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Monte Carlo Corrections for Bayesian Methods

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ABSTRACT

I present a rigorous treatment for truncated or biased datasets by combining Bayesian analysis methods with Monte Carlo simulations.

1 INTRODUCTION

Find some treatment of biased or truncated data before. Maybe Gull (1989).

2 THE PERFECT WORLD

In a perfect world, data is neither biased nor truncated. The data is perfect. Uncertainties are well quantified and normally distributed around true values. Presumably everything is also spherical and in a vacuum. Let us create a mock model in this perfect world. Let us observe a series of iid events $\vec{x}$, which is known perfectly and drawn from a normal distribution such that

$$\vec{x} \sim \mathcal{N}(\mu, \sigma) \tag{1}$$

If, having collected our observations of $\vec{x}$, we wanted to constrain μ and σ, this would be a simple task of modelling the posterior surface. Taking uniform priors on both parameters we simply wish to map the surface

$$P(\mu, \sigma|\vec{x}) \propto P(\vec{x}|\mu, \sigma)P(\mu, \sigma) \tag{2}$$

$$P(\mu, \sigma|\vec{x}) \propto \prod_{i=1}^{N} \mathcal{N}(x_i|\mu, \sigma). \tag{3}$$

Generating a hundred data points with $\mu = 100$, $\sigma = 10$, we can recover our input parameters easily, as shown in Figure 1.

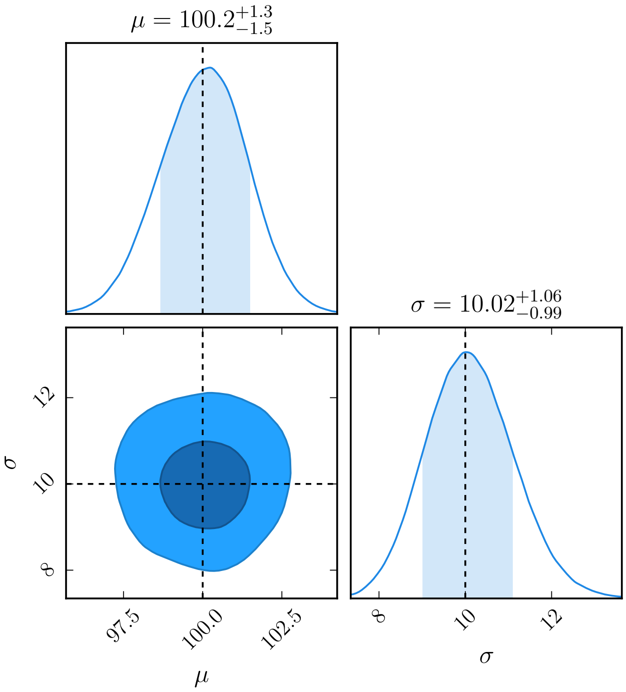

Figure 1. A systematic test of our perfect model, done by stacking the output chains from fitting 100 independent realisations of our 100 data points. Posterior surface mapped out using emcee.

3 THE IMPERFECT WORLD

In a slightly imperfect world we may have to deal with something like truncated data. For an example, consider the previous model, but with an instrumentation deficiency such that we can only observe events above a certain threshold, such that $x > \alpha$, assigning a value $\alpha = 85$ for convenience. If we don't take this truncation into account, we will recovered biased parameter estimates, as shown in Figure 2. However, we can correct for this truncation. If restate our likelihood to take into account some selection effect S, our likelihood for a single event can be stated as

$$\mathcal{L} = P(x|\theta, S) \tag{4}$$

$$= \frac{P(S|x, \theta)P(x|\theta)}{P(S|\theta)} \tag{5}$$

$$= \frac{P(S|x, \theta)P(x|\theta)}{\int P(S, D|\theta)\, dD} \tag{6}$$

$$= \frac{P(S|x, \theta)P(x|\theta)}{\int P(S|D, \theta)P(D|\theta)\, dD}, \tag{7}$$

where D is a potential observable. In our example, the selection efficiency is a step function, having observed x,

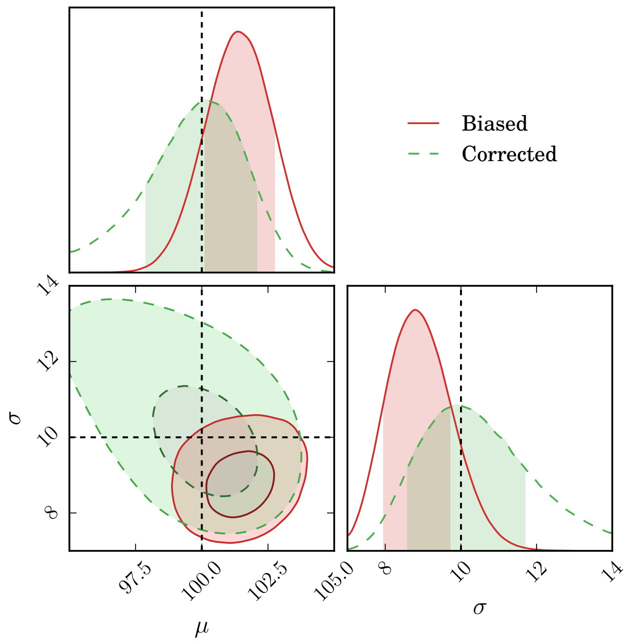

Figure 2. A systematic test of our imperfect model, done by stacking the output chains from fitting 100 independent realisations of our 100 data points, subject to our thresholding.

$P(S|x,\theta) = 1$. To substitute in our normal model,

$$\mathcal{L} = \frac{\mathcal{N}(x|\mu,\sigma)}{\int_{-\infty}^{\infty} \mathcal{H}(D-\alpha)\mathcal{N}(D|\mu,\sigma)\,dD} \tag{8}$$

$$= \frac{\mathcal{N}(x|\mu,\sigma)}{\int_{\alpha}^{\infty} \mathcal{N}(D|\mu,\sigma)\,dD} \tag{9}$$

$$= \frac{\mathcal{N}(x|\mu,\sigma)}{\frac{1}{2}\mathrm{erfc}\left[\frac{\alpha-\mu}{\sqrt{2}\sigma}\right]}, \tag{10}$$

if $\mu > \alpha$. We can add this correction to our model, and note that we now recover unbiased parameter estimates, also shown in Figure 2.

4 THE REAL WORLD

Unfortunately it is a rare scenario when dealing with nature and all her faults for us to be able to have a perfect step function or for selection efficiency to be wholly encapsulated by a single parameter. A more realistic scenario involves a selection efficiency which cannot be conveniently described by an analytic function and instead is a high dimensional non-analytic function. And its probably stochastic too, just to throw another wrench in the works.

5 CONCLUSION

Still da bes

ACKNOWLEDGMENTS

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REFERENCES

Gull S. F., 1989, in , Maximum Entropy and Bayesian Methods. Springer, pp 511–518

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