SIMULATION OF A MARKOV CHAIN USING THE METROPOLIS ALGORITHM

PROJECT PRESENTATION

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Outline

INTRODUCTION

Definition

- A Markov chain $\{X_t\}$ is a sequence of dependent random variables

$$\{X_0, X_1, \ldots\}$$

  such that the probability distribution of X_t given the past variables depends only on X_{t-1}.
- Markov chain Monte Carlo methods generates correlated variables from a Markov chain.
- The chain (values) generated by the Monte Carlo Markov Chain methods enjoy a very strong stability property, i.e., a stationary probability distribution exists by construction of those chains

OBJECTIVE OF STUDY

- The objective of this study is to simulate a Markov chain using the metropolis independence sampler algorithm to simulate a Markov chain where the Cauchy distribution C(0,1) is used as the target distribution.
- The normal distribution with mean 0 and variance 2
- The t distribution with degree of freedom 20 are selected as the proposed distribution
- How the simulated Markov chain depends on the variance of the proposed distribution.

METROPOLIS ALGORITHM

ALGORITHM

- The proposal distribution in the independence sampling algorithm does not depend on the previous value of the chain that is the random variable Y is independent of X_t
- The Metropolis-algorithm generates the Markov chain $\{X_0, X_1, \dots\}$ as follows:

- Choose a proposal distribution $g(\cdot|X_t)$ with the same support set as the target distribution.
- Generate X_0 from a distribution g.
- Repeat (until the chain has converged to a stationary distribution according to some criteria):

METROPOLIS ALGORITHM

ALGORITHM

- Generate Y from a distribution $g(\cdot|X_t)$.
- Generate U from $Uniform(0,1)$.
- If

$$U \le \frac{f(Y)g(X_t|Y)}{f(X_t)g(Y|X_t)}$$

accept Y and set $X_{t+1} = Y$; otherwise set $X_{t+1} = X_t$.

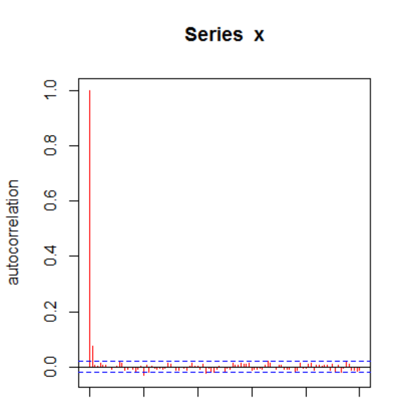
- Increment t.

Note in third step candidate point Y is accepted with probability

$$\alpha(X_t, Y) = \min\left(1, \frac{f(Y)g(X_t)}{f(X_t)g(Y)}\right) \tag{1}$$

Implementing algorithm using the normal distribution as the proposed distribution

- The algorithm is implemented using R and the probability of acceptance is calculated to be 0.5580673



METROPOLIS ALGORITHM

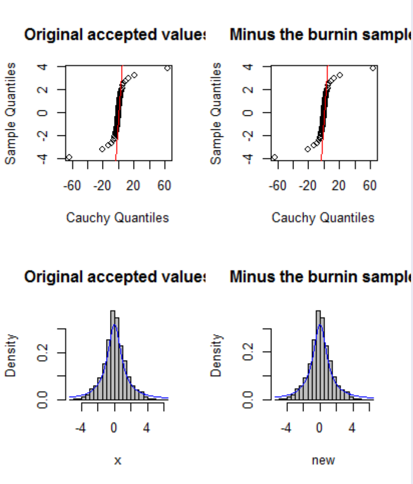
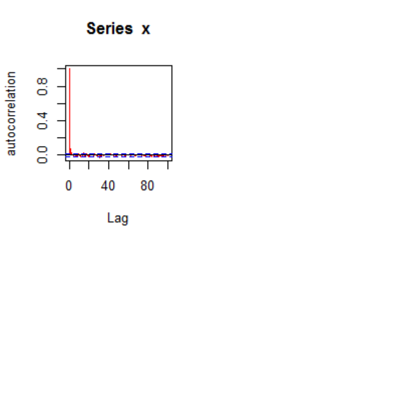


Figure: qqplot and histogram

Implementing algorithm using the t distribution as the proposed distribution

- The algorithm is implemented using R and the probability of acceptance is calculated to be 0.0.8824849



METROPOLIS ALGORITHM

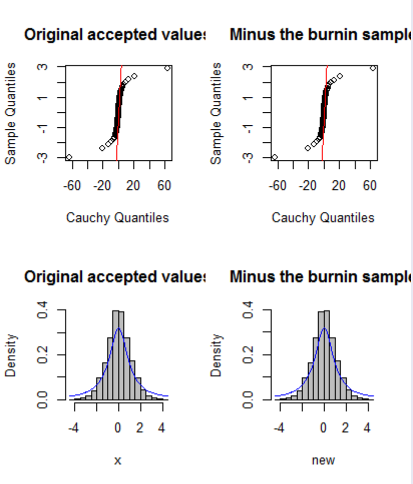
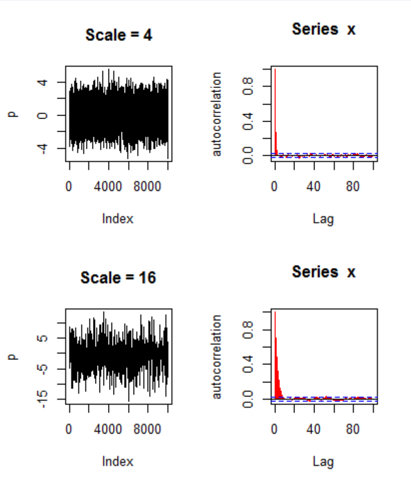


Figure: qqplot and histogram

EFFECT OF VARIANCE ON SAMPLER FUNCTION

- Consider different scales for the proposed distribution to investigate how the simulated Markov chain depends on the variance.



- The Cauchy distribution c(0,1) was used as the the target distribution to simulate a Markov chain using the metropolis independence sampler algorithm with normal distribution with mean 0 and variance 2 and the t distribution with degree of freedom 20 used as the proposed distribution
- The scale parameter was varied to monitor the acceptance rate and the variance.
- The variance of the sampler is inversely related to the auto correlation. With a small variance in the sampler function the series has high auto correlation but large acceptance rates. Thus the entire sample space runs in a short amount of runs. With a large variance in the sampler function the series has low auto correlation and low acceptance rates. This means it takes many runs to sample from the whole sample space.

LIMITATION

- The choice of variance in the sampler function is highly important. If the variance is quite large then the acceptance rate is low so the algorithm converges slowly. On the other hand if it is too small, the algorithm moves slowly throughout the sample space and converges slowly.

THANK YOU !!!!!!!!