Stat 589, Fall 2017, Homework 1

Your Name

Due Date: August 31, 2017, 20 points

Directions

- Download the (`.Rmd`) file version of this homework from Github. Right click the link and "Save as..." to your desired folder.
- Change the file extension to `.Rmd`. This is important otherwise your file will not be converted to word document.
- This RMarkdown `(.Rmd)` file type is used widely to share R codes and reproduce statistical results. More information is found on this page.
  - Open this file using RStudio.
  - Write your name on the header configuration at the top. Replace also the file output into `pdf_document` so that you can generate a pdf document instead.
  - Write your R code inside the code chunks after each question.
  - Write your comments after the `#` sign.
  - To generate the word document output, click the button `Knit` and wait for the word document to appear.
  - RStudio will prompt you to install the `knitr` package.
- Submit your completed laboratory exercise using Blackboard's Turnitin feature. Your Turnitin upload link is found on your Blackboard course shell under the Laboratory folder.

Question 1. Given the random vector

$$\mathbf{X} = [X_1, X_2, X_3, X_4, X_5]'$$

with mean

$$\mu = [2, 4, -1, 3, 0]'$$

and variance-covariance matrix

$$\Sigma = \begin{bmatrix} 4 & -1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ -1 & 3 & 1 & -1 & 0 \\ \frac{1}{2} & 1 & 6 & 1 & -1 \\ -\frac{1}{2} & -1 & 1 & 4 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{bmatrix}$$

Let $\mathbf{X}^{(1)} = [X_1, X_2]'$ and $\mathbf{X}^{(2)} = [X_3, X_4, X_5]'$. Also, let

$$\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \text{ and. } \mathbf{B} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

Use R to find the following
i) Σ^{-1} and $\Sigma^{1/2}$
ii) eigenvalvalues and eigenvectors of Σ
iii) $E(\mathbf{A}\mathbf{X}^{(1)})$
iv) $Cov(\mathbf{A}\mathbf{X}^{(1)})$

v) $E(\mathbf{BX}^{(2)})$
vi) $Cov(\mathbf{BX}^{(2)})$
vii) $Cov(\mathbf{AX}^{(1)}, \mathbf{BX}^{(2)})$

Code chunk

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# Insert your code for this question after this line


# last R code line for this question
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Question 2. Using the vector $\mathbf{b} = [-4, 1]'$ and $\mathbf{d} = [1, 1]'$, verify the extended Cauchy-Scwarz inequality $(\mathbf{b}'\mathbf{d})^2 \leq (\mathbf{b}'\mathbf{B}\mathbf{b})(\mathbf{d}'\mathbf{B}^{-1}\mathbf{d})$ if

$$B = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}.$$

Code chunk

```
# Insert your code for this question after this line


# last R code line for this question
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Question 3. Refer to the data set `T1-7.DAT`. The data consists of average ratings over the course of treatment for patients undergoing radiotherapy.

variable	details
x_1	number of symptoms, such as sore throat or nausea
x_2	amount of activity, on a 1-5 scale
x_3	amount of sleep, on a 1-5 scale
x_4	amount of food consumed, on a 1-3 scale
x_5	appetite, on a 1-5 scale
x_6	skin reaction, on a 0-3 scale

i) Check the structure of the data. Display the first 4 rows and last 5 rows.

ii) Describe the data by summarizing each variable numerically.

iii) Construct a scatterplot matrix for all the variables in the data. Describe any possible linear relationships between pairs of variables. Use the function `GGally::ggpairs`.

iv) Find the sample covariance matrix $\mathbf{S}_n$ and $\mathbf{R}$ arrays. Interpret the pairwise correlations.

Code chunk

```
# Insert your code for this question after this line


# last R code line for this question
```
