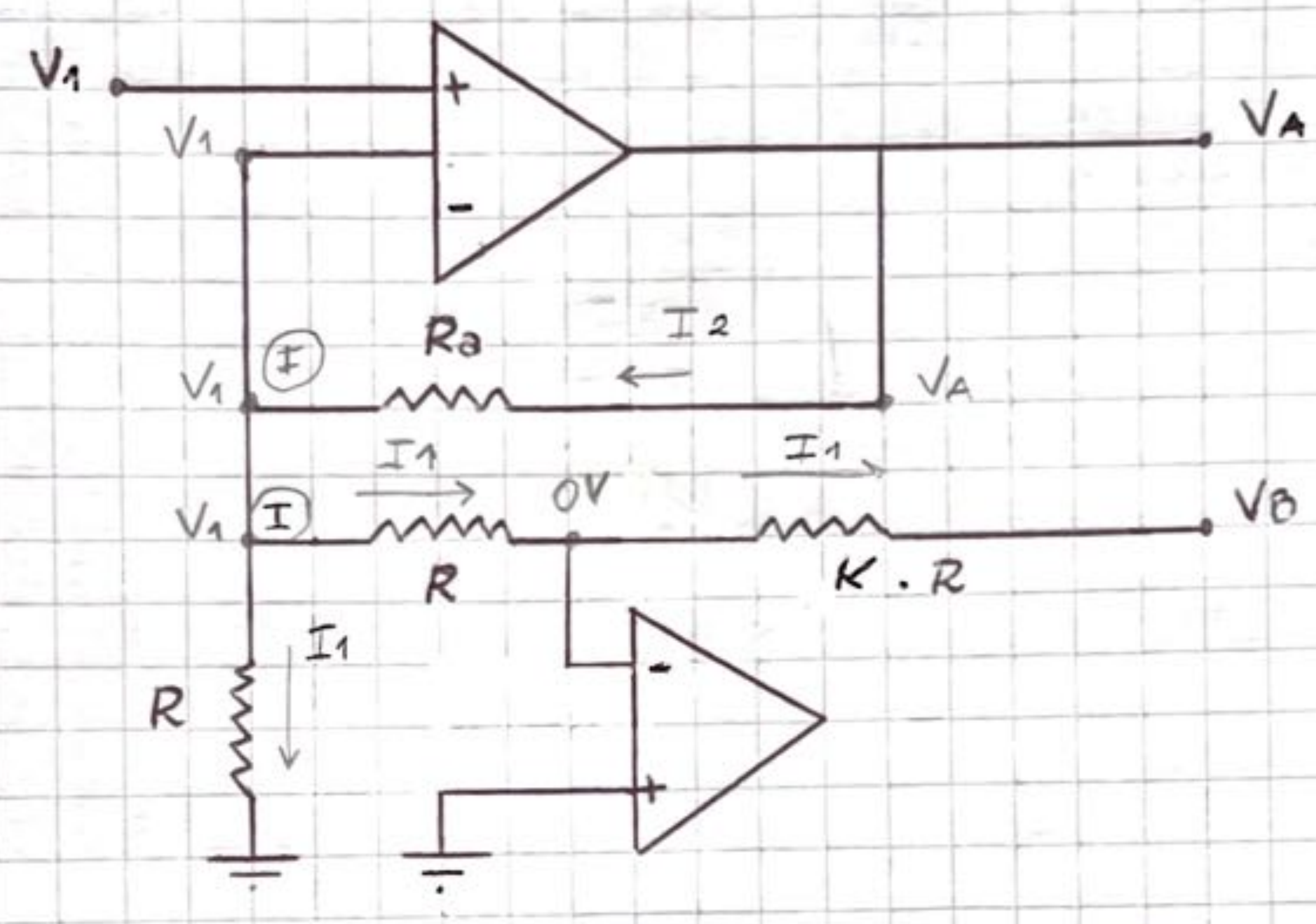
② Amplificador con salida diferencial obtener la expresión de V_{AB}. Utilizar el valor de $R_a = \frac{R}{2}(K-1)$.



- $I_1 = \frac{V_1}{R} = \frac{-V_B}{K.R} \longrightarrow \boxed{V_B = -V_1 K}$

- $I_2 = \frac{V_A - V_1}{Ra}$

Plantiamos nodos.

Ⓘ $(V_A - V_1).G_a = 2V_1 G$

$V_A.G_a - V_1 G_a = 2V_1 G$

$V_A . G_a = 2V_1 (G + G_a)$

$V_A = 2V_1 \frac{(G+G_a)}{G_a} \longrightarrow \boxed{V_A = 2V_1 \left(\frac{\frac{1}{R} + \frac{1}{\frac{R}{2}(K-1)}}{\frac{1}{\frac{R}{2}(K-1)}} \right)}$

$V_A = 2V_1 \left(\frac{R}{2}(K-1)\right)\left(\frac{1}{R} + \frac{1}{\frac{R}{2}(K-1)}\right)$

$V_{AB} = V_A - V_B = 2V_1 \left(\frac{K-1}{2} + 1\right) + V_1 K = V_1(K-1) + 2V_1 + V_1 K$

$V_{AB} = V_1 K - \cancel{V_1} + \cancel{2}V_1 + V_1 K = 2V_1 K + V_1 = V_1(2K+1)$

$\boxed{V_{AB} = V_1(2K+1)}$

7) Desarrollo algebraico:

A)

$$\frac{V_1 - V_2}{R_1 + R_2} = \frac{V_1 - \frac{V_1 \cdot R_3}{R_3 + 1/Cs}}{R_1} = \frac{1}{R_1}\left(V_1 - \frac{V_1 \cdot sCR_3}{sCR_3 + 1}\right) = \frac{V_1}{R_1}\left(1 - \frac{sCR_3}{sCR_3 + 1}\right)$$

$$\frac{V_1 - V_2}{R_1 + R_2} = \frac{V_1}{R_1}\left(1 - \frac{sCR_3}{CR_3\left(s + \frac{1}{CR_3}\right)}\right) = \frac{V_1}{R_1}\left(1 - \frac{s}{s + \frac{1}{CR_3}}\right) = \frac{V_1}{R_1}\left(\frac{1/CR_3}{s + \frac{1}{CR_3}}\right)$$

$$\frac{V_1 - V_2}{R_1 + R_2} = V_1 \cdot \frac{1}{CR_1 \cdot R_3} \cdot \frac{1}{s + \frac{1}{CR_3}}$$

Despejamos V_2.

$$-V_2 = \left(V_1 \cdot \frac{1}{CR_1R_3} \cdot \frac{1}{s + \frac{1}{CR_3}}\right)(R_1 + R_2) - V_1$$

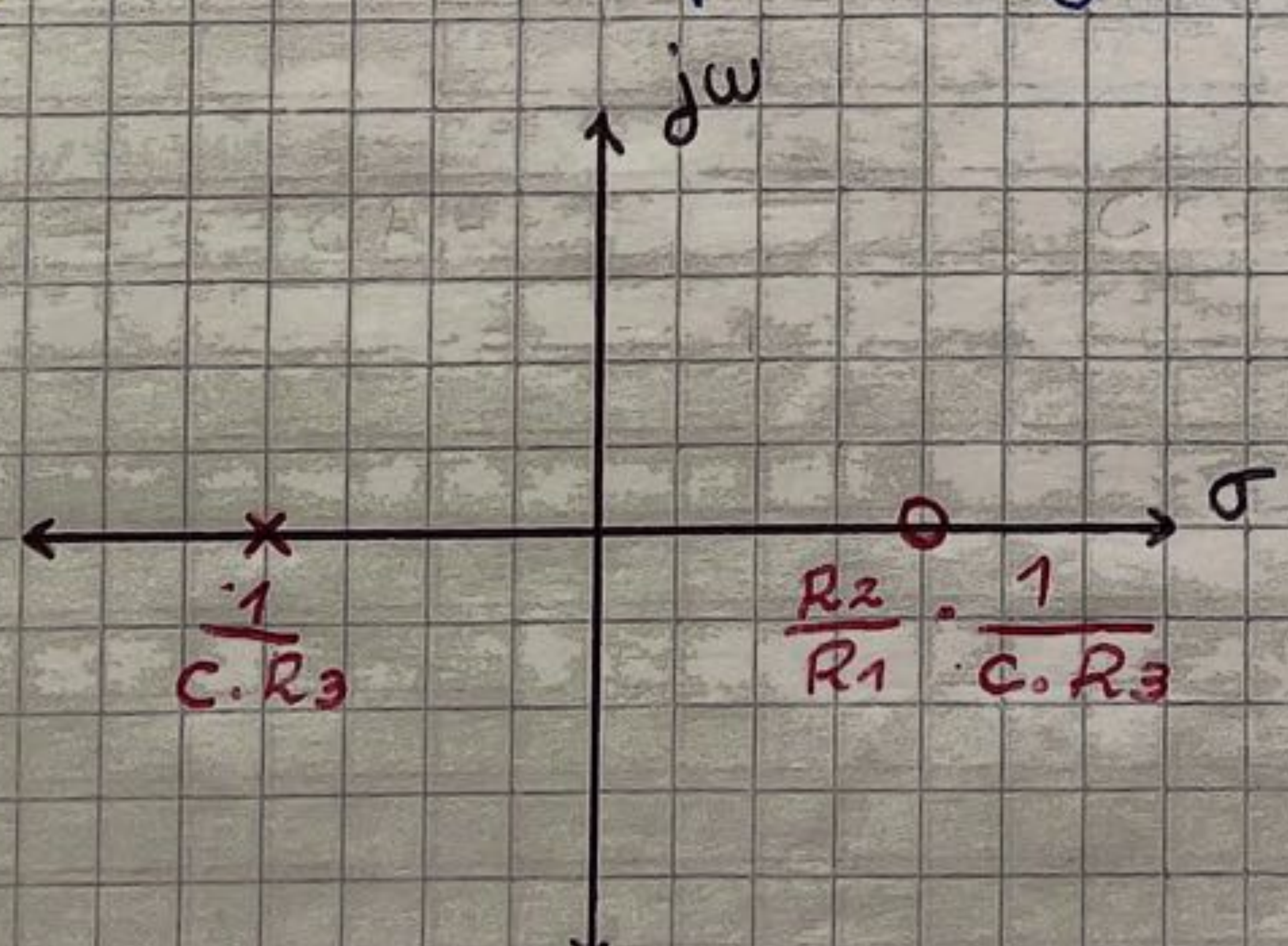
$$V_2 = V_1\left(1 - \frac{R_1 + R_2}{R_1(sCR_3 + 1)}\right) = V_1\left(\frac{R_1(sCR_3 + 1) - R_1 - R_2}{R_1(sCR_3 + 1)}\right)$$

$$V_2 = V_1\left(\frac{sCR_3 + 1 - 1 - R_2/R_1}{sCR_3 + 1}\right)$$

Finalmente:

$$\boxed{\frac{V_2}{V_1} = \frac{sCR_3 - R_2/R_1}{sCR_3 + 1} = \frac{CR_3\,(s - R_2/R_1CR_3)}{CR_3\,(s + 1/CR_3)} = \frac{s - R_2/R_1 \cdot CR_3}{s + 1/CR_3}}$$

· Diagrama de polos y ceros:

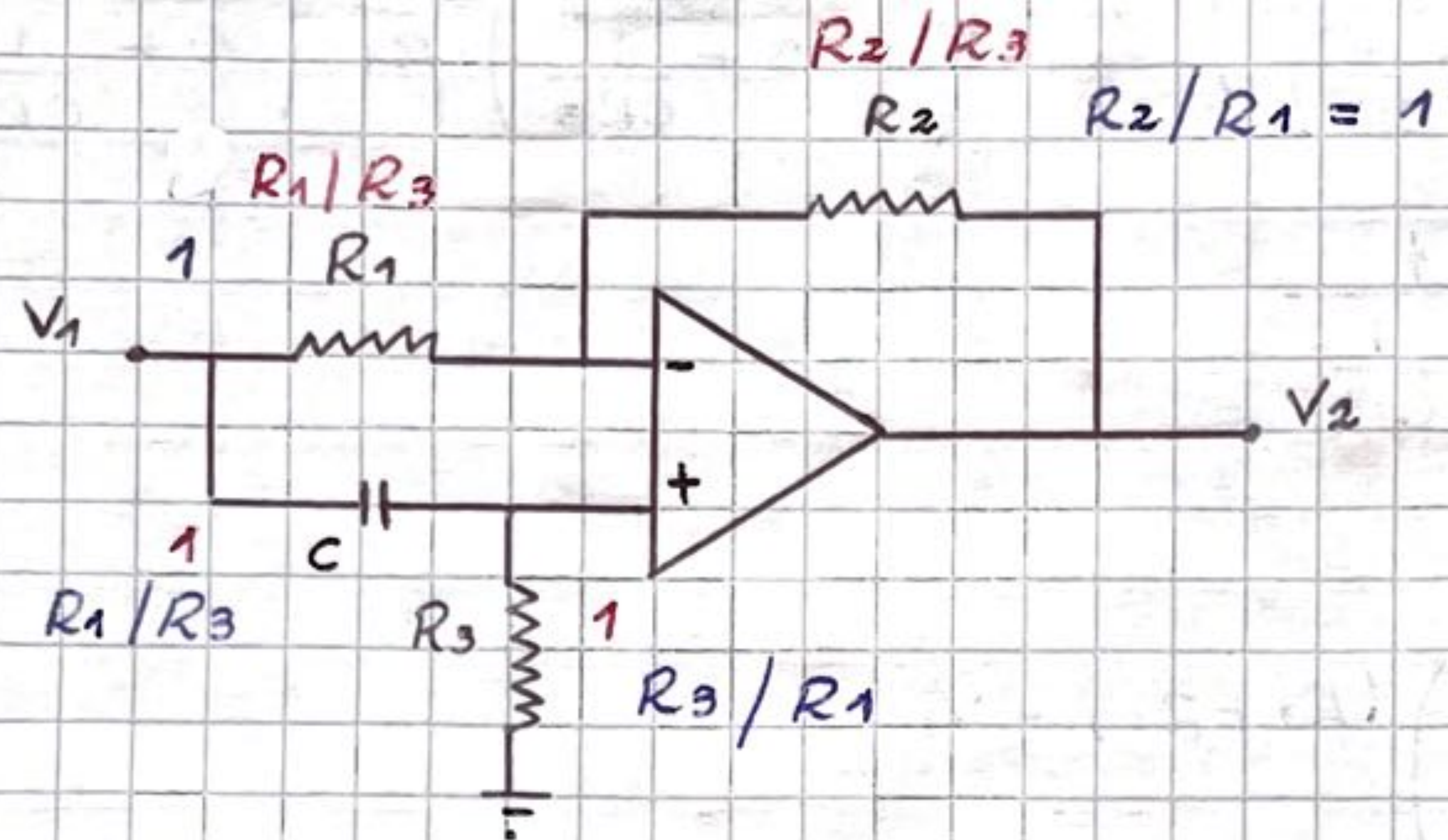


NOTA

Normalización:

- $\Omega_w = \dfrac{1}{R_3 \cdot C}$ → Recordamos que debemos multiplicar por estos parámetros para normalizar.

- $\Omega_z =$ → R_1, R_3



Recordamos que:

$$\frac{V_2}{V_1} = \frac{SCR_3 + R_2/R_1}{S.CR_3 + 1}$$

Normalizamos. $s' = s/\Omega_w \rightarrow s = s' \cdot \Omega_w = s' \cdot \dfrac{1}{R_3 . C}$

- $$\frac{V_2}{V_1} = \frac{s' - R_2/R_1}{s' + 1}$$

Y $c' = \Omega_w . C \rightarrow C' = \dfrac{1}{R_3 . C} . C = \dfrac{1}{R_3}$

Y en impedancia:

$$\begin{cases} R_1' = R_1/\Omega_z = R_1/R_3 \\ R_2' = R_2/\Omega_z = R_2/R_3 \\ R_3' = R_3/\Omega_z = 1 \\ C'' = C' . \Omega_z = 1/R_3 . R_3 = 1 \end{cases}$$

$$\begin{cases} R_1' = R_1/\Omega_z = 1 \\ R_2' = R_2/\Omega_z = R_2/R_1 \\ R_3' = R_3/\Omega_z = R_3/R_1 \\ C'' = C' . \Omega_z = R_1/R_3 \end{cases}$$

Expresión de módulo y fase:

Partimos de la transferencia: $\frac{V_2}{V_1} = \frac{s - \frac{R_2}{R_1} \cdot \frac{1}{C \cdot R_3}}{s + \frac{1}{C \cdot R_3}}$

· Módulo:

$$\frac{V_2}{V_1}(s)\bigg|_{s=j\omega} = \frac{j\omega - \frac{R_2}{R_1} \cdot \frac{1}{C \cdot R_3}}{j\omega + \frac{1}{C \cdot R_3}}$$

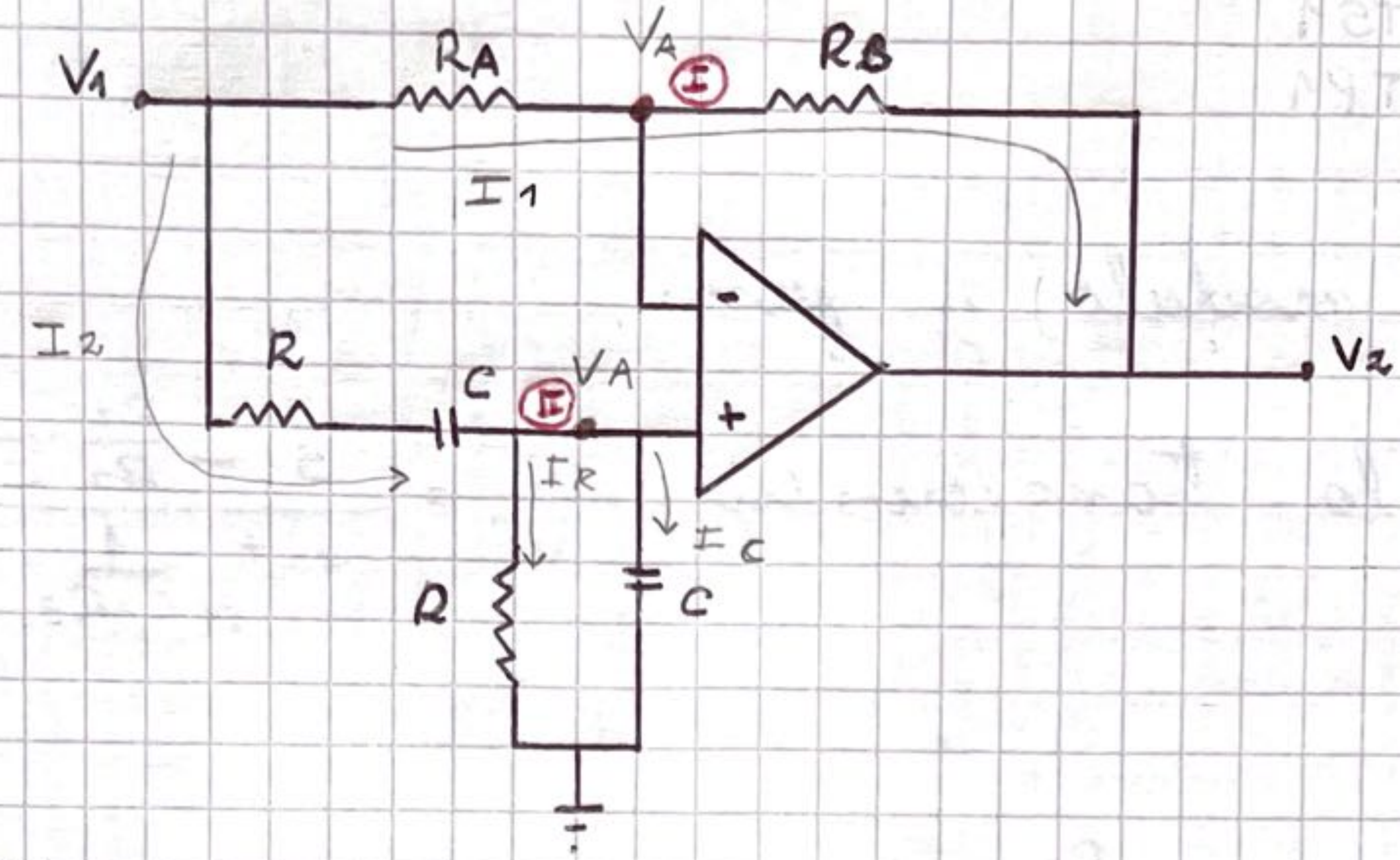
$$\boxed{\left|\frac{V_2}{V_1}(j\omega)\right|^2 = \frac{\omega^2 + \left(\frac{R_2}{R_1} \cdot \frac{1}{C R_3}\right)^2}{\omega^2 + \left(\frac{1}{C R_3}\right)^2}}$$

· Fase: $\phi(\omega) = \sum_1^m tg^{-1} \frac{\omega \pm \beta_z}{\alpha_z} - \sum_1^n tg^{-1} \frac{\omega \pm \beta_p}{\alpha_p}$

$$\boxed{\phi(\omega) = tg^{-1}\left(\frac{\omega}{-\frac{R_2}{R_1} \cdot \frac{1}{C \cdot R_3}}\right) - tg^{-1}\left(\frac{\omega}{\frac{1}{C R_3}}\right)}$$

E) Este circuito es útil como rotador de fase de orden 1 si se pretende no modificar la amplitud de la señal.

Ⓑ



$$\begin{cases} R_A / R_B = 5 \\ R = 1 k\Omega \\ C = 1 \mu F \end{cases}$$

Planteamos:

- $I_1 = (V_1 - V_a)\, G_A = (V_A - V_2) . G_B$
- $I_2 = (V_1 - V_A)\underbrace{\left(\frac{1}{R + \frac{1}{SC}}\right)}_{Y_1} = (V_1 - V_A)\, \frac{SC}{SCR + 1}$
- $I_R = V_A . G$
- $I_C = V_A . \underbrace{SC}_{Y_2}$

Planteamos nodos:

Ⓘ $(V_1 - V_A)\, G_A = (V_A - V_2) . G_B$

$V_1 . G_A - V_A . G_A = V_A . G_B - V_2 . G_B$

$V_1 . G_A + V_2 . G_B = V_A . G_B + V_A . G_A \longrightarrow \boxed{V_A = \frac{V_1 G_A + V_2 G_B}{G_B + G_A}}$

Ⓘⓘ $(V_1 - V_A)\left(\frac{SC}{SRC + 1}\right) = V_A . G + V_A . Y_2 = V_A\, (G + Y_2)$

$$V_1\left(\frac{SC}{SRC + 1}\right) - V_A\left(\frac{SC}{SRC + 1}\right) = V_A\, (G + Y_2)$$

$$V_1\left(\frac{SC}{SRC + 1}\right) = V_A\, (G + Y_2) + V_A\left(\frac{SC}{SRC + 1}\right)$$

$$V_1 . Y_1 = V_A \left[(G + Y_2) + Y_1 \right]$$

Reemplazamos con V_A

$$V_1 . Y_1 = \left(\frac{V_1 . G_A + V_2 . G_B}{G_B + G_A}\right)(G + Y_2 + Y_1) = \left(\frac{V_1 G_A}{G_B + G_A} + \frac{V_2 G_B}{G_B + G_A}\right)(G + Y_1 + Y_2)$$

NOTA

$$V_1 \cdot Y_1 = \left(\frac{V_1}{1+\frac{G_B}{G_A}} + \frac{V_2}{1+\frac{G_A}{G_B}} \right)(G + Y_1 + Y_2)$$

$$V_1 \cdot Y_1 = \left(V_1 \frac{1}{1+\frac{G_B}{G_A}} + \frac{V_2}{1+\frac{G_A}{G_B}} \right)\left(\frac{1}{R} + \frac{SC}{SCR+1} + SC \right)$$

$$\cdot\ \frac{1}{R} + \frac{SC}{SRC+1} + SC = \frac{SCR+1+SCR+S^2C^2R^2+SCR}{R(SRC+1)} = \frac{S^2C^2R^2+3SCR+1}{SCR^2+R}$$

Entonces:

$$V_1 \cdot \frac{SC}{SCR+1} = \frac{V_1}{1+\frac{R_A}{R_B}} \left(\frac{S^2C^2R^2+3SCR+1}{SCR^2+R} \right) + \frac{V_2}{1+\frac{R_B}{R_A}} \left(\frac{S^2C^2R^2+3SCR+1}{SCR^2+R} \right)$$

$$V_1 \left(\frac{SC}{SCR+1} - \frac{S^2C^2R^2+3SCR+1}{\left(1+\frac{R_A}{R_B}\right)(SCR^2+R)} \right) = \left(\frac{S^2C^2R^2+3SCR+1}{\left(1+\frac{R_B}{R_A}\right)(SCR^2+R)} \right) V_2$$

$$\frac{SC\left(1+\frac{R_A}{R_B}\right)R - (S^2C^2R^2+3SCR+1)}{\left(1+\frac{R_A}{R_B}\right)(SCR+1)R} = \frac{\cancel{SCR} + SCR\left(\frac{R_A}{R_B}\right) - S^2C^2R^2 - 3\cancel{SCR} - 1}{\left(1+\frac{R_A}{R_B}\right)(SCR+1)R}$$

$$= \frac{-S^2C^2R^2 + SCR\left(\frac{R_A}{R_B}-2\right) - 1}{\left(1+\frac{R_A}{R_B}\right)(SCR^2+R)}$$

Finalmente:

$$\frac{V_2}{V_1} = \frac{-S^2C^2R^2+SCR\left(\frac{R_A}{R_B}-2\right)-1}{\left(1+\frac{R_A}{R_B}\right)\cancel{(SCR^2+R)}} \cdot \frac{\left(1+\frac{R_B}{R_A}\right)\cancel{(SCR^2+R)}}{S^2C^2R^2+3SCR+1}$$

$$\frac{V_2}{V_1} = \frac{1+R_B/R_A}{1+R_A/R_B} \cdot \frac{-S^2C^2R^2+SCR\left(\frac{R_A}{R_B}-2\right)-1}{S^2C^2R^2+3SCR+1}$$

↓ Reemplazamos con $R_A/R_B = 5$.

$$\frac{V_2}{V_1} = \frac{-S^2C^2R^2+SCR(5-2)-1}{S^2C^2R^2+3SCR+1} \cdot \frac{1+1/5}{1+5} = \frac{1}{5}\left(\frac{-S^2C^2R^2+3SCR-1}{S^2C^2R^2+3SCR+1} \right)$$

$$\frac{V_2}{V_1} = \left(-\frac{1}{5}\right) \cdot \left(\frac{S^2C^2R^2 - 3SCR + 1}{S^2C^2R^2 + 3SCR + 1}\right)$$

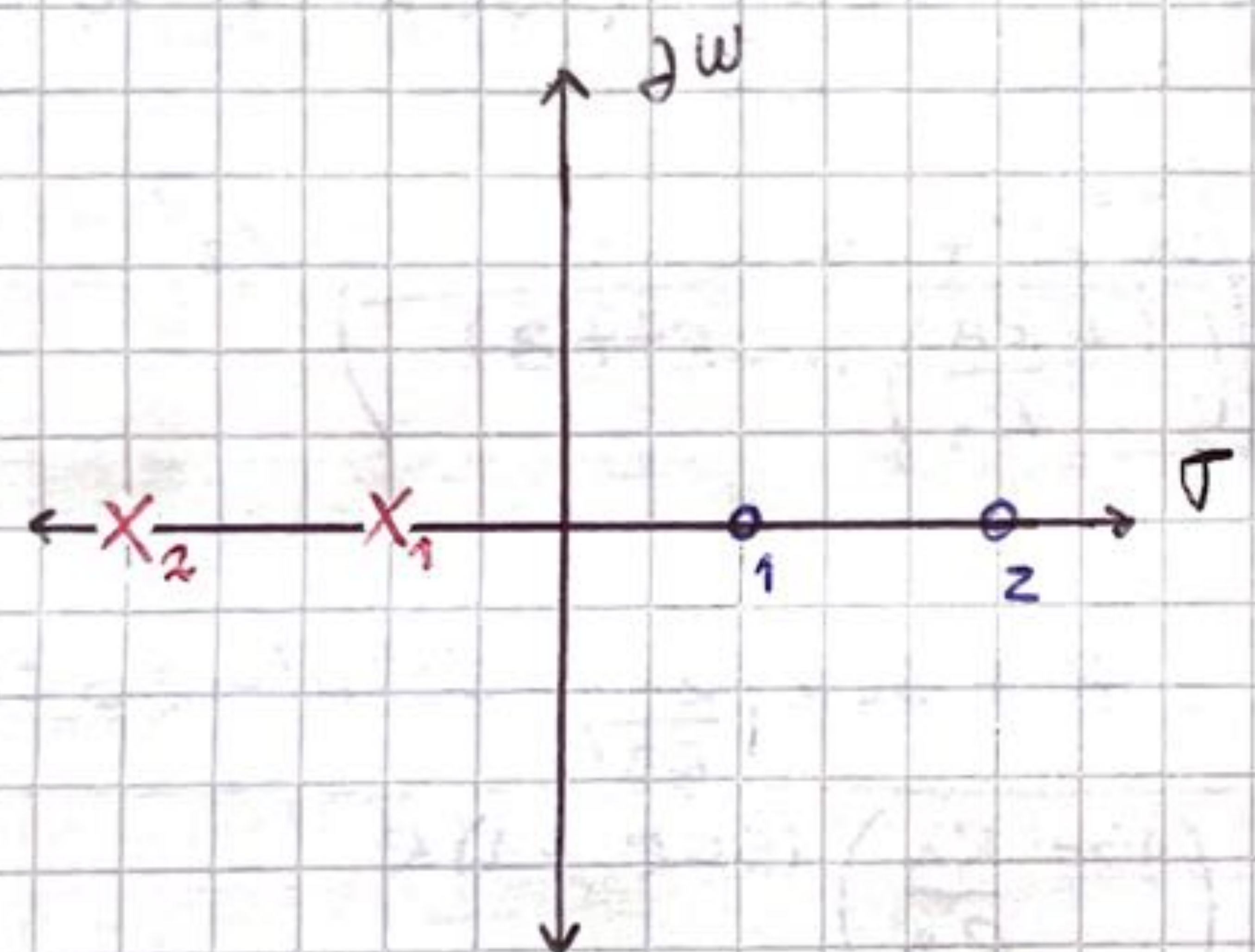
De forma mónica:

$$\boxed{\frac{V_2}{V_1} = \left(\frac{-1}{5}\right)\left(\frac{S^2 - 3/CR + 1/C^2R^2}{S^2 + 3/CR + 1/C^2R^2}\right)}$$

Reemplazando con valores: $R = 1K\Omega$ y $C = 1\mu F$:

$$\frac{V_2}{V_1} = \left(-\frac{1}{5}\right)\left(\frac{S^2 - 3000S + 10^6}{S^2 + 3000S + 10^6}\right)$$

• Diagrama de polos y ceros:



O_1: 381,966

O_2: 2618,034

X_1: −381,966

X_2: −2618,034

• Módulo : $\frac{V_2}{V_1}(S)\Big|_{S=j\omega} = \left(\frac{-1}{5}\right)\left(\frac{(j\omega)^2 - 3000j\omega + 10^6}{(j\omega)^2 + 3000j\omega + 10^6}\right) = \left(\frac{-1}{5}\right)\frac{-3000j\omega + (10^6 - \omega^2)}{+3000j\omega + (10^6 - \omega^2)}$

$$\boxed{\left|\frac{V_2}{V_1}(j\omega)\right|^2 = \frac{(10^6 - \omega^2)^2 + (3000\omega)^2}{(10^6 - \omega^2)^2 + (3000\omega)^2}\left(-\frac{1}{5}\right)^2}$$

• Fase : $\phi(\omega) = \sum_1^m tg^{-1}\frac{\omega \pm \beta_z}{\alpha_z} - \sum_1^n tg^{-1}\frac{\omega \pm \beta_p}{\alpha_p}$

$$\boxed{\phi(\omega) = \left[tg^{-1}\left(\frac{\omega}{381,966}\right) + tg^{-1}\left(\frac{\omega}{2618}\right)\right] - \left[tg^{-1}\left(\frac{\omega}{-381,966}\right) + tg^{-1}\left(\frac{\omega}{-2618}\right)\right]}$$

NOTA

B) Normalización:

Partimos de: $\frac{V_2}{V_1}(s) = \left(-\frac{1}{5}\right)\left(\frac{s^2C^2R^2 - 3sCR + 1}{s^2C^2R^2 + 3sCR + 1}\right)$

Entonces: $s'^2 = s^2C^2R^2 \rightarrow s = \frac{s'}{CR} \rightarrow \Omega_\omega = \frac{1}{CR}$

Adoptamos: $\boxed{\Omega_\omega = \frac{1}{CR} \rightarrow \Omega_\omega^2 = \frac{1}{C^2R^2}}$

Luego, nuestra función normalizada en frecuencia:

$$\boxed{\frac{V_2}{V_1}(s) = \left(-\frac{1}{5}\right)\cdot\left(\frac{s'^2 - 3s' + 1}{s'^2 + 3s' + 1}\right)}$$

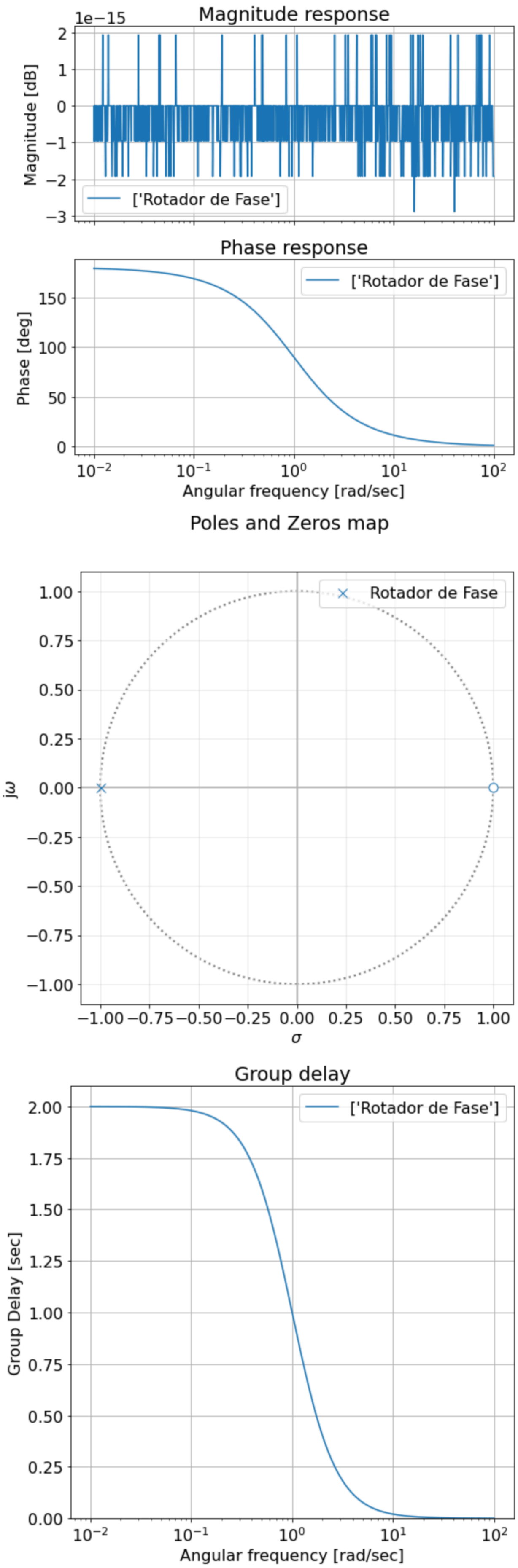
La interpretación circuital es que es la pulsación de resonancia del circuito.

E) Este circuito podría ser útil si lo que se busca es un rotador de fase de 180° que a su vez atenúe a un 20% a la señal de entrada.

Ejercicio 7) Circuito A) Punto c.

- Simule la función transferencia normalizada (Python, Matlab, etc.).

$$\frac{s+-1}{s+1}$$



Ejercicio 7) Circuito B) Punto c.

- Simule la función transferencia normalizada (Python, Matlab, etc.).

$$\frac{s^2 \ -0.2 + s \ 0.6 + -0.2}{s^2 + s \ 3 + 1}$$