Problem Set 8

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1. Explanation for the Code

First, I would like to explain the code and then reply to the questions. Besides importing necessary packages and setting the parameter values, at the beginning of the code, several functions are defined. First, agg function defines the matrix multiplication cell by cell and addition of each cell to get the aggregate labor demand, investment, output and adjustment cost. For the value function iteration over z, k and kprime, `VFI_loop` function is defined, where `V_prime` is V matrix iterated one period by being multiplied with pi (transition) matrix. Likewise, `SD_loop` function is defined in order to get the stationary distribution of firms. These last two functions are defined right after the numba in order to increase the efficiency of the code. To report the degree of efficiency, the time spent on running this specific function decreased by more than 100 times.

As the second step, in order to approximate the continuous AR(1) process of natural logarithm of z as a discrete process, random error term, epsilon, is drawn from a normal distribution. Then, AR(1) process of z is put into the code and then 9 grid points of z are defined. However, in order to get the grid points for z instead of log(z), we took the exponential of grid points of z.

As the third step, transition matrix pi is calculated via using the integrand function as we did in the class. Pi function is 9x9 matrix since we have 9 grid points for z (productivity shock).

We included all the next steps except the plot of the final graphs into the loop which is defined as MKclear function for labor market clearing. As the fourth step, we defined 285 kgrid points for the capital stock. We used the same kstar (steady state of capital stock) as there is no effect of productivity shock and adjustment cost since in the steady state, adjustment costs almost converge to zero. Then, as the fifth step, we defined the operating profits as op, which is iterated over z and k and also defined firm cash flow as e, which is iterated over z, k and kprime. It is important to note that the labor demand choice of firms is coming from the intratemporal decision and then, this optimal choice of labor supply is plugged into the operating profits, pi, function as follows:

$$l = (\frac{\alpha_l}{w})^{\frac{1}{1-\alpha_l}} z^{\frac{1}{1-\alpha_l}} k^{\frac{\alpha_k}{1-\alpha_l}}$$

$$\pi(z,k) = (1-\alpha_l)(\frac{\alpha_l}{w})^{\frac{\alpha_l}{1-\alpha_l}} z^{\frac{1}{1-\alpha_l}} k^{\frac{\alpha_k}{1-\alpha_l}}$$

After finding operating profits, the earnings or per period cash flow function is as follows:

$$e(z,k,k^{'}) = \pi(z,k) + s - d - (k^{'} - (1-\delta)k) - \frac{\psi}{2}\frac{(k^{'} - (1-\delta)k)^2)}{k}$$

As the sixth step, the necessary parameters and matrices for value function iteration are set. Then, by starting the time, the iteration is performed and the policy function, as kprime being a function of k and z, is stored as 9x285 (sizezxsizek) matrix under PF variable. The optimal value function in the last iteration is stored under the matrix VF. Bellman equation is the following:

$$V(z,k) = \max_{k^{'},s,d} \pi(z,k) + s - d - (k^{'} - (1-\delta)k) - \frac{\psi}{2}\frac{(k^{'} - (1-\delta)k)^2)}{k} + \frac{1}{1+r}E_{z^{'}|z}V(z^{'},k^{'})$$

As the seventh step, optimal amount of investment, capital stock and labor demand are calculated. After this step, the stationary distribution of the firms are found via `SD_loop` function by iteration as Gamma.

As the eighth step, optimal aggregate labor demand (optALD), optimal aggregate investment (optAI), optimal aggregate adjustment cost (optAADJC) and optimal aggregate output (optAY) are calculated by using the Agg function defined at the beginning. Furthermore, using the aggregate resource constraint of the economy, optimal aggregate consumption (optCON) is calculated. Then, by using the first order condition of households with respect to labor supply, we calculated the optimal aggregate labor supply (optALS).

$$\bar{w_i} = -\frac{U_2(\bar{C}, \bar{L})}{U_1(\bar{C}, \bar{L})} = -\frac{-h\bar{L}}{\frac{1}{\bar{c}}} = h\bar{c}\bar{L}$$

$$\bar{L_s} = \frac{\bar{w_i}}{h\bar{c}}$$

Then, as the next step, we can now check the absolute value of the difference between optimal aggregate labor demand and supply, which is defined as mkclear.

As the ninth step, the stationary distribution for firms is graphed besides with the plot for the policy function where krpime is a function of k and z. Then, this mkclear function ends by returning the mkclear value.

As the last step, the Nelder-Mead method is used to find the minimum value of mkclear and ensure that the labor markets clear. As in the Walras' law, if n-1 markets clear, then the remaining market clears as well. By using the aggregate resource constraint of the economy as a condition above, we already stated that goods market clear and via the loop, labor markets clear. Thus, capital markets clear as well. The minimum of the MKclear function is returned as `GE_results` as the optimal labor wage (`opt_w`).

2. Reporting the Results

The optimal wage rate in the equilibrium is found as 0.90 (I did not want to report more than two digits for the decimal to show a clear result).

Figure 1 shows the 3-dimensional graph of the stationary distribution of firms, where one axis displays natural logarithm of the productivity and the other one shows the capital stock. The density can be seen from the third axis. If the firms got low productivity shock and had low amount of capital stock in the last period, then the probability of them

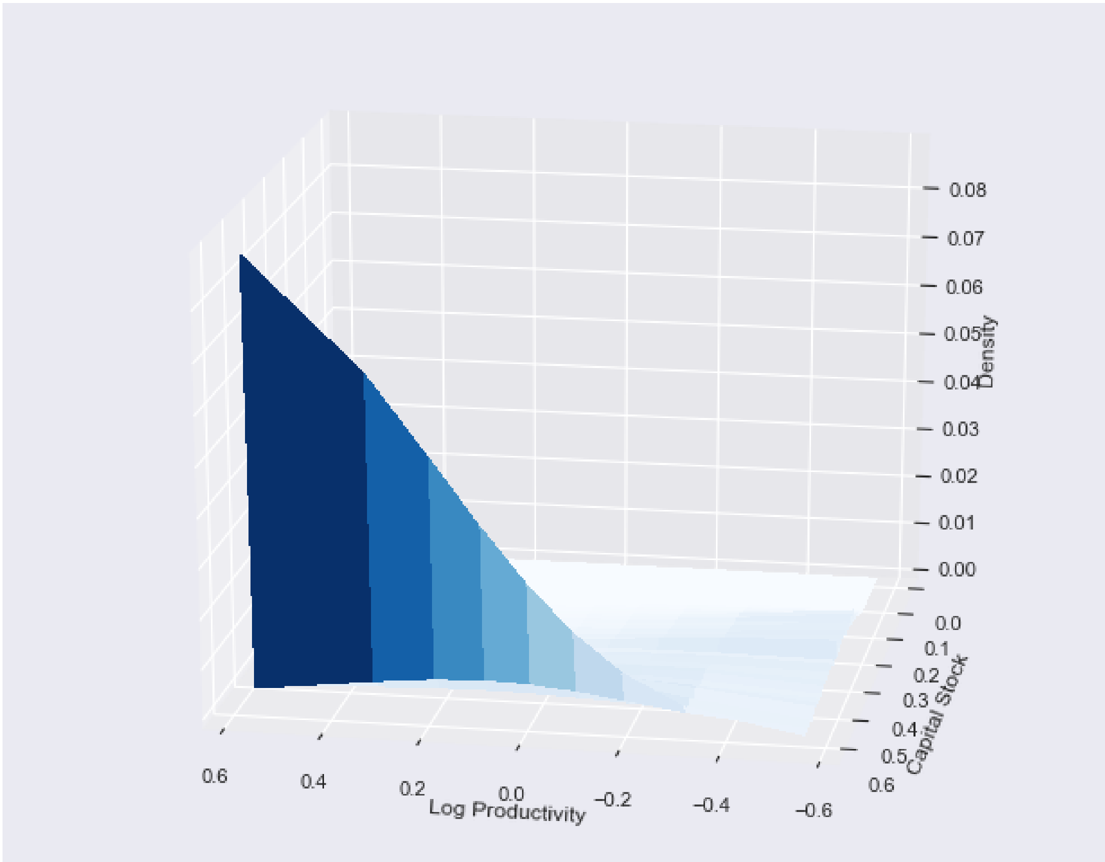

Figure 1: Stationary Distribution of Firms

(or the frequency of firms) having high productivity and high capital stock is low in the next period as seen from the lighter blue of the graph in this area. However, the darker blue part shows that the density is higher there. In other words, when firms received high productivity shock and had high amount of capital stock in the last period, the probability of them (or the frequency of firms) having high productivity and high capital stock in the next period is high as the density is higher here. As seen from the figure, also as firms get higher productivity shocks, frequency of firms having higher capital stock levels increase.

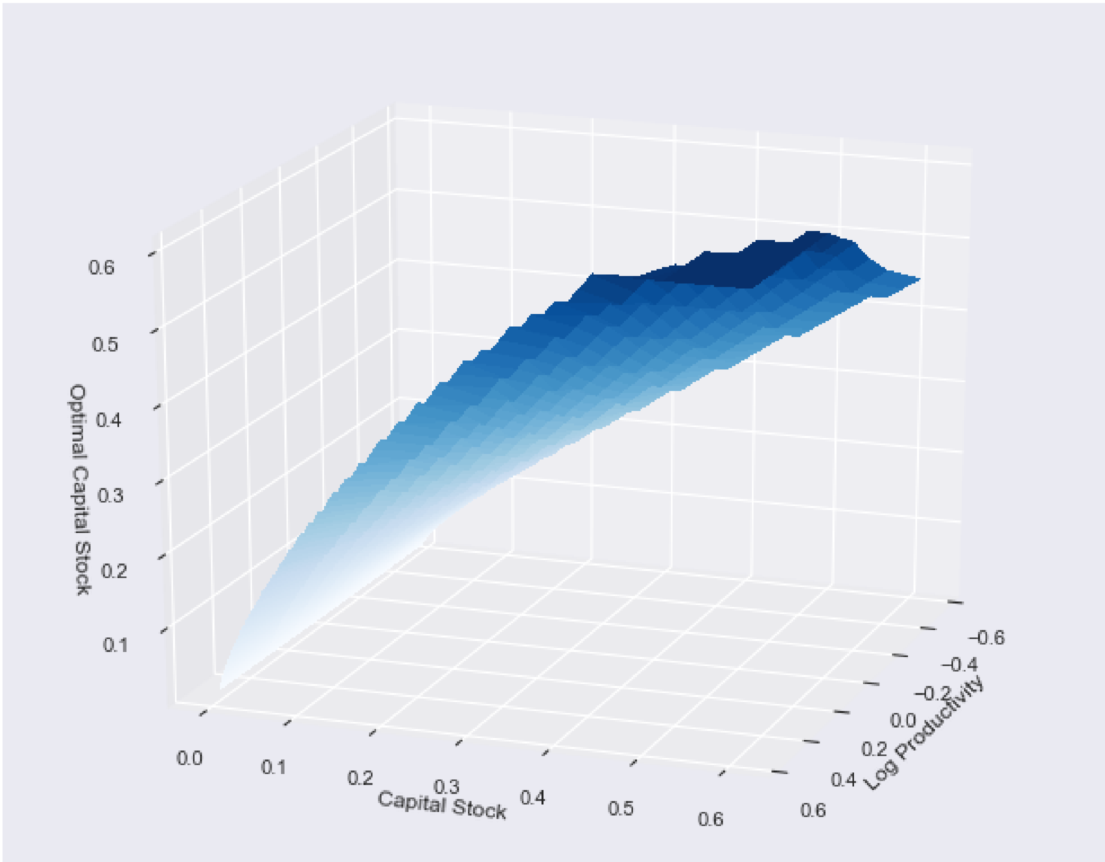

Figure 2: Policy Function

Figure 2 shows the 3-dimensional graph of the optimal policy function, where capital stock in the next period (vertical axis) is a function of the capital stock in this period (the first horizontal axis) and the productivity shock received in this period (the second horizontal axis). As seen in the graph, optimal capital stock is concave with respect to the combination of capital stock and log of productivity. As the color of the graph transitions from lighter to darker blue, the capital stock also follows a transition from low to high levels in terms of the next period's optimal capita stock. The darkest area shows the steady state level of capital stock given the capital stock in the last period and the productivity shock received. The transition states that if firms have low levels of initial capital stock and received high productivity shock, then their optimal level of capital stock next period goes higher up to a point due to the decreasing marginal product of capital stock (or the concavity, decreasing returns to scale property of the production function).