$$F = \frac{Gm_1m_2}{r^2}$$

have similar mathematical structure.

Many of the results derived for gravitational field, potential and potential energy may, therefore, be used for the corresponding electrical quantities. We state some of the useful results for a spherical charge distribution of radius R.

(a) The electric field due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre of the shell, $E = Q/(4\pi\varepsilon_0 r^2)$.

(b) The electric field due to a uniformly charged thin spherical shell at an internal point is zero.

(c) The electric field due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre of the sphere.

(d) The electric field due to a uniformly charged sphere at an internal point is proportional to the distance of the point from the centre of the sphere. Thus, it is zero at the centre and increases linearly as one moves out towards the surface.

(e) The electric potential due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre, $V = Q/(4\pi\varepsilon_0 r)$.

(f) The electric potential due to a uniformly charged, thin spherical shell at an internal point is the same everywhere and is equal to that at the surface, $V = Q/(4\pi\varepsilon_0 R)$.

(g) The electric potential due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre, $V = Q/(4\pi\varepsilon_0 r)$.

Electric Potential Energy of a Uniformly Charged Sphere

Consider a uniformly charged sphere of radius R having a total charge Q. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere. Let us assume that at some instant, charge is assembled up to a radius x. In the next step, we bring some charge from infinity and put it on this sphere to increase the radius from x to $x + dx$. The entire sphere is assembled as x varies from 0 to R.

The charge density is

$$\rho = \frac{3Q}{4\pi R^3}.$$

When the radius of the sphere is x, the charge contained in it is,

$$q = \frac{4}{3}\pi x^3 \rho = \frac{Q}{R^3} x^3.$$

The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 x} = \frac{Q}{4\pi\varepsilon_0 R^3} x^2.$$

The charge needed to increase the radius from x to $x + dx$ is

$$dq = (4\pi x^2 dx)\rho = \frac{3Q}{R^3} x^2 dx.$$

The work done in bringing the charge dq from infinity to the surface of the sphere of radius x is

$$dW = V(dq) = \frac{3Q^2}{4\pi\varepsilon_0 R^6} x^4 dx.$$

The total work done in assembling the charged sphere of radius R is

$$W = \frac{3Q^2}{4\pi\varepsilon_0 R^6} \int_0^R x^4 dx = \frac{3Q^2}{20\pi\varepsilon_0 R}.$$

This is the electric potential energy of the charged sphere.

Electric Potential Energy of a Uniformly Charged, Thin Spherical Shell

Consider a uniformly charged, thin spherical shell of radius R having a total charge Q. The electric potential energy is equal to the work done in bringing charges from infinity and put them on the shell. Suppose at some instant, a charge q is placed on the shell. The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R}.$$

The work done in bringing a charge dq from infinity to this shell is

$$dW = V(dq) = \frac{q\,dq}{4\pi\varepsilon_0 R}.$$

The total work done in assembling the charge on the shell is

$$W = \int_0^Q \frac{q dq}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

This is the electric potential energy of the charged spherical shell.

30.6 EARTHING A CONDUCTOR

The earth is a good conductor of electricity. If we assume that the earth is uncharged, its potential will be zero. In fact, the earth's surface has a negative charge of about 1 nC m^{-2} and hence is at a constant potential V. All conductors which are not given any external charge, are also very nearly at the same potential. In turns out that for many practical calculations, we can ignore the charge on the earth. The potential of the earth can then be taken as the same as that of a point far away from all charges, i.e., at infinity. So, the potential of the earth is often taken to be zero. Also, if a small quantity of charge is given to the earth or is taken away from it, the potential does not change by any appreciable extent. This is because of the large size of the earth.

If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e., zero. If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

When a conductor is connected to the earth, the conductor is said to be *earthed* or *grounded*. Figure (30.22a) shows the symbol for earthing.

Suppose a spherical conductor of radius R is given a charge Q. The charge will be distributed uniformly on the surface. So it is equivalent to a uniformly charged, thin spherical shell. Its potential will, therefore, become $Q/(4\pi\varepsilon_0 R)$. If this conductor is connected to the earth, the charge Q will be transferred to the earth so that the potential will become zero.

Next suppose, a charge $+Q$ is placed at the centre of a spherical conducting shell. A charge $-Q$ will appear on its inner surface and $+Q$ on its outer surface (figure 30.22b). The potential of the sphere due to the charge at the centre and that due to the charge at the inner surface are $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{-Q}{4\pi\varepsilon_0 R}$ respectively. The potential due to the charge on the outer surface is $\frac{Q}{4\pi\varepsilon_0 R}$. The net potential of the sphere is, therefore, $\frac{Q}{4\pi\varepsilon_0 R}$. If this sphere is now connected to the earth (figure 30.22c), the charge Q on the outer surface flows to the earth and the potential of the sphere becomes zero.

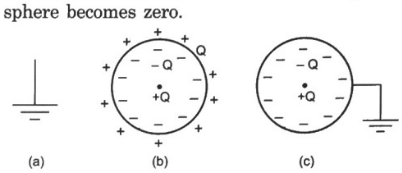

Figure 30.22

Earthing a conductor is a technical job. A thick metal plate is buried deep into the earth and wires are drawn from this plate. The electric wiring in our houses has three wires: live, neutral and earth. The live and neutral wires carry electric currents which come from the power station. The earth wire is connected to the metal plate buried in the earth. The metallic bodies of electric appliances such as electric iron, refrigerator, etc. are connected to the earth wire. This ensures that the metallic body remains at zero potential while an appliance is being used. If by any fault, the live wire touches the metallic body, charge flows to the earth and the potential of the metallic body remains zero. If it is not connected to the earth, the user may get an electric shock.

Worked Out Examples

1. *A uniform electric field of magnitude* $E = 100\ \mathrm{N\,C^{-1}}$ *exists in the space in x-direction. Calculate the flux of this field through a plane square area of edge* 10 cm *placed in the y–z plane. Take the normal along the positive x-axis to be positive.*

Solution : The flux $\Phi = \int E\cos\theta\, dS$. As the normal to the area points along the electric field, $\theta = 0$. Also, E is uniform, so

$$\Phi = E\,\Delta S$$

$$= (100\ \mathrm{N\,C^{-1}})\,(0{\cdot}10\ \mathrm{m})^2 = 1{\cdot}0\ \mathrm{N\,m^2C^{-1}}.$$

2. *A large plane charge sheet having surface charge density* $\sigma = 2{\cdot}0\times10^{-6}\ \mathrm{C\,m^{-2}}$ *lies in the x–y plane. Find the flux of the electric field through a circular area of radius* 1 cm *lying completely in the region where x, y, z are all positive and with its normal making an angle of* 60° *with the z-axis.*

Solution : The electric field near the plane charge sheet is $E = \sigma/2\varepsilon_0$ in the direction away from the sheet. At the given area, the field is along the z-axis.

The area $= \pi r^2 = 3{\cdot}14\times1\ \mathrm{cm}^2 = 3{\cdot}14\times10^{-4}\ \mathrm{m}^2$.

The angle between the normal to the area and the field is 60°.

Hence, the flux $= \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\theta = \frac{\sigma}{2\varepsilon_0}\pi r^2\cos 60°$

$$= \frac{2{\cdot}0\times10^{-6}\ \mathrm{C\,m^{-2}}}{2\times8{\cdot}85\times10^{-12}\ \mathrm{C^2\,N\,m^{-2}}}\times(3{\cdot}14\times10^{-4}\ \mathrm{m}^2)\frac{1}{2}$$

$$= 17{\cdot}5\ \mathrm{N\,m^2\,C^{-1}}.$$

3. *A charge of* $4\times10^{-8}\ \mathrm{C}$ *is distributed uniformly on the surface of a sphere of radius* 1 cm. *It is covered by a concentric, hollow conducting sphere of radius* 5 cm. *(a) Find the electric field at a point* 2 cm *away from the centre. (b) A charge of* $6\times10^{-8}\ \mathrm{C}$ *is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.*

Solution :

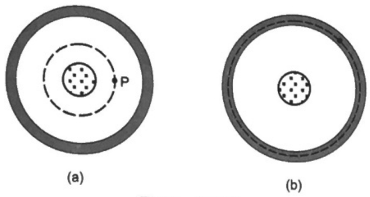

Figure 30-W1

(a) Let us consider figure (30-W1a). Suppose, we have to find the field at the point P. Draw a concentric spherical surface through P. All the points on this surface are equivalent and by symmetry, the field at all these points will be equal in magnitude and radial in direction.

The flux through this surface $= \oint \vec{E} \cdot d\vec{S}$

$$= \oint E\, dS = E \oint dS$$

$$= 4\pi x^2 E,$$

where $\quad x = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}.$

From Gauss's law, this flux is equal to the charge q contained inside the surface divided by ε_0. Thus,

$$4\pi x^2 E = q/\varepsilon_0$$

or,
$$E = \frac{q}{4\pi\varepsilon_0 x^2}$$

$$= (9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times \frac{4 \times 10^{-8} \text{ C}}{4 \times 10^{-4} \text{ m}^2}$$

$$= 9 \times 10^5 \text{ N C}^{-1}.$$

(b) See figure (30-W1b). Take a Gaussian surface through the material of the hollow sphere. As the electric field in a conducting material is zero, the flux $\oint \vec{E} \cdot d\vec{S}$ through this Gaussian surface is zero. Using Gauss's law, the total charge enclosed must be zero. Hence, the charge on the inner surface of the hollow sphere is -4×10^{-8} C. But the total charge given to this hollow sphere is 6×10^{-8} C. Hence, the charge on the outer surface will be 10×10^{-8} C.

4. *Figure (30-W2a) shows three concentric thin spherical shells A, B and C of radii a, b and c respectively. The shells A and C are given charges q and –q respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.*

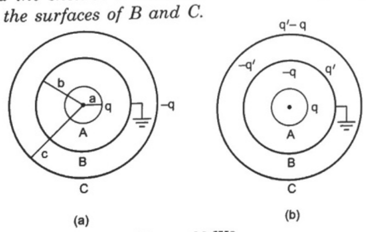

Figure 30-W2

Solution :

As shown in the previous worked out example, the inner surface of B must have a charge $-q$ from the Gauss's law. Suppose, the outer surface of B has a charge q'. The inner surface of C must have a charge $-q'$ from the Gauss's law. As the net charge on C must be $-q$, its outer surface should have a charge $q' - q$. The charge distribution is shown in figure (30-W2b).

The potential at B due to the charge q on A

$$= \frac{q}{4\pi\varepsilon_0 b},$$

due to the charge $-q$ on the inner surface of B

$$= \frac{-q}{4\pi\varepsilon_0 b},$$

due to the charge q' on the outer surface of B

$$= \frac{q'}{4\pi\varepsilon_0 b},$$

due to the charge $-q'$, on the inner surface of C

$$= \frac{-q'}{4\pi\varepsilon_0 c}$$

and due to the charge $q' - q$ on the outer surface of C

$$= \frac{q' - q}{4\pi\varepsilon_0 c}.$$

The net potential is

$$V_B = \frac{q'}{4\pi\varepsilon_0 b} - \frac{q}{4\pi\varepsilon_0 c}.$$

This should be zero as the shell B is earthed. Thus,

$$q' = \frac{b}{c} q.$$

The charges on various surfaces are as shown in figure (30-W3).

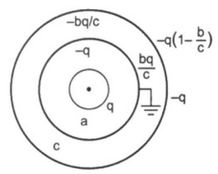

Figure 30-W3

5. *An electric dipole consists of charges* $\pm 2{\cdot}0 \times 10^{-8}$ C *separated by a distance of* $2{\cdot}0 \times 10^{-3}$ m. *It is placed near a long line charge of linear charge density* $4{\cdot}0 \times 10^{-4}$ C m^{-1} *as shown in figure (30-W4), such that the negative charge is at a distance of* $2{\cdot}0$ cm *from the line charge. Find the force acting on the dipole.*

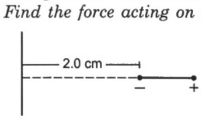

Figure 30-W4

Solution : The electric field at a distance r from the line charge of linear density λ is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

Hence, the field at the negative charge is

$$E_1 = \frac{(4{\cdot}0 \times 10^{-4}\ \mathrm{C\ m^{-1}})\,(2 \times 9 \times 10^{9}\ \mathrm{N\ m^2\ C^{-2}})}{0{\cdot}02\ \mathrm{m}}$$

$$= 3{\cdot}6 \times 10^{8}\ \mathrm{N\ C^{-1}}.$$

The force on the negative charge is

$$F_1 = (3{\cdot}6 \times 10^{8}\ \mathrm{N\ C^{-1}})\,(2{\cdot}0 \times 10^{-8}\ \mathrm{C}) = 7{\cdot}2\ \mathrm{N}$$

towards the line charge.

Similarly, the field at the positive charge, i.e., at $r = 0{\cdot}022$ m is

$$E_2 = 3{\cdot}3 \times 10^{8}\ \mathrm{N\ C^{-1}}.$$

The force on the positive charge is

$$F_2 = (3{\cdot}3 \times 10^{8}\ \mathrm{N\ C^{-1}}) \times (2{\cdot}0 \times 10^{-8}\ \mathrm{C})$$

$$= 6{\cdot}6\ \mathrm{N}\ \text{ away from the line charge}.$$

Hence, the net force on the dipole $= (7{\cdot}2 - 6{\cdot}6)$ N

$= 0{\cdot}6$ N towards the line charge.

6. *The electric field in a region is radially outward with magnitude $E = Ar$. Find the charge contained in a sphere of radius a centred at the origin. Take $A = 100\ \mathrm{V\ m^{-2}}$ and $a = 20{\cdot}0$ cm.*

Solution : The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

$$\Phi = \oint E\,dS\cos\theta = Aa(4\pi a^2).$$

The charge contained in the sphere is, from Gauss's law,

$$Q_{inside} = \varepsilon_0\,\Phi = 4\pi\varepsilon_0\,Aa^3$$

$$= \left(\frac{1}{9 \times 10^{9}}\ \mathrm{C^2\ N^{-1}\ m^{-2}}\right)(100\ \mathrm{V\ m^{-2}})\,(0{\cdot}20\ \mathrm{m})^3$$

$$= 8{\cdot}89 \times 10^{-11}\ \mathrm{C}.$$

7. *A particle of mass 5×10^{-6} g is kept over a large horizontal sheet of charge of density $4{\cdot}0 \times 10^{-6}\ \mathrm{C\ m^{-2}}$ (figure 30-W5). What charge should be given to this particle so that if released, it does not fall down? How many electrons are to be removed to give this charge? How much mass is decreased due to the removal of these electrons?*

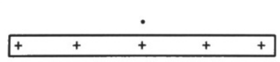

Figure 30-W5

Solution : The electric field in front of the sheet is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{4{\cdot}0 \times 10^{-6}\ \mathrm{C\ m^{-2}}}{2 \times 8{\cdot}85 \times 10^{-12}\ \mathrm{C^2\ N^{-1}\ m^{-2}}}$$

$$= 2{\cdot}26 \times 10^{5}\ \mathrm{N\ C^{-1}}.$$

If a charge q is given to the particle, the electric force qE acts in the upward direction. It will balance the weight of the particle if

$$q \times 2{\cdot}26 \times 10^{5}\ \mathrm{N\ C^{-1}} = 5 \times 10^{-9}\ \mathrm{kg} \times 9{\cdot}8\ \mathrm{m\ s^{-2}}$$

or, $$q = \frac{4{\cdot}9 \times 10^{-8}}{2{\cdot}26 \times 10^{5}}\ \mathrm{C}$$

$$= 2{\cdot}21 \times 10^{-13}\ \mathrm{C}.$$

The charge on one electron is $1{\cdot}6 \times 10^{-19}$ C. The number of electrons to be removed

$$= \frac{2{\cdot}21 \times 10^{-13}\ \mathrm{C}}{1{\cdot}6 \times 10^{-19}\ \mathrm{C}} = 1{\cdot}4 \times 10^{6}.$$

Mass decreased due to the removal of these electrons

$$= 1{\cdot}4 \times 10^{6} \times 9{\cdot}1 \times 10^{-31}\ \mathrm{kg}$$

$$= 1{\cdot}3 \times 10^{-24}\ \mathrm{kg}.$$

8. *Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Find the distribution of charges on the four surfaces.*

Solution : Consider a Gaussian surface as shown in figure (30-W6a). Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore, zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

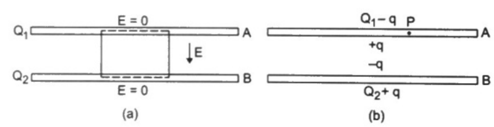

Figure 30-W6

The distribution should be like the one shown in figure (30-W6b). To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation $E = \sigma/(2\varepsilon_0)$, the electric field at P

due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward),

due to the charge $+q = \dfrac{q}{2A\varepsilon_0}$ (upward),

due to the charge $-q = \dfrac{q}{2A\varepsilon_0}$ (downward), and due to the charge $Q_2 + q = \frac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$\frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}.$$

As the point P is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - Q_2 - q = 0$$

or, $$q = \frac{Q_1 - Q_2}{2}. \quad \dots \text{(i)}$$

Thus, $$Q_1 - q = \frac{Q_1 + Q_2}{2} \quad \dots \text{(ii)}$$

and $$Q_2 + q = \frac{Q_1 + Q_2}{2}.$$

Using these equations, the distribution shown in the figure (30-W6) can be redrawn as in figure (30-W7).

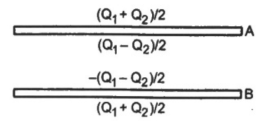

Figure 30-W7

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

□

QUESTIONS FOR SHORT ANSWER

1. A small plane area is rotated in an electric field. In which orientation of the area is the flux of electric field through the area maximum? In which orientation is it zero?
2. A circular ring of radius r made of a nonconducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through 180°. Does the flux of electric field change? If yes, does it decrease or increase?
3. A charge Q is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell? If a point charge is brought close to the shell, will the field at the centre change? Does your answer depend on whether the shell is conducting or nonconducting?
4. A spherical shell made of plastic, contains a charge Q distributed uniformly over its surface. What is the electric field inside the shell? If the shell is hammered to deshape it without altering the charge, will the field inside be changed? What happens if the shell is made of a metal?
5. A point charge q is placed in a cavity in a metal block. If a charge Q is brought outside the metal, will the charge q feel an electric force?
6. A rubber balloon is given a charge Q distributed uniformly over its surface. Is the field inside the balloon zero everywhere if the balloon does not have a spherical surface?
7. It is said that any charge given to a conductor comes to its surface. Should all the protons come to the surface? Should all the electrons come to the surface? Should all the free electrons come to the surface?

OBJECTIVE I

1. A charge Q is uniformly distributed over a large plastic plate. The electric field at a point P close to the centre of the plate is $10\ \text{V m}^{-1}$. If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge Q, the electric field at the point P will become
(a) zero (b) $5\ \text{V m}^{-1}$ (c) $10\ \text{V m}^{-1}$ (d) $20\ \text{V m}^{-1}$.
2. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be
(a) towards the plate (b) away from the plate
(c) parallel to the plate (d) zero.
3. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in figure (30-Q1). All the three charges are positive. The force on the charge at the centre is

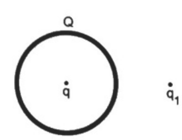

Figure 30-Q1

(a) towards left (b) towards right
(c) upward (d) zero.

4. Consider the situation of the previous problem. The force on the central charge due to the shell is
(a) towards left (b) towards right
(c) upward (d) zero.

5. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V m. The flux over a concentric sphere of radius 20 cm will be
(a) 25 V m (b) 50 V m (c) 100 V m (d) 200 V m.

6. Figure (30-Q2a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length L moves towards left at a small but constant speed v. At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in figure (30-Q2b) represents the flux of the electric field through the cube as the rod goes through it ?

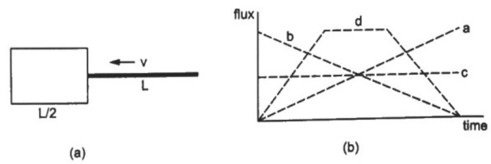

Figure 30-Q2

7. A charge q is placed at the centre of the open end of a cylindrical vessel (figure 30-Q3). The flux of the electric field through the surface of the vessel is
(a) zero (b) q/ε_0 (c) $q/2\varepsilon_0$ (d) $2q/\varepsilon_0$.

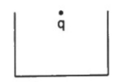

Figure 30-Q3

OBJECTIVE II

1. Mark the correct options:
(a) Gauss's law is valid only for symmetrical charge distributions.
(b) Gauss's law is valid only for charges placed in vacuum.
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

2. A positive point charge Q is brought near an isolated metal cube.
(a) The cube becomes negatively charged.
(b) The cube becomes positively charged.
(c) The interior becomes positively charged and the surface becomes negatively charged.
(d) The interior remains charge free and the surface gets nonuniform charge distribution.

3. A large nonconducting sheet M is given a uniform charge density. Two uncharged small metal rods A and B are placed near the sheet as shown in figure (30-Q4).
(a) M attracts A. (b) M attracts B.
(c) A attracts B. (d) B attracts A.

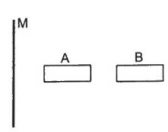

Figure 30-Q4

4. If the flux of the electric field through a closed surface is zero,
(a) the electric field must be zero everywhere on the surface
(b) the electric field may be zero everywhere on the surface
(c) the charge inside the surface must be zero
(d) the charge in the vicinity of the surface must be zero.

5. An electric dipole is placed at the centre of a sphere. Mark the correct options:
(a) The flux of the electric field through the sphere is zero.
(b) The electric field is zero at every point of the sphere.
(c) The electric field is not zero anywhere on the sphere.
(d) The electric field is zero on a circle on the sphere.

6. Figure (30-Q5) shows a charge q placed at the centre of a hemisphere. A second charge Q is placed at one of the positions A, B, C and D. In which position(s) of this second charge, the flux of the electric field through the hemisphere remains unchanged ?
(a) A (b) B (c) C (d) D.

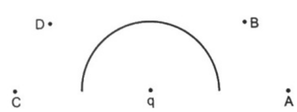

Figure 30-Q5

7. A closed surface S is constructed around a conducting wire connected to a battery and a switch (figure 30-Q6). As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface S is equal to the number of electrons leaving it. On closing

the switch, the flux of the electric field through the closed surface
(a) is increased
(b) is decreased
(c) remains unchanged
(d) remains zero.

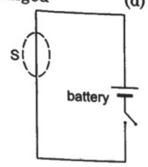

Figure 30-Q6

8. Figure (30-Q7) shows a closed surface which intersects a conducting sphere. If a positive charge is placed at the point P, the flux of the electric field through the closed surface
(a) will remain zero
(b) will become positive
(c) will become negative
(d) will become undefined.

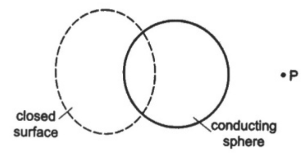

Figure 30-Q7

EXERCISES

1. The electric field in a region is given by $\vec{E} = \frac{3}{5} E_0 \vec{i} + \frac{4}{5} E_0 \vec{j}$ with $E_0 = 2{\cdot}0 \times 10^3$ N C^{-1}. Find the flux of this field through a rectangular surface of area 0·2 m^2 parallel to the y–z plane.

2. A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

3. Show that there can be no net charge in a region in which the electric field is uniform at all points.

4. The electric field in a region is given by $\vec{E} = \frac{E_0 x}{l} \vec{i}$. Find the charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3$ N C^{-1}, $l = 2$ cm and $a = 1$ cm.

5. A charge Q is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

6. A charge Q is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge a as shown in figure (30-E1). Find the flux of the electric field through the square surface.

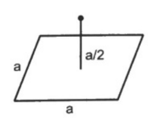

Figure 30-E1

7. Find the flux of the electric field through a spherical surface of radius R due to a charge of 10^{-7} C at the centre and another equal charge at a point $2R$ away from the centre (figure 30-E2).

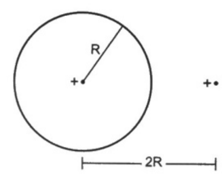

Figure 30-E2

8. A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure 30-E3).

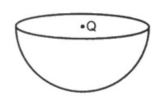

Figure 30-E3

9. A spherical volume contains a uniformly distributed charge of density $2{\cdot}0 \times 10^{-4}$ C m^{-3}. Find the electric field at a point inside the volume at a distance 4·0 cm from the centre.

10. The radius of a gold nucleus ($Z = 79$) is about $7{\cdot}0 \times 10^{-15}$ m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at (a) the surface of the nucleus and (b) at the middle point of a radius. Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface ?

11. A charge Q is distributed uniformly within the material of a hollow sphere of inner and outer radii r_1 and r_2 (figure 30-E4). Find the electric field at a point P a

distance x away from the centre for $r_1 < x < r_2$. Draw a rough graph showing the electric field as a function of x for $0 < x < 2r_2$ (figure 30-E4).

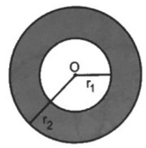

Figure 30-E4

12. A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a. (a) Find the surface charge density on the inner surface and on the outer surface. (b) If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces? (c) Find the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

13. Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius 10^{-15} m. The two $1s$ electrons make a spherical charge cloud at an average distance of $1{\cdot}3 \times 10^{-11}$ m from the nucleus, whereas the two $2s$ electrons make another spherical cloud at an average distance of $5{\cdot}2 \times 10^{-11}$ m from the nucleus. Find the electric field at (a) a point just inside the $1s$ cloud and (b) a point just inside the $2s$ cloud.

14. Find the magnitude of the electric field at a point 4 cm away from a line charge of density 2×10^{-6} C m^{-1}.

15. A long cylindrical wire carries a positive charge of linear density $2{\cdot}0 \times 10^{-8}$ C m^{-1}. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. Find the electric field at a point P inside the cylindrical volume at a distance x from its axis (figure 30-E5)

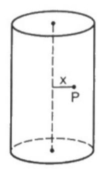

Figure 30-E5

17. A nonconducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate, at a distance x from the central plane. Draw a qualitative graph of E against x for $0 < x < d$.

18. A charged particle having a charge of $-2{\cdot}0 \times 10^{-6}$ C is placed close to a nonconducting plate having a surface charge density $4{\cdot}0 \times 10^{-6}$ C m^{-2}. Find the force of attraction between the particle and the plate.

19. One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of $4{\cdot}0 \times 10^{-6}$ C. In equilibrium, the thread makes an angle of 60° with the vertical. Find the surface charge density on the plate.

20. Consider the situation of the previous problem. (a) Find the tension in the string in equilibrium. (b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

21. Two large conducting plates are placed parallel to each other with a separation of 2·00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2·00 microseconds. Find the surface charge density on the inner surfaces.

22. Two large conducting plates are placed parallel to each other and they carry equal and opposite charges with surface density σ as shown in figure (30-E6). Find the electric field (a) at the left of the plates, (b) in between the plates and (c) at the right of the plates.

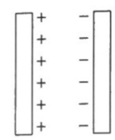
Figure 30-E6

23. Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure (30-E7). The plate X is given a charge Q whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate X, (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric field at a point to the right of the plates.

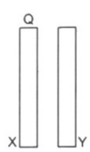

Figure 30-E7

24. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure (30-E8). The leftmost plate is given a charge Q, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.

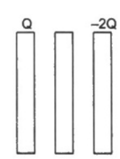

Figure 30-E8

ANSWERS

OBJECTIVE I

1. (c) 2. (a) 3. (d) 4. (b) 5. (a) 6. (d)
7. (c)

OBJECTIVE II

1. (d) 2. (d) 3. all
4. (b), (c) 5. (a), (c) 6. (a), (c)
7. (c), (d) 8. (b)

EXERCISES

1. $240\ \mathrm{N\,m^2\,C^{-1}}$
2. $Q/(2\varepsilon_0)$
4. $2{\cdot}2\times10^{-12}\ \mathrm{C}$
5. Q/ε_0
6. $Q/(6\varepsilon_0)$
7. $1{\cdot}1\times10^{4}\ \mathrm{N\,m^{-2}\,C^{-1}}$
8. $Q/(2\varepsilon_0)$
9. $3{\cdot}0\times10^{5}\ \mathrm{N\,C^{-1}}$
10. (a) $2{\cdot}32\times10^{21}\ \mathrm{N\,C^{-1}}$ (b) $1{\cdot}16\times10^{21}\ \mathrm{N\,C^{-1}}$
11. $\dfrac{Q(x^3-r_1^3)}{4\pi\varepsilon_0 x^2(r_2^3-r_1^3)}$
12. (a) $-\dfrac{Q}{4\pi a^2}, \dfrac{Q}{4\pi a^2}$ (b) $-\dfrac{Q}{4\pi a^2}, \dfrac{Q+q}{4\pi a^2}$
(c) $\dfrac{Q}{4\pi\varepsilon_0 x^2}$ in both situations
13. (a) $3{\cdot}4\times10^{13}\ \mathrm{N\,C^{-1}}$ (b) $1{\cdot}1\times10^{12}\ \mathrm{N\,C^{-1}}$
14. $9\times10^{5}\ \mathrm{N\,C^{-1}}$
15. $2{\cdot}88\times10^{-17}\ \mathrm{J}$
16. $\rho x/(2\varepsilon_0)$
17. $\rho x/\varepsilon_0$
18. $0{\cdot}45\ \mathrm{N}$
19. $7{\cdot}5\times10^{-7}\ \mathrm{C\,m^{-2}}$
20. (a) $0{\cdot}20\ \mathrm{N}$ (b) $0{\cdot}45\ \mathrm{s}$
21. $0{\cdot}505\times10^{-12}\ \mathrm{C\,m^{-2}}$
22. (a) zero (b) σ/ε_0 (c) zero
23. (a) $\dfrac{Q}{2A}$ (b) $\dfrac{Q}{2A\varepsilon_0}$ towards left (c) $\dfrac{Q}{2A\varepsilon_0}$ towards right
(d) $\dfrac{Q}{2A\varepsilon_0}$ towards right
24. $-Q/2$

□

CHAPTER 31

CAPACITORS

31.1 CAPACITOR AND CAPACITANCE

A combination of two conductors placed close to each other is called a *capacitor*. One of the conductors is given a positive charge and the other is given an equal negative charge. The conductor with the positive charge is called the *positive plate* and the other is called the *negative plate*. The charge on the positive plate is called *the charge on the capacitor* and the potential difference between the plates is called the *potential of the capacitor*. Figure (31.1a) shows two conductors. One of the conductors has a positive charge $+Q$ and the other has an equal, negative charge $-Q$. The first one is at a potential V_+ and the other is at a potential V_-. The charge on the capacitor is Q and the potential of the capacitor is $V = V_+ - V_-$. Note that the term *charge on a capacitor* does not mean the total charge given to the capacitor. This total charge is $+Q - Q = 0$. Figure (31.1b) shows the symbol used to represent a capacitor.

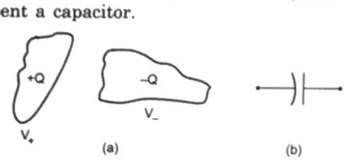

Figure 31.1

For a given capacitor, the charge Q on the capacitor is proportional to the potential difference V between the plates

Thus, $\quad Q \propto V$

or, $\quad Q = CV. \quad \ldots (31.1)$

The proportionality constant C is called the *capacitance* of the capacitor. It depends on the shape, size and geometrical placing of the conductors and the medium between them.

The SI unit of capacitance is coulomb per volt which is written as farad. The symbol F is used for it. This is a large unit on normal scales and microfarad (μF) is used more frequently.

To put equal and opposite charges on the two conductors, they may be connected to the terminals of a *battery*. We shall discuss in somewhat greater detail about the battery in the next chapter. Here we state the following properties of an ideal battery.

(a) A battery has two terminals.

(b) The potential difference V between the terminals is constant for a given battery. The terminal with higher potential is called the *positive terminal* and that with lower potential is called the *negative terminal*.

(c) The value of this fixed potential difference is equal to the *electromotive force* or *emf* of the battery. If a conductor is connected to a terminal of a battery, the potential of the conductor becomes equal to the potential of the terminal. When the two plates of a capacitor are connected to the terminals of a battery, the potential difference between the plates of the capacitor becomes equal to the emf of the battery.

(d) The total charge in a battery always remains zero. If its positive terminal supplies a charge Q, its negative terminal supplies an equal, negative charge $-Q$.

(e) When a charge Q passes through a battery of emf $\mathcal{E}$ from the negative terminal to the positive terminal, an amount $Q\mathcal{E}$ of work is done by the battery.

An ideal battery is represented by the symbol shown in figure (31.2). The potential difference between the facing parallel lines is equal to the emf $\mathcal{E}$ of the battery. The longer line is at the higher potential.

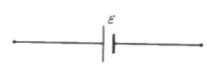

Figure 31.2

Example 31.1

A capacitor gets a charge of 60 μC *when it is connected to a battery of emf* 12 V. *Calculate the capacitance of the capacitor.*

Solution : The potential difference between the plates is the same as the emf of the battery which is 12 V. Thus,