accelerating universe (Riess et al. 1998; Perlmutter et al. 1999). Since that time, of the number of observed Type Ia supernovae (SN Ia) have increased by more than an order of magnitude thanks to modern surveys at both low redshift (Bailey et al. 2008; Freedman et al. 2009; Hicken et al. 2009; Contreras et al. 2010; Conley et al. 2011), and higher redshift (Astier et al. 2006; Wood-Vasey et al. 2007; Balland et al. 2009; Amanullah et al. 2010). Cosmological analysis of these supernova samples (Kowalski et al. 2008; Conley et al. 2011; Suzuki et al. 2012; Betoule et al. 2014; Rest et al. 2014) have been combined with complimentary probes of large scale structure (Alam et al. 2017) and the CMB (Hinshaw et al. 2013; Planck Collaboration et al. 2013), and yet, despite these prodigious efforts, the nature of dark energy remains an unsolved mystery.

In attempts to tease out the nature of dark energy, currently running and planned surveys are once again ramping up their statistical power. The Dark Energy Survey (DES, Bernstein et al. 2012; Abbott et al. 2016) will be observing thousands of Type Ia supernova, attaining both spectroscopic and photometric confirmation. The Large Synoptic Survey Telescope (LSST, Ivezic et al. 2008; LSST Science Collaboration et al. 2009) will produce scores of thousands of photometrically classified supernovae. Such increased statistical power demands a similarly increased fidelity and flexibility in modelling the supernovae for cosmological purposes, as systematic uncertainty will prove to be the limiting factor in our analyses.

As such, staggering effort is being put into developing more sophisticated supernovae analyses. Scolnic & Kessler (2016) and Kessler & Scolnic (2017) explore sophisticated simulation corrections to traditional analyses. Approximate Bayesian computation methods also make use of simulations, trading traditional likelihoods and analytic approximations for more robust models with only the cost of increased computational time (Weyant et al. 2013; Jennings et al. 2016). Hierarchical Bayesian Models abound (Mandel et al. 2009; March et al. 2011, 2014a; Rubin et al. 2015; Shariff et al. 2016; Roberts et al. 2017), however often face difficulties finding sufficient analytic approximations for complicated effects such as Malmquist bias.

In this paper, we lay out a new hierarchical model that extends on past work. Section 2 is dedicated to a quick review of the supernovae cosmology. In Section 3 we outline our methodology and apply it to simulated datasets. Forecasts for the impending DES three year spectroscopic supernova survey are contain in Section 4. Section 5 investigates the effect of various systematics on our model, and Section 6 provides details on potential areas of improvement and unsuccessful methodologies.

2 REVIEW

Whilst supernova observations take the form of time-series photometric measurements of brightness in many photometric bands, most analyses do not work from these measurements of apparent magnitude and colour. Instead, most techniques fit these observations of magnitude (along with red-

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shift) to a supernova model, with the most widely used being that of the empirical SALT2 model (Guy et al. 2007, 2010). This model is trained separately before fitting the supernovae light curves for the cosmology selected supernova sample (Guy et al. 2010; Mosher et al. 2014). The resulting output from the model is, for each supernova, a characterised amplitude x_0 (which can be converted into apparent magnitude $m_B = -2.5\log(x_0)$), a stretch term x_1 and colour term c, along with a covariance matrix describing the uncertainty on these summary statistics. As such, the product at the end is a (redshift dependent) population of m_B, x_1 and c.

The underlying actual supernova population is not as clear cut, and indeed accurately characterising this population, its evolution over redshift and effects from environment is one of the challenges of supernova cosmology. However, given some modelled underlying population that lives in the redshift dependent space M_B, x_1 and c, the introduction of cosmology into the model is simple – it is encoded in the functional map between those two populations, from apparent magnitude space to absolute magnitude. Specifically, for any given supernova our functional map may take the traditional form:

$$M_B = m_B + \alpha x_1 - \beta c - \mu(z) + \text{corrections}, \tag{1}$$

where α is the stretch correction (Phillips 1993), and β is the colour correction (Tripp 1998) that respectively encapsulate the empirical relation that broader and bluer supernovae are brighter. The corrections term at the end often includes corrections for host galaxy environment, as this has statistically significant effects on supernova properties (Kelly et al. 2010; Lampeitl et al. 2010; Sullivan et al. 2010; Rigault et al. 2013; Uddin et al. 2017). The cosmological term, $\mu(z)$ represents the distance modulus, and is precisely known given cosmological parameters and an input redshift.

2.1 Traditional Analyses

Traditional χ^2 analyses such as that found in Kowalski et al. (2008); Conley et al. (2011); Betoule et al. (2014), minimise the difference in distance modulus between the cosmologically predicted values μ_C and the observed distance modulus $\mu_{\rm obs}$, shown respectively below:

$$\mu_C = 5\log\left[\frac{(1+z)r}{10}\right] \tag{2}$$

$$r = \frac{c}{H_0}\int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_k(1+z')^2 + \Omega_\Lambda(1+z')^{3(1+w)}}} \tag{3}$$

$$\mu_{\rm obs} = m_B + \alpha x_1 - \beta c - M_B \tag{4}$$

The minimising function is then given as

$$\chi^2 = (\mu_{\rm obs} - \mu_C)^\dagger C^{-1}(\mu_{\rm obs} - \mu_C) \tag{5}$$

where C^{-1} is an uncertainty matrix which combined the uncertainty from the SALT2 fits, intrinsic dispersion, calibration, dust, peculiar velocity and many other factors (see Betoule et al. (2014) for a review). The benefit this analysis methodology provides is speed - for samples of hundreds of supernova, efficient matrix inversion algorithms allow the likelihood to be evaluated quickly. The speed comes with two costs. Firstly, formulating a χ^2 likelihood requires a loss of model flexibility by building into the model assumptions of uncertainty Gaussianity. Secondly, the computational efficiency is dependent on inverting a covariance matrix with dimensionality linearly proportional to the number of supernovae. As this number increases, the cost of inversion rises quickly, and is not viable for samples with thousands of supernovae.

Selection efficiency, such as the well known Malmquist bias (Malmquist K. G. 1922) is accounted for by correcting data. Simulations following survey observational strategies and geometry and used to calculate the expected bias in distance modulus, which is then added onto the observational data.

2.2 Approximate Bayesian Computation

To try and escape the limitations of the traditional analysis methodology, several recent methods have adopted Approximate Bayesian Computation, where supernova samples are forward modelled in parameter space and compared to observed distributions. Weyant et al. (2013) provides an introduction into ABC methods for supernova cosmology in the context of the SDSS-II results (Sako et al. 2014) and Flat ΛCDM cosmology, whilst Jennings et al. (2016) demonstrates their *superABC* method on simulated first season Dark Energy Survey samples, described in Kessler et al. (2015). In both examples, the supernova simulation package SNANA (Kessler et al. 2009) is used to forward model the data at each point in parameter space.

By building the systematic uncertainties and selection effects into the simulation package, there is vastly more freedom in how to treat and model those effects. Data does not need to be corrected, analytic approximations do not need to be applied, we are free to incorporate algorithms which simply cannot be expressed in a tractable likelihood. This freedom comes with a cost – computation. The classical χ^2 method's most computationally expensive step in a fit is matrix inversion. For ABC methods, we must instead simulate an entire supernova population in its entirety – drawing from underlying supernova populations, modelling light curves, applying selection effects, fitting light curves and applying data cuts. This is an intensive process. Luckily, efficient sampling algorithms that have walkers which rely on the Markov properties of groups instead of individual walkers, such as Ensemble sampling (Foreman-Mackey et al. 2013) allow easy parallelisation of parameter fits, such as used in the BAMBIS framework CITE RACHEL.

One final benefit of ABC methods is that they can move past the traditional treatment of supernovae with summary statistics (m_B, x_1 and c). Jennings et al. (2016) presents both a metric used to compare forward modelled summary statistic populations (denoted the 'Tripp' metric) and a metric directly applicable to the observed supernova light curves themselves, however evaluation of systematic uncertainty was only performed using the Tripp metric.

2.3 Hierarchical Bayesian Models

Sitting comfortably between the traditional models simplicity and the complexity of forward modelling lies Hierarchical Bayesian Models. With the introduction of multiple layers in our model, we can add far more flexibility than a traditional analysis whilst still maintaining most of the computational benefits that come from having a tractable likelihood. Mandel et al. (2009) and Mandel et al. (2011) construct a hierarchical model which they apply to the light curve fitting for supernova. March et al. (2011, 2014b); Karpenka (2015) derive a hierarchical model and simplify it by analytically marginalising over nuisance parameters provide a model which offers increased flexibility with reduced uncertainty over the traditional method. The recent BAHAMAS model (Shariff et al. 2016) builds off this and reanalyses the JLA dataset, whilst including extra freedom in the correction factors α and β, finding evidence for redshift dependence on β. Ma et al. (2016) also performed a reanalysis of the JLA dataset, finding significant difference in α and β values from the original as well. Notably, these methods rely on data that is bias corrected, however the UNITY framework given by Rubin et al. (2015) incorporates selection efficiency analytically in the model, and is applied to the Union 2.1 dataset (Suzuki et al. 2012). The well known BEAMS (Bayesian estimation applied to multiple species) methodology from Kunz et al. (2007) has been extended and applied in several works (Hlozek et al. 2012), mostly lately to include redshift uncertainty for photometric redshift application (Roberts et al. 2017), as zBEAMS.

The flexibility afforded by a hierarchical model allows for investigations into different treatments of underlying populations, rates, redshift distributions, mass corrections and redshift evolution, each of will will be discussed further in the outline of our model below.

3 OUR METHOD

We construct our Bayesian Hierarchical Model with several goals in mind: creation of a redshift dependent correlated underlying supernova population, increased treatment of systematics, and analytic correction of selection effects. As this is closest to the UNITY method from Rubin et al. (2015), we follow a similar model scaffold, and construct the model in Stan (Carpenter et al. 2017; Stan Development Team 2017) which uses automatic differentiation and the no-U-turn Sampler (NUTS), which is a variant of Hamiltonian Monte Carlo, to efficiently sample high dimensional parameter space.

At the most fundamental level, a supernova analysis is simply a mapping from an underlying population onto an observed population, where cosmology is encoded directly in the mapping function. The difficulty arises in adding sufficient, physically motivated flexibility in both of these populations whilst not adding *too* much flexibility, such that model fitting becomes pathological due to increasing parameter degeneracies within the model.

3.1 Observed and Latent Population

Like most of the BHM methods introduced previously, we work from the summary statistics as well, where each observed supernova has an apparent magnitude $\hat{m_B}$, stretch $\hat{x_1}$ and colour $\hat{c}$, with uncertainty C^{-1} on those values. Additionally, each supernova has an observed redshift $\hat{z}$ which we assume is precisely known as we are focused on the spectroscopically confirmed DES supernovae for this iteration of the method. Finally, each supernova also has a host galaxy mass associated with it $\hat{m}$.

The first layer of the hierarchy represents the latent population of m_B, x_1 and c. This can be thought of as the 'true' values for the supernova, and is normally distributed around the observed values such that

$$P(m_B, x_1, c | \hat{m_B}, \hat{x_1}, \hat{c}, C^{-1}) = \mathcal{N}(\{m_B, x_1, c\} | \{\hat{m_B}, \hat{x_1}, \hat{c}\}, C^{-1}) \tag{6}$$

3.2 Underlying Population

3.3 Cosmological Map

Mapping population of observables on a population of underlying SN, where the map function encodes cosmology. Difficulty is creating an underlying SN population that is flexible enough to not introduce bias whilst still being physically motivated.

Observables -> Transformation function (latent, mass, cosmology, systematics) -> Underlying pop (and outlier)

3.4 Applied to Spectroscopic Sample

Minimal outliers

4 APPLICATION TO DES

4.1 Simulating DES SN data

4.2 Model validation

appoximate_simple_test.py
multisim
bulk

4.3 Results on simulated data (ie projections)

4.3.1 Spectroscopic Sample

4.3.2 Photometric sample

4.4 Comparison with bells and whistles fixed

5 SYSTEMATICS STRENGTH TEST

systematics test

6 INTERESTING IMPLEMENTATION DETAILS

Anything interesting.

Also talk about non-analytic correction factors (and their failure - mc integration, GP, NNGP)

7 CONCLUSIONS

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APPENDIX A: PAPERS

This paper has been typeset from a TEX/LATEX file prepared by the author.