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Concepts of Physics

2

H C Verma

CONCEPTS OF PHYSICS

[VOLUME 2]

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Dedicated to
Indian Philosophy & Way of Life
of which
my parents were
an integral part

FOREWORD

A few years ago I had an occasion to go through the book *Calculus* by L V Terasov. It unravels intricacies of the subject through a dialogue between Teacher and Student. I thoroughly enjoyed reading it. For me this seemed to be one of the few books which teach a difficult subject through inquisition, and using programmed concept for learning. After that book, Dr Harish Chandra Verma's book on physics, *CONCEPTS OF PHYSICS* is another such attempt, even though it is not directly in the dialogue form. I have thoroughly appreciated it. It is clear that Dr Verma has spent considerable time in formulating the structure of the book, besides its contents. I think he has been successful in this attempt. Dr Verma's book has been divided into two parts because of the size of the total manuscript. There have been several books on this subject, each one having its own flavour. However, the present book is a totally different attempt to teach physics, and I am sure it will be extremely useful to the undergraduate students. The exposition of each concept is extremely lucid. In carefully formatted chapters, besides problems and short questions, a number of objective questions have also been included. This book can certainly be extremely useful not only as a textbook, but also for preparation of various competitive examinations.

Those who have followed Dr Verma's scientific work always enjoyed the outstanding contributions he has made in various research areas. He was an outstanding student of Physics Department of IIT Kanpur during his academic career. An extremely methodical, sincere person as a student, he has devoted himself to the task of educating young minds and inculcating scientific temper amongst them. The present venture in the form of these two volumes is another attempt in that direction. I am sure that young minds who would like to *learn physics in an appropriate manner* will find these volumes extremely useful.

I must heartily congratulate Dr Harish Chandra Verma for the magnificent job he has done.

Y R Waghmare
Professor of Physics
IIT Kanpur.

PREFACE

Why a new book ?

Excellent books exist on physics at an introductory college level so why a new one ? Why so many books exist at the same level, in the first place, and why each of them is highly appreciated ? It is because each of these books has the privilege of having an author or authors who have *experienced* physics and have their own method of communicating with the students. During my years as a physics teacher, I have developed a somewhat different methodology of presenting physics to the students. *Concepts of Physics* is a translation of this methodology into a textbook.

Prerequisites

The book presents a calculus-based physics course which makes free use of algebra, trigonometry and co-ordinate geometry. The level of the latter three topics is quite simple and high school mathematics is sufficient. Calculus is generally done at the introductory college level and I have assumed that the student is enrolled in a concurrent first calculus course. The relevant portions of calculus have been discussed in Chapter 2 so that the student may start using it from the beginning.

Almost no knowledge of physics is a prerequisite. I have attempted to start each topic from the zero level. A receptive mind is all that is needed to use this book.

Basic philosophy of the book

The motto underlying the book is *physics is enjoyable.*

Being a description of the nature around us, physics is our best friend from the day of our existence. I have extensively used this aspect of physics to introduce the physical principles starting with common day occurrences and examples. The subject then appears to be friendly and enjoyable. I have taken care that numerical values of different quantities used in problems correspond to real situations to further strengthen this approach.

Teaching and training

The basic aim of physics teaching has been to let the student know and understand the principles and equations of physics and their applications in real life.

However, to be able to use these principles and equations correctly in a given physical situation, one needs further training. A large number of *questions and solved and unsolved problems* are given for this purpose. Each question or problem has a specific purpose. It may be there to bring out a subtle point which might have passed unnoticed while doing the text portion. It may be a further elaboration of a concept developed in the text. It may be there to make the student react when several concepts introduced in different chapters combine and show up as a physical situation and so on. Such tools have been used to develop a culture: *analyse the situation, make a strategy to invoke correct principles and work it out.*

Conventions

I have tried to use symbols, names, etc., which are popular nowadays. SI units have been consistently used throughout the book. SI prefixes such as *micro, milli, mega,* etc., are used whenever they make the presentation more readable. Thus, $20\ \mu F$ is preferred over 20×10^{-6} F. Co-ordinate sign convention is used in geometrical optics. Special emphasis has been given to dimensions of physical quantities. Numerical values of physical quantities have been mentioned with the units even in equations to maintain dimensional consistency.

I have tried my best to keep errors out of this book. I shall be grateful to the readers who point out any errors and/or make other constructive suggestions.

H C Verma

ACKNOWLEDGEMENTS

The work on this book started in 1984. Since then, a large number of teachers, students and physics lovers have made valuable suggestions which I have incorporated in this work. It is not possible for me to acknowledge all of them individually. I take this opportunity to express my gratitude to them. However, to Dr S B Mathur, who took great pains in going through the entire manuscript and made valuable comments, I am specially indebted. I am also beholden to my colleagues Dr A Yadav, Dr Deb Mukherjee, Mr M M R Akhtar, Dr Arjun Prasad, Dr S K Sinha and others who gave me valuable advice and were good enough to find time for fruitful discussions. To Dr T K Dutta of B E College, Sibpur I am grateful for having taken time to go through portions of the book and making valuable comments.

I thank my student Mr Shailendra Kumar who helped me in checking the answers. I am grateful to Dr B C Rai, Mr Sunil Khijwania & Mr Tejaswi Khijwania for helping me in the preparation of rough sketches for the book.

Finally, I thank the members of my family for their support and encouragement.

H C Verma

TO THE STUDENTS

Here is a brief discussion on the organisation of the book which will help you in using the book most effectively. The book contains 47 chapters divided in two volumes. Though I strongly believe in the underlying unity of physics, a broad division may be made in the book as follows:

Chapters 1–14: Mechanics
15–17: Waves including wave optics
18–22: Optics
23–28: Heat and thermodynamics
29–40: Electric and magnetic phenomena
41–47: Modern physics

Each chapter contains a description of the physical principles related to that chapter. It is well supported by mathematical derivations of equations, descriptions of laboratory experiments, historical background, etc. There are "in-text" solved examples. These examples explain the equation just derived or the concept just discussed. These will help you in fixing the ideas firmly in your mind. Your teachers may use these in-text examples in the classroom to encourage students to participate in discussions.

After the theory section, there is a section on *Worked Out Examples.* These numerical examples correspond to various thinking levels and often use several concepts introduced in that chapter or even in previous chapters. You should read the statement of a problem and try to solve it yourself. In case of difficulty, look at the solution given in the book. Even if you solve the problem successfully, you should look into the solution to compare it with your method of solution. You might have thought of a better method, but knowing more than one method is always beneficial.

Then comes the part which tests your understanding as well as develops it further. *Questions for Short Answer* generally touch very minute points of your understanding. It is not necessary that you answer these questions in a single sitting. They have great potential to initiate very fruitful dicussions. So, freely discuss these questions with your friends and see if they agree with your answer. Answers to these questions are not given for the simple reason that the answers could have cut down the span of such discussions and that would have sharply reduced the utility of these questions.

There are two sections on multiple-choice questions, namely OBJECTIVE I and OBJECTIVE II. There are four options following each of these questions. Only one option is correct for OBJECTIVE I questions. Any number of options, zero to four, may be correct for OBJECTIVE II questions. Answers to all these questions are provided.

Finally, a set of numerical problems are given for your practice. Answers to these problems are also provided. The problems are generally arranged according to the sequence of the concepts developed in the chapter but they are not grouped under section-headings. I don't want to bias your ideas beforehand by telling you that this problem belongs to that section and hence use that particular equation. You should yourself look into the problem and decide which equations or which methods should be used to solve it. Many of the problems use several concepts developed in different sections of the chapter. Many of them even use the concepts from the previous chapters. Hence, you have to plan out the strategy after understanding the problem.

Remember, no problem is difficult. Once you understand the theory, each problem will become easy. So, don't jump to exercise problems before you have gone through the theory, the worked-out problems and the objectives. Once you feel confident in theory, do the exercise problems. The exercise problems are so arranged that they gradually require more thinking.

I hope you will enjoy *Concepts of Physics.*

H C Verma

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CHAPTER 29

ELECTRIC FIELD AND POTENTIAL

29.1 WHAT IS ELECTRIC CHARGE ?

Matter is made of certain elementary particles. With the advancement in technology, we have discovered hundreds of elementary particles. Many of them are rare and of no concern to us in the present course. The three most common elementary particles are electrons, protons and neutrons having masses $m_e = 9{\cdot}10940 \times 10^{-31}$ kg, $m_p = 1{\cdot}67262 \times 10^{-27}$ kg and $m_n = 1{\cdot}67493 \times 10^{-27}$ kg. Because of their mass these particles attract each other by gravitational forces. Thus, an electron attracts another electron, placed 1 cm away, with a gravitational force

$$F = \frac{Gm_1 m_2}{r^2}$$

$$= \frac{(6{\cdot}67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2}) \times (9{\cdot}1 \times 10^{-31}\ \text{kg})^2}{(10^{-2}\ \text{m})^2}$$

$$= 5{\cdot}5 \times 10^{-67}\ \text{N}.$$

However, an electron is found to repel another electron at 1 cm with a force of $2{\cdot}3 \times 10^{-24}$ N. This extra force is called the *electric force*. The electric force is very large as compared to the gravitational force. The electrons must have some additional property, apart from their mass, which is responsible for the electric force. We call this property *charge*. Just as masses are responsible for the gravitational force, charges are responsible for the electric force. Two protons placed at a distance of 1 cm also repel each other with a force of $2{\cdot}3 \times 10^{-24}$ N. Thus, protons also have charge. Two neutrons placed at a distance of 1 cm attract each other with a force of $1{\cdot}9 \times 10^{-60}$ N which is equal to $\frac{Gm_1 m_2}{r^2}$. Thus, neutrons exert only gravitational force on each other and experience no electric force. The neutrons have mass but no charge.

Two Kinds of Charges

As mentioned above, the electric force between two electrons is the same as the electric force between two protons placed at the same separation. We may guess that the amount of charge on an electron is the same as that on a proton. However, if a proton and an electron are placed 1 cm apart, they attract each other with a force of $2{\cdot}3 \times 10^{-24}$ N. Certainly this force is electric, but it is attractive and not repulsive. The charge on an electron repels the charge on another electron but attracts the charge on a proton. Thus, although the charge on an electron and that on a proton have the same strength, they are of two different nature. Also, if we pack a proton and an electron together in a small volume, the combination does not attract or repel another electron or proton placed at a distance. The net charge on the proton–electron system seems to be zero. It is, therefore, convenient to define one charge as positive and the other as negative. We arbitrarily call the charge on a proton as positive and that on an electron as negative. This assignment of positive and negative signs to the proton charge and the electron charge is purely a convention. It does not mean that the charge on an electron is "less" than the charge on a proton.

Unit of Charge

The above discussion suggests that charge is a basic property associated with the elementary particles and its definition is as difficult as the definition of mass or time or length. We can measure the charge on a system by comparing it with the charge on a standard body but we do not know what exactly it is that we intend to measure. The SI unit of charge is coulomb abbreviated as C. 1 coulomb is the charge flowing through a wire in 1 *s* if the electric current in it is 1 A. The charge on a proton is

$$e = 1{\cdot}60218 \times 10^{-19}\ \text{C}.$$

The charge on an electron is the negative of this value.

Charge is Quantized

If protons and electrons are the only charge carriers in the universe, all observable charges must be integral multiples of e. If an object contains n_1 protons and n_2 electrons, the net charge on the object is

$$n_1(e) + n_2(-e) = (n_1 - n_2)e.$$

Indeed, there are elementary particles other than protons and electrons, which carry charge. However, they all carry charges which are integral multiples of e. Thus, the charge on any object is always an integral multiple of e and can be changed only in steps of e, i.e., charge is quantized.

The step size e is usually so small that we can easily neglect the quantization. If we rub a glass rod with a silk cloth, typically charges of the order of a microcoulomb appear on the rubbed objects. Now, 1 μC contains n units of basic charge e where

$$n = \frac{1\ \mu\text{C}}{1{\cdot}6 \times 10^{-19}\ \text{C}} \approx 6 \times 10^{12}.$$

The step size is thus very small as compared to the charges usually found and in many cases we can assume a continuous charge variation.

Charge is Conserved

The charge of an isolated system is conserved. It is possible to create or destroy charged particles but it is not possible to create or destroy *net charge*. In a beta decay process, a neutron converts itself into a proton and a fresh electron is created. The charge however, remains zero before and after the event.

Frictional Electricity : Induction

The simplest way to experience electric charges is to rub certain solid bodies against each other. Long ago, around 600 BC, the Greeks knew that when amber is rubbed with wool, it acquires the property of attracting light objects such as small pieces of paper. This is because amber becomes electrically charged. If we pass a comb through dry hair, the comb becomes electrically charged and can attract small pieces of paper. An automobile becomes charged when it travels through the air. A paper sheet becomes charged when it passes through a printing machine. A gramophone record becomes charged when cleaned with a dry cloth.

The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body may pass on to the other body. The body that receives the extra electrons, becomes negatively charged. The body that donates the electrons, becomes positively charged because it has more protons than electrons. Thus, when a glass rod is rubbed with a silk cloth, electrons are transferred from the glass rod to the silk cloth. The glass rod becomes positively charged and the silk cloth becomes negatively charged.

If we take a positively charged glass rod near small pieces of paper, the rod attracts the pieces. Why does the rod attract paper pieces which are uncharged ? This is because the positively charged rod attracts the electrons of a paper piece towards itself. Some of the electrons accumulate at that edge of the paper piece which is closer to the rod. At the farther end of the piece there is a deficiency of electrons and hence positive charge appears there. Such a redistribution of charge in a material, due to the presence of a nearby charged body, is called *induction*. The rod exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charges. This is because the negative charges are closer to the rod. Hence, there is a net attraction between the rod and the paper piece.

29.2 COULOMB'S LAW

The experiments of Coulomb and others established that the force exerted by a charged particle on the other is given by

$$F = \frac{kq_1q_2}{r^2}, \qquad \ldots \text{(29.1)}$$

where q_1 and q_2 are the charges on the particles, r is the separation between them and k is a constant. The force is attractive if the charges are of opposite signs and is repulsive if they are of the same sign. We can write Coulomb's law as

$$\vec{F} = \frac{kq_1q_2\vec{r}}{r^3},$$

where $\vec{r}$ is the position vector of the force-experiencing particle with respect to the force-exerting particle. In this form, the equation includes the direction of the force.

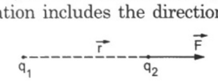

Figure 29.1

As F, q_1, q_2 and r are all independently defined quantities, the constant k can be measured experimentally. In SI units, the constant k is measured to be $8{\cdot}98755 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$.

The constant k is often written as $\frac{1}{4\pi\varepsilon_0}$ so that equation (29.1) becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}. \qquad \ldots \text{(29.2)}$$

The constant ε_0 is called the *permittivity of free space* and its value is

$$\varepsilon_0 = \frac{1}{4\pi k} = 8{\cdot}85419 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}.$$

29.3 ELECTRIC FIELD

We have already discussed in the chapter on gravitation that a particle cannot directly interact with another particle kept at a distance. A particle creates a gravitational field around it and this field exerts force on another particle placed in it. The electric force between two charged particles is also seen as a two-step process. A charge produces something called an *electric field* in the space around it and this electric field exerts a force on any charge (except the source charge itself) placed in it. The electric field has its own existence and is present even if there is no additional charge to experience the force. The field takes finite time to propagate. Thus, if a charge is displaced from its position, the field at a distance r will change after a time $t = r/c$, where c is the speed of light. We define the *intensity of electric field* at a point as follows:

Bring a charge q at the given point without disturbing any other charge that has produced the field. If the charge q experiences an electric force $\vec{F}$, we define the intensity of electric field at the given point as

$$\vec{E} = \frac{\vec{F}}{q} \cdot \qquad \dots (29.3)$$

The charge q used to define $\vec{E}$ is called a *test charge*.

One way to ensure that the test charge q does not disturb other charges is to keep its magnitude very small. If this magnitude is not small, the positions of the other charges may change. Equation (29.3) then gives the electric field due to the charges in the changed positions. The intensity of electric field is often abbreviated as *electric field*.

The electric field at a point is a vector quantity. Suppose, $\vec{E}_1$ is the field at a point due to a charge Q_1 and $\vec{E}_2$ is the field at the same point due to a charge Q_2. The resultant field when both the charges are present, is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

Electric Field due to a Point Charge

Consider a point charge Q placed at a point A (figure 29.2). We are interested in the electric field $\vec{E}$ at a point P at a distance r from Q. Let us imagine a test charge q placed at P. The charge Q creates a field $\vec{E}$ at P and this field exerts a force $\vec{F} = q\vec{E}$ on the charge q. But, from Coulomb's law the force on the charge q in the given situation is

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

along AP. The electric field at P is, therefore,

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad \dots (29.4)$$

along AP.

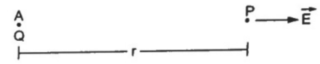

Figure 29.2

The electric field due to a set of charges may be obtained by finding the fields due to each individual charge and then adding these fields according to the rules of vector addition.

Example 29.1

Two charges $10\,\mu C$ *and* $-10\,\mu C$ *are placed at points A and B separated by a distance of* 10 cm. *Find the electric field at a point P on the perpendicular bisector of AB at a distance of* 12 cm *from its middle point.*

Solution :

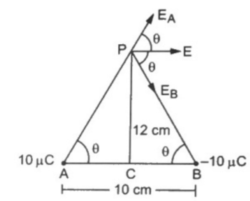

Figure 29.3

The situation is shown in figure (29.3). The distance $AP = BP = \sqrt{(5\text{ cm})^2 + (12\text{ cm})^2} = 13$ cm.

The field at the point P due to the charge $10\,\mu C$ is

$$E_A = \frac{10\,\mu C}{4\pi\varepsilon_0 (13\text{ cm})^2} = \frac{(10 \times 10^{-6}\text{ C}) \times (9 \times 10^9\text{ N m}^2\text{ C}^{-2})}{169 \times 10^{-4}\text{ m}^2}$$

$$= 5{\cdot}3 \times 10^6\text{ N C}^{-1}.$$

This field is along AP. The field due to $-10\,\mu C$ at P is $E_B = 5{\cdot}3 \times 10^6\text{ N C}^{-1}$ along PB. As E_A and E_B are equal in magnitude, the resultant will bisect the angle between the two. The geometry of the figure shows that this resultant is parallel to the base AB. The magnitude of the resultant field is

$$E = E_A \cos\theta + E_B \cos\theta$$

$$= 2 \times (5{\cdot}3 \times 10^6\text{ N C}^{-1}) \times \frac{5}{13}$$

$$= 4{\cdot}1 \times 10^6\text{ N C}^{-1}.$$

If a given charge distribution is continuous, we can use the technique of integration to find the resultant electric field at a point. A small element dQ is chosen in the distribution and the field $d\vec{E}$ due to dQ is calculated. The resultant field is then calculated by integrating the components of $d\vec{E}$ under proper limits.

Example 29.2

A ring of radius a contains a charge q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance x from the centre.

Solution :

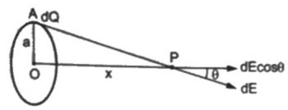

Figure 29.4

Figure (29.4) shows the situation. Let us consider a small element of the ring at the point A having a charge dQ. The field at P due to this element is

$$dE = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}.$$

By symmetry, the field at P will be along the axis OP. The component of dE along this direction is

$$dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}\left(\frac{OP}{AP}\right)$$

$$= \frac{x\,dQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}.$$

The net field at P is

$$E = \int dE\cos\theta = \int \frac{x\,dQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}$$

$$= \frac{x}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}} \int dQ = \frac{xQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}.$$

29.4 LINES OF ELECTRIC FORCE

The electric field in a region can be graphically represented by drawing certain curves known as *lines of electric force* or *electric field lines*. Lines of force are drawn in such a way that the tangent to a line of force gives the direction of the resultant electric field there. Thus, the electric field due to a positive point charge is represented by straight lines originating from the charge (figure 29.5a). The electric field due to a negative point charge is represented by straight lines terminating at the charge (figure 29.5b). If we draw the lines isotropically (the lines are drawn uniformly in all directions, originating from the point charge), we can compare the intensities of the field at two points by just looking at the distribution of the lines of force.

Consider two points P_1 and P_2 in figure (29.5). Draw equal small areas through P_1 and P_2 perpendicular to the lines. More number of lines pass through the area at P_1 and less number of lines pass through the area at P_2. Also, the intensity of electric field is more at P_1 than at P_2. In fact, the electric field is proportional to the lines per unit area if the lines originate isotropically from the charge.

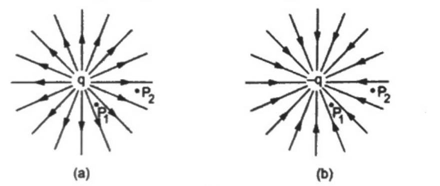

Figure 29.5

We can draw the lines of force for a charge distribution containing more than one charge. From each charge we can draw the lines isotropically. The lines may not be straight as one moves away from a charge. Figure (29.6) shows the shapes of these lines for some charge distributions.

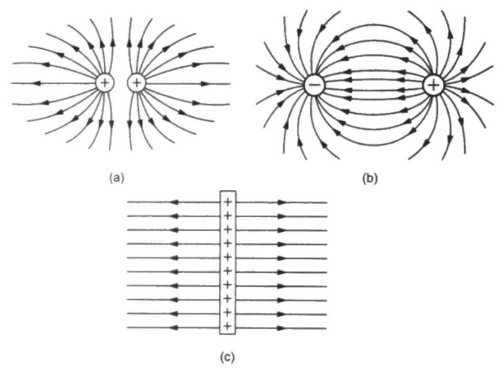

Figure 29.6

The lines of force are purely a geometrical construction which help us to visualise the nature of electric field in a region. They have no physical existence.

29.5 ELECTRIC POTENTIAL ENERGY

Consider a system of charges. The charges of the system exert electric forces on each other. If the position of one or more charges is changed, work may be done by these electric forces. We define *change in electric potential energy* of the system as negative of the work done by the electric forces as the configuration of the system changes.

Consider a system of two charges q_1 and q_2. Suppose, the charge q_1 is fixed at a point A and the charge q_2 is taken from a point B to a point C along the line ABC (figure 29.7).

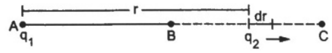

Figure 29.7

Let the distance $AB = r_1$ and the distance $AC = r_2$.

Consider a small displacement of the charge q_2 in which its distance from q_1 changes from r to $r + dr$. The electric force on the charge q_2 is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \text{ towards } \overrightarrow{AB}.$$

The work done by this force in the small displacement dr is

$$dW = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr.$$

The total work done as the charge q_2 moves from B to C is

$$W = \int_{r_1}^{r_2} \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

No work is done by the electric force on the charge q_1 as it is kept fixed. The change in potential energy $U(r_2) - U(r_1)$ is, therefore,

$$U(r_2) - U(r_1) = -W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \quad \ldots (29.5)$$

We choose the potential energy of the two-charge system to be zero when they have infinite separation (that means when they are widely separated). This means $U(\infty) = 0$. The potential energy when the separation is r is

$$U(r) = U(r) - U(\infty)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}. \quad \ldots (29.6)$$

The above equation is derived by assuming that one of the charges is fixed and the other is displaced. However, the potential energy depends essentially on the separation between the charges and is independent of the spatial location of the charged particles. Equations (29.5) and (29.6) are, therefore, general.

Equation (29.6) gives the electric potential energy of a pair of charges. If there are three charges q_1, q_2 and q_3, there are three pairs. Similarly for an N-particle system, the potential energy of the system is equal to the sum of the potential energies of the N pairs of charged particles.

Example 29.3

Three particles, each having a charge of $10\,\mu C$, *are placed at the vertices of an equilateral triangle of side* 10 cm. *Find the work done by a person in pulling them apart to infinite separations.*

Solution : The potential energy of the system in the initial condition is

$$U = \frac{3 \times (10\,\mu C) \times (10\,\mu C)}{4\pi\varepsilon_0 (10\text{ cm})}$$

$$= \frac{(3 \times 10^{-10}\text{ C}^2) \times (9 \times 10^9\text{ N m}^2\text{ C}^{-2})}{0{\cdot}1\text{ m}} = 27\text{ J}.$$

When the charges are infinitely separated, the potential energy is reduced to zero. If we assume that the charges do not get kinetic energy in the process, the total mechanical energy of the system decreases by 27 J. Thus, the work done by the person on the system is –27 J.

29.6 ELECTRIC POTENTIAL

The electric field in a region of space is described by assigning a vector quantity $\vec{E}$ at each point. The same field can also be described by assigning a scalar quantity V at each point. We now define this scalar quantity known as *electric potential.*

Suppose, a test charge q is moved in an electric field from a point A to a point B while all the other charges in question remain fixed. If the electric potential energy changes by $U_B - U_A$ due to this displacement, we define the *potential difference* between the point A and the point B as

$$V_B - V_A = \frac{U_B - U_A}{q}. \quad \ldots (29.7)$$

Conversely, if a charge q is taken through a potential difference $V_B - V_A$, the electric potential energy is increased by $U_B - U_A = q(V_B - V_A)$. This equation defines potential difference between any two points in an electric field. We can define absolute electric potential at any point by choosing a reference point P and saying that the potential at this point is zero. The electric potential at a point A is then given by (equation 29.7)

$$V_A = V_A - V_P = \frac{U_A - U_P}{q}. \quad \ldots (29.8)$$

So, *the potential at a point A is equal to the change in electric potential energy per unit test charge when it is moved from the reference point to the point A.*

Suppose, the test charge is moved in an electric field without changing its kinetic energy. The total work done on the charge should be zero from the work–energy theorem. If W_{ext} and W_{el} be the work done by the external agent and by the electric field as the charge moves, we have,

$$W_{ext} + W_{el} = 0$$

or, $$W_{ext} = -W_{el} = \Delta U,$$

where ΔU is the change in electric potential energy. Using this equation and equation (29.8), the potential at a point A may also be defined as follows:

The potential at a point A is equal to the work done per unit test charge by an external agent in moving the test charge from the reference point to the point A (without changing its kinetic energy).

The choice of reference point is purely ours. Generally, a point widely separated from all charges in question is taken as the reference point. Such a point is assumed to be at infinity.

As potential energy is a scalar quantity, potential is also a scalar quantity. Thus, if V_1 is the potential at a given point due to a charge q_1 and V_2 is the potential at the same point due to a charge q_2, the potential due to both the charges is $V_1 + V_2$.

29.7 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

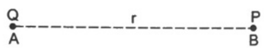

Figure 29.8

Consider a point charge Q placed at a point A (figure 29.8). We have to find the electric potential at a point P where $AP = r$. Let us take the reference point at $r = \infty$. Suppose, a test charge q is moved from $r = \infty$ to the point P. The change in electric potential energy of the system is, from equation (29.6),

$$U_P - U_\infty = \frac{Qq}{4\pi\varepsilon_0 r}.$$

The potential at P is, from equation (29.8),

$$V_P = \frac{U_P - U_\infty}{q} = \frac{Q}{4\pi\varepsilon_0 r}. \quad \dots \text{(29.9)}$$

The electric potential due to a system of charges may be obtained by finding potentials due to the individual charges using equation (29.9) and then adding them. Thus,

$$V = \frac{1}{4\pi\varepsilon_0} \sum \frac{Q_i}{r_i}.$$

Example 29.4

Two charges $+10\,\mu C$ *and* $+20\,\mu C$ *are placed at a separation of* 2 cm. *Find the electric potential due to the pair at the middle point of the line joining the two charges.*

Solution : Using the equation $V = \frac{Q}{4\pi\varepsilon_0 r}$, the potential due to $+10\,\mu C$ is

$$V_1 = \frac{(10 \times 10^{-6}\,\text{C}) \times (9 \times 10^{9}\,\text{N m}^2\text{C}^{-2})}{1 \times 10^{-2}\,\text{m}} = 9\,\text{MV}.$$

The potential due to $+20\,\mu C$ is

$$V_2 = \frac{(20 \times 10^{-6}\,\text{C}) \times (9 \times 10^{9}\,\text{N m}^2\text{C}^{-2})}{1 \times 10^{-2}\,\text{m}} = 18\,\text{MV}.$$

The net potential at the given point is

$$9\,\text{MV} + 18\,\text{MV} = 27\,\text{MV}.$$

If the charge distribution is continuous, we may use the technique of integration to find the electric potential.

29.8 RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Suppose, the electric field at a point $\vec{r}$ due to a charge distribution is $\vec{E}$ and the electric potential at the same point is V. Suppose, a point charge q is displaced slightly from the point $\vec{r}$ to $\vec{r} + \vec{dr}$. The force on the charge is

$$\vec{F} = q\vec{E}$$

and the work done by the electric field during the displacement is

$$dW = \vec{F}\cdot\vec{dr} = q\vec{E}\cdot\vec{dr}.$$

The change in potential energy is

$$dU = -\,dW = -\,q\vec{E}\cdot\vec{dr}.$$

The change in potential is

$$dV = \frac{dU}{q}$$

or,
$$dV = -\,\vec{E}\cdot\vec{dr}. \quad \dots \text{(29.10)}$$

Integrating between the points $\vec{r_1}$ and $\vec{r_2}$, we get

$$V_2 - V_1 = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E}\cdot\vec{dr} \quad \dots \text{(29.11)}$$

where V_1 and V_2 are the potentials at $\vec{r_1}$ and $\vec{r_2}$ respectively. If we choose $\vec{r_1}$, at the reference point (say at infinity) and $\vec{r_2}$ at $\vec{r}$, equation (29.11) becomes

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot\vec{dr}. \quad \dots \text{(29.12)}$$

Example 29.5

Figure (29.9) shows two metallic plates A and B placed parallel to each other at a separation d. A uniform electric field E exists between the plates in the direction from plate B to plate A. Find the potential difference between the plates.

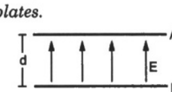

Figure 29.9

Solution : Let us take the origin at plate A and x-axis along the direction from plate A to plate B. We have

$$V_B - V_A = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{E}\cdot d\vec{r} = -\int_0^d - E\, dx = Ed.$$

If we work in Cartesian coordinate system

$$E = E_x \vec{i} + E_y \vec{j} + E_z \vec{k}$$

and $\quad d\vec{r} = dx\, \vec{i} + dy\, \vec{j} + dz\, \vec{k}.$

Thus, from (29.10)

$$dV = -E_x\, dx - E_y\, dy - E_z\, dz. \qquad \text{... (i)}$$

If we change x to $x + dx$ keeping y and z constant, $dy = dz = 0$ and from (i),

$$E_x = -\frac{\partial V}{\partial x}\cdot$$

Similarly, $$E_y = -\frac{\partial V}{\partial y} \qquad \text{... (29.13)}$$

and $$E_z = -\frac{\partial V}{\partial z}\cdot$$

The symbols $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$, etc., are used to indicate that while differentiating with respect to one coordinate, the others are kept constant.

If we know the electric field in a region, we can find the electric potential using equation (29.12) and if we know the electric potential in a region, we can find the electric field using (29.13).

Equation (29.10) may also be written as

$$dV = -E\, dr \cos\theta$$

where θ is the angle between the field $\vec{E}$ and the small displacement $d\vec{r}$. Thus,

$$-\frac{dV}{dr} = E \cos\theta. \qquad \text{... (29.14)}$$

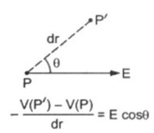

Figure 29.10

We see that, $-\frac{dV}{dr}$ gives the component of the electric field in the direction of displacement $d\vec{r}$. In figure (29.10), we show a small displacement $PP' = dr$. The electric field is E making an angle θ with PP'. We have

$$dV = V(P') - V(P)$$

so that $\quad \frac{V(P) - V(P')}{dr} = E\cos\theta.$

This gives us a method to get the component of the electric field in any given direction if we know the potential. Move a small distance dr in the given direction and see the change dV in the potential. The component of electric field along that direction is $-\frac{dV}{dr}$.

If we move a distance dr in the direction of the field, θ is zero and $-\frac{dV}{dr} = E$ is maximum. Thus, *the electric field is along the direction in which the potential decreases at the maximum rate.*

If a small displacement $d\vec{r}$ perpendicular to the electric field is considered, $\theta = 90°$ and $dV = -\vec{E}\cdot d\vec{r} = 0$. The potential does not vary in a direction perpendicular to the electric field.

Equipotential Surfaces

If we draw a surface in such a way that the electric potential is the same at all the points of the surface, it is called an *equipotential surface*. The component of electric field parallel to an equipotential surface is zero, as the potential does not change in this direction. Thus, the electric field is perpendicular to the equipotential surface at each point of the surface. For a point charge, the electric field is radial and the equipotential surfaces are concentric spheres with centres at the charge (figure 29.11).

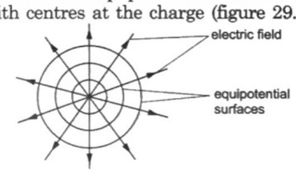

Figure 29.11

29.9 ELECTRIC DIPOLE

A combination of two charges $+q$ and $-q$ separated by a small distance d constitutes an *electric dipole*. The *electric dipole moment* of this combination is defined as a vector

$$\vec{p} = q\vec{d}, \qquad \text{... (29.15)}$$

where $\vec{d}$ is the vector joining the negative charge to the positive charge. The line along the direction of the dipole moment is called the *axis of the dipole.*

Electric Potential due to a Dipole

Suppose, the negative charge $-q$ is placed at a point A and the positive charge q is placed at a point

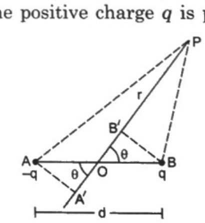

Figure 29.12

B (figure 29.12), the separation $AB = d$. The middle point of AB is O. The potential is to be evaluated at a point P where $OP = r$ and $\angle POB = \theta$. Also, let $r >> d$.

Let AA' be the perpendicular from A to PO and BB' be the perpendicular from B to PO. As d is very small compared to r,

$$AP \approx A'P = OP + OA'$$

$$= OP + AO\cos\theta = r + \frac{d}{2}\cos\theta.$$

Similarly, $BP \approx B'P = OP - OB'$

$$= r - \frac{d}{2}\cos\theta.$$

The potential at P due to the charge $-q$ is

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} \approx -\frac{1}{4\pi\varepsilon_0}\frac{q}{r + \frac{d}{2}\cos\theta}$$

and that due to the charge $+q$ is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{r - \frac{d}{2}\cos\theta}.$$

The net potential at P due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r - \frac{d}{2}\cos\theta} - \frac{q}{r + \frac{d}{2}\cos\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q\,d\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}$$

$$\approx \frac{1}{4\pi\varepsilon_0}\frac{q\,d\cos\theta}{r^2}$$

or, $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}. \quad \dots (29.16)$$

Generalised Definition of Electric Dipole

The potential at a distance r from a point charge q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

It is inversely proportional to r and is independent of direction. The potential due to a dipole is inversely proportional to r^2 and depends on direction as shown by the term $\cos\theta$ in equation (29.16). In general, any charge distribution that produces electric potential given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

is called an electric dipole. The constant p is called its dipole moment and the direction from which the angle θ is measured to get the above equation is called the direction of the dipole moment.

Electric Field due to a Dipole

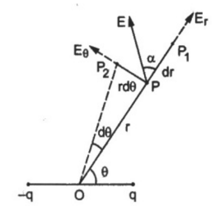

Figure 29.13

We can find the electric field due to an electric dipole using the expression (29.16) for the electric potential. In figure (29.13), PP_1 is a small displacement in the direction of OP and PP_2 is a small displacement perpendicular to OP. Thus, PP_1 is in radial direction and PP_2 is in transverse direction. In going from P to P_1, the angle θ does not change and the distance OP changes from r to $r + dr$. Thus, $PP_1 = dr$. In going from P to P_2, the angle θ changes from θ to $\theta + d\theta$ while the distance r remains almost constant. Thus, $PP_2 = r\,d\theta$. From equation (29.14), the component of the electric field at P in the radial direction PP_1 is

$$E_r = -\frac{dV}{PP_1} = -\frac{dV}{dr} = -\frac{\partial V}{\partial r}. \quad \dots (i)$$

The symbol ∂ specifies that θ should be treated as constant while differentiating with respect to r.

Similarly, the component of the electric field at P in the transverse direction PP_2 is

$$E_\theta = -\frac{dV}{PP_2} = -\frac{dV}{r d\theta} = -\frac{1}{r}\frac{\partial V}{\partial\theta}. \quad \dots (ii)$$

As $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2},$$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}(p\cos\theta)\frac{d}{dr}\left(\frac{1}{r^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \quad \dots (iii)$$

and $$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = -\frac{1}{r}\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial\theta}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\frac{d}{d\theta}(\cos\theta)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}. \quad \dots (iv)$$

The resultant electric field at P (figure 29.13) is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{p\sin\theta}{r^3}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1}. \quad \dots (29.17)$$

If the resultant field makes an angle α with the radial direction OP, we have

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{p\sin\theta/r^3}{2p\cos\theta/r^3} = \frac{1}{2}\tan\theta$$

or, $$\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right). \quad \dots (29.18)$$

Special Cases

(a) $\theta = 0$

In this case, the point P is on the axis of the dipole. From equation (29.16), the electric potential is $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$.

The field at such a point is, from equation (29.17), $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ along the axis. Such a position of the point P is called an *end-on position.*

(b) $\theta = 90°$

In this case the point P is on the perpendicular bisector of the dipole. The potential here is zero while the field is, from equation (29.17), $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$.

The angle α is given by

$$\tan\alpha = \frac{\tan\theta}{2} = \infty$$

or, $$\alpha = 90°.$$

The field is antiparallel to the dipole axis. Such a position of the point P is called a *broadside-on position.*

29.10 TORQUE ON AN ELECTRIC DIPOLE PLACED IN AN ELECTRIC FIELD

Consider an electric dipole placed in a uniform electric field $\vec{E}$. The dipole consists of charges $-q$ placed at A and $+q$ placed at B (figure 29.14). The mid-point of AB is O and the length $AB = d$. Suppose the axis of the dipole AB makes an angle θ with the electric field at a certain instant.

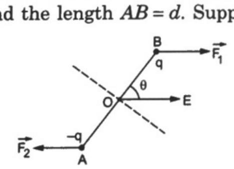

Figure 29.14

The force on the charge $+q$ is $\vec{F}_1 = q\vec{E}$ and the force on the charge $-q$ is $\vec{F}_2 = -q\vec{E}$. Let us calculate the torques $(\vec{r} \times \vec{F})$ of these forces about O.

The torque of $\vec{F}_1$ about O is

$$\vec{\Gamma}_1 = \overrightarrow{OB} \times \vec{F}_1 = q(\overrightarrow{OB} \times \vec{E})$$

and the torque of $\vec{F}_2$ about O is

$$\vec{\Gamma}_2 = \overrightarrow{OA} \times \vec{F}_2 = -q(\overrightarrow{OA} \times \vec{E}) = q(\overrightarrow{AO} \times \vec{E}).$$

The net torque acting on the dipole is

$$\vec{\Gamma} = \vec{\Gamma}_1 + \vec{\Gamma}_2$$

$$= q(\overrightarrow{OB} \times \vec{E}) + q(\overrightarrow{AO} \times \vec{E})$$

$$= q(\overrightarrow{OB} + \overrightarrow{AO}) \times \vec{E}$$

$$= q\,\overrightarrow{AB} \times \vec{E} = \vec{p} \times \vec{E}. \quad \dots (29.19)$$

The direction of the torque is perpendicular to the plane containing the dipole axis and the electric field. In figure (29.14), this is perpendicular to the plane of paper and is going into the page. The magnitude is $\Gamma = |\vec{\Gamma}| = pE\sin\theta$.

29.11 POTENTIAL ENERGY OF A DIPOLE PLACED IN A UNIFORM ELECTRIC FIELD

When an electric dipole is placed in an electric field $\vec{E}$, a torque $\vec{\Gamma} = \vec{p} \times \vec{E}$ acts on it (figure 29.14). If we rotate the dipole through a small angle $d\theta$, the work done by the torque is

$$dW = \Gamma d\theta$$

$$= -pE\sin\theta\, d\theta.$$

The work is negative as the rotation $d\theta$ is opposite to the torque.

The change in electric potential energy of the dipole is, therefore,

$$dU = -\,dW = pE\sin\theta\, d\theta.$$

If the angle θ is changed from 90° to θ, the change in potential energy is

$$U(\theta) - U(90°) = \int_{90°}^{\theta} pE\sin\theta\, d\theta$$

$$= pE\,[-\cos\theta]_{90°}^{\theta}$$

$$= -pE\cos\theta = -\vec{p}\cdot\vec{E}.$$

If we choose the potential energy of the dipole to be zero when $\theta = 90°$ (dipole axis is perpendicular to the field), $U(90°) = 0$ and the above equation becomes

$$U(\theta) = -\vec{p}\cdot\vec{E}. \quad \dots (29.20)$$

29.12 CONDUCTORS, INSULATORS AND SEMICONDUCTORS

Any piece of matter of moderate size contains millions and millions of atoms or molecules. Each atom contains a positively charged nucleus and several electrons going round it.

In gases, the atoms or molecules almost do not interact with each other. In solids and liquids, the interaction is comparatively stronger. It turns out that the materials may be broadly divided into three categories according to their behaviour when they are placed in an electric field.

In some materials, the outer electrons of each atom or molecule are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called *free electrons*. They are also known as *conduction electrons*. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called *conductors*.

Another class of materials is called *insulators* in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no *free electrons*. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called *dielectrics*.

In *semiconductors*, the behaviour is like an insulator at the temperature 0 K. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

We shall learn more about conductivity in later chapters. At the moment we accept the simple approximate model described above. The conductors have large number of free electrons everywhere in the material whereas the insulators have none. The discussion of semiconductors is deferred to a separate chapter.

Roughly speaking, the metals are conductors and the nonmetals are insulators. The above discussion may be extended to liquids and gases. Some of the liquids, such as mercury, and ionized gases are conductors.

29.13 THE ELECTRIC FIELD INSIDE A CONDUCTOR

Consider a conducting plate placed in a region. Initially, there is no electric field and the conduction electrons are almost uniformly distributed within the plate (shown by dots in figure 29.15a). In any small volume (which contains several thousand molecules) the number of electrons is equal to the number of protons in the nuclei. The net charge in the volume is zero.

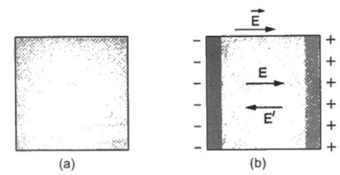

Figure 29.15

Now, suppose an electric field $\vec{E}$ is created in the direction left to right (figure 29.15b). This field exerts force on the free electrons from right to left. The electrons move towards left, the number of electrons on the left face increases and the number on the right face decreases. The left face becomes negatively charged and the right face becomes positively charged. These extra charges produce an extra electric field E' inside the plate from right to left. The electrons continue to drift and the internal field $\vec{E'}$ becomes stronger and stronger. A situation comes when the field $\vec{E'}$ due to the redistribution of free electrons becomes equal in magnitude to $\vec{E}$. The net electric field inside the plate is then zero. The free electrons there do not experience any net force and the process of further drifting stops. Thus, a steady state is reached in which some positive and negative charges appear at the surface of the plate and there is no electric field inside the plate.

Whenever a conductor is placed in an electric field some of the free electrons redistribute themselves on the surface of the conductor. The redistribution takes place in such a way that the electric field is zero at all the points inside the conductor. The redistribution takes a time which is, in general, less than a millisecond. Thus, *there can be no electric field inside a conductor in electrostatics.*

Worked Out Examples

1. *Charges* $5{\cdot}0 \times 10^{-7}$ C, $-2{\cdot}5 \times 10^{-7}$ C *and* $1{\cdot}0 \times 10^{-7}$ C *are held fixed at the three corners A, B, C of an equilateral triangle of side* 5·0 cm. *Find the electric force on the charge at C due to the rest two.*

Solution :

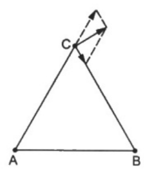

Figure 29-W1

The force on C due to A

$$= \frac{1}{4\pi\varepsilon_0} \frac{(5 \times 10^{-7}\ \text{C})\ (1 \times 10^{-7}\ \text{C})}{(0{\cdot}05\ \text{m})^2}$$

$$= 9 \times 10^9\ \text{Nm}^2\text{C}^{-2} \times \frac{5 \times 10^{-14}\ \text{C}^2}{25 \times 10^{-4}\ \text{m}^2} = 0{\cdot}18\ \text{N}.$$

This force acts along AC. The force on C due to B

$$= \frac{1}{4\pi\varepsilon_0} \frac{(2{\cdot}5 \times 10^{-7}\ \text{C})\ (1 \times 10^{-7}\ \text{C})}{(0{\cdot}05\ \text{m})^2} = 0{\cdot}09\ \text{N}.$$

This attractive force acts along CB. As the triangle is equilateral, the angle between these two forces is 120°. The resultant electric force on C is

$$[(0{\cdot}18\ \text{N})^2 + (0{\cdot}09\ \text{N})^2 + 2(0{\cdot}18\ \text{N})\ (0{\cdot}09\ \text{N})\ (\cos 120°)]^{1/2}$$

$= 0{\cdot}16$ N.

The angle made by this resultant with CB is

$$\tan^{-1} \frac{0{\cdot}18 \sin 120°}{0{\cdot}09 + 0{\cdot}18 \cos 120°} = 90°.$$

2. *Two particles A and B having charges* $8{\cdot}0 \times 10^{-6}$ C *and* $-2{\cdot}0 \times 10^{-6}$ C *respectively are held fixed with a separation of* 20 cm. *Where should a third charged particle be placed so that it does not experience a net electric force?*

Solution : As the net electric force on C should be equal to zero, the force due to A and B must be opposite in direction. Hence, the particle should be placed on the line AB. As A and B have charges of opposite signs, C cannot be between A and B. Also, A has larger magnitude of charge than B. Hence, C should be placed closer to B than A. The situation is shown in figure (29-W2).

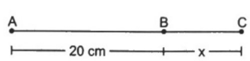

Figure 29-W2

Suppose $BC = x$ and the charge on C is Q.

The force due to $A = \dfrac{(8{\cdot}0 \times 10^{-6}\ \text{C})Q}{4\pi\varepsilon_0(20\ \text{cm} + x)^2}$.

The force due to $B = \dfrac{(2{\cdot}0 \times 10^{-6}\ \text{C})Q}{4\pi\varepsilon_0 x^2}$.

They are oppositely directed and to have a zero resultant, they should be equal in magnitude. Thus,

$$\frac{8}{(20\ \text{cm} + x)^2} = \frac{2}{x^2}$$

or, $$\frac{20\ \text{cm} + x}{x} = 2, \text{ giving } x = 20\ \text{cm}.$$

3. *Three equal charges, each having a magnitude of* $2{\cdot}0 \times 10^{-6}$ C, *are placed at the three corners of a right-angled triangle of sides* 3 cm, 4 cm *and* 5 cm. *Find the force on the charge at the right-angle corner.*

Solution :

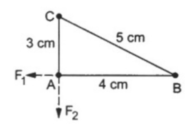

Figure 29-W3

The situation is shown in figure (29-W3). The force on A due to B is

$$F_1 = \frac{(2{\cdot}0 \times 10^{-6}\ \text{C})\ (2{\cdot}0 \times 10^{-6}\ \text{C})}{4\pi\varepsilon_0\ (4\ \text{cm})^2}$$

$$= 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2} \times 4 \times 10^{-12}\ \text{C}^2 \times \frac{1}{16 \times 10^{-4}\ \text{m}^2}$$

$= 22{\cdot}5$ N.

This force acts along BA. Similarly, the force on A due to C is $F_2 = 40$ N in the direction of CA. Thus, the net electric force on A is

$$F = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{(22{\cdot}5\ \text{N})^2 + (40\ \text{N})^2} = 45{\cdot}9\ \text{N}.$$

This resultant makes an angle θ with BA where

$$\tan\theta = \frac{40}{22{\cdot}5} = \frac{16}{9}.$$

4. *Two small iron particles, each of mass* 280 mg, *are placed at a distance* 10 cm *apart. If* 0·01% *of the electrons of one particle are transferred to the other, find the electric force between them. Atomic weight of iron is* 56 g mol^{-1} *and there are* 26 *electrons in each atom of iron.*

Solution : The atomic weight of iron is 56 g mol^{-1}. Thus, 56 g of iron contains 6×10^{23} atoms and each atom contains 26 electrons. Hence, 280 mg of iron contains

$$\frac{280\text{ mg}\times 6\times 10^{23}\times 26}{56\text{ g}} = 7{\cdot}8\times 10^{22}\text{ electrons.}$$

The number of electrons transferred from one particle to another

$$= \frac{0{\cdot}01}{100}\times 7{\cdot}8\times 10^{22} = 7{\cdot}8\times 10^{18}.$$

The charge transferred is, therefore,

$$1{\cdot}6\times 10^{-19}\text{ C}\times 7{\cdot}8\times 10^{18} = 1{\cdot}2\text{ C}.$$

The electric force between the particles is

$$(9\times 10^{9}\text{ N m}^{2}\text{ C}^{-2})\frac{(1{\cdot}2\text{ C})^{2}}{(10\times 10^{-2}\text{ m})^{2}}$$

$$= 1{\cdot}3\times 10^{12}\text{ N}.$$

This equals the load of approximately 2000 million grown-up persons !

5. *A charge Q is to be divided on two objects. What should be the values of the charges on the objects so that the force between the objects can be maximum ?*

Solution : Suppose one object receives a charge q and the other $Q-q$. The force between the objects is

$$F = \frac{q(Q-q)}{4\pi\varepsilon_0 d^{2}},$$

where d is the separation between them. For F to be maximum, the quantity

$$y = q(Q-q) = Qq - q^{2}$$

should be maximum. This is the case when,

$$\frac{dy}{dq} = 0 \text{ or, } Q - 2q = 0 \text{ or, } q = Q/2.$$

Thus, the charge should be divided equally on the two objects.

6. *Two particles, each having a mass of* 5 g *and charge* $1{\cdot}0\times 10^{-7}$ C, *stay in limiting equilibrium on a horizontal table with a separation of* 10 cm *between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient.*

Solution : The electric force on one of the particles due to the other is

$$F = (9\times 10^{9}\text{ N m}^{2}\text{ C}^{-2})\times(1{\cdot}0\times 10^{-7}\text{C})^{2}\times\frac{1}{(0{\cdot}10\text{ m})^{2}}$$

$$= 0{\cdot}009\text{ N}.$$

The frictional force in limiting equilibrium

$$f = \mu\times(5\times 10^{-3}\text{ kg})\times 9{\cdot}8\text{ m s}^{-2}$$

$$= (0{\cdot}049\,\mu)\text{ N}.$$

As these two forces balance each other,

$$0{\cdot}049\,\mu = 0{\cdot}009$$

or, $$\mu = 0{\cdot}18.$$

7. *A vertical electric field of magnitude* $4{\cdot}00\times 10^{5}\text{ N C}^{-1}$ *just prevents a water droplet of mass* $1{\cdot}00\times 10^{-4}$ kg *from falling. Find the charge on the droplet.*

Solution : The forces acting on the droplet are

(i) the electric force $q\vec{E}$ and

(ii) the force of gravity $m\vec{g}$.

To just prevent from falling, these two forces should be equal and opposite. Thus,

$$q(4{\cdot}00\times 10^{5}\text{ N C}^{-1}) = (1{\cdot}00\times 10^{-4}\text{ kg})\times(9{\cdot}8\text{ m s}^{-2})$$

or, $$q = 2{\cdot}45\times 10^{-9}\text{ C}.$$

8. *Three charges, each equal to q, are placed at the three corners of a square of side a. Find the electric field at the fourth corner.*

Solution :

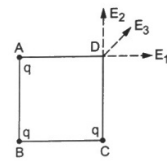

Figure 29-W4

Let the charges be placed at the corners A, B and C (figure 29-W4). We shall calculate the electric field at the fourth corner D. The field E_1 due to the charge at A will have the magnitude $\frac{q}{4\pi\varepsilon_0 a^{2}}$ and will be along AD. The field E_2 due to the charge at C will have the same magnitude and will be along CD.

The field E_3 due to the charge at B will have the magnitude $\frac{q}{4\pi\varepsilon_0(\sqrt{2}a)^{2}}$ and will be along BD. As E_1 and E_2 are equal in magnitude, their resultant will be along the bisector of the angle between E_1, E_2 and hence along E_3. The magnitude of this resultant is $\sqrt{E_1^{2}+E_2^{2}}$ as the angle between E_1 and E_2 is $\pi/2$. The resultant electric field at D is, therefore, along E_3 and has magnitude

$$\sqrt{E_1^{2}+E_2^{2}} + E_3$$

$$= \sqrt{\left(\frac{q}{4\pi\varepsilon_0 a^{2}}\right)^{2} + \left(\frac{q}{4\pi\varepsilon_0 a^{2}}\right)^{2}} + \frac{q}{4\pi\varepsilon_0(\sqrt{2}a)^{2}}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\sqrt{2}}{a^{2}} + \frac{1}{2a^{2}}\right] = (2\sqrt{2}+1)\frac{q}{8\pi\varepsilon_0 a^{2}}.$$

9. *A charged particle of mass* 1·0 g *is suspended through a silk thread of length* 40 cm *in a horizontal electric field of* $4{\cdot}0\times 10^{4}\text{ N C}^{-1}$. *If the particle stays at a distance of* 24 cm *from the wall in equilibrium, find the charge on the particle.*

Solution :

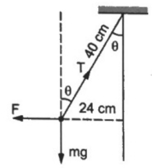

Figure 29-W5

The situation is shown in figure (29-W5).
The forces acting on the particle are
(i) the electric force $F = qE$ horizontally,
(ii) the force of gravity mg downward and
(iii) the tension T along the thread.
As the particle is at rest, these forces should add to zero. Taking components along horizontal and vertical,

$$T\cos\theta = mg \text{ and } T\sin\theta = F$$

or, $$F = mg\tan\theta \qquad \ldots \text{(i)}$$

From the figure,

$$\sin\theta = \frac{24}{40} = \frac{3}{5}\cdot$$

Thus, $\tan\theta = \frac{3}{4}\cdot$ From (i),

$$q(4{\cdot}0\times10^{4}\ \text{N C}^{-1}) = (1{\cdot}0\times10^{-3}\ \text{kg})\ (9{\cdot}8\ \text{m s}^{-2})\frac{3}{4},$$

giving $$q = 1{\cdot}8\times10^{-7}\ \text{C}.$$

10. *A particle A having a charge of* $5{\cdot}0\times10^{-7}$ C *is fixed in a vertical wall. A second particle B of mass* 100 g *and having equal charge is suspended by a silk thread of length* 30 cm *from the wall. The point of suspension is* 30 cm *above the particle A. Find the angle of the thread with the vertical when it stays in equilibrium.*

Solution :

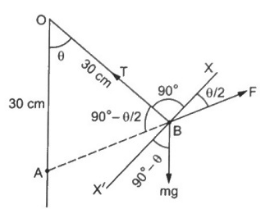

Figure 29-W6

The situation is shown in figure (29-W6). Suppose the point of suspension is O and let θ be the angle between the thread and the vertical. Forces on the particle B are
(i) weight mg downward
(ii) tension T along the thread and
(iii) electric force of repulsion F along AB.

For equilibrium, these forces should add to zero. Let $X'BX$ be the line perpendicular to OB. We shall take the components of the forces along BX. This will give a relation between F, mg and θ.

The various angles are shown in the figure. As

$$OA = OB,\ \angle OBA = \angle OAB = 90^\circ - \frac{\theta}{2}\cdot$$

The other angles can be written down directly.
Taking components along BX, we get

$$F\cos\frac{\theta}{2} = mg\cos(90^\circ - \theta)$$

$$= 2\,mg\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

or, $$\sin\frac{\theta}{2} = \frac{F}{2\,mg}\cdot \qquad \ldots \text{(i)}$$

Now, $$F = (9\times10^{9}\ \text{N m}^2\ \text{C}^{-2})\times(5{\cdot}0\times10^{-7}\ \text{C})^2\times\frac{1}{AB^2}$$

and $$AB = 2(OA)\sin\frac{\theta}{2}\cdot$$

Thus, $$F = \frac{9\times10^{9}\times25\times10^{-14}}{4\times(30\times10^{-2})^2\times\sin^2\frac{\theta}{2}}\ \text{N}. \qquad \ldots \text{(ii)}$$

From (i) and (ii),

$$\sin\frac{\theta}{2} = \frac{F}{2\,mg} = \frac{9\times10^{9}\times25\times10^{-14}\ \text{N}}{4\times(30\times10^{-2})^2\times\sin^2\frac{\theta}{2}}\cdot\frac{1}{2\,mg}$$

or,

$$\sin^3\frac{\theta}{2} = \frac{9\times10^{9}\times25\times10^{-14}\ \text{N}}{4\times9\times10^{-2}\times2\times(100\times10^{-3}\ \text{kg})\times9{\cdot}8\ \text{m s}^{-2}}$$

$$= 0{\cdot}0032$$

or, $$\sin\frac{\theta}{2} = 0{\cdot}15,\ \text{giving}\ \theta = 17^\circ.$$

11. *Four particles, each having a charge q, are placed on the four vertices of a regular pentagon. The distance of each corner from the centre is a. Find the electric field at the centre of the pentagon.*

Solution :

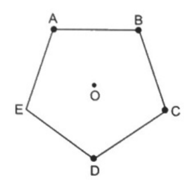

Figure 29-W7

Let the charges be placed at the vertices A, B, C and D of the pentagon $ABCDE$. If we put a charge q at the corner E also, the field at O will be zero by symmetry. Thus, the field at the centre due to the charges at A, B,

C and D is equal and opposite to the field due to the charge q at E alone.

The field at O due to the charge q at E is

$$\frac{q}{4\pi\varepsilon_0 a^2} \text{ along } EO.$$

Thus, the field at O due to the given system of charges is $\frac{q}{4\pi\varepsilon_0 a^2}$ along OE.

12. *Find the electric field at a point P on the perpendicular bisector of a uniformly charged rod. The length of the rod is L, the charge on it is Q and the distance of P from the centre of the rod is a.*

Solution :

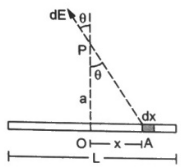

Figure 29-W8

Let us take an element of length dx at a distance x from the centre of the rod (figure 29-W8). The charge on this element is

$$dQ = \frac{Q}{L}dx.$$

The electric field at P due to this element is

$$dE = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}.$$

By symmetry, the resultant field at P will be along OP (if the charge is positive). The component of dE along OP is

$$dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}\cdot\frac{OP}{AP} = \frac{a\,Q\,dx}{4\pi\varepsilon_0 L(a^2+x^2)^{3/2}}$$

Thus, the resultant field at P is

$$E = \int dE\cos\theta$$

$$= \frac{aQ}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2}\frac{dx}{(a^2+x^2)^{3/2}}. \qquad \dots \text{(i)}$$

We have $x = a\tan\theta$ or $dx = a\sec^2\theta\, d\theta$.

Thus, $\displaystyle\int\frac{dx}{(a^2+x^2)^{3/2}} = \int\frac{a\sec^2\theta\, d\theta}{a^3\sec^3\theta}$

$$= \frac{1}{a^2}\int\cos\theta\, d\theta = \frac{1}{a^2}\sin\theta = \frac{1}{a^2}\frac{x}{(x^2+a^2)^{1/2}}.$$

From (i),

$$E = \frac{aQ}{4\pi\varepsilon_0 La^2}\left[\frac{x}{(x^2+a^2)^{1/2}}\right]_{-L/2}^{L/2}$$

$$= \frac{aQ}{4\pi\varepsilon_0 La^2}\left[\frac{2L}{(L^2+4a^2)^{1/2}}\right]$$

$$= \frac{Q}{2\pi\varepsilon_0 a\sqrt{L^2+4a^2}}.$$

13. *A uniform electric field E is created between two parallel, charged plates as shown in figure (29-W9). An electron enters the field symmetrically between the plates with a speed v_0. The length of each plate is l. Find the angle of deviation of the path of the electron as it comes out of the field.*

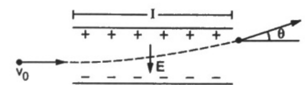

Figure 29-W9

Solution : The acceleration of the electron is $a = \frac{eE}{m}$ in the upward direction. The horizontal velocity remains v_0 as there is no acceleration in this direction. Thus, the time taken in crossing the field is

$$t = \frac{l}{v_0}. \qquad \dots \text{(i)}$$

The upward component of the velocity of the electron as it emerges from the field region is

$$v_y = at = \frac{eEl}{mv_0}.$$

The horizontal component of the velocity remains

$$v_x = v_0.$$

The angle θ made by the resultant velocity with the original direction is given by

$$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv_0^2}.$$

Thus, the electron deviates by an angle

$$\theta = \tan^{-1}\frac{eEl}{mv_0^2}.$$

14. *In a circuit, 10 C of charge is passed through a battery in a given time. The plates of the battery are maintained at a potential difference of 12 V. How much work is done by the battery?*

Solution : By definition, the work done to transport a charge q through a potential difference V is qV. Thus, work done by the battery

$$= 10\text{ C} \times 12\text{ V} = 120\text{ J}.$$

15. *Charges $2{\cdot}0\times10^{-6}$ C and $1{\cdot}0\times10^{-6}$ C are placed at corners A and B of a square of side 5·0 cm as shown in figure (29-W10). How much work will be done against the electric field in moving a charge of $1{\cdot}0\times10^{-6}$ C from C to D?*

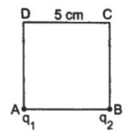

Figure 29-W10

Solution : The electric potential at C

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{AC} + \frac{q_2}{BC}\right)$$

$$= 9\times 10^9\ \mathrm{Nm^{2}C^{-2}}\left(\frac{2\cdot0\times10^{-6}\ \mathrm{C}}{\sqrt{2}\times 0\cdot05\ \mathrm{m}} + \frac{1\cdot0\times10^{-6}\ \mathrm{C}}{0\cdot05\ \mathrm{m}}\right)$$

$$= (9000\ \mathrm{V})\left(\frac{2+\sqrt{2}}{\sqrt{2}\times 0\cdot05}\right).$$

The electric potential at D

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{AD} + \frac{q_2}{BD}\right)$$

$$= 9\times 10^9\ \mathrm{Nm^{2}C^{-2}}\left(\frac{2\cdot0\times10^{-6}\ \mathrm{C}}{0\cdot05\ \mathrm{m}} + \frac{1\cdot0\times10^{-6}\ \mathrm{C}}{\sqrt{2}\times 0\cdot05\ \mathrm{m}}\right)$$

$$= (9000\ \mathrm{V})\left(\frac{2\sqrt{2}+1}{\sqrt{2}\times 0\cdot05}\right).$$

The work done against the electric field in moving the charge $1\cdot0\times10^{-6}$ C from C to D is $q(V_D - V_C)$

$$= (1\cdot0\times10^{-6}\ \mathrm{C})\,(9000\ \mathrm{V})\left(\frac{2\sqrt{2}+1-2-\sqrt{2}}{\sqrt{2}\times 0\cdot05}\right)$$

$$= 0\cdot053\ \mathrm{J}.$$

16. *The electric field in a region is given by $\vec{E} = (A/x^3)\,\vec{i}$. Write a suitable SI unit for A. Write an expression for the potential in the region assuming the potential at infinity to be zero.*

Solution : The SI unit of electric field is $\mathrm{N\,C^{-1}}$ or $\mathrm{V\,m^{-1}}$. Thus, the unit of A is $\mathrm{N\,m^3\,C^{-1}}$ or $\mathrm{V\,m^{2}}$.

$$V(x, y, z) = -\int_{\infty}^{(x,y,z)} \vec{E}\cdot\vec{dr}$$

$$= -\int_{\infty}^{(x,y,z)} \frac{A\,dx}{x^3} = \frac{A}{2x^2}.$$

17. *Three point charges q, 2q and 8q are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the charge q due to the other two charges?*

Solution : The maximum contribution may come from the charge $8q$ forming pairs with others. To reduce its effect, it should be placed at a corner and the smallest charge q in the middle. This arrangement shown in figure (29-W11) ensures that the charges in the strongest pair $2q$, $8q$ are at the largest separation.

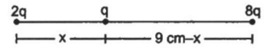

Figure 29-W11

The potential energy is

$$U = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{2}{x} + \frac{16}{9\ \mathrm{cm}} + \frac{8}{9\ \mathrm{cm} - x}\right].$$

This will be minimum if

$$A = \frac{2}{x} + \frac{8}{9\ \mathrm{cm} - x} \text{ is minimum.}$$

For this, $$\frac{dA}{dx} = -\frac{2}{x^2} + \frac{8}{(9\ \mathrm{cm} - x)^2} = 0 \qquad \dots \text{(i)}$$

or, $9\ \mathrm{cm} - x = 2x$ or, $x = 3$ cm.

The electric field at the position of charge q is

$$\frac{q}{4\pi\varepsilon_0}\left(\frac{2}{x^2} - \frac{8}{(9\ \mathrm{cm} - x)^2}\right)$$

$$= 0 \qquad \text{from (i).}$$

18. *An HCl molecule has a dipole moment of $3\cdot4\times10^{-30}$ Cm. Assuming that equal and opposite charges lie on the two atoms to form a dipole, what is the magnitude of this charge? The separation between the two atoms of HCl is $1\cdot0\times10^{-10}$ m.*

Solution : If the charges on the two atoms are q, $-q$,

$$q(1\cdot0\times10^{-10}\ \mathrm{m}) = 3\cdot4\times10^{-30}\ \mathrm{Cm}$$

or, $$q = 3\cdot4\times10^{-20}\ \mathrm{C}.$$

Note that this is less than the charge of a proton. Can you explain, how such a charge can appear on an atom?

19. *Figure (29-W12) shows an electric dipole formed by two particles fixed at the ends of a light rod of length l. The mass of each particle is m and the charges are –q and +q. The system is placed in such a way that the dipole axis is parallel to a uniform electric field E that exists in the region. The dipole is slightly rotated about its centre and released. Show that for small angular displacement, the motion is angular simple harmonic and find its time period.*

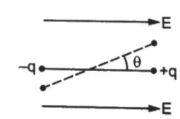

Figure 29-W12

Solution : Suppose, the dipole axis makes an angle θ with the electric field at an instant. The magnitude of the torque on it is

$$|\tau| = |\vec{p} \times \vec{E}|$$
$$= qlE \sin\theta.$$

This torque will tend to rotate the dipole back towards the electric field. Also, for small angular displacement $\sin\theta \approx \theta$ so that

$$\tau = -qlE\theta.$$

The moment of inertia of the system about the axis of rotation is

$$I = 2 \times m\left(\frac{l}{2}\right)^2 = \frac{ml^2}{2}.$$

Thus, the angular acceleration is

$$\alpha = \frac{\tau}{I} = -\frac{2qE}{ml}\theta = -\omega^2\theta$$

where $\omega^2 = \dfrac{2qE}{ml}$.

Thus, the motion is angular simple harmonic and the time period is $T = 2\pi\sqrt{\dfrac{ml}{2qE}}$.

□

QUESTIONS FOR SHORT ANSWER

1. The charge on a proton is $+1.6 \times 10^{-19}$ C and that on an electron is -1.6×10^{-19} C. Does it mean that the electron has a charge 3.2×10^{-19} C less than the charge of a proton?
2. Is there any lower limit to the electric force between two particles placed at a separation of 1 cm?
3. Consider two particles A and B having equal charges and placed at some distance. The particle A is slightly displaced towards B. Does the force on B increase as soon as the particle A is displaced? Does the force on the particle A increase as soon as it is displaced?
4. Can a gravitational field be added vectorially to an electric field to get a total field?
5. Why does a phonograph-record attract dust particles just after it is cleaned?
6. Does the force on a charge due to another charge depend on the charges present nearby?
7. In some old texts it is mentioned that 4π lines of force originate from each unit positive charge. Comment on the statement in view of the fact that 4π is not an integer.
8. Can two equipotential surfaces cut each other?
9. If a charge is placed at rest in an electric field, will its path be along a line of force? Discuss the situation when the lines of force are straight and when they are curved.
10. Consider the situation shown in figure (29-Q1). What are the signs of q_1 and q_2? If the lines are drawn in proportion to the charge, what is the ratio q_1/q_2?

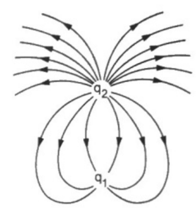

Figure 29-Q1

11. A point charge is taken from a point A to a point B in an electric field. Does the work done by the electric field depend on the path of the charge?
12. It is said that the separation between the two charges forming an electric dipole should be small. Small compared to what?
13. The number of electrons in an insulator is of the same order as the number of electrons in a conductor. What is then the basic difference between a conductor and an insulator?
14. When a charged comb is brought near a small piece of paper, it attracts the piece. Does the paper become charged when the comb is brought near it?

OBJECTIVE I

1. Figure (29-Q2) shows some of the electric field lines corresponding to an electric field. The figure suggests that

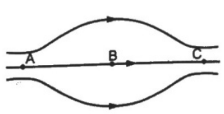

Figure 29-Q2

(a) $E_A > E_B > E_C$ (b) $E_A = E_B = E_C$
(c) $E_A = E_C > E_B$ (d) $E_A = E_C < E_B$.

2. When the separation between two charges is increased, the electric potential energy of the charges
(a) increases (b) decreases
(c) remains the same (d) may increase or decrease.

3. If a positive charge is shifted from a low-potential region to a high-potential region, the electric potential energy

(a) increases (b) decreases
(c) remains the same (d) may increase or decrease.

4. Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving from A to B. The potential
(a) continuously increases
(b) continuously decreases
(c) increases then decreases
(d) decreases then increases.

5. The electric field at the origin is along the positive x-axis. A small circle is drawn with the centre at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, $(0, -a)$ respectively. Out of the points on the periphery of the circle, the potential is minimum at
(a) A (b) B (c) C (d) D.

6. If a body is charged by rubbing it, its weight
(a) remains precisely constant
(b) increases slightly
(c) decreases slightly
(d) may increase slightly or may decrease slightly.

7. An electric dipole is placed in a uniform electric field. The net electric force on the dipole
(a) is always zero
(b) depends on the orientation of the dipole
(c) can never be zero
(d) depends on the strength of the dipole.

8. Consider the situation of figure (29-Q3). The work done in taking a point charge from P to A is W_A, from P to B is W_B and from P to C is W_C.
(a) $W_A < W_B < W_C$ (b) $W_A > W_B > W_C$
(c) $W_A = W_B = W_C$ (d) None of these

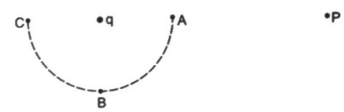

Figure 29-Q3

9. A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is
(a) zero (b) positive (c) negative
(d) zero if the charge Q is at the centre and nonzero otherwise.

OBJECTIVE II

1. Mark out the correct options.
(a) The total charge of the universe is constant.
(b) The total positive charge of the universe is constant.
(c) The total negative charge of the universe is constant.
(d) The total number of charged particles in the universe is constant.

2. A point charge is brought in an electric field. The electric field at a nearby point
(a) will increase if the charge is positive
(b) will decrease if the charge is negative
(c) may increase if the charge is positive
(d) may decrease if the charge is negative.

3. The electric field and the electric potential at a point are E and V respectively.
(a) If $E = 0$, V must be zero.
(b) If $V = 0$, E must be zero.
(c) If $E \neq 0$, V cannot be zero.
(d) If $V \neq 0$, E cannot be zero.

4. The electric potential decreases uniformly from 120 V to 80 V as one moves on the x-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin
(a) must be equal to 20 Vcm^{-1}
(b) may be equal to 20 Vcm^{-1}
(c) may be greater than 20 Vcm^{-1}
(d) may be less than 20 Vcm^{-1}.

5. Which of the following quantities do not depend on the choice of zero potential or zero potential energy?
(a) Potential at a point
(b) Potential difference between two points
(c) Potential energy of a two-charge system
(d) Change in potential energy of a two-charge system.

6. An electric dipole is placed in an electric field generated by a point charge.
(a) The net electric force on the dipole must be zero.
(b) The net electric force on the dipole may be zero.
(c) The torque on the dipole due to the field must be zero.
(d) The torque on the dipole due to the field may be zero.

7. A proton and an electron are placed in a uniform electric field.
(a) The electric forces acting on them will be equal.
(b) The magnitudes of the forces will be equal.
(c) Their accelerations will be equal.
(d) The magnitudes of their accelerations will be equal.

8. The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero,
(a) it is uniform in the region
(b) it is proportional to r
(c) it is proportional to r^2
(d) it increases as one goes away from the origin.

EXERCISES

1. Find the dimensional formula of ε_0.
2. A charge of 1·0 C is placed at the top of your college building and another equal charge at the top of your house. Take the separation between the two charges to be 2·0 km. Find the force exerted by the charges on each other. How many times of your weight is this force ?
3. At what separation should two equal charges, 1·0 C each, be placed so that the force between them equals the weight of a 50 kg person ?
4. Two equal charges are placed at a separation of 1·0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person ?
5. Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = 10^{-15} m). The protons in a nucleus remain at a separation of this order.
6. Two charges 2.0×10^{-6} C and 1.0×10^{-6} C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges ?
7. Suppose the second charge in the previous problem is -1.0×10^{-6} C. Locate the position where a third charge will not experience a net force.
8. Two charged particles are placed at a distance 1·0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge ?
9. Estimate the number of electrons in 100 g of water. How much is the total negative charge on these electrons ?
10. Suppose all the electrons of 100 g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively charged particle. If these two particles are placed 10·0 cm away from each other, find the force of attraction between them. Compare it with your weight.
11. Consider a gold nucleus to be a sphere of radius 6·9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion ?
12. Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0·1 N, how many electrons were transferred from one sphere to the other during rubbing ?
13. NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between the ions to be 2.75×10^{-8} cm, find the force of attraction between them. State the assumptions (if any) that you have made.
14. Find the ratio of the electric and gravitational forces between two protons.
15. Suppose an attractive nuclear force acts between two protons which may be written as $F = Ce^{-kr}/r^2$. (a) Write down the dimensional formulae and appropriate SI units of C and k. (b) Suppose that $k = 1$ fermi^{-1} and that the repulsive electric force between the protons is just balanced by the attractive nuclear force when the separation is 5 fermi. Find the value of C.
16. Three equal charges, 2.0×10^{-6} C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.
17. Four equal charges 2.0×10^{-6} C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.
18. A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0·53 angstrom (1 angstrom = 10^{-10} m and is abbreviated as Å) with the proton at the centre. The hydrogen atom is said to be in the ground state in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.
19. Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.
20. Ten positively charged particles are kept fixed on the x-axis at points $x = 10$ cm, 20 cm, 30 cm, ..., 100 cm. The first particle has a charge 1.0×10^{-8} C, the second 8×10^{-8} C, the third 27×10^{-8} C and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the electric force acting on a 1 C charge placed at the origin.
21. Two charged particles having charge 2.0×10^{-8} C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.
22. Two identical balls, each having a charge of 2.00×10^{-7} C and a mass of 100 g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5·0 cm apart and then released. Find (a) the electric force on one of the charged balls (b) the components of the resultant force on it along and perpendicular to the string (c) the tension in the string (d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.
23. Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude 2.0×10^{-8} C.
24. Two small spheres, each having a mass of 20 g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at

equilibrium is found to be 4 cm. Find the charge on each sphere.

25. Two identical pith balls, each carrying a charge q, are suspended from a common point by two strings of equal length l. Find the mass of each ball if the angle between the strings is 2θ in equilibrium.

26. A particle having a charge of 2.0×10^{-4} C is placed directly below and at a separation of 10 cm from the bob of a simple pendulum at rest. The mass of the bob is 100 g. What charge should the bob be given so that the string becomes loose?

27. Two particles A and B having charges q and $2q$ respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped?

28. Two identically charged particles are fastened to the two ends of a spring of spring constant 100 N m^{-1} and natural length 10 cm. The system rests on a smooth horizontal table. If the charge on each particle is 2.0×10^{-8} C, find the extension in the length of the spring. Assume that the extension is small as compared to the natural length. Justify this assumption after you solve the problem.

29. A particle A having a charge of 2.0×10^{-6} C is held fixed on a horizontal table. A second charged particle of mass 80 g stays in equilibrium on the table at a distance of 10 cm from the first charge. The coefficient of friction between the table and this second particle is $\mu = 0.2$. Find the range within which the charge of this second particle may lie.

30. A particle A having a charge of 2.0×10^{-6} C and a mass of 100 g is placed at the bottom of a smooth inclined plane of inclination 30°. Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium?

31. Two particles A and B, each having a charge Q, are placed a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? What is the magnitude of this maximum force?

32. Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having mass m and charge q is kept at the middle point of the line AB. (a) If it is displaced through a distance x perpendicular to AB, what would be the electric force experienced by it. (b) Assuming $x << d$, show that this force is proportional to x. (c) Under what conditions will the particle C execute simple harmonic motion if it is released after such a small displacement? Find the time period of the oscillations if these conditions are satisfied.

33. Repeat the previous problem if the particle C is displaced through a distance x along the line AB.

34. The electric force experienced by a charge of 1.0×10^{-6} C is 1.5×10^{-3} N. Find the magnitude of the electric field at the position of the charge.

35. Two particles A and B having charges of $+2.00 \times 10^{-6}$ C and of -4.00×10^{-6} C respectively are held fixed at a separation of 20.0 cm. Locate the point(s) on the line AB where (a) the electric field is zero (b) the electric potential is zero.

36. A point charge produces an electric field of magnitude 5.0 N C^{-1} at a distance of 40 cm from it. What is the magnitude of the charge?

37. A water particle of mass 10.0 mg and having a charge of 1.50×10^{-6} C stays suspended in a room. What is the magnitude of electric field in the room? What is its direction?

38. Three identical charges, each having a value 1.0×10^{-8} C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field and potential at the centre of the triangle.

39. Positive charge Q is distributed uniformly over a circular ring of radius R. A particle having a mass m and a negative charge q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming $x << R$, find the time period of oscillation of the particle if it is released from there.

40. A rod of length L has a total charge Q distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

41. A 10-cm long rod carries a charge of +50 μC distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both the ends of the rod.

42. Consider a uniformly charged ring of radius R. Find the point on the axis where the electric field is maximum.

43. A wire is bent in the form of a regular hexagon and a total charge q is distributed uniformly on it. What is the electric field at the centre? You may answer this part without making any numerical calculations.

44. A circular wire-loop of radius a carries a total charge Q distributed uniformly over its length. A small length dL of the wire is cut off. Find the electric field at the centre due to the remaining wire.

45. A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

46. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a uniform, horizontal electric field of 20 kVm^{-1}. Find the tension in the thread.

47. A particle of mass m and charge q is thrown at a speed u against a uniform electric field E. How much distance will it travel before coming to momentary rest?

48. A particle of mass 1 g and charge 2.5×10^{-4} C is released from rest in an electric field of $1.2 \times 10^{4} \text{ N C}^{-1}$. (a) Find the electric force and the force of gravity acting on this particle. Can one of these forces be neglected in comparison with the other for approximate analysis? (b) How long will it take for the particle to travel a distance of 40 cm? (c) What will be the speed of the particle after travelling this distance? (d) How much is the work done by the electric force on the particle during this period?

49. A ball of mass 100 g and having a charge of $4{\cdot}9 \times 10^{-5}$ C is released from rest in a region where a horizontal electric field of $2{\cdot}0 \times 10^{4}$ N C^{-1} exists. (a) Find the resultant force acting on the ball. (b) What will be the path of the ball ? (c) Where will the ball be at the end of 2 s ?

50. The bob of a simple pendulum has a mass of 40 g and a positive charge of $4{\cdot}0 \times 10^{-6}$ C. It makes 20 oscillations in 45 s. A vertical electric field pointing upward and of magnitude $2{\cdot}5 \times 10^{4}$ N C^{-1} is switched on. How much time will it now take to complete 20 oscillations ?

51. A block of mass m having a charge q is placed on a smooth horizontal table and is connected to a wall through an unstressed spring of spring constant k as shown in figure (29-E1). A horizontal electric field E parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block.

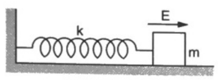

Figure 29-E1

52. A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure (29-E2). The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion ?

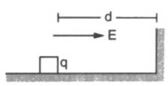

Figure 29-E2

53. A uniform electric field of 10 N C^{-1} exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.

54. 12 J of work has to be done against an existing electric field to take a charge of 0·01 C from A to B. How much is the potential difference $V_B - V_A$?

55. Two equal charges, $2{\cdot}0 \times 10^{-7}$ C each, are held fixed at a separation of 20 cm. A third charge of equal magnitude is placed midway between the two charges. It is now moved to a point 20 cm from both the charges. How much work is done by the electric field during the process ?

56. An electric field of 20 N C^{-1} exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by,
(a) $A = (0,\ 0)$; $B = (4\text{ m},\ 2\text{ m})$
(b) $A = (4\text{ m},\ 2\text{ m})$; $B = (6\text{ m},\ 5\text{ m})$
(c) $A = (0,\ 0)$; $B = (6\text{ m},\ 5\text{ m})$.
Do you find any relation between the answers of parts (a), (b) and (c) ?

57. Consider the situation of the previous problem. A charge of $-2{\cdot}0 \times 10^{-4}$ C is moved from the point A to the point B. Find the change in electrical potential energy $U_B - U_A$ for the cases (a), (b) and (c).

58. An electric field $\vec{E} = (\vec{i}\,20 + \vec{j}\,30)$ N C^{-1} exists in the space. If the potential at the origin is taken to be zero, find the potential at (2 m, 2 m).

59. An electric field $\vec{E} = \vec{i}Ax$ exists in the space, where $A = 10$ V m^{-2}. Take the potential at (10 m, 20 m) to be zero. Find the potential at the origin.

60. The electric potential existing in space is $V(x,\ y,\ z) = A(xy + yz + zx)$. (a) Write the dimensional formula of A. (b) Find the expression for the electric field. (c) If A is 10 SI units, find the magnitude of the electric field at (1 m, 1 m, 1 m).

61. Two charged particles, having equal charges of $2{\cdot}0 \times 10^{-5}$ C each, are brought from infinity to within a separation of 10 cm. Find the increase in the electric potential energy during the process.

62. Some equipotential surfaces are shown in figure (29-E3). What can you say about the magnitude and the direction of the electric field ?

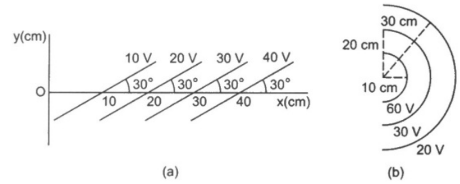

Figure 29-E3

63. Consider a circular ring of radius r, uniformly charged with linear charge density λ. Find the electric potential at a point on the axis at a distance x from the centre of the ring. Using this expression for the potential, find the electric field at this point.

64. An electric field of magnitude 1000 N C^{-1} is produced between two parallel plates having a separation of 2·0 cm as shown in figure (29-E4). (a) What is the potential difference between the plates ? (b) With what minimum speed should an electron be projected from the lower plate in the direction of the field so that it may reach the upper plate ? (c) Suppose the electron is projected from the lower plate with the speed calculated in part (b). The direction of projection makes an angle of 60° with the field. Find the maximum height reached by the electron.

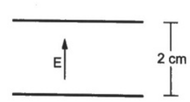

Figure 29-E4

65. A uniform field of 2·0 N C^{-1} exists in space in x-direction. (a) Taking the potential at the origin to be zero, write an expression for the potential at a general point (x, y, z). (b) At which points, the potential is 25 V ? (c) If the potential at the origin is taken to be 100 V, what will be the expression for the potential at a general point ? (d) What will be the potential at the origin if the potential at infinity is taken to be zero ? Is it practical to choose the potential at infinity to be zero ?

66. How much work has to be done in assembling three charged particles at the vertices of an equilateral triangle as shown in figure (29-E5) ?

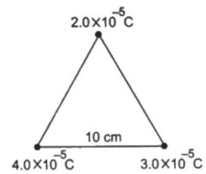

Figure 29-E5

67. The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.

68. Two identical particles, each having a charge of $2{\cdot}0 \times 10^{-4}$ C and mass of 10 g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large ?

69. Two particles have equal masses of 5·0 g each and opposite charges of $+4{\cdot}0 \times 10^{-5}$ C and $-4{\cdot}0 \times 10^{-5}$ C. They are released from rest with a separation of 1·0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.

70. A sample of HCl gas is placed in an electric field of $2{\cdot}5 \times 10^{4}$ N C^{-1}. The dipole moment of each HCl molecule is $3{\cdot}4 \times 10^{-30}$ Cm. Find the maximum torque that can act on a molecule.

71. Two particles A and B, having opposite charges $2{\cdot}0 \times 10^{-6}$ C and $-2{\cdot}0 \times 10^{-6}$ C, are placed at a separation of 1·0 cm. (a) Write down the electric dipole moment of this pair. (b) Calculate the electric field at a point on the axis of the dipole 1·0 m away from the centre. (c) Calculate the electric field at a point on the perpendicular bisector of the dipole and 1·0 m away from the centre.

72. Three charges are arranged on the vertices of an equilateral triangle as shown in figure (29-E6). Find the dipole moment of the combination.

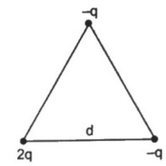

Figure 29-E6

73. Find the magnitude of the electric field at the point P in the configuration shown in figure (29-E7) for $d >> a$. Take $2qa = p$.

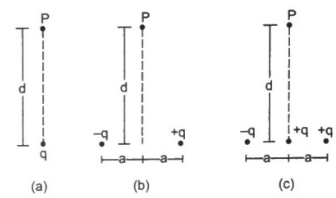

Figure 29-E7

74. Two particles, carrying charges $-q$ and $+q$ and having equal masses m each, are fixed at the ends of a light rod of length a to form a dipole. The rod is clamped at an end and is placed in a uniform electric field E with the axis of the dipole along the electric field. The rod is slightly tilted and then released. Neglecting gravity find the time period of small oscillations.

75. Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6·4 g (take the atomic weight of copper to be 64 g mol^{-1}).

□

ANSWERS

OBJECTIVE I

1. (c) 2. (d) 3. (a) 4. (d) 5. (a) 6. (d)
7. (a) 8. (c) 9. (a)

OBJECTIVE II

1. (a) 2. (c), (d) 3. none
4. (b), (c) 5. (b), (d) 6. (d)
7. (b) 8. (c)

EXERCISES

1. $I^2M^{-1}L^{-3}T^4$
2. $2{\cdot}25 \times 10^3$ N
3. $4{\cdot}3 \times 10^3$ m
4. $2{\cdot}3 \times 10^{-4}$ C
5. 230 N
6. 5·9 cm from the larger charge in between the two charges
7. 34·1 cm from the larger charge on the line joining the charge in the side of the smaller charge
8. $2{\cdot}3 \times 10^{-24}$ N
9. $3{\cdot}35 \times 10^{25}$, $5{\cdot}35 \times 10^6$ C
10. $2{\cdot}56 \times 10^{25}$ N
11. 1·2 N
12. 2×10^{11}
13. $3{\cdot}05 \times 10^{-9}$ N
14. $1{\cdot}23 \times 10^{36}$
15. (a) ML^3T^{-2}, L^{-1}, N m^2, m^{-1} (b) $3{\cdot}4 \times 10^{-26}$ N m^2
16. 24·9 N at 30° with the extended sides from the charge under consideration
17. 27·5 N at 45° with the extended sides of the square from the charge under consideration
18. $8{\cdot}2 \times 10^{-8}$ N
19. $2{\cdot}18 \times 10^6$ m s^{-1}
20. $4{\cdot}95 \times 10^5$ N
21. $3{\cdot}6 \times 10^{-6}$ N
22. (a) 0·144 N
 (b) zero, 0·095 N away from the other charge
 (c) 0·986 N and (d) 0·95 m s^{-2} perpendicular to the string and going away from the other charge
23. 8·2 g, $8{\cdot}2 \times 10^{-2}$ N
24. $4{\cdot}17 \times 10^{-8}$ C
25. $\dfrac{q^2 \cot\theta}{16\pi\varepsilon_0 g l^2 \sin^2\theta}$
26. $5{\cdot}4 \times 10^{-9}$ C
27. $-(6 - 4\sqrt{2})\,q$, between q and $2q$ at a distance of $(\sqrt{2} - 1)\,d$ from q
28. $3{\cdot}6 \times 10^{-6}$ m
29. between $\pm 8{\cdot}71 \times 10^{-8}$ C
30. 27 cm from the bottom
31. $d/2\sqrt{2}$, $3{\cdot}08 \dfrac{Qq}{4\pi\varepsilon_0 d^2}$
32. (a) $\dfrac{Qqx}{2\pi\varepsilon_0\left(x^2 + \dfrac{d^2}{4}\right)^{3/2}}$ (c) $\left[\dfrac{m\pi^3\varepsilon_0 d^3}{Qq}\right]^{\frac{1}{2}}$
33. time period $= \left[\dfrac{\pi^3\varepsilon_0 m d^3}{2Qq}\right]^{\frac{1}{2}}$
34. $1{\cdot}5 \times 10^3$ N C^{-1}
35. (a) 48·3 cm from A along BA
 (b) 20 cm from A along BA and $\dfrac{20}{3}$ cm from A along AB
36. $8{\cdot}9 \times 10^{-11}$ C
37. 65·3 N C^{-1}, upward
38. zero, $2{\cdot}3 \times 10^3$ V
39. $\left[\dfrac{16\pi^3\varepsilon_0 m R^3}{Qq}\right]^{1/2}$
40. $\dfrac{Q}{2\varepsilon_0 L^2}$
41. $5{\cdot}2 \times 10^7$ N C^{-1}
42. $R/\sqrt{2}$
43. zero
44. $\dfrac{QdL}{8\pi^2\varepsilon_0 a^3}$
45. $\dfrac{q}{4\pi\varepsilon_0 d^2}$ towards the charge q
46. $8{\cdot}8 \times 10^{-4}$ N
47. $\dfrac{mu^2}{2qE}$
48. (a) 3·0 N, $9{\cdot}8 \times 10^{-3}$ N (b) $1{\cdot}63 \times 10^{-2}$ s
 (c) 49·0 m s^{-1} (d) 1·20 J
49. (a) 1·4 N making an angle of 45° with $\vec{g}$ and $\vec{E}$
 (b) straight line along the resultant force
 (c) 28 m from the starting point on the line of motion
50. 52 s
51. qE/k
52. $\sqrt{\dfrac{8\,md}{qE}}$
53. 5 V
54. 1200 volts
55. $3{\cdot}6 \times 10^{-3}$ J
56. (a) −80 V (b) −40 V (c) −120 V
57. 0·016 J, 0·008 J, 0·024 J
58. −100 V
59. 500 V
60. (a) $MT^{-3}I^{-1}$ (b) $-A\,\{\vec{i}(y+z) + \vec{j}(z+x) + \vec{k}(x+y)\}$
 (c) 35 N C^{-1}
61. 36 J
62. (a) 200 V m^{-1} making an angle 120° with the x-axis
 (b) radially outward, decreasing with distance as $E = \dfrac{6\text{ Vm}}{r^2}$.
63. $\dfrac{r\lambda}{2\varepsilon_0(r^2 + x^2)^{1/2}}$, $\dfrac{r\lambda x}{2\varepsilon_0(r^2 + x^2)^{3/2}}$

64. (a) 20 V (b) $2{\cdot}65 \times 10^{6}$ m s^{-1} (c) 0·50 cm

65. (a) $-(2{\cdot}0 \text{ V m}^{-1})\,x$
  (b) points on the plane $x = -12{\cdot}5$ m
  (c) $100 \text{ V} - (2{\cdot}0 \text{ V m}^{-1})\,x$
  (d) infinity

66. 234 J

67. 0·1 C

68. 600 m s^{-1}

69. 54 m s^{-1} for each particle

70. $8{\cdot}5 \times 10^{-26}$ Nm

71. (a) $2{\cdot}0 \times 10^{-8}$ Cm (b) 360 N C^{-1} (c) 180 N C^{-1}

72. $qd\sqrt{3}$, along the bisector of the angle at $2q$, away from the triangle

73. (a) $\frac{q}{4\pi\varepsilon_0 d^2}$ (b) $\frac{p}{4\pi\varepsilon_0 d^3}$ (c) $\frac{1}{4\pi\varepsilon_0 d^3}\sqrt{q^2 d^2 + p^2}$

74. $2\pi\sqrt{\frac{ma}{qE}}$

75. 6×10^{22}

□

CHAPTER 30

GAUSS'S LAW

Gauss's law is one of the fundamental laws of physics. It relates the electric field to the charge distribution which has produced this field. In section (30.1) we define the flux of an electric field and in the next section we discuss the concept of a solid angle. These will be needed to state and understand Gauss's law.

30.1 FLUX OF AN ELECTRIC FIELD THROUGH A SURFACE

Consider a hypothetical plane surface of area ΔS and suppose a uniform electric field $\vec{E}$ exists in the space (figure 30.1). Draw a line perpendicular to the surface and call one side of it, the positive normal to the surface. Suppose, the electric field $\vec{E}$ makes an angle θ with the positive normal.

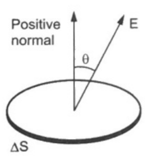

Figure 30.1

The quantity

$$\Delta\Phi = E\,\Delta S\cos\theta$$

is called *the flux of the electric field through the chosen surface*. If we draw a vector of magnitude ΔS along the positive normal, it is called the *area-vector* $\Delta\vec{S}$ corresponding to the area ΔS. One can then write

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S}.$$

Remember, the direction of the area-vector is always along the normal to the corresponding surface. If the field $\vec{E}$ is perpendicular to the surface, it is parallel to the area-vector. If $\vec{E}$ is along the positive normal, $\theta = 0$, and $\Delta\Phi = E\,\Delta S$. If it is opposite to the positive normal, $\theta = \pi$, and $\Delta\Phi = -E\,\Delta S$. If the electric field is parallel to the surface, $\theta = \pi/2$, and $\Delta\Phi = 0$.

Flux is a scalar quantity and may be added using the rules of scalar addition. Thus, if the surface ΔS has two parts ΔS_1 and ΔS_2, the flux through ΔS equals the flux through ΔS_1 plus the flux through ΔS_2. This gives us a clue to define the flux through surfaces which are not plane, as well as the flux when the field is not uniform. We divide the given surface into smaller parts so that each part is approximately plane and the variation of electric field over each part can be neglected. We calculate the flux through each part separately, using the relation $\Delta\Phi = \vec{E}\cdot\Delta\vec{S}$ and then add the flux through all the parts. Using the techniques of integration, the flux is

$$\Phi = \int \vec{E}\cdot d\vec{S}$$

where integration has to be performed over the entire surface through which the flux is required.

The surface under consideration may be a closed one, enclosing a volume, such as a spherical surface. A hemispherical surface is an open surface. A cylindrical surface is also open. A cylindrical surface plus two plane surfaces perpendicular to the axis enclose a volume and these three taken together form a closed surface. When flux through a closed surface is required, we use a small circular sign on the integration symbol;

$$\Phi = \oint \vec{E}\cdot d\vec{S}.$$

It is customary to take the outward normal as positive in this case.

Example 30.1

A square frame of edge 10 cm *is placed with its positive normal making an angle of* 60° *with a uniform electric field of* $20\ \mathrm{V\,m^{-1}}$. *Find the flux of the electric field through the surface bounded by the frame.*

Solution : The surface considered is plane and the electric field is uniform (figure 30.2). Hence, the flux is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S}$$

$$= E\,\Delta S\cos 60°$$

$$= (20 \text{ V m}^{-1})(0{\cdot}01 \text{ m}^2)\left(\frac{1}{2}\right) = 0{\cdot}1 \text{ Vm}.$$

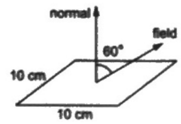

Figure 30.2

Example 30.2

A charge q is placed at the centre of a sphere. Taking outward normal as positive, find the flux of the electric field through the surface of the sphere due to the enclosed charge.

Solution :

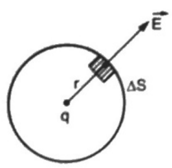

Figure 30.3

Let us take a small element ΔS on the surface of the sphere (Figure 30.3). The electric field here is radially outward and has the magnitude

$$\frac{q}{4\pi\varepsilon_0 r^2},$$

where r is the radius of the sphere. As the positive normal is also outward, $\theta = 0$ and the flux through this part is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S.$$

Summing over all the parts of the spherical surface,

$$\Phi = \sum\Delta\Phi = \frac{q}{4\pi\varepsilon_0 r^2}\sum\Delta S = \frac{q}{4\pi\varepsilon_0 r^2}4\pi r^2 = \frac{q}{\varepsilon_0}.$$

Example 30.3

A uniform electric field exists in space. Find the flux of this field through a cylindrical surface with the axis parallel to the field.

Solution :

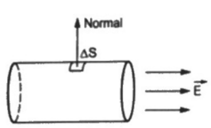

Figure 30.4

Consider figure (30.4) and take a small area ΔS on the cylindrical surface. The normal to this area will be perpendicular to the axis of the cylinder. But the electric field is parallel to the axis and hence

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos(\pi/2) = 0.$$

This is true for each small part of the cylindrical surface. Summing over the entire surface, the total flux is zero.

30.2 SOLID ANGLE

Solid angle is a generalisation of the plane angle. In figure (30.5a) we show a plane curve AB. The end points A and B are joined to the point O. We say that the curve AB subtends an *angle* or a *plane angle* at O. An angle is formed at O by the two lines OA and OB passing through O.

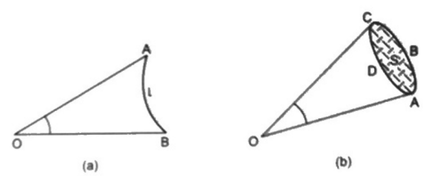

Figure 30.5

To construct a solid angle, we start with a surface S (figure 30.5b) and join all the points on the periphery such as A, B, C, D, etc., with the given point O. We then say that a *solid angle* is formed at O and that the surface S has subtended the solid angle. The solid angle is formed by the lines joining the points on the periphery with O. The whole figure looks like a cone. As a typical example, think of the paper containers used by Moongfaliwalas.

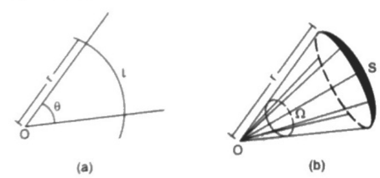

Figure 30.6

How do we measure a solid angle ? Let us consider how do we measure a plane angle. See figure (30.6a). We draw a circle of any radius r with the centre at O and measure the length l of the arc intercepted by the angle. The angle θ is then defined as $\theta = l/r$. In order to measure a solid angle at the point O (figure 30.6b), we draw a sphere of any radius r with O as the centre and measure the area S of the part of the sphere intercepted by the cone. The solid angle Ω is then defined as

$$\Omega = S/r^2.$$

Note that this definition makes the solid angle a dimensionless quantity. It is independent of the radius of the sphere drawn.

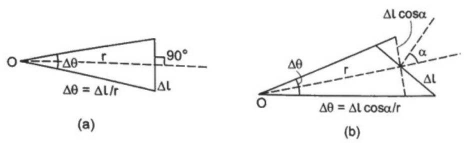

Figure 30.7

Next, consider a plane angle subtended at a point O by a small line segment Δl (figure 30.7a). Suppose, the line joining O to the middle point of Δl is perpendicular to Δl. As the segment is small, we can approximately write

$$\Delta\theta = \frac{\Delta l}{r}.$$

As Δl gets smaller, the approximation becomes better. Now suppose, the line joining O to Δl is not perpendicluar to Δl (figure 30.7b). Suppose, this line makes an angle α with the perpendicular to Δl. The angle subtended by Δl at O is

$$\Delta\theta = \frac{\Delta l \cos\alpha}{r}.$$

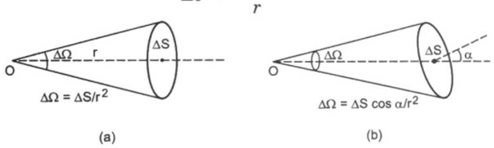

Figure 30.8

Similarly, if a small plane area ΔS (figure 30.8a) subtends a solid angle $\Delta\Omega$ at O in such a way that the line joining O to ΔS is normal to ΔS, we can write $\Delta\Omega = \Delta S / r^2$. But if the line joining O to ΔS makes an angle α with the normal to ΔS (figure 30.8b), we should write

$$\Delta\Omega = \frac{\Delta S \cos\alpha}{r^2}.$$

A complete circle subtends an angle

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi$$

at the centre. In fact, any closed curve subtends an angle 2π at any of the internal points. Similarly, a complete sphere subtends a solid angle

$$\Omega = \frac{S}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

at the centre. Also, any closed surface subtends a solid angle 4π at any internal point.

How much is the angle subtended by a closed plane curve at an external point ?

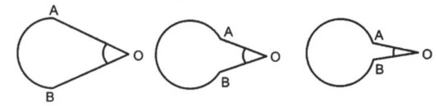

Figure 30.9

See figure (30.9). As we gradually close the curve, the angle finally diminishes to zero. A closed curve subtends zero angle at an external point. Similarly, a closed surface subtends zero solid angle at an external point.

30.3 GAUSS'S LAW AND ITS DERIVATION FROM COULOMB'S LAW

The statement of the Gauss's law may be written as follows:

The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by ε_0. In symbols,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} \qquad \dots (30.1)$$

where q_{in} is the net charge enclosed by the surface through which the flux is calculated.

It should be carefully noted that the electric field on the left-hand side of equation (30.1) is the resultant electric field due to all the charges existing in the space, whereas, the charge appearing on the right-hand side includes only those which are inside the closed surface.

Gauss's law is taken as a fundamental law of nature, a law whose validity is shown by experiments. However, historically Coulomb's law was discovered before Gauss's law and it is possible to derive Gauss's law from Coulomb's law.

Proof of Gauss's Law (Assuming Coulomb's Law)

Flux due to an internal charge

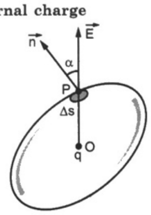

Figure 30.10

Suppose a charge q is placed at a point O inside a "closed" surface (figure 30.10). Take a point P on the surface and consider a small area ΔS on the surface around P. Let $OP = r$. The electric field at P due to the charge q is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

along the line OP. Suppose this line OP makes an angle α with the outward normal to ΔS. The flux of the electric field through ΔS is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\alpha$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\Delta S\cos\alpha$$

$$= \frac{q}{4\pi\varepsilon_0}\Delta\Omega$$

where $\Delta\Omega = \frac{\Delta S\cos\alpha}{r^2}$ is the solid angle subtended by ΔS at O. The flux through the entire surface is

$$\Phi = \Sigma\frac{q}{4\pi\varepsilon_0}\Delta\Omega = \frac{q}{4\pi\varepsilon_0}\Sigma\Delta\Omega.$$

The sum over $\Delta\Omega$ is the total solid angle subtended by the closed surface at the internal point O and hence is equal to 4π.

The total flux of the electric field due to the internal charge q through the closed surface is, therefore,

$$\Phi = \frac{q}{4\pi\varepsilon_0}4\pi = \frac{q}{\varepsilon_0}\cdot \qquad \ldots \text{(i)}$$

Flux due to an external charge

Now, suppose a charge q is placed at a point O outside the closed surface. The flux of the electric field due to q through the small area ΔS is again

$$\Delta\Phi = \frac{q}{4\pi\varepsilon_0}\frac{\Delta S\cos\alpha}{r^2} = \frac{q}{4\pi\varepsilon_0}\Delta\Omega.$$

When we sum over all the small area elements of the closed surface we get $\Sigma\Delta\Omega = 0$ as this is the total solid angle subtended by the closed surface at an external point. Hence,

$$\Phi = 0. \qquad \ldots \text{(ii)}$$

Flux due to a combination of charges

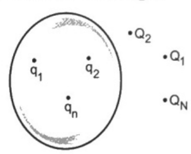

Figure 30.11

Finally, consider a general situation (figure 30.11) where charges q_1, q_2, ..., q_n are inside a closed surface and charges Q_1, Q_2, ..., Q_N are outside it. The resultant electric field at any point is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n + \vec{E'}_1 + \vec{E'}_2 + \ldots + \vec{E'}_N$$

where E_i and E'_i are the fields due to q_i and Q_i respectively. Thus, the flux of the resultant electric field through the closed surface is

$$\Phi = \oint\vec{E}\cdot d\vec{S} = \oint\vec{E}_1\cdot d\vec{S} + \oint\vec{E}_2\cdot d\vec{S} + \ldots + \oint\vec{E}_n\cdot d\vec{S}$$

$$+ \oint\vec{E'}_1\cdot d\vec{S} + \oint\vec{E'}_2\cdot d\vec{S} + \ldots + \oint\vec{E'}_N\cdot d\vec{S}. \qquad \ldots \text{(iii)}$$

Now, $\oint\vec{E}_i\cdot d\vec{S}$ is the flux of the electric field due to the charge q_i only. As this charge is inside the closed surface, from (i), it is equal to q_i/ε_0. Also, $\oint\vec{E'}_i\cdot d\vec{S}$ is the flux of the electric field due to the charge Q_i which is outside the closed surface. This flux is, therefore, zero from (ii). Using these results in (iii),

$$\Phi = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \ldots + \frac{q_n}{\varepsilon_0} + 0 + \ldots + 0$$

or, $$\Phi = \frac{1}{\varepsilon_0}\Sigma q_i$$

or, $$\oint\vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}.$$

This completes the derivation of Gauss's law (equation 30.1).

We once again emphasise that the electric field appearing in the Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charge q_{in} appearing in the law is only the charge contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied, is sometimes called the *Gaussian surface*.

Example 30.4

A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring (figure 30.12). Find the flux of the electric field through the surface of the sphere.

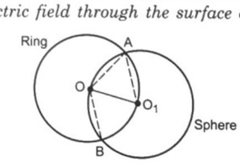

Figure 30.12

Solution : From the geometry of the figure, $OA = OO_1$ and $O_1A = O_1O$. Thus, OAO_1 is an equilateral triangle. Hence $\angle AOO_1 = 60°$ or $\angle AOB = 120°$.

The arc AO_1B of the ring subtends an angle 120° at the centre O. Thus, one third of the ring is inside the sphere.

The charge enclosed by the sphere $= \frac{Q}{3}$. From Gauss's law, the flux of the electric field through the surface of the sphere is $\frac{Q}{3\varepsilon_0}$.

30.4 APPLICATIONS OF GAUSS'S LAW

(A) Charged Conductor

As discussed earlier, an electric conductor has a large number of free electrons and when placed in an electric field, these electrons redistribute themselves to make the field zero at all the points inside the conductor.

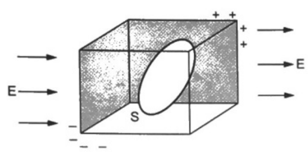

Figure 30.13

Consider a charged conductor placed in an electric field (figure 30.13). We assume that the redistribution of free electrons (if any) is complete. Draw a closed surface S going through the interior points only. As the electric field at all the points of this surface is zero (they are interior points), the flux $\oint \vec{E} \cdot d\vec{S}$ is also zero. But from Gauss's law, this equals the charge contained inside the surface divided by ε_0. Hence, the charge enclosed by the surface is zero. This shows that any volume completely inside a conductor is electrically neutral. If a charge is injected anywhere in the conductor, it must come over to the surface of the conductor so that the interior is always charge free. Also, if the conductor has a cavity, the charge must come over to the outer surface.

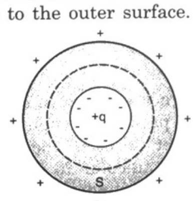

Figure 30.14

However, if a charge is placed within the cavity as in figure (30.14), the inner surface cannot be charge free. Taking the Gaussian surface S as shown, $\vec{E} = 0$ at all the points of this surface and hence $\oint \vec{E} \cdot d\vec{S} = 0$. This ensures that the charge contained in S is zero and if a charge $+q$ is placed in the cavity, there must be a charge $-q$ on the inner surface of the conductor. If the conductor is neutral, i.e., no charge is placed on it, a charge $+q$ will appear on the outer surface.

If there is a cavity in the conductor and no charge is placed in the cavity, the electric field at all the points in the cavity is zero. This can be proved using a little more advanced mathematics.

(B) Electric Field due to a Uniformly Charged Sphere

Suppose a total charge Q is uniformly distributed in a spherical volume of radius R and we are required to find the electric field at a point P which is at a distance r from the centre of the charge distribution.

Field at an outside point

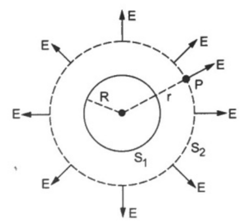

Figure 30.15

For a point P outside the charge distribution (figure 30.15), we have $r > R$. Draw a spherical surface passing through the point P and concentric with the charge distribution. Take this to be the Gaussian surface. The electric field is radial by symmetry and if Q is positive the field is outward. Also, its magnitude at all the points of the Gaussian surface must be equal. Let this magnitude be E. This is also the magnitude of the field at P. As the field $\vec{E}$ is normal to the surface element everywhere, $\vec{E} \cdot d\vec{S} = E\,dS$ for each element. The flux of the electric field through this closed surface is

$$\Phi = \oint \vec{E} \cdot d\vec{S}$$
$$= \oint E\,dS = E \oint dS = E\,4\pi r^2.$$

This should be equal to the charge contained inside the Gaussian surface divided by ε_0. As the entire charge Q is contained inside the Gaussian surface, we get

$$E\,4\pi r^2 = Q/\varepsilon_0$$

or, $$E = \frac{Q}{4\pi\varepsilon_0 r^2}. \qquad \ldots (30.2)$$

The electric field due to a uniformly charged sphere at a point outside it, is identical with the field due to an equal point charge placed at the centre.

Notice the use of the argument of symmetry. All the points of the sphere through P are equivalent. No point on this surface has any special property which a different point does not have. That is why we could say that the field has the same magnitude E at all these points. Also, the field is radial at all the points. We have wisely chosen the Gaussian surface which has these properties. We could then easily evaluate the flux $\oint \vec{E} \cdot d\vec{S} = E\, 4\pi r^2$.

Field at an internal point

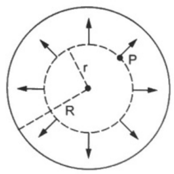

Figure 30.16

Suppose, we wish to find the electric field at a point P inside the spherical charge distribution (figure 30.16). We draw a spherical surface passing through P and concentric with the given charge distribution. The radius of this sphere will be r. All the points of this sphere are equivalent. By symmetry, the field is radial at all the points of this surface and has a constant magnitude E. The flux through this spherical surface is

$$\Phi = \oint \vec{E} \cdot d\vec{S}$$

$$= \oint E\, dS = E \oint dS = E\, 4\pi r^2. \qquad \dots \text{(i)}$$

Let us now calculate the total charge contained inside this spherical surface. As the charge is uniformly distributed within the given spherical volume, the charge per unit volume is $\frac{Q}{\frac{4}{3}\pi R^3}$. The volume enclosed by the Gaussian surface, through which the flux is calculated, is $\frac{4}{3}\pi r^3$. Hence, the charge enclosed is

$$\frac{Q}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi r^3 = \frac{Q r^3}{R^3}.$$

Using Gauss's law and (i),

$$E\, 4\pi r^2 = \frac{Q r^3}{\varepsilon_0 R^3}$$

or,
$$E = \frac{Qr}{4\pi\varepsilon_0 R^3}. \qquad \dots (30.3)$$

The electric field due to a uniformly charged sphere at an internal point is $\frac{Qr}{4\pi\varepsilon_0 R^3}$ in radial direction.

At the centre, $r = 0$ and hence $E = 0$. This is clear from the symmetry arguments as well. At the centre, all directions are equivalent. If the electric field is not zero, what can be its direction ? You cannot choose a unique direction. The field has to be zero. It is proportional to the distance r from the centre for the internal points. Equations (30.2) and (30.3) give the same value of the field at the surface, where $r = R$.

(C) Electric Field due to a Linear

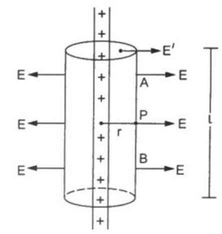

Figure 30.17

Charge Distribution

Consider a long line charge with a linear charge density (that is, charge per unit length) λ. We have to calculate the electric field at a point P which is at a distance r from the line charge (figure 30.17). What can be the direction of the electric field at P? Can it be along PA ? If yes, then why not along PB ? PA and PB are equivalent to each other. In fact, the only unique direction through P is along the perpendicular to the line charge. The electric field must be along this direction. If the charge is positive, the field will be outward.

Now, we construct a Gaussian surface. We draw a cylinder of length l passing through P and coaxial with the line charge. Let us close the cylinder with two plane surfaces perpendicular to the line charge. The curved surface of the cylinder together with the two plane parallel surfaces constitutes a closed surface as shown in figure (30.17). We use this surface as the Gaussian surface.

All the points on the curved part of this Gaussian surface are at the same perpendicular distance from the line charge. All these points are equivalent. The electric field at all these points will have the same magnitude E as that at P. Also, the direction of the field at any point on the curved surface is normal to the line and hence normal to the cylindrical surface element there. The flux through the curved part is, therefore,

$$\int \vec{E} \cdot d\vec{S} = \int E\, dS = E \int dS = E\, 2\pi r l.$$

Now, consider the flat parts of the Gaussian surface, that is, the lids of the cylinder. The electric field at any point is perpendicular to the line charge. The normal to any element on these plane surfaces is parallel to the line charge. Hence, the field and the area-vector make an angle of 90° with each other so that $\int \vec{E} \cdot d\vec{S} = 0$ on these parts. The total flux through the closed Gaussian surface is, therefore,

$$\oint \vec{E} \cdot d\vec{S} = E\, 2\pi r l. \qquad \ldots \text{(i)}$$

The charge enclosed in the Gaussian surface is λl as a length l of the line charge is inside the closed surface. Using Gauss's law and (i),

$$E\, 2\pi r l = (\lambda l)/\varepsilon_0$$

or,
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}. \qquad \ldots (30.4)$$

This is the field at a distance r from the line. It is directed away from the line if the charge is positive and towards the line if the charge is negative.

(D) Electric Field due to a Plane Sheet of Charge

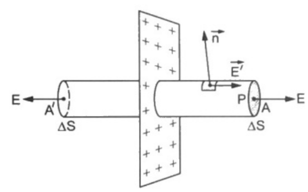

Figure 30.18

Consider a large plane sheet of charge with surface charge density (charge per unit area) σ. We have to find the electric field E at a point P in front of the sheet (Figure 30.18). What can be the direction of the electric field at P? The only unique direction we can identify is along the perpendicular to the plane. The field must be along this line. If the charge is positive, the field is away from the plane. Same is true for all points near the plane provided the sheet is large and the charge density is uniform. If these conditions are not fulfilled, the argument of symmetry may fail.

To calculate the field E at P, we choose a Gaussian surface as follows. Draw a plane surface A passing through P and parallel to the charge sheet. Draw a cylinder with this surface as a cross section and extend it to the other side of the plane charge sheet. Close the cylinder on the other side by a cross section A' such that A and A' are equidistant from the sheet. Also, A and A' have equal area ΔS. The cylinder together with its cross sectional areas forms a closed surface and we apply Gauss's law to this surface.

The electric field at all the points of A has the same magnitude E. The direction is along the positive normal to A. Thus, the flux of the electric field through A is

$$\Phi = \vec{E} \cdot \Delta\vec{S} = E\,\Delta S.$$

Note that the two sides of the charge sheet are equivalent in all respect. As A and A' are equidistant from the sheet, the electric field at any point of A' is also equal to E and is along the positive normal (that is, the outward normal) to A'. Hence, the flux of the electric field through A' is also $E\,\Delta S$. At the points on the curved surface, the field and the outward normal make an angle of 90° with each other and hence $\vec{E} \cdot \Delta\vec{S} = 0$. The total flux through the closed surface is

$$\Phi = \oint \vec{E} \cdot d\vec{S} = E\,\Delta S + E\,\Delta S + 0 = 2\,E\,\Delta S.$$

The area of the sheet enclosed in the cylinder is ΔS. The charge contained in the cylinder is, therefore, $\sigma\,\Delta S$. Hence from Gauss's law,

$$2\,E\Delta S = \frac{\sigma\,\Delta S}{\varepsilon_0}$$

or,
$$E = \frac{\sigma}{2\varepsilon_0}. \qquad \ldots (30.5)$$

We see that the field is uniform and does not depend on the distance from the charge sheet. This is true as long as the sheet is large as compared to its distance from P.

(E) Electric Field near a Charged Conducting Surface

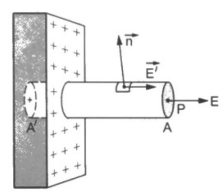

Figure 30.19

In figure (30.19), we show a large, plane conducting sheet. The surface on right has a uniform surface charge density σ. We have to find the electric field at a point P near this surface and outside the conductor. As we know, the conducting surface is an equipotential surface and the electric field near the surface is perpendicular to the surface. For positive charge on the surface, the field is away from the surface. To find the electric field, we construct a Gaussian surface as follows. Take a small plane surface A passing through the point P and parallel to the given conducting surface. Draw a cylinder with A as a cross section and terminate it with another plane surface A' parallel to A and lying in the interior of the conducting sheet.

If the electric field at P is E, the flux through the plane surface A is

$$\Phi = E\,\Delta S,$$

where ΔS is the area of A. At the curved parts of the cylinder, the electric field is either zero (inside the conductor) or is parallel to the curved surface (outside the conductor). The field $\vec{E}$ and the area-vector $\Delta\vec{S}$ are perpendicular to each other making $\vec{E}\cdot\Delta\vec{S} = 0$ at these outside points. The flux on the curved part is, therefore, zero. Also, the flux on A' is zero as the field inside the conductor is zero.

The total flux through the Gaussian surface constructed is, therefore, $E\,\Delta S$. The charge enclosed inside the closed surface is $\sigma\,\Delta S$ and hence from Gauss's law,

$$E\,\Delta S = \frac{\sigma\,\Delta S}{\varepsilon_0}$$

or, $$E = \frac{\sigma}{\varepsilon_0}. \qquad \dots (30.6)$$

The electric field near a charged conducting surface is σ/ε_0 *and it is normal to the surface.*

Compare this result with the field due to a plane sheet of charge of surface density σ (equation 30.5). The field E had a magnitude $\sigma/(2\varepsilon_0)$ in that case. Apparently, for a conductor also we have a plane sheet of charge of the same density but the field derived is σ/ε_0 and not $\sigma/(2\varepsilon_0)$. Consider the conducting sheet shown in figure (30.20). In fact, the field due to the charge on the right surface is indeed $\frac{\sigma}{2\varepsilon_0}$ at P. Where does the extra $\frac{\sigma}{2\varepsilon_0}$ field come from ?

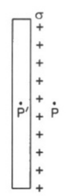

Figure 30.20

Consider a point P' inside the sheet as shown in figure (30.20). The electric field at this point due to the charge sheet on the right surface of the conductor is $\sigma/(2\varepsilon_0)$ towards left. But P' is a point inside the conductor and hence the field here must be zero. This means that the charge distribution shown in figure (30.20) is not complete as it does not ensure zero field inside the conductor. Apart from the surface charge of density σ shown in the figure, there must be other charges nearby. These other nearby charges must create a field at P' towards right so that the resultant field at P' becomes zero. These other charges also create a field $\sigma/(2\varepsilon_0)$ towards right at P which adds to the field due to the surface charge shown in figure (30.20). Thus, the field at P becomes

$$\frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

towards right. As Gauss's law gives the net field, equation (30.6) gives $E = \sigma/\varepsilon_0$ which is the actual field due to all the charges and not the field due to the surface charge only. As examples, we give in figure (30.21) some of the possible complete charge distributions which ensure zero field inside the conductor.

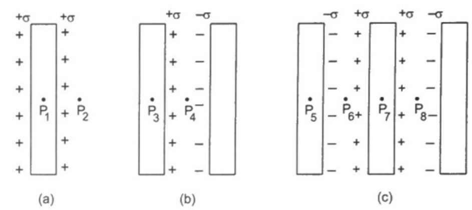

Figure 30.21

Calculate the electric field at the points indicated using the formula $E = \sigma/(2\varepsilon_0)$ for each charged surface. Verify that the field at each of the points P_1, P_3, P_5 and P_7 is zero and at each of the points P_2, P_4, P_6 and P_8 is σ/ε_0.

Note that electric field changes discontinuously at the surface of a conductor. Just outside the conductor it is σ/ε_0 and inside the conductor it is zero. In fact, the field gradually decreases from σ/ε_0 to zero in a small thickness of about 4–5 atomic layers at the surface. When we say 'the surface of a conductor' we actually mean this small thickness.

30.5 SPHERICAL CHARGE DISTRIBUTIONS

We have seen that the electric field due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre. Similar result was obtained for the gravitational field due to a uniform sphere. This similarity is expected because the Coulomb force

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0\, r^2}$$

and the gravitational force

$$F = \frac{Gm_1m_2}{r^2}$$

have similar mathematical structure.

Many of the results derived for gravitational field, potential and potential energy may, therefore, be used for the corresponding electrical quantities. We state some of the useful results for a spherical charge distribution of radius R.

(a) The electric field due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre of the shell, $E = Q/(4\pi\varepsilon_0 r^2)$.

(b) The electric field due to a uniformly charged thin spherical shell at an internal point is zero.

(c) The electric field due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre of the sphere.

(d) The electric field due to a uniformly charged sphere at an internal point is proportional to the distance of the point from the centre of the sphere. Thus, it is zero at the centre and increases linearly as one moves out towards the surface.

(e) The electric potential due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre, $V = Q/(4\pi\varepsilon_0 r)$.

(f) The electric potential due to a uniformly charged, thin spherical shell at an internal point is the same everywhere and is equal to that at the surface, $V = Q/(4\pi\varepsilon_0 R)$.

(g) The electric potential due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre, $V = Q/(4\pi\varepsilon_0 r)$.

Electric Potential Energy of a Uniformly Charged Sphere

Consider a uniformly charged sphere of radius R having a total charge Q. The electric potential energy of this sphere is equal to the work done in bringing the charges from infinity to assemble the sphere. Let us assume that at some instant, charge is assembled up to a radius x. In the next step, we bring some charge from infinity and put it on this sphere to increase the radius from x to $x + dx$. The entire sphere is assembled as x varies from 0 to R.

The charge density is

$$\rho = \frac{3Q}{4\pi R^3}.$$

When the radius of the sphere is x, the charge contained in it is,

$$q = \frac{4}{3}\pi x^3 \rho = \frac{Q}{R^3}x^3.$$

The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 x} = \frac{Q}{4\pi\varepsilon_0 R^3}x^2.$$

The charge needed to increase the radius from x to $x + dx$ is

$$dq = (4\pi x^2 dx)\rho = \frac{3Q}{R^3}x^2 dx.$$

The work done in bringing the charge dq from infinity to the surface of the sphere of radius x is

$$dW = V(dq) = \frac{3Q^2}{4\pi\varepsilon_0 R^6}x^4 dx.$$

The total work done in assembling the charged sphere of radius R is

$$W = \frac{3Q^2}{4\pi\varepsilon_0 R^6}\int_0^R x^4 dx = \frac{3Q^2}{20\pi\varepsilon_0 R}.$$

This is the electric potential energy of the charged sphere.

Electric Potential Energy of a Uniformly Charged, Thin Spherical Shell

Consider a uniformly charged, thin spherical shell of radius R having a total charge Q. The electric potential energy is equal to the work done in bringing charges from infinity and put them on the shell. Suppose at some instant, a charge q is placed on the shell. The potential at the surface is

$$V = \frac{q}{4\pi\varepsilon_0 R}.$$

The work done in bringing a charge dq from infinity to this shell is

$$dW = V(dq) = \frac{q\,dq}{4\pi\varepsilon_0 R}.$$

The total work done in assembling the charge on the shell is

$$W = \int_0^Q \frac{q\,dq}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

This is the electric potential energy of the charged spherical shell.

30.6 EARTHING A CONDUCTOR

The earth is a good conductor of electricity. If we assume that the earth is uncharged, its potential will be zero. In fact, the earth's surface has a negative charge of about 1 nC m^{-2} and hence is at a constant potential V. All conductors which are not given any external charge, are also very nearly at the same potential. In turns out that for many practical calculations, we can ignore the charge on the earth. The potential of the earth can then be taken as the same as that of a point far away from all charges, i.e., at infinity. So, the potential of the earth is often taken to be zero. Also, if a small quantity of charge is given to the earth or is taken away from it, the potential does not change by any appreciable extent. This is because of the large size of the earth.

If a conductor is connected to the earth, the potential of the conductor becomes equal to that of the earth, i.e., zero. If the conductor was at some other potential, charges will flow from it to the earth or from the earth to it to bring its potential to zero.

When a conductor is connected to the earth, the conductor is said to be *earthed* or *grounded*. Figure (30.22a) shows the symbol for earthing.

Suppose a spherical conductor of radius R is given a charge Q. The charge will be distributed uniformly on the surface. So it is equivalent to a uniformly charged, thin spherical shell. Its potential will, therefore, become $Q/(4\pi\varepsilon_0 R)$. If this conductor is connected to the earth, the charge Q will be transferred to the earth so that the potential will become zero.

Next suppose, a charge $+Q$ is placed at the centre of a spherical conducting shell. A charge $-Q$ will appear on its inner surface and $+Q$ on its outer surface (figure 30.22b). The potential of the sphere due to the charge at the centre and that due to the charge at the inner surface are $\frac{Q}{4\pi\varepsilon_0 R}$ and $\frac{-Q}{4\pi\varepsilon_0 R}$ respectively. The potential due to the charge on the outer surface is $\frac{Q}{4\pi\varepsilon_0 R}\cdot$ The net potential of the sphere is, therefore, $\frac{Q}{4\pi\varepsilon_0 R}\cdot$ If this sphere is now connected to the earth (figure 30.22c), the charge Q on the outer surface flows to the earth and the potential of the sphere becomes zero.

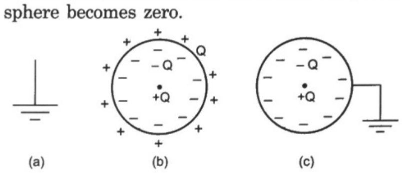

Figure 30.22

Earthing a conductor is a technical job. A thick metal plate is buried deep into the earth and wires are drawn from this plate. The electric wiring in our houses has three wires: live, neutral and earth. The live and neutral wires carry electric currents which come from the power station. The earth wire is connected to the metal plate buried in the earth. The metallic bodies of electric appliances such as electric iron, refrigerator, etc. are connected to the earth wire. This ensures that the metallic body remains at zero potential while an appliance is being used. If by any fault, the live wire touches the metallic body, charge flows to the earth and the potential of the metallic body remains zero. If it is not connected to the earth, the user may get an electric shock.

Worked Out Examples

1. *A uniform electric field of magnitude* $E = 100\ \mathrm{N\ C^{-1}}$ *exists in the space in x-direction. Calculate the flux of this field through a plane square area of edge* 10 cm *placed in the y–z plane. Take the normal along the positive x-axis to be positive.*

Solution : The flux $\Phi = \int E\cos\theta\, dS$. As the normal to the area points along the electric field, $\theta = 0$. Also, E is uniform, so

$$\Phi = E\,\Delta S$$

$$= (100\ \mathrm{N\ C^{-1}})\ (0{\cdot}10\ \mathrm{m})^2 = 1{\cdot}0\ \mathrm{N\ m^2 C^{-1}}.$$

2. *A large plane charge sheet having surface charge density* $\sigma = 2{\cdot}0 \times 10^{-6}\ \mathrm{C\ m^{-2}}$ *lies in the x–y plane. Find the flux of the electric field through a circular area of radius* 1 cm *lying completely in the region where x, y, z are all positive and with its normal making an angle of* 60° *with the z-axis.*

Solution : The electric field near the plane charge sheet is $E = \sigma/2\varepsilon_0$ in the direction away from the sheet. At the given area, the field is along the z-axis.

The area $= \pi r^2 = 3{\cdot}14 \times 1\ \mathrm{cm}^2 = 3{\cdot}14 \times 10^{-4}\ \mathrm{m}^2$.

The angle between the normal to the area and the field is 60°.

Hence, the flux $= \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\theta = \frac{\sigma}{2\varepsilon_0}\pi r^2\cos 60°$

$$= \frac{2{\cdot}0 \times 10^{-6}\ \mathrm{C\ m^{-2}}}{2 \times 8{\cdot}85 \times 10^{-12}\ \mathrm{C^2\ N\ m^{-2}}} \times (3{\cdot}14 \times 10^{-4}\ \mathrm{m^2})\frac{1}{2}$$

$$= 17{\cdot}5\ \mathrm{N\ m^2\ C^{-1}}.$$

3. *A charge of* $4 \times 10^{-8}\ \mathrm{C}$ *is distributed uniformly on the surface of a sphere of radius* 1 cm. *It is covered by a concentric, hollow conducting sphere of radius* 5 cm. *(a) Find the electric field at a point* 2 cm *away from the centre. (b) A charge of* $6 \times 10^{-8}\ \mathrm{C}$ *is placed on the hollow sphere. Find the surface charge density on the outer surface of the hollow sphere.*

Solution **:**

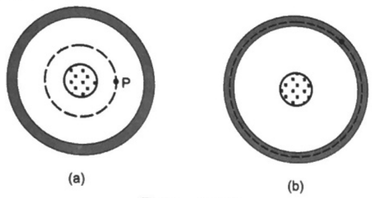

Figure 30-W1

(a) Let us consider figure (30-W1a). Suppose, we have to find the field at the point P. Draw a concentric spherical surface through P. All the points on this surface are equivalent and by symmetry, the field at all these points will be equal in magnitude and radial in direction.

The flux through this surface $= \oint \vec{E} \cdot d\vec{S}$

$$= \oint E\, dS = E \oint dS$$

$$= 4\pi x^2 E,$$

where $\quad x = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}.$

From Gauss's law, this flux is equal to the charge q contained inside the surface divided by ε_0. Thus,

$$4\pi x^2 E = q/\varepsilon_0$$

or,

$$E = \frac{q}{4\pi\varepsilon_0 x^2}$$

$$= (9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}) \times \frac{4 \times 10^{-8} \text{ C}}{4 \times 10^{-4} \text{ m}^2}$$

$$= 9 \times 10^5 \text{ N C}^{-1}.$$

(b) See figure (30-W1b). Take a Gaussian surface through the material of the hollow sphere. As the electric field in a conducting material is zero, the flux $\oint \vec{E} \cdot d\vec{S}$ through this Gaussian surface is zero. Using Gauss's law, the total charge enclosed must be zero. Hence, the charge on the inner surface of the hollow sphere is -4×10^{-8} C. But the total charge given to this hollow sphere is 6×10^{-8} C. Hence, the charge on the outer surface will be 10×10^{-8} C.

4. *Figure (30-W2a) shows three concentric thin spherical shells A, B and C of radii a, b and c respectively. The shells A and C are given charges q and –q respectively and the shell B is earthed. Find the charges appearing on the surfaces of B and C.*

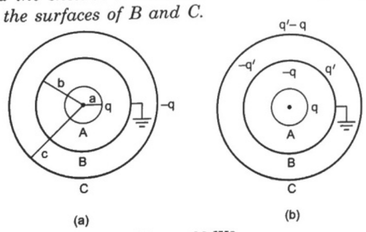

Figure 30-W2

Solution **:**

As shown in the previous worked out example, the inner surface of B must have a charge $-q$ from the Gauss's law. Suppose, the outer surface of B has a charge q'. The inner surface of C must have a charge $-q'$ from the Gauss's law. As the net charge on C must be $-q$, its outer surface should have a charge $q' - q$. The charge distribution is shown in figure (30-W2b).

The potential at B due to the charge q on A

$$= \frac{q}{4\pi\varepsilon_0 b},$$

due to the charge $-q$ on the inner surface of B

$$= \frac{-q}{4\pi\varepsilon_0 b},$$

due to the charge q' on the outer surface of B

$$= \frac{q'}{4\pi\varepsilon_0 b},$$

due to the charge $-q'$, on the inner surface of C

$$= \frac{-q'}{4\pi\varepsilon_0 c}$$

and due to the charge $q' - q$ on the outer surface of C

$$= \frac{q' - q}{4\pi\varepsilon_0 c}.$$

The net potential is

$$V_B = \frac{q'}{4\pi\varepsilon_0 b} - \frac{q}{4\pi\varepsilon_0 c}.$$

This should be zero as the shell B is earthed. Thus,

$$q' = \frac{b}{c} q.$$

The charges on various surfaces are as shown in figure (30-W3).

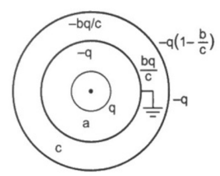

Figure 30-W3

5. *An electric dipole consists of charges* $\pm 2{\cdot}0 \times 10^{-8}$ C *separated by a distance of* $2{\cdot}0 \times 10^{-3}$ m. *It is placed near a long line charge of linear charge density* $4{\cdot}0 \times 10^{-4}$ C m^{-1} *as shown in figure (30-W4), such that the negative charge is at a distance of* $2{\cdot}0$ cm *from the line charge. Find the force acting on the dipole.*

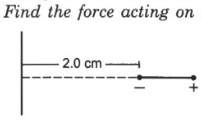

Figure 30-W4

Solution **:** The electric field at a distance r from the line charge of linear density λ is given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

Hence, the field at the negative charge is

$$E_1 = \frac{(4{\cdot}0 \times 10^{-4}\ \text{C m}^{-1})\,(2 \times 9 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2})}{0{\cdot}02\ \text{m}}$$

$$= 3{\cdot}6 \times 10^{8}\ \text{N C}^{-1}.$$

The force on the negative charge is

$$F_1 = (3{\cdot}6 \times 10^{8}\ \text{N C}^{-1})\,(2{\cdot}0 \times 10^{-8}\ \text{C}) = 7{\cdot}2\ \text{N}$$

towards the line charge.

Similarly, the field at the positive charge, i.e., at $r = 0{\cdot}022$ m is

$$E_2 = 3{\cdot}3 \times 10^{8}\ \text{N C}^{-1}.$$

The force on the positive charge is

$$F_2 = (3{\cdot}3 \times 10^{8}\ \text{N C}^{-1}) \times (2{\cdot}0 \times 10^{-8}\ \text{C})$$

$$= 6{\cdot}6\ \text{N away from the line charge}.$$

Hence, the net force on the dipole $= (7{\cdot}2 - 6{\cdot}6)$ N

$= 0{\cdot}6$ N towards the line charge.

6. *The electric field in a region is radially outward with magnitude $E = Ar$. Find the charge contained in a sphere of radius a centred at the origin. Take $A = 100\ \text{V m}^{-2}$ and $a = 20{\cdot}0$ cm.*

Solution : The electric field at the surface of the sphere is Aa and being radial it is along the outward normal. The flux of the electric field is, therefore,

$$\Phi = \oint E\, dS \cos\theta = Aa(4\pi a^2).$$

The charge contained in the sphere is, from Gauss's law,

$$Q_{inside} = \varepsilon_0 \Phi = 4\pi\varepsilon_0 Aa^3$$

$$= \left(\frac{1}{9 \times 10^{9}}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}\right)(100\ \text{V m}^{-2})\ (0{\cdot}20\ \text{m})^3$$

$$= 8{\cdot}89 \times 10^{-11}\ \text{C}.$$

7. *A particle of mass 5×10^{-6} g is kept over a large horizontal sheet of charge of density $4{\cdot}0 \times 10^{-6}\ \text{C m}^{-2}$ (figure 30-W5). What charge should be given to this particle so that if released, it does not fall down ? How many electrons are to be removed to give this charge ? How much mass is decreased due to the removal of these electrons ?*

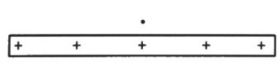

Figure 30-W5

Solution : The electric field in front of the sheet is

$$E = \frac{\sigma}{2\varepsilon_0} = \frac{4{\cdot}0 \times 10^{-6}\ \text{C m}^{-2}}{2 \times 8{\cdot}85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}}$$

$$= 2{\cdot}26 \times 10^{5}\ \text{N C}^{-1}.$$

If a charge q is given to the particle, the electric force qE acts in the upward direction. It will balance the weight of the particle if

$$q \times 2{\cdot}26 \times 10^{5}\ \text{N C}^{-1} = 5 \times 10^{-9}\ \text{kg} \times 9{\cdot}8\ \text{m s}^{-2}$$

or,
$$q = \frac{4{\cdot}9 \times 10^{-8}}{2{\cdot}26 \times 10^{5}}\ \text{C}$$

$$= 2{\cdot}21 \times 10^{-13}\ \text{C}.$$

The charge on one electron is $1{\cdot}6 \times 10^{-19}$ C. The number of electrons to be removed

$$= \frac{2{\cdot}21 \times 10^{-13}\ \text{C}}{1{\cdot}6 \times 10^{-19}\ \text{C}} = 1{\cdot}4 \times 10^{6}.$$

Mass decreased due to the removal of these electrons

$$= 1{\cdot}4 \times 10^{6} \times 9{\cdot}1 \times 10^{-31}\ \text{kg}$$

$$= 1{\cdot}3 \times 10^{-24}\ \text{kg}.$$

8. *Two conducting plates A and B are placed parallel to each other. A is given a charge Q_1 and B a charge Q_2. Find the distribution of charges on the four surfaces.*

Solution : Consider a Gaussian surface as shown in figure (30-W6a). Two faces of this closed surface lie completely inside the conductor where the electric field is zero. The flux through these faces is, therefore, zero. The other parts of the closed surface which are outside the conductor are parallel to the electric field and hence the flux on these parts is also zero. The total flux of the electric field through the closed surface is, therefore, zero. From Gauss's law, the total charge inside this closed surface should be zero. The charge on the inner surface of A should be equal and opposite to that on the inner surface of B.

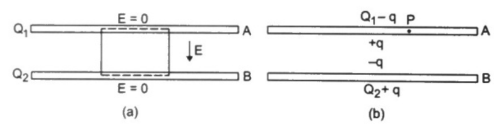

Figure 30-W6

The distribution should be like the one shown in figure (30-W6b). To find the value of q, consider the field at a point P inside the plate A. Suppose, the surface area of the plate (one side) is A. Using the equation $E = \sigma/(2\varepsilon_0)$, the electric field at P

due to the charge $Q_1 - q = \dfrac{Q_1 - q}{2A\varepsilon_0}$ (downward),

due to the charge $+q = \dfrac{q}{2A\varepsilon_0}$ (upward),

due to the charge $-q = \dfrac{q}{2A\varepsilon_0}$ (downward), and due to the charge $Q_2 + q = \frac{Q_2 + q}{2A\varepsilon_0}$ (upward).

The net electric field at P due to all the four charged surfaces is (in the downward direction)

$$\frac{Q_1 - q}{2A\varepsilon_0} - \frac{q}{2A\varepsilon_0} + \frac{q}{2A\varepsilon_0} - \frac{Q_2 + q}{2A\varepsilon_0}.$$

As the point P is inside the conductor, this field should be zero. Hence,

$$Q_1 - q - Q_2 - q = 0$$

or, $$q = \frac{Q_1 - Q_2}{2}. \quad \dots \text{(i)}$$

Thus, $$Q_1 - q = \frac{Q_1 + Q_2}{2} \quad \dots \text{(ii)}$$

and $$Q_2 + q = \frac{Q_1 + Q_2}{2}.$$

Using these equations, the distribution shown in the figure (30-W6) can be redrawn as in figure (30-W7).

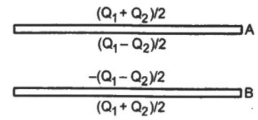

Figure 30-W7

This result is a special case of the following result. When charged conducting plates are placed parallel to each other, the two outermost surfaces get equal charges and the facing surfaces get equal and opposite charges.

□

QUESTIONS FOR SHORT ANSWER

1. A small plane area is rotated in an electric field. In which orientation of the area is the flux of electric field through the area maximum ? In which orientation is it zero ?
2. A circular ring of radius r made of a nonconducting material is placed with its axis parallel to a uniform electric field. The ring is rotated about a diameter through 180°. Does the flux of electric field change ? If yes, does it decrease or increase ?
3. A charge Q is uniformly distributed on a thin spherical shell. What is the field at the centre of the shell ? If a point charge is brought close to the shell, will the field at the centre change ? Does your answer depend on whether the shell is conducting or nonconducting ?
4. A spherical shell made of plastic, contains a charge Q distributed uniformly over its surface. What is the electric field inside the shell ? If the shell is hammered to deshape it without altering the charge, will the field inside be changed ? What happens if the shell is made of a metal ?
5. A point charge q is placed in a cavity in a metal block. If a charge Q is brought outside the metal, will the charge q feel an electric force ?
6. A rubber balloon is given a charge Q distributed uniformly over its surface. Is the field inside the balloon zero everywhere if the balloon does not have a spherical surface ?
7. It is said that any charge given to a conductor comes to its surface. Should all the protons come to the surface ? Should all the electrons come to the surface ? Should all the free electrons come to the surface ?

OBJECTIVE I

1. A charge Q is uniformly distributed over a large plastic plate. The electric field at a point P close to the centre of the plate is 10 V m^{-1}. If the plastic plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge Q, the electric field at the point P will become
   (a) zero (b) 5 V m^{-1} (c) 10 V m^{-1} (d) 20 V m^{-1}.
2. A metallic particle having no net charge is placed near a finite metal plate carrying a positive charge. The electric force on the particle will be
   (a) towards the plate (b) away from the plate
   (c) parallel to the plate (d) zero.
3. A thin, metallic spherical shell contains a charge Q on it. A point charge q is placed at the centre of the shell and another charge q_1 is placed outside it as shown in figure (30-Q1). All the three charges are positive. The force on the charge at the centre is

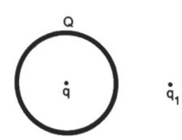

Figure 30-Q1

(a) towards left (b) towards right
(c) upward (d) zero.

4. Consider the situation of the previous problem. The force on the central charge due to the shell is
(a) towards left (b) towards right
(c) upward (d) zero.

5. Electric charges are distributed in a small volume. The flux of the electric field through a spherical surface of radius 10 cm surrounding the total charge is 25 V m. The flux over a concentric sphere of radius 20 cm will be
(a) 25 V m (b) 50 V m (c) 100 V m (d) 200 V m.

6. Figure (30-Q2a) shows an imaginary cube of edge $L/2$. A uniformly charged rod of length L moves towards left at a small but constant speed v. At $t = 0$, the left end just touches the centre of the face of the cube opposite it. Which of the graphs shown in figure (30-Q2b) represents the flux of the electric field through the cube as the rod goes through it ?

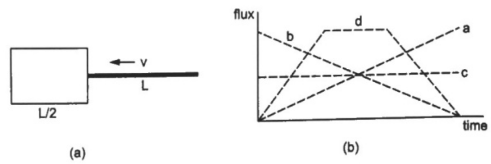

Figure 30-Q2

7. A charge q is placed at the centre of the open end of a cylindrical vessel (figure 30-Q3). The flux of the electric field through the surface of the vessel is
(a) zero (b) q/ε_0 (c) $q/2\varepsilon_0$ (d) $2q/\varepsilon_0$.

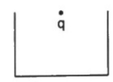

Figure 30-Q3

OBJECTIVE II

1. Mark the correct options:
(a) Gauss's law is valid only for symmetrical charge distributions.
(b) Gauss's law is valid only for charges placed in vacuum.
(c) The electric field calculated by Gauss's law is the field due to the charges inside the Gaussian surface.
(d) The flux of the electric field through a closed surface due to all the charges is equal to the flux due to the charges enclosed by the surface.

2. A positive point charge Q is brought near an isolated metal cube.
(a) The cube becomes negatively charged.
(b) The cube becomes positively charged.
(c) The interior becomes positively charged and the surface becomes negatively charged.
(d) The interior remains charge free and the surface gets nonuniform charge distribution.

3. A large nonconducting sheet M is given a uniform charge density. Two uncharged small metal rods A and B are placed near the sheet as shown in figure (30-Q4).
(a) M attracts A. (b) M attracts B.
(c) A attracts B. (d) B attracts A.

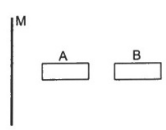

Figure 30-Q4

4. If the flux of the electric field through a closed surface is zero,
(a) the electric field must be zero everywhere on the surface
(b) the electric field may be zero everywhere on the surface
(c) the charge inside the surface must be zero
(d) the charge in the vicinity of the surface must be zero.

5. An electric dipole is placed at the centre of a sphere. Mark the correct options:
(a) The flux of the electric field through the sphere is zero.
(b) The electric field is zero at every point of the sphere.
(c) The electric field is not zero anywhere on the sphere.
(d) The electric field is zero on a circle on the sphere.

6. Figure (30-Q5) shows a charge q placed at the centre of a hemisphere. A second charge Q is placed at one of the positions A, B, C and D. In which position(s) of this second charge, the flux of the electric field through the hemisphere remains unchanged ?
(a) A (b) B (c) C (d) D.

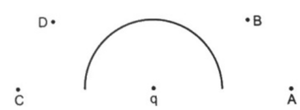

Figure 30-Q5

7. A closed surface S is constructed around a conducting wire connected to a battery and a switch (figure 30-Q6). As the switch is closed, the free electrons in the wire start moving along the wire. In any time interval, the number of electrons entering the closed surface S is equal to the number of electrons leaving it. On closing

the switch, the flux of the electric field through the closed surface
(a) is increased (b) is decreased
(c) remains unchanged (d) remains zero.

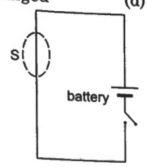

Figure 30-Q6

8. Figure (30-Q7) shows a closed surface which intersects a conducting sphere. If a positive charge is placed at the point P, the flux of the electric field through the closed surface
(a) will remain zero (b) will become positive
(c) will become negative (d) will become undefined.

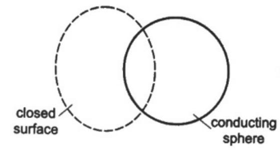

Figure 30-Q7

EXERCISES

1. The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\vec{i} + \frac{4}{5}E_0\vec{j}$ with $E_0 = 2{\cdot}0 \times 10^3\ \text{N C}^{-1}$. Find the flux of this field through a rectangular surface of area $0{\cdot}2\ \text{m}^2$ parallel to the y–z plane.

2. A charge Q is uniformly distributed over a rod of length l. Consider a hypothetical cube of edge l with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

3. Show that there can be no net charge in a region in which the electric field is uniform at all points.

4. The electric field in a region is given by $\vec{E} = \frac{E_0 x}{l}\vec{i}$. Find the charge contained inside a cubical volume bounded by the surfaces $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$ and $z = a$. Take $E_0 = 5 \times 10^3\ \text{N C}^{-1}$, $l = 2$ cm and $a = 1$ cm.

5. A charge Q is placed at the centre of a cube. Find the flux of the electric field through the six surfaces of the cube.

6. A charge Q is placed at a distance $a/2$ above the centre of a horizontal, square surface of edge a as shown in figure (30-E1). Find the flux of the electric field through the square surface.

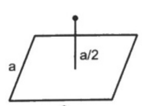

Figure 30-E1

7. Find the flux of the electric field through a spherical surface of radius R due to a charge of 10^{-7} C at the centre and another equal charge at a point $2R$ away from the centre (figure 30-E2).

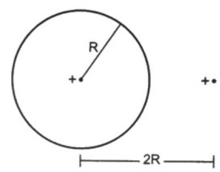

Figure 30-E2

8. A charge Q is placed at the centre of an imaginary hemispherical surface. Using symmetry arguments and the Gauss's law, find the flux of the electric field due to this charge through the surface of the hemisphere (figure 30-E3).

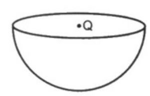

Figure 30-E3

9. A spherical volume contains a uniformly distributed charge of density $2{\cdot}0 \times 10^{-4}\ \text{C m}^{-3}$. Find the electric field at a point inside the volume at a distance 4·0 cm from the centre.

10. The radius of a gold nucleus ($Z = 79$) is about $7{\cdot}0 \times 10^{-15}$ m. Assume that the positive charge is distributed uniformly throughout the nuclear volume. Find the strength of the electric field at (a) the surface of the nucleus and (b) at the middle point of a radius. Remembering that gold is a conductor, is it justified to assume that the positive charge is uniformly distributed over the entire volume of the nucleus and does not come to the outer surface ?

11. A charge Q is distributed uniformly within the material of a hollow sphere of inner and outer radii r_1 and r_2 (figure 30-E4). Find the electric field at a point P a

distance x away from the centre for $r_1 < x < r_2$. Draw a rough graph showing the electric field as a function of x for $0 < x < 2r_2$ (figure 30-E4).

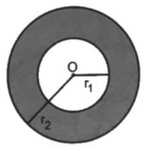

Figure 30-E4

12. A charge Q is placed at the centre of an uncharged, hollow metallic sphere of radius a. (a) Find the surface charge density on the inner surface and on the outer surface. (b) If a charge q is put on the sphere, what would be the surface charge densities on the inner and the outer surfaces? (c) Find the electric field inside the sphere at a distance x from the centre in the situations (a) and (b).

13. Consider the following very rough model of a beryllium atom. The nucleus has four protons and four neutrons confined to a small volume of radius 10^{-15} m. The two $1s$ electrons make a spherical charge cloud at an average distance of $1{\cdot}3 \times 10^{-11}$ m from the nucleus, whereas the two $2s$ electrons make another spherical cloud at an average distance of $5{\cdot}2 \times 10^{-11}$ m from the nucleus. Find the electric field at (a) a point just inside the $1s$ cloud and (b) a point just inside the $2s$ cloud.

14. Find the magnitude of the electric field at a point 4 cm away from a line charge of density 2×10^{-6} C m^{-1}.

15. A long cylindrical wire carries a positive charge of linear density $2{\cdot}0 \times 10^{-8}$ C m^{-1}. An electron revolves around it in a circular path under the influence of the attractive electrostatic force. Find the kinetic energy of the electron. Note that it is independent of the radius.

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. Find the electric field at a point P inside the cylindrical volume at a distance x from its axis (figure 30-E5)

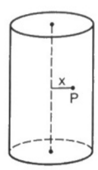

Figure 30-E5

17. A nonconducting sheet of large surface area and thickness d contains uniform charge distribution of density ρ. Find the electric field at a point P inside the plate, at a distance x from the central plane. Draw a qualitative graph of E against x for $0 < x < d$.

18. A charged particle having a charge of $-2{\cdot}0 \times 10^{-6}$ C is placed close to a nonconducting plate having a surface charge density $4{\cdot}0 \times 10^{-6}$ C m^{-2}. Find the force of attraction between the particle and the plate.

19. One end of a 10 cm long silk thread is fixed to a large vertical surface of a charged nonconducting plate and the other end is fastened to a small ball having a mass of 10 g and a charge of $4{\cdot}0 \times 10^{-6}$ C. In equilibrium, the thread makes an angle of 60° with the vertical. Find the surface charge density on the plate.

20. Consider the situation of the previous problem. (a) Find the tension in the string in equilibrium. (b) Suppose the ball is slightly pushed aside and released. Find the time period of the small oscillations.

21. Two large conducting plates are placed parallel to each other with a separation of 2·00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2·00 microseconds. Find the surface charge density on the inner surfaces.

22. Two large conducting plates are placed parallel to each other and they carry equal and opposite charges with surface density σ as shown in figure (30-E6). Find the electric field (a) at the left of the plates, (b) in between the plates and (c) at the right of the plates.

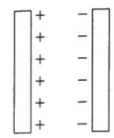

Figure 30-E6

23. Two conducting plates X and Y, each having large surface area A (on one side), are placed parallel to each other as shown in figure (30-E7). The plate X is given a charge Q whereas the other is neutral. Find (a) the surface charge density at the inner surface of the plate X, (b) the electric field at a point to the left of the plates, (c) the electric field at a point in between the plates and (d) the electric field at a point to the right of the plates.

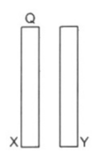

Figure 30-E7

24. Three identical metal plates with large surface areas are kept parallel to each other as shown in figure (30-E8). The leftmost plate is given a charge Q, the rightmost a charge $-2Q$ and the middle one remains neutral. Find the charge appearing on the outer surface of the rightmost plate.

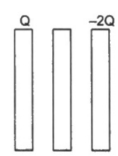

Figure 30-E8

ANSWERS

OBJECTIVE I

1. (c) 2. (a) 3. (d) 4. (b) 5. (a) 6. (d)
7. (c)

OBJECTIVE II

1. (d) 2. (d) 3. all
4. (b), (c) 5. (a), (c) 6. (a), (c)
7. (c), (d) 8. (b)

EXERCISES

1. $240 \text{ N m}^2 \text{ C}^{-1}$
2. $Q/(2\varepsilon_0)$
4. $2{\cdot}2 \times 10^{-12}$ C
5. Q/ε_0
6. $Q/(6\varepsilon_0)$
7. $1{\cdot}1 \times 10^4 \text{ N m}^{-2} \text{ C}^{-1}$
8. $Q/(2\varepsilon_0)$
9. $3{\cdot}0 \times 10^5 \text{ N C}^{-1}$
10. (a) $2{\cdot}32 \times 10^{21} \text{ N C}^{-1}$ (b) $1{\cdot}16 \times 10^{21} \text{ N C}^{-1}$
11. $\dfrac{Q(x^3 - r_1^3)}{4\pi\varepsilon_0 x^2 (r_2^3 - r_1^3)}$
12. (a) $-\dfrac{Q}{4\pi a^2}, \dfrac{Q}{4\pi a^2}$ (b) $-\dfrac{Q}{4\pi a^2}, \dfrac{Q+q}{4\pi a^2}$
    (c) $\dfrac{Q}{4\pi\varepsilon_0 x^2}$ in both situations
13. (a) $3{\cdot}4 \times 10^{13} \text{ N C}^{-1}$ (b) $1{\cdot}1 \times 10^{12} \text{ N C}^{-1}$
14. $9 \times 10^5 \text{ N C}^{-1}$
15. $2{\cdot}88 \times 10^{-17}$ J
16. $\rho x/(2\varepsilon_0)$
17. $\rho x/\varepsilon_0$
18. 0·45 N
19. $7{\cdot}5 \times 10^{-7} \text{ C m}^{-2}$
20. (a) 0·20 N (b) 0·45 s
21. $0{\cdot}505 \times 10^{-12} \text{ C m}^{-2}$
22. (a) zero (b) σ/ε_0 (c) zero
23. (a) $\dfrac{Q}{2A}$ (b) $\dfrac{Q}{2A\varepsilon_0}$ towards left (c) $\dfrac{Q}{2A\varepsilon_0}$ towards right
    (d) $\dfrac{Q}{2A\varepsilon_0}$ towards right
24. $-Q/2$

□

CHAPTER 31

CAPACITORS

31.1 CAPACITOR AND CAPACITANCE

A combination of two conductors placed close to each other is called a *capacitor*. One of the conductors is given a positive charge and the other is given an equal negative charge. The conductor with the positive charge is called the *positive plate* and the other is called the *negative plate*. The charge on the positive plate is called *the charge on the capacitor* and the potential difference between the plates is called the *potential of the capacitor*. Figure (31.1a) shows two conductors. One of the conductors has a positive charge $+Q$ and the other has an equal, negative charge $-Q$. The first one is at a potential V_+ and the other is at a potential V_-. The charge on the capacitor is Q and the potential of the capacitor is $V = V_+ - V_-$. Note that the term *charge on a capacitor* does not mean the total charge given to the capacitor. This total charge is $+Q - Q = 0$. Figure (31.1b) shows the symbol used to represent a capacitor.

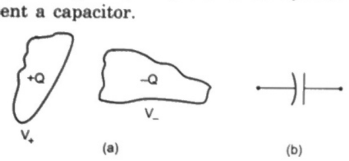

Figure 31.1

For a given capacitor, the charge Q on the capacitor is proportional to the potential difference V between the plates

Thus, $\quad Q \propto V$

or, $\quad Q = CV. \qquad \dots (31.1)$

The proportionality constant C is called the *capacitance* of the capacitor. It depends on the shape, size and geometrical placing of the conductors and the medium between them.

The SI unit of capacitance is coulomb per volt which is written as farad. The symbol F is used for it. This is a large unit on normal scales and microfarad (μF) is used more frequently.

To put equal and opposite charges on the two conductors, they may be connected to the terminals of a *battery*. We shall discuss in somewhat greater detail about the battery in the next chapter. Here we state the following properties of an ideal battery.

(a) A battery has two terminals.

(b) The potential difference V between the terminals is constant for a given battery. The terminal with higher potential is called the *positive terminal* and that with lower potential is called the *negative terminal*.

(c) The value of this fixed potential difference is equal to the *electromotive force* or *emf* of the battery. If a conductor is connected to a terminal of a battery, the potential of the conductor becomes equal to the potential of the terminal. When the two plates of a capacitor are connected to the terminals of a battery, the potential difference between the plates of the capacitor becomes equal to the emf of the battery.

(d) The total charge in a battery always remains zero. If its positive terminal supplies a charge Q, its negative terminal supplies an equal, negative charge $-Q$.

(e) When a charge Q passes through a battery of emf $\mathcal{E}$ from the negative terminal to the positive terminal, an amount $Q\mathcal{E}$ of work is done by the battery.

An ideal battery is represented by the symbol shown in figure (31.2). The potential difference between the facing parallel lines is equal to the emf $\mathcal{E}$ of the battery. The longer line is at the higher potential.

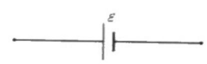

Figure 31.2

Example 31.1

A capacitor gets a charge of 60 μC *when it is connected to a battery of emf* 12 V. *Calculate the capacitance of the capacitor.*

Solution : The potential difference between the plates is the same as the emf of the battery which is 12 V. Thus,

Solution : The capacitance is

$$C = \frac{\varepsilon_0 A}{d}$$

$$= \frac{8{\cdot}85 \times 10^{-12}\ \text{F m}^{-1} \times 400 \times 10^{-4}\ \text{m}^2}{1 \times 10^{-3}\ \text{m}}$$

$$= 3{\cdot}54 \times 10^{-10}\ \text{F} \approx 350\ \text{pF}.$$

Spherical Capacitor

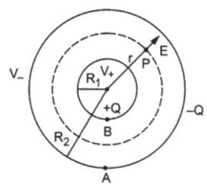

Figure 31.5

A spherical capacitor consists of a solid or a hollow spherical conductor surrounded by another concentric hollow spherical conductor. Suppose, the inner sphere has a radius R_1 and the outer sphere has a radius R_2. Suppose, the inner sphere is given a positive charge Q and the outer is given a negative charge $-Q$.

The field at any point P between the spheres is radially outward and its magnitude depends only on its distance r from the centre. Let us draw a sphere through P concentric with the given system. The flux of the electric field through this sphere is

$$\Phi = \oint \vec{E}\cdot d\vec{S} = \oint E\, dS$$

$$= E \oint dS = E\, 4\pi r^2.$$

The charge enclosed in this sphere is Q. Thus, from Gauss's law,

$$E\, 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

or,
$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

The potential difference between the two conductors is

$$V = V_+ - V_- = -\int_A^B \vec{E}\cdot d\vec{r}$$

$$= -\int_{R_2}^{R_1} \frac{Q}{4\pi\varepsilon_0 r^2}\, dr$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q(R_2 - R_1)}{4\pi\varepsilon_0 R_1 R_2}.$$

The capacitance of the spherical capacitor is

$$C = \frac{Q}{V}$$

$$= \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}. \qquad \dots (31.3)$$

Isolated sphere

If we assume that the outer sphere is at infinity, we get an isolated single sphere of radius R_1. The capacitance of such a single sphere can be obtained from equation (31.3) by taking the limit as $R_2 \to \infty$. Then

$$C = \frac{4\pi\varepsilon_0 R_1 R_2}{R_2 - R_1}$$

$$\approx \frac{4\pi\varepsilon_0 R_1 R_2}{R_2} = 4\pi\varepsilon_0 R_1.$$

If a charge Q is placed on this sphere, its potential (with zero potential at infinity) becomes

$$V = \frac{Q}{C} = \frac{Q}{4\pi\varepsilon_0 R_1}.$$

Parallel limit

If both R_1 and R_2 are made large but $R_2 - R_1 = d$ is kept fixed, we can write

$$4\pi R_1 R_2 \approx 4\pi R^2 = A$$

where R is approximately the radius of each sphere and A is the area. Equation (31.3) then becomes

$$C = \frac{\varepsilon_0 A}{d}$$

which is the same as the equation for the capacitance of a parallel-plate capacitor.

Cylindrical Capacitor

A cylindrical capacitor consists of a solid or a hollow cylindrical conductor surrounded by another coaxial hollow cylindrical conductor. Let the length of the cylinders be l and the radii of the inner and outer cylinders be R_1 and R_2 respectively. Suppose, a positive charge Q is placed on the inner cylinder and a negative charge $-Q$ is placed on the outer cylinder. If the cylinders are long as compared to the separation between them, the electric field at a point between the cylinders will be radial and its magnitude will depend only on the distance of the point from the axis. Let P be a point between the cylinders at a distance r from the axis (figure 31.6).

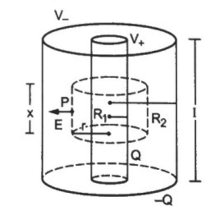

Figure 31.6

To calculate the electric field at the point P, let us draw a coaxial cylinder of length x through the point P. This cylinder together with its two cross sections forms a Gaussian surface. The flux through the cross sections is zero because the electric field is radial wherever it exists and hence is parallel to the cross sections. The flux through the curved part is

$$\Phi = \oint \vec{E} \cdot d\vec{S}$$
$$= \int E \, dS$$
$$= E \int dS = E \, 2\pi r x.$$

The charge enclosed by the Gaussian surface is

$$Q_{in} = \frac{Q}{l} x.$$

Thus, from Gauss's law,

$$E \, 2\pi r x = \left(\frac{Q}{l} x \right) / \varepsilon_0$$

or, $$E = \frac{Q}{2\pi\varepsilon_0 \, r l}.$$

The potential difference between the cylinders is

$$V = V_+ - V_-$$
$$= -\int_A^B \vec{E} \cdot d\vec{r} = -\int_{R_2}^{R_1} E \, dr$$
$$= -\int_{R_2}^{R_1} \frac{Q}{2\pi\varepsilon_0 \, r l} dr$$
$$= \frac{Q}{2\pi\varepsilon_0 \, l} \ln \frac{R_2}{R_1}.$$

The capacitance is

$$C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \, l}{\ln(R_2 / R_1)}. \quad \ldots (31.4)$$

31.3 COMBINATION OF CAPACITORS

Two or more capacitors may be connected in a number of ways. The combination should have two points which may be connected to a battery to apply a potential difference. The battery supplies positive and negative charges to the system. If V be the potential difference between the points and Q be the magnitude of the charge supplied by either terminal of the battery, we define *equivalent capacitance* of the combination *between the two points* to be

$$C = \frac{Q}{V}.$$

If the combination is replaced by a single capacitor of this capacitance, the single capacitor will store the same amount of charge for a given potential difference as the combination does.

Two special methods of combination are frequently used, one known as *series* combination and the other as *parallel* combination.

Series Combination

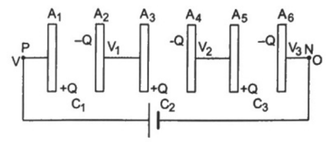

Figure 31.7

Figure (31.7) shows three capacitors connected in series. The capacitances are C_1, C_2 and C_3. The points P and N serve as the points through which a potential difference may be applied and a charge may be supplied to the combination. Let us connect the point P to the positive terminal and the point N to the negative terminal of a battery. The battery supplies a charge $+Q$ to the plate A_1 and a charge $-Q$ to the plate A_6. The charge $+Q$ given by the battery appears on the right surface of the plate A_1. The facing surface of A_2 must have a charge $-Q$ on it.

The plates A_2 and A_3 are connected and they together are isolated from everything else. The charge $-Q$ appearing on A_2 comes from the electrons drifted from the plate A_3 to A_2. This leaves a positive charge $+Q$ on the plate A_3. The facing surface of A_4 gets a charge $-Q$ from A_5 and a charge $+Q$ appears on the right surface of A_5. The facing surface of A_6 gets a charge $-Q$ from the battery. This completes the charge distribution. *In series combination, each capacitor has equal charge for any value of capacitances.*

Let us take the potential of the point N to be zero. The potential of the plate A_6 is also zero as it is connected to N by a conducting wire. The potential of the point P as well as that of the plate A_1 is V. The plates A_2 and A_3 are at the same potential, say, V_1. Similarly, A_4 and A_5 are at the same potential, say, V_2.

The charge on the first capacitor is Q and the potential difference is $V - V_1$. As the capacitance of this capacitor is C_1, we have

$$Q = C_1(V - V_1)$$

or, $$V - V_1 = \frac{Q}{C_1}. \quad \ldots \text{(i)}$$

Similarly, considering the other capacitors,

$$V_1 - V_2 = \frac{Q}{C_2} \quad \ldots \text{(ii)}$$

and $$V_2 - 0 = \frac{Q}{C_3}. \quad \ldots \text{(iii)}$$

Adding (i), (ii) and (iii);

$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right). \quad \dots \text{(iv)}$$

If the equivalent capacitance of the combination between the points P and N is C, we have

$$C = \frac{Q}{V}$$

and equation (iv) becomes

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

The above analysis may be extended to any number of capacitors, the equivalent capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \quad \dots (31.5)$$

Example 31.4

Calculate the charge on each capacitor shown in figure (31.8).

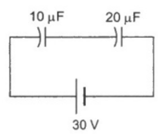

Figure 31.8

Solution : The two capacitors are joined in series. Their equivalent capacitance is given by $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

or, $$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(10\,\mu\text{F})\,(20\,\mu\text{F})}{30\,\mu\text{F}} = \frac{20}{3}\,\mu\text{F}.$$

The charge supplied by the battery is

$$Q = CV$$

$$= \left(\frac{20}{3}\,\mu\text{F}\right)(30\text{ V}) = 200\,\mu\text{C}.$$

In series combination, each capacitor has equal charge and this charge equals the charge supplied by the battery. Thus, each capacitor has a charge of 200 μC.

Parallel Combination

Figure (31.9) shows three capacitors connected in parallel. The capacitances are C_1, C_2 and C_3. The points

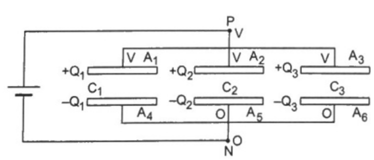

Figure 31.9

P and N are the two points through which a potential difference can be applied and charge can be supplied. Let us connect the point P to the positive terminal of a battery and the point N to its negative terminal. The battery supplies a charge $+Q$ which is distributed on the three positive plates A_1, A_2 and A_3 of the capacitors. Let the charges on the three plates A_1, A_2 and A_3 be Q_1, Q_2 and Q_3 respectively. The battery also supplies a charge $-Q$ which is distributed on the three plates A_4, A_5 and A_6. These plates must receive charges $-Q_1$, $-Q_2$ and $-Q_3$ respectively because the facing surfaces must have equal and opposite charges. We have

$$Q = Q_1 + Q_2 + Q_3. \quad \dots \text{(i)}$$

Let us take the potential of the point N to be zero. The potentials of the plates A_4, A_5 and A_6 are also zero as they are all connected to N by conducting wires. Let the potential of the point P be V. This will also be the potential of the plates A_1, A_2 and A_3. Thus, *the potential differences of the capacitors connected in parallel are equal for any value of capacitances.* Using the equation $Q = CV$ for the three capacitors,

$$Q_1 = C_1 V \quad \dots \text{(ii)}$$

$$Q_2 = C_2 V \quad \dots \text{(iii)}$$

and $$Q_3 = C_3 V. \quad \dots \text{(iv)}$$

Adding (ii), (iii) and (iv) and using (i),

$$Q = (C_1 + C_2 + C_3)V$$

or, $$\frac{Q}{V} = C_1 + C_2 + C_3.$$

But Q/V is the equivalent capacitance of the given combination. Thus,

$$C = C_1 + C_2 + C_3. \quad \dots (31.6)$$

In parallel combination, all the positive plates are at the same potential and all the negative plates are at the same potential. The potential difference on each capacitor is the same in parallel combination but the charges on the capacitors may be different. In series combination, the charges on the capacitors are equal, the potential differences may be different.

Example 31.5

Find the equivalent capacitance of the combination shown in figure (31.10) between the points P and N.

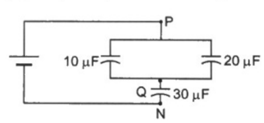

Figure 31.10

Solution : The 10 μF and 20 μF capacitors are connected in parallel. Their equivalent capacitance is

$10\,\mu F + 20\,\mu F = 30\,\mu F$. We can replace the $10\,\mu F$ and the $20\,\mu F$ capacitors by a single capacitor of capacitance $30\,\mu F$ between P and Q. This is connected in series with the given $30\,\mu F$ capacitor. The equivalent capacitance C of this combination is given by

$$\frac{1}{C} = \frac{1}{30\,\mu F} + \frac{1}{30\,\mu F} \quad \text{or,} \quad C = 15\,\mu F.$$

We have used series–parallel combination to solve the above example. Sometimes it may not be easy to find the equivalent capacitance of a combination using the equations for series–parallel combinations. We may then use the general method which was applied to derive the equivalent capacitance in series and parallel combinations. For any given combination, one may proceed as follows:

Step 1

Identify the two points between which the equivalent capacitance is to be calculated. Call any one of them as P and the other as N.

Step 2

Connect (mentally) a battery between P and N with the positive terminal connected to P and the negative terminal to N. Send a charge $+Q$ from the positive terminal of the battery and $-Q$ from the negative terminal of the battery.

Step 3

Write the charges appearing on each of the plates of the capacitors. The charge conservation principle may be used. The facing surfaces of a capacitor will always have equal and opposite charges. Assume variables Q_1, Q_2 ..., etc., for charges wherever needed.

Step 4

Take the potential of the negative terminal N to be zero and that of the positive terminal P to be V. Write the potential of each of the plates. If necessary, assume variables V_1, V_2 ... .

Step 5

Write the capacitor equation $Q = CV$ for each capacitor. Eliminate Q_1, Q_2, ... and V_1, V_2, ..., etc., to obtain the equivalent capacitance $C = Q/V$.

Example 31.6

Find the equivalent capacitance of the combination shown in figure (31.11a) between the points P and N.

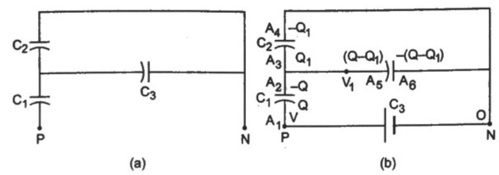

Figure 31.11

Solution : Let us connect a battery between the points P and N. The charges and the potentials are shown in figure (31.11b). The positive terminal of the battery supplies a charge $+Q$ which appears on the plate A_1. The facing plate A_2 gets a charge $-Q$. The plates A_2, A_3 and A_5 taken together form an isolated system. The total charge on these three plates should be zero. Let a charge Q_1 appear on A_3, then a charge $Q - Q_1$ will appear on A_5 to make the total charge zero on the three plates. The plate A_4 will get a charge $-Q_1$ (facing plate of A_3) and A_6 will get a charge $-(Q - Q_1)$ (facing plate of A_5). The total charge $-Q$ on A_4 and A_6 is supplied by the negative terminal of the battery. This completes the charge distribution.

Next, suppose the potential at the point N is zero and at P it is V. The potential of the plates A_4 and A_6 is also zero. The potential of the plate A_1 is V. The plates A_2, A_3 and A_5 are at the same potential. Let this common potential be V_1. This completes the potential distribution.

Applying the capacitor equation $Q = CV$ to the three capacitors,

$$Q = C_1(V - V_1) \quad \text{... (i)}$$

$$Q_1 = C_2 V_1 \quad \text{... (ii)}$$

and $$Q - Q_1 = C_3 V_1. \quad \text{... (iii)}$$

Adding (ii) and (iii),

$$Q = (C_2 + C_3)V_1$$

or, $$\frac{Q}{C_2 + C_3} = V_1. \quad \text{... (iv)}$$

From (i), $$\frac{Q}{C_1} = V - V_1. \quad \text{... (v)}$$

Adding (iv) and (v),

$$\frac{Q}{C_2 + C_3} + \frac{Q}{C_1} = V$$

or, $$\frac{(C_1 + C_2 + C_3)Q}{C_1(C_2 + C_3)} = V$$

or, $$C = \frac{Q}{V} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}.$$

It may be noted that the above example could be solved by using the equations for series–parallel combinations. However, the general method was used to demonstrate its application.

Symmetry arguments play important role in simplifying the algebra involved in the problem. The use of symmetry arguments in writing the charges on different plates will be demonstrated later in the section of worked out examples.

31.4 FORCE BETWEEN THE PLATES OF A CAPACITOR

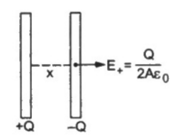

Figure 31.12

Consider a parallel-plate capacitor with plate area A. Suppose a positive charge $+Q$ is given to one plate and a negative charge $-Q$ to the other plate. The electric field due to only the positive plate is

$$E_+ = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

at all points if the plate is large. The negative charge $-Q$ finds itself in the field of this positive charge. The force on $-Q$ is, therefore,

$$F = -QE_+$$

$$= (-Q)\frac{Q}{2A\varepsilon_0} = -\frac{Q^2}{2A\varepsilon_0}.$$

The magnitude of the force is

$$F = \frac{Q^2}{2A\varepsilon_0}.$$

This is the force with which the positive plate attracts the negative plate. This is also the force of attraction on the positive plate by the negative plate. Thus, the plates of a parallel-plate capacitor attract each other with a force

$$F = \frac{Q^2}{2A\varepsilon_0}. \qquad \text{... (31.7)}$$

31.5 ENERGY STORED IN A CAPACITOR AND ENERGY DENSITY IN ELECTRIC FIELD

Let us consider a parallel-plate capacitor of plate area A (figure 31.13). Suppose the plates of the capacitor are almost touching each other and a charge Q is given to the capacitor. One of the plates, say a, is kept fixed and the other, say b, is slowly pulled away from a to increase the separation from zero to d. The attractive force on the plate b at any instant due to the first plate is, from equation (31.7),

$$F = \frac{Q^2}{2A\varepsilon_0}.$$

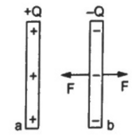

Figure 31.13

The person pulling the plate b must apply an equal force F in the opposite direction if the plate is only slowly moved.

The work done by the person during the displacement of the second plate is

$$W = Fd$$

$$= \frac{Q^2 d}{2A\varepsilon_0} = \frac{Q^2}{2\,C}$$

where C is the capacitance of the capacitor in the final position. The work done by the person must be equal to the increase in the energy of the system. Thus, a capacitor of capacitance C has a stored energy

$$U = \frac{Q^2}{2\,C} \qquad \text{... (31.8)}$$

where Q is the charge given to it. Using $Q = CV$, the above equation may also be written as

$$U = \frac{1}{2}CV^2 \qquad \text{... (31.9)}$$

or,

$$U = \frac{1}{2}QV. \qquad \text{... (31.10)}$$

Example 31.7

Find the energy stored in a capacitor of capacitance 100 μF *when it is charged to a potential difference of* 20 V.

Solution : The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(100\ \mu\text{F})\,(20\ \text{V})^2 = 0{\cdot}02\ \text{J}.$$

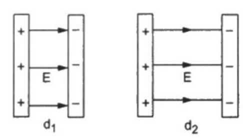

Figure 31.14

The energy stored in a capacitor is electrostatic potential energy. When we pull the plates of a capacitor apart, we have to do work against the electrostatic attraction between the plates. In which region of space is the energy stored ? When we increase the separation between the plates from d_1 to d_2, an amount $\frac{Q^2}{2A\varepsilon_0}(d_2 - d_1)$ of work is performed by us and this much energy goes into the capacitor. On the other hand, new electric field is created in a volume $A(d_2 - d_1)$ (figure 31.14). We conclude that the energy $\frac{Q^2}{2A\varepsilon_0}(d_2 - d_1)$ is stored in the volume $A(d_2 - d_1)$ which is now filled with the electric field. Thus, an electric field has energy associated with it. The energy stored per unit volume in the electric field is

$$u = \frac{\dfrac{Q^2(d_2 - d_1)}{2A\varepsilon_0}}{A(d_2 - d_1)} = \frac{Q^2}{2A^2\varepsilon_0}$$

$$= \frac{1}{2}\varepsilon_0\left(\frac{Q}{A\varepsilon_0}\right)^2 = \frac{1}{2}\varepsilon_0 E^2$$

where E is the intensity of the electric field.

Once it is established that a region containing electric field E has energy $\frac{1}{2}\varepsilon_0 E^2$ per unit volume, the result can be used for any electric field whether it is due to a capacitor or otherwise.

31.6 DIELECTRICS

In dielectric materials, effectively there are no free electrons. The monatomic materials are made of atoms. Each atom consists of a positively charged nucleus surrounded by electrons. In general, the centre of the negative charge coincides with the centre of the positive charge. Polyatomic materials, on the other hand, are made of molecules. The centre of the negative charge distribution in a molecule may or may not coincide with the centre of the positive charge distribution. If it does not coincide, each molecule has a permanent dipole moment $\vec{p}$. Such materials are known as *polar materials.* However, different molecules have different directions of the dipole moment because of the random thermal agitation in the material. In any volume containing a large number of molecules (say more than a thousand), the net dipole moment is zero. If such a material is placed in an electric field, the individual dipoles experience torque due to the field and they try to align along the field. On the other hand, thermal agitation tries to randomise the orientation and hence, there is a partial alignment. As a result, we get a net dipole moment in any volume of the material.

In nonpolar materials, the centre of the positive charge distribution in an atom or a molecule coincides with the centre of the negative charge distribution. The atoms or the molecules do not have any permanent dipole moment. If such a material is placed in an electric field, the electron charge distribution is slightly shifted opposite to the electric field. This induces dipole moment in each atom or molecule and thus, we get a dipole moment in any volume of the material.

Thus, when a dielectric material is placed in an electric field, dipole moment appears in any volume in it. This fact is known as *polarization* of the material. The *polarization vector* $\vec{P}$ is defined as the dipole moment per unit volume. Its magnitude P is often referred to as the polarization.

Consider a rectangular slab of a dielectric. The individual dipole moments are randomly oriented (figure 31.15a). In any volume containing a large number of molecules, the net charge is zero. When an electric field is applied, the dipoles get aligned along the field. Figure (31.15b) and (31.15c) show the effect of dipole alignment when a field is applied from left to right. We see that the interior is still charge free but the left surface of the slab gets negative charge and the right surface gets positive charge. The situation may be represented as in figure (31.15d). The charge appearing on the surface of a dielectric when placed in an electric field is called *induced charge.* As the induced charge appears due to a shift in the electrons bound to the nuclei, this charge is also called *bound charge.*

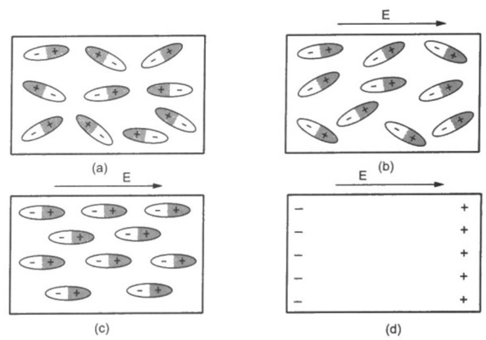

Figure 31.15

The surface charge density of the induced charge has a simple relationship with the polarization P. Suppose, the rectangular slab of figure (31.15) has a length l and area of cross-section A. Let σ_p be the magnitude of the induced charge per unit area on the faces. The dipole moment of the slab is then $(\sigma_p A)l = \sigma_p(Al)$. The polarization is dipole moment induced per unit volume. Thus,

$$P = \frac{\sigma_p\,(Al)}{Al} = \sigma_p. \qquad \dots (31.11)$$

Although this result is deduced for a rectangular slab, it is true in general. The induced surface charge density is equal in magnitude to the polarization P.

Dielectric Constant

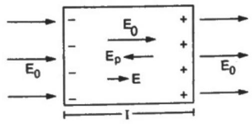

Figure 31.16

Because of the induced charges, an extra electric field is produced inside the material. Let $\vec{E}_0$ be the applied field due to external sources and $\vec{E}_p$ be the field due to polarization (figure 31.16). The resultant field is $\vec{E}=\vec{E}_0+\vec{E}_p$. For homogeneous and isotropic dielectrics, the direction of $\vec{E}_p$ is opposite to the direction of $\vec{E}_0$. The resultant field $\vec{E}$ is in the same direction as the applied field $\vec{E}_0$ but its magnitude is reduced. We can write

$$\vec{E}=\frac{\vec{E}_0}{K}$$

where K is a constant for the given dielectric which has a value greater than one. This constant K is called the *dielectric constant* or *relative permittivity* of the dielectric. For vacuum, there is no polarization and hence $\vec{E}=\vec{E}_0$ and $K=1$.

If a very high electric field is created in a dielectric, the outer electrons may get detached from their parent atoms. The dielectric then behaves like a conductor. This phenomenon is known as *dielectric breakdown*. The minimum field at which the breakdown occurs is called the *dielectric strength* of the material. Table (31.1) gives dielectric constants and dielectric strengths for some of the dielectrics.

31.7. PARALLEL-PLATE CAPACITOR WITH A DIELECTRIC

Consider a parallel-plate capacitor with plate area A and separation d between the plates (figure 31.17). A dielectric slab of dielectric constant K is inserted in the space between the plates. Suppose, the slab almost completely fills the space between the plates. A charge Q is given to the positive plate and $-Q$ to the negative plate of the capacitor. The electric field polarizes the dielectric so that induced charges $+Q_p$ and $-Q_p$ appear on the two faces of the slab.

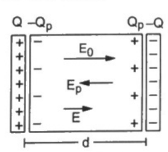

Figure 31.17

Table 31.1 : *Dielectric constants and dielectric strengths*

Material	Dielectric constant	Dielectric strength (kVmm^{-1})
Vacuum	1	∞
Pyrex Glass	5·6	≈ 14
Mica	3–6	12
Neoprene rubber	6·9	12
Bakelite	4·9	24
Plexiglas	3·40	40
Fused quartz	3·8	8
Paper	3·5	14
Polystyrene	2·6	25
Teflon	2·1	60
Strontium titanate	310	8
Titanium dioxide	100	6
Water	80	–
Glycerin	42·5	–
Benzene	2·3	–
Air (1 atm)	1·00059	3
Air (100 atm)	1·0548	–

The electric field at a point between the plates due to the charges $+Q$, $-Q$ on the capacitor plates is

$$E_0=\frac{\sigma}{\varepsilon_0}=\frac{Q}{A\varepsilon_0} \qquad \dots \text{(i)}$$

in a direction left to right in the figure (31.17).

From the definition of dielectric constant, the resultant field is

$$E=\frac{E_0}{K}=\frac{Q}{\varepsilon_0 AK}\cdot \qquad \dots \text{(ii)}$$

The potential difference between the plates is

$$V=Ed$$
$$=\frac{Qd}{\varepsilon_0 AK}\cdot$$

The capacitance is

$$C=\frac{Q}{V}=\frac{K\varepsilon_0 A}{d}=KC_0 \qquad \dots (31.12)$$

where $C_0=\frac{\varepsilon_0 A}{d}$ is the capacitance without the dielectric. Thus,

The capacitance of a capacitor is increased by a factor of K when the space between the plates is filled with a dielectric of dielectric constant K.

This result is often taken as the definition of the dielectric constant.

Magnitude of the Induced Charge

From (i), the electric field at a point between the plates due to the charges $+Q$, $-Q$ is

$$E_0=\frac{Q}{A\varepsilon_0}\cdot$$

The field due to the charges Q_p, $-Q_p$ is directed oppositely and has magnitude

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{Q_p}{A\varepsilon_0}.$$

The resultant field is

$$E = E_0 - E_p$$

$$= \frac{Q - Q_p}{A\varepsilon_0}. \qquad \dots \text{(iii)}$$

From equations (ii) and (iii),

$$\frac{Q - Q_p}{\varepsilon_0 A} = \frac{Q}{\varepsilon_0 AK}$$

or, $$Q - Q_p = \frac{Q}{K}$$

or, $$Q_p = Q\left(1 - \frac{1}{K}\right). \qquad \dots (31.13)$$

Example 31.8

Two parallel-plate capacitors, each of capacitance 40 μF, are connected in series. The space between the plates of one capacitor is filled with a dielectric material of dielectric constant K = 4. Find the equivalent capacitance of the system.

Solution : The capacitance of the capacitor with the dielectric is

$$C_1 = KC_0 = 4 \times 40\ \mu\text{F} = 160\ \mu\text{F}.$$

The other capacitor has capacitance $C_2 = 40\ \mu\text{F}$. As they are connected in series, the equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{(160\ \mu\text{F})\ (40\ \mu\text{F})}{200\ \mu\text{F}} = 32\ \mu\text{F}.$$

Example 31.9

A parallel-plate capacitor has plate area A and plate separation d. The space between the plates is filled up to a thickness x (< d) with a dielectric of dielectric constant K. Calculate the capacitance of the system.

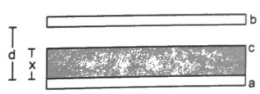

Figure 31.18

Solution :

The situation is shown in figure (31.18). The given system is equivalent to the series combination of two capacitors, one between a and c and the other between c and b. Here c represents the upper surface of the dielectric. This is because the potential at the upper surface of the dielectric is constant and we can imagine a thin metal plate being placed there.

The capacitance of the capacitor between a and c is

$$C_1 = \frac{K\varepsilon_0 A}{x}$$

and that between c and b is

$$C_2 = \frac{\varepsilon_0 A}{d - x}.$$

The equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{K\varepsilon_0 A}{Kd - x(K - 1)}.$$

31.8 AN ALTERNATIVE FORM OF GAUSS'S LAW

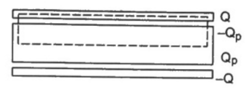

Figure 31.19

Let us again consider a parallel-plate capacitor with a charge Q. The space between the plates is filled with a dielectric slab of dielectric constant K. Let us consider a Gaussian surface as shown in figure (31.19). The charge enclosed by the surface is $Q - Q_p$. From Gauss's law,

$$\oint \vec{E} \cdot d\vec{S} = \frac{Q - Q_p}{\varepsilon_0} \qquad \dots \text{(i)}$$

$$= \frac{1}{\varepsilon_0}\left[Q - Q\left(1 - \frac{1}{K}\right)\right] = \frac{Q}{\varepsilon_0 K}$$

or, $$\oint K\vec{E} \cdot d\vec{S} = \frac{Q_{free}}{\varepsilon_0}. \qquad \dots (31.14)$$

Q_{free} is used in place of Q to emphasise that it is the free charge given to the plates and does not include the bound charge appearing due to polarization.

Equation (31.14) is taken as another form of Gauss's law. This form differs from the usual form of Gauss's law in two respects. Firstly, the charge Q_{free} appearing on the right-hand side is not the total charge inside the Gaussian surface. It is the free charge or external charge inside the Gaussian surface. The bound charge Q_p appearing due to polarization of the dielectric is left out. Secondly, an extra factor K appears on the left-hand side. The two differences compensate the effects of each other and the two forms of Gauss's law are identical. Either of the two may be used in any case.

Though we derived this result for a special case of parallel-plate capacitor, it is true in any situation where the dielectric used is homogeneous and isotropic. Let us now write Gauss's law in yet another form valid for any case.

Displacement Vector

The field due to the polarization is

$$E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$$

where P is the polarization (the dipole moment per unit volume). As the direction of E_p is opposite to the polarization vector $\vec{P}$, we write

$$\vec{E}_p = -\frac{\vec{P}}{\varepsilon_0}.$$

Now, $$\vec{E} = \vec{E}_0 + \vec{E}_p$$

or, $$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{\varepsilon_0}$$

or, $$\varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E}_0 \qquad \dots \text{(i)}$$

or, $$\oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S} = \oint \varepsilon_0 \vec{E}_0 \cdot d\vec{S}$$

over any closed surface. As $\vec{E}_0$ is the field produced by the free charge Q_{free}, $\oint \varepsilon_0 \vec{E}_0 \cdot d\vec{S} = Q_{free}$ from Gauss's law. Thus,

$$\oint (\varepsilon_0 \vec{E} + \vec{P}) \cdot d\vec{S} = Q_{free}. \qquad \dots \text{(ii)}$$

The quantity $\varepsilon_0 \vec{E} + \vec{P}$ is known as the *electric displacement vector* $\vec{D}$. Equation (ii) above may be written in terms of $\vec{D}$ as

$$\oint \vec{D} \cdot d\vec{S} = Q_{free} \qquad \dots (31.15)$$

which is another form of Gauss's law.

If there is no polarization, $\vec{D} = \varepsilon_0 \vec{E}$ and Q_{free} is equal to the total charge inside the Gaussian surface. Equation (31.15) then reduces to the usual form of Gauss's law.

In case of homogeneous and isotropic dielectrics, $\vec{E}_0 = K\vec{E}$ so that equation (i) above gives $\vec{D} = \varepsilon_0 K\vec{E}$ and equation (31.15) reduces to (31.14).

31.9 ELECTRIC FIELD DUE TO A POINT CHARGE q PLACED IN AN INFINITE DIELECTRIC

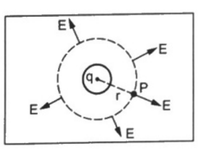

Figure 31.20

Suppose, a point charge q is placed inside an infinite dielectric and we wish to calculate the electric field at a point P at a distance r from the charge q (figure 31.20). We draw a spherical surface through P with the centre at q. From Gauss's law,

$$\oint K\, \vec{E} \cdot d\vec{S} = \frac{q}{\varepsilon_0}$$

or, $$KE\, 4\pi r^2 = \frac{q}{\varepsilon_0}$$

or, $$E = \frac{q}{4\pi\varepsilon_0 K r^2}. \qquad \dots (31.16)$$

The field is radially away from the charge. Note that q is the total *free charge* inside the Gaussian surface.

It should be clear that the field $\frac{q}{4\pi\varepsilon_0 K r^2}$ is due to the free charge q and the polarization charges induced in the dielectric medium. Because of the radially outward field (assuming q to be positive), negative charges shift inward. This produces an induced charge $-q\left(1 - \frac{1}{K}\right)$ on the surface of the cavity in the dielectric in which the charge q is residing. The effective charge is, therefore, $q - q\left(1 - \frac{1}{K}\right) = q/K$ and hence the field is $\frac{q}{4\pi\varepsilon_0 K r^2}$.

31.10 ENERGY IN THE ELECTRIC FIELD IN A DIELECTRIC

Consider a parallel-plate capacitor filled with a dielectric of dielectric constant K. The energy stored in the capacitor is $U = \frac{1}{2} CV^2$. The energy density in the volume between the plates is

$$u = \frac{U}{Ad} = \frac{\frac{1}{2}\left(\frac{K\varepsilon_0 A}{d}\right)V^2}{Ad} = \frac{1}{2} K\varepsilon_0 \left(\frac{V}{d}\right)^2 = \frac{1}{2} K\varepsilon_0 E^2$$

where $E = V/d$ is the electric field between the plates.

We see that the energy density in dielectrics is greater than that in vacuum for the same electric field. The dipole moments interact with each other so as to give this additional energy.

31.11 CORONA DISCHARGE

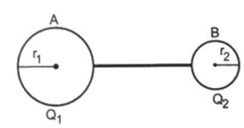

Figure 31.21

Let us consider two conducting spheres A and B connected to each other by a conducting wire. The radius of A is r_1 which is larger than the radius r_2 of B. A charge Q is given to this system. Suppose a part

Q_1 resides on the surface of A and the rest Q_2 on the surface of B. The potential of the sphere A is

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r_1}$$

and that of the sphere B is

$$V_2 = \frac{Q_2}{4\pi\varepsilon_0 r_2}.$$

As the two spheres are connected by a conducting wire, their potentials must be the same. Thus,

$$\frac{Q_1}{4\pi\varepsilon_0 r_1} = \frac{Q_2}{4\pi\varepsilon_0 r_2}$$

or, $$\sigma_1 r_1 = \sigma_2 r_2$$

or, $$\frac{\sigma_1}{\sigma_2} = \frac{r_2}{r_1} \qquad \dots (31.17)$$

where σ_1 and σ_2 are charge densities on the two spheres. We see that the sphere with smaller radius has larger charge density to maintain the same potential.

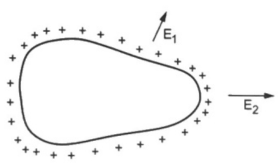

Figure 31.22

Now consider a single conductor with a nonspherical shape. If a charge is given to this conductor (figure 31.22), the charge density will not be uniform on the entire surface. A portion where the surface is more "flat" may be considered as part of a sphere of larger radius. The charge density at such a portion will be smaller from equation (31.17). At portions where the surface is more curved, the charge density will be larger. More precisely, the charge density will be larger where the radius of curvature is small.

The electric field just outside the surface of a conductor is σ/ε_0. Thus, the electric field near the portions of small radius of curvature (more curved part) is large as compared to the field near the portions of large radius of curvature (flatter part). If a conductor has a pointed shape like a needle and a charge is given to it, the charge density at the pointed end will be very high. Correspondingly, the electric field near these pointed ends will be very high which may cause dielectric breakdown in air. The charge may jump from the conductor to the air because of increased conductivity of the air. Often this discharge of air is accompanied by a visible glow surrounding the pointed end. This phenomenon is called *corona discharge.*

31.12 HIGH-VOLTAGE GENERATOR

In 1929, Robert J van de Graaff designed a machine which could produce large electrostatic potential difference, of the order of 10^7 volts. This machine, known as *van de Graaff generator,* is now described.

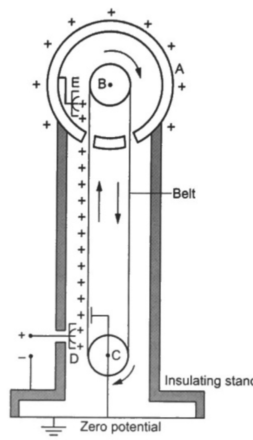

Figure 31.23

A hollow, metallic sphere A is mounted on an insulating stand. A pulley B is mounted at the centre of the sphere and another pulley C is mounted near the bottom. A belt of insulating material (such as silk) goes over the pulleys. The pulley C is continuously driven by an electric motor, or by hand for a smaller machine used for demonstration. The belt, therefore, continuously moves. Two comb-shaped conductors D and E, having a number of metallic needles, are mounted near the pulleys. The needles point towards the belt. The lower comb D is maintained at a positive potential of the order of 10^4 volts by a power supply system. The upper comb E is connected to the metallic sphere A.

Because of the high electric field near the needles of D, the air becomes conducting (corona discharge). The negative charges in the air move towards the needles and the positive charges towards the belt. This positive charge sticks to the belt. The negative charge neutralises some of the positive charge on the comb D. The power supply maintains the positive potential of the needles by supplying more positive charge to it. Effectively, positive charge is transferred from the power supply to the belt. As the belt moves, this positive charge is physically carried upwards. When it reaches near the upper comb E, corona discharge takes place and the air becomes conducting. The negative charges of the air move towards the belt and the positive charges towards the needles of the comb. The negative charges neutralise the positive charge on the belt. The positive charges of the air which have moved to the comb are transferred to the sphere. Effectively, the positive charge on the belt is transferred to the sphere. This positive charge quickly goes to the outer surface of the sphere.

The machine, thus, continuously transfers positive charge to the sphere. The potential of the sphere keeps on increasing. The main limiting factor on the value of this high potential is the radius of the sphere. If the electric field just outside the sphere is sufficient for dielectric breakdown of air, no more charge can be transferred to it. The dielectric strength of air is 3×10^6 V m^{-1}. For a conducting sphere, the electric field just outside the sphere is $E = \frac{Q}{4\pi\varepsilon_0 R^2}$ and the potential of the sphere is $V = \frac{Q}{4\pi\varepsilon_0 R}$. Thus, $V = ER$. To have a field of 3×10^6 V m^{-1} with a sphere of radius 1 m, its potential should be 3×10^6 V. Thus, the potential of a sphere of radius 1 m can be raised to 3×10^6 V by this method. The potential can be increased by enclosing the sphere in a highly evacuated chamber.

Worked Out Examples

1. *A parallel-plate capacitor has plates of area* 200 cm^2 *and separation between the plates* 1·00 mm. *What potential difference will be developed if a charge of* 1·00 nC (*i.e.,* $1{\cdot}00 \times 10^{-9}$ C) *is given to the capacitor? If the plate separation is now increased to* 2·00 mm, *what will be the new potential difference?*

Solution : The capacitance of the capacitor is $C = \frac{\varepsilon_0 A}{d}$

$$= 8{\cdot}85 \times 10^{-12}\ \mathrm{Fm}^{-1} \times \frac{200 \times 10^{-4}\ \mathrm{m}^2}{1 \times 10^{-3}\ \mathrm{m}}$$

$$= 0{\cdot}177 \times 10^{-9}\ \mathrm{F} = 0{\cdot}177\ \mathrm{nF}.$$

The potential difference between the plates is

$$V = \frac{Q}{C} = \frac{1\ \mathrm{nC}}{0{\cdot}177\ \mathrm{nF}} = 5{\cdot}65\ \text{volts}.$$

If the separation is increased from 1·00 mm to 2·00 mm, the capacitance is decreased by a factor of 2. If the charge remains the same, the potential difference will increase by a factor of 2. Thus, the new potential difference will be

$$5{\cdot}65\ \text{volts} \times 2 = 11{\cdot}3\ \text{volts}.$$

2. *An isolated sphere has a capacitance of* 50 pF. *(a) Calculate its radius. (b) How much charge should be placed on it to raise its potential to* 10^4 V?

Solution : (a) The capacitance of an isolated sphere is $C = 4\pi\varepsilon_0 R$. Thus,

$$50 \times 10^{-12}\ \mathrm{F} = \frac{R}{9 \times 10^9\ \mathrm{mF}^{-1}}$$

or, $\quad R = 50 \times 10^{-12} \times 9 \times 10^9\ \mathrm{m} = 45\ \mathrm{cm}.$

(b) $\quad Q = CV$

$$= 50 \times 10^{-12}\ \mathrm{F} \times 10^4\ \mathrm{V} = 0{\cdot}5\ \mu\mathrm{C}.$$

3. *Consider the connections shown in figure (31-W1). (a) Find the capacitance between the points A and B. (b) Find the charges on the three capacitors. (c) Taking the potential at the point B to be zero, find the potential at the point D.*

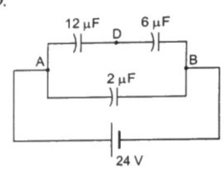

Figure 31-W1

Solution : (a) The 12 μF and 6 μF capacitors are joined in series. The equivalent of these two will have a capacitance given by

$$\frac{1}{C} = \frac{1}{12\ \mu\mathrm{F}} + \frac{1}{6\ \mu\mathrm{F}},$$

or, $\quad C = 4\ \mu\mathrm{F}.$

The combination of these two capacitors is joined in parallel with the 2 μF capacitor. Thus, the equivalent capacitance between A and B is

$$4\ \mu\mathrm{F} + 2\ \mu\mathrm{F} = 6\ \mu\mathrm{F}.$$

(b) The charge supplied by the battery is

$$Q = CV = 6\ \mu\mathrm{F} \times 24\ \mathrm{V} = 144\ \mu\mathrm{C}.$$

The potential difference across the 2 μF capacitor is 24 V. The charge on this capacitor is, therefore,

$$2\ \mu\mathrm{F} \times 24\ \mathrm{V} = 48\ \mu\mathrm{C}.$$

The charge on the 12 μF and 6 μF capacitor is, therefore,

$$144\ \mu\mathrm{C} - 48\ \mu\mathrm{C} = 96\ \mu\mathrm{C}.$$

(c) The potential difference across the 6 μF capacitor is

$$\frac{96\ \mu\mathrm{C}}{6\ \mu\mathrm{F}} = 16\ \mathrm{V}.$$

As the potential at the point B is taken to be zero, the potential at the point D is 16 V.

4. *If 100 volts of potential difference is applied between a and b in the circuit of figure (31-W2a), find the potential difference between c and d.*

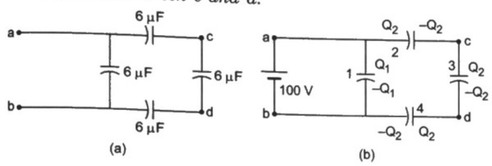

Figure 31-W2

Solution : The charge distribution on different plates is shown in figure (31-W2b). Suppose charge $Q_1 + Q_2$ is given by the positive terminal of the battery, out of which Q_1 resides on the positive plate of capacitor (1) and Q_2 on that of (2). The remaining plates will have charges as shown in the figure.

Take the potential at the point b to be zero. The potential at a will be 100 V. Let the potentials at points c and d be V_c and V_d respectively. Writing the equation $Q = CV$ for the four capacitors, we get,

$$Q_1 = 6\ \mu\text{F} \times 100\ \text{V} = 600\ \mu\text{C} \quad \text{... (i)}$$

$$Q_2 = 6\ \mu\text{F} \times (100\ \text{V} - V_c) \quad \text{... (ii)}$$

$$Q_2 = 6\ \mu\text{F} \times (V_c - V_d) \quad \text{... (iii)}$$

$$Q_2 = 6\ \mu\text{F} \times V_d. \quad \text{... (iv)}$$

From (ii) and (iii),

$$100\ \text{V} - V_c = V_c - V_d$$

or, $$2\,V_c - V_d = 100\ \text{V} \quad \text{... (v)}$$

and from (iii) and (iv),

$$V_c - V_d = V_d$$

or, $$V_c = 2\,V_d. \quad \text{... (vi)}$$

From (v) and (vi),

$$V_d = \frac{100}{3}\ \text{V} \ \text{ and } \ V_c = \frac{200}{3}\ \text{V}$$

so that $V_c - V_d = \dfrac{100}{3}$ V.

5. *Find the charges on the three capacitors shown in figure (31-W3a).*

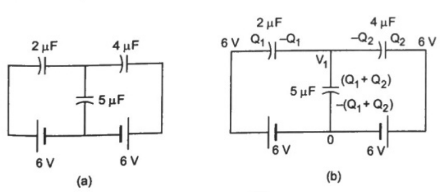

Figure 31-W3

Solution : Take the potential at the junction of the batteries to be zero. Let the left battery supply a charge Q_1 and the right battery a charge Q_2. The charge on the 5 μF capacitor will be $Q_1 + Q_2$. Let the potential at the junction of the capacitors be V_1. The charges at different plates and potentials at different points are shown in figure (31-W3b).

Note that the charges on the three plates which are in contact add to zero. It should be so, because, these plates taken together form an isolated system which cannot receive charges from the batteries. Applying the equation $Q = CV$ to the three capacitors, we get,

$$Q_1 = 2\ \mu\text{F}(6\ \text{V} - V_1) \quad \text{... (i)}$$

$$Q_2 = 4\ \mu\text{F}(6\ \text{V} - V_1) \quad \text{... (ii)}$$

and $$Q_1 + Q_2 = 5\ \mu\text{F}(V_1 - 0). \quad \text{... (iii)}$$

From (i) and (ii),

$$2\,Q_1 - Q_2 = 0 \ \text{ or, } \ Q_2 = 2\,Q_1.$$

From (ii) and (iii),

$$5\,Q_2 + 4(Q_1 + Q_2) = 20\ \mu\text{F} \times 6\ \text{V}$$

or, $$4\,Q_1 + 9\,Q_2 = 120\ \mu\text{C}$$

or, $$4\,Q_1 + 18\,Q_1 = 120\ \mu\text{C}$$

or, $Q_1 = 5{\cdot}45\ \mu\text{C}$ and $Q_2 = 10{\cdot}9\ \mu\text{C}$.

Thus, the charges on the 2 μF, 4 μF and 5 μF capacitors are 5·45 μC, 10·9 μC and 16·35 μC respectively.

6. *Find the equivalent capacitance of the system shown in figure (31-W4a) between the points a and b.*

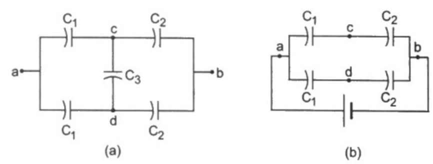

Figure 31-W4

Solution : Suppose, the capacitor C_3 is removed from the given system and a battery is connected between a and b. The remaining system is shown in figure (31-W4b). From the symmetry of the figure, the potential at c will be the same as the potential at d. Thus, if the capacitor C_3 is connected between c and d, it will have no charge. The charges of all the remaining four capacitors will remain unchanged. Thus, the system of capacitors in figure (31-W4a) is equivalent to that in the figure (31-W4b). The equivalent capacitance of the system in figure (31-W4b) can be calculated by applying the formulae for series and parallel combinations. C_1 and C_2 are connected in series. Their equivalent capacitance is

$$C = \frac{C_1 C_2}{C_1 + C_2}.$$

Two such capacitors are joined in parallel. So the equivalent capacitance of the given system is

$$2C = \frac{2\,C_1C_2}{C_1 + C_2}.$$

7. *Find the equivalent capacitance between the point A and B in figure (31-W5a).*

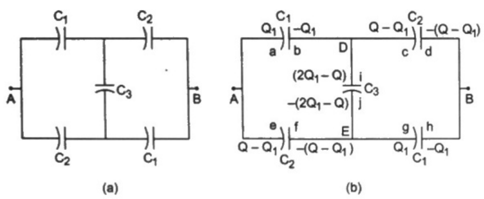

Figure 31-W5

Solution : Let us connect a battery between the points A and B. The charge distribution is shown in figure (31-W5b). Suppose the positive terminal of the battery supplies a charge $+Q$ and the negative terminal a charge $-Q$. The charge Q is divided between plates a and e. A charge Q_1 goes to the plate a and the rest $Q - Q_1$ goes to the plate e. The charge $-Q$ supplied by the negative terminal is divided between plates d and h. Using the symmetry of the figure, charge $-Q_1$ goes to the plate h and $-(Q - Q_1)$ to the plate d. This is because if you look into the circuit from A or from B, the circuit looks identical. The division of charge at A and at B should, therefore, be similar. The charges on the other plates may be written easily. The charge on the plate i is $2\,Q_1 - Q$ which ensures that the total charge on plates b, c and i remains zero as these three plates form an isolated system.

We have,

$$V_A - V_B = (V_A - V_D) + (V_D - V_B)$$

$$= \frac{Q_1}{C_1} + \frac{Q - Q_1}{C_2} \qquad \text{... (i)}$$

Also,

$$V_A - V_B = (V_A - V_D) + (V_D - V_E) + (V_E - V_B)$$

$$= \frac{Q_1}{C_1} + \frac{2\,Q_1 - Q}{C_3} + \frac{Q_1}{C_1}. \qquad \text{... (ii)}$$

We have to eliminate Q_1 from these equations to get the equivalent capacitance $Q/(V_A - V_B)$.

The first equation may be written as

$$V_A - V_B = Q_1\left(\frac{1}{C_1} - \frac{1}{C_2}\right) + \frac{Q}{C_2}$$

or, $$\frac{C_1C_2}{C_2 - C_1}(V_A - V_B) = Q_1 + \frac{C_1}{C_2 - C_1}Q. \qquad \text{... (iii)}$$

The second equation may be written as

$$V_A - V_B = 2\,Q_1\left(\frac{1}{C_1} + \frac{1}{C_3}\right) - \frac{Q}{C_3}$$

or, $$\frac{C_1C_3}{2(C_1 + C_3)}(V_A - V_B) = Q_1 - \frac{C_1}{2(C_1 + C_3)}Q. \qquad \text{... (iv)}$$

Subtracting (iv) from (iii),

$$(V_A - V_B)\left[\frac{C_1C_2}{C_2 - C_1} - \frac{C_1C_3}{2(C_1 + C_3)}\right]$$

$$= \left[\frac{C_1}{C_2 - C_1} + \frac{C_1}{2(C_1 + C_3)}\right]Q$$

or, $$(V_A - V_B)\,[2\,C_1C_2(C_1 + C_3) - C_1C_3(C_2 - C_1)]$$

$$= C_1[2\,(C_1 + C_3) + (C_2 - C_1)]\,Q$$

or, $$C = \frac{Q}{V_A - V_B} = \frac{2\,C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2 + 2\,C_3}.$$

8. *Twelve capacitors, each having a capacitance C, are connected to form a cube (figure 31-W6a). Find the equivalent capacitance between the diagonally opposite corners such as A and B.*

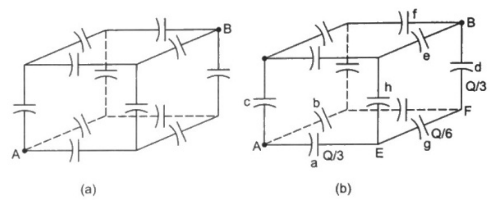

Figure 31-W6

Solution : Suppose the points A and B are connected to a battery. The charges appearing on some of the capacitors are shown in figure (31-W6b). Suppose the positive terminal of the battery supplies a charge $+Q$ through the point A. This charge is divided on the three plates connected to A. Looking from A, the three sides of the cube have identical properties and hence, the charge will be equally distributed on the three plates. Each of the capacitors a, b and c will receive a charge $Q/3$.

The negative terminal of the battery supplies a charge $-Q$ through the point B. This is again divided equally on the three plates connected to B. Each of the capacitors d, e and f gets equal charge $Q/3$.

Now consider the capacitors g and h. As the three plates connected to the point E form an isolated system, their total charge must be zero. The negative plate of the capacitor a has a charge $-Q/3$. The two plates of g and h connected to E should have a total charge $Q/3$. By symmetry, these two plates should have equal charges and hence each of these has a charge $Q/6$.

The capacitors a, g and d have charges $Q/3$, $Q/6$ and $Q/3$ respectively.

We have,

$$V_A - V_B = (V_A - V_E) + (V_E - V_F) + (V_F - V_B)$$

$$= \frac{Q/3}{C} + \frac{Q/6}{C} + \frac{Q/3}{C} = \frac{5Q}{6C}$$

or, $$C_{eq} = \frac{Q}{V_A - V_B} = \frac{6}{5}C.$$

9. *The negative plate of a parallel plate capacitor is given a charge of -20×10^{-8} C. Find the charges appearing on the four surfaces of the capacitor plates.*

***Solution* :**

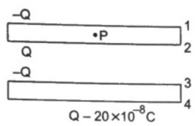

Figure 31-W7

Let the charge appearing on the inner surface of the negative plate be $-Q$. The charge on its outer surface will be $Q - 20 \times 10^{-8}$ C.

The charge on the inner surface of the positive plate will be $+Q$ from Gauss's law and that on the outer surface will be $-Q$ as the positive plate is electrically neutral. The distribution is shown in figure (31-W7).

To obtain the value of Q, consider the electric field at a point P inside the upper plate.

Field due to surface (1) $= \frac{Q}{2\varepsilon_0 A}$ upward,

due to surface (2) $= \frac{Q}{2\varepsilon_0 A}$ upward,

due to surface (3) $= \frac{Q}{2\varepsilon_0 A}$ downward

and due to surface (4) $= \frac{Q - 20 \times 10^{-8}\text{ C}}{2\varepsilon_0 A}$ upward.

As P is a point inside the conductor, the field here must be zero. Thus,

$$Q = -Q + 20 \times 10^{-8}\text{ C}$$

or, $$Q = 10 \times 10^{-8}\text{ C}.$$

The charges on the four surfaces may be written immediately from figure (31-W7).

10. *Three capacitors of capacitances 2 μF, 3 μF and 6 μF are connected in series with a 12 V battery. All the connecting wires are disconnected, the three positive plates are connected together and the three negative plates are connected together. Find the charges on the three capacitors after the reconnection.*

Solution : The equivalent capacitance of the three capacitors joined in series is given by

$$\frac{1}{C} = \frac{1}{2\,\mu\text{F}} + \frac{1}{3\,\mu\text{F}} + \frac{1}{6\,\mu\text{F}}$$

or, $$C = 1\,\mu\text{F}.$$

The charge supplied by the battery $= 1\,\mu\text{F} \times 12\text{ V}$
$= 12\,\mu\text{C}$.

As the capacitors are connected in series, $12\,\mu$C charge appears on each of the positive plates and $-12\,\mu$C on each of the negative plates. The charged capacitors are now connected as shown in figure (31-W8).

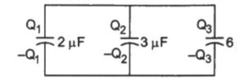

Figure 31-W8

The $36\,\mu$C charge on the three positive plates now redistribute as Q_1, Q_2 and Q_3 on the three connected positive plates. Similarly, $-36\,\mu$C redistributes as $-Q_1$, $-Q_2$ and $-Q_3$. The three positive plates are now at a common potential and the three negative plates are also at a common potential. Let the potential difference across each capacitor be V. Then

$$Q_1 = (2\,\mu\text{F})\,V,$$
$$Q_2 = (3\,\mu\text{F})\,V,$$

and $$Q_3 = (6\,\mu\text{F})\,V.$$

Also, $$Q_1 + Q_2 + Q_3 = 36\,\mu\text{C}.$$

Solving these equations,

$$Q_1 = \frac{72}{11}\,\mu\text{C},\ Q_2 = \frac{108}{11}\,\mu\text{C and } Q_3 = \frac{216}{11}\,\mu\text{C}.$$

11. *The connections shown in figure (31-W9a) are established with the switch S open. How much charge will flow through the switch if it is closed?*

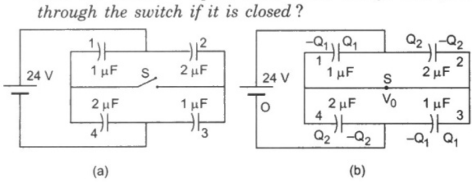

Figure 31-W9

Solution : When the switch is open, capacitors (2) and (3) are in series. Their equivalent capacitance is $\frac{2}{3}\,\mu$F. The charge appearing on each of these capacitors is, therefore, $24\text{ V} \times \frac{2}{3}\,\mu\text{F} = 16\,\mu\text{C}$.

The equivalent capacitance of (1) and (4), which are also connected in series, is also $\frac{2}{3}\,\mu$F and the charge on each of these capacitors is also $16\,\mu$C. The total charge on the two plates of (1) and (4) connected to the switch is, therefore, zero.

The situation when the switch S is closed is shown in figure (31-W9b). Let the charges be distributed as shown in the figure. Q_1 and Q_2 are arbitrarily chosen for the positive plates of (1) and (2). The same magnitude of charges will appear at the negative plates of (3) and (4).

Take the potential at the negative terminal to be zero and at the switch to be V_0.

Writing equations for the capacitors (1), (2), (3) and (4),

$$Q_1 = (24\text{ V} - V_0) \times 1\ \mu\text{F} \quad \ldots \text{(i)}$$

$$Q_2 = (24\text{ V} - V_0) \times 2\ \mu\text{F} \quad \ldots \text{(ii)}$$

$$Q_1 = V_0 \times 1\ \mu\text{F} \quad \ldots \text{(iii)}$$

$$Q_2 = V_0 \times 2\ \mu\text{F}. \quad \ldots \text{(iv)}$$

From (i) and (iii), $V_0 = 12$ V.

Thus, from (iii) and (iv),

$$Q_1 = 12\ \mu\text{C} \text{ and } Q_2 = 24\ \mu\text{C}.$$

The charge on the two plates of (1) and (4) which are connected to the switch is, therefore, $Q_2 - Q_1 = 12\ \mu$C.

When the switch was open, this charge was zero. Thus, 12 μC of charge has passed through the switch after it was closed.

12. *Each of the three plates shown in figure (31-W10a) has an area of 200 cm^2 on one side and the gap between the adjacent plates is 0·2 mm. The emf of the battery is 20 V. Find the distribution of charge on various surfaces of the plates. What is the equivalent capacitance of the system between the terminal points?*

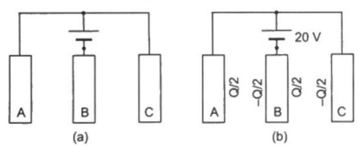

Figure 31-W10

Solution : Suppose the negative terminal of the battery gives a charge $-Q$ to the plate B. As the situation is symmetric on the two sides of B, the two faces of the plate B will share equal charges $-Q/2$ each. From Gauss's law, the facing surfaces will have charges $Q/2$ each. As the positive terminal of the battery has supplied just this much charge ($+Q$) to A and C, the outer surfaces of A and C will have no charge. The distribution will be as shown in figure (31-W10b).

The capacitance between the plates A and B is

$$8{\cdot}85 \times 10^{-12}\text{ F m}^{-1} \times \frac{200 \times 10^{-4}\text{ m}^2}{2 \times 10^{-4}\text{ m}}$$

$$= 8{\cdot}85 \times 10^{-10}\text{ F} = 0{\cdot}885\text{ nF}.$$

Thus, $\quad \frac{Q}{2} = 0{\cdot}885\text{ nF} \times 20\text{ V} = 17{\cdot}7\text{ nC}.$

The distribution of charge on various surfaces may be written from figure (31-W10b).

The equivalent capacitance is

$$\frac{Q}{20\text{ V}} = 1{\cdot}77\text{ nF}.$$

13. *Find the capacitance of the infinite ladder shown in figure (31-W11).*

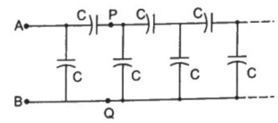

Figure 31-W11

Solution : As the ladder is infinitely long, the capacitance of the ladder to the right of the points P, Q is the same as that of the ladder to the right of the points A, B. If the equivalent capacitance of the ladder is C_1, the given ladder may be replaced by the connections shown in figure (31-W12).

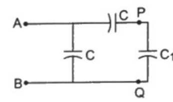

Figure 31-W12

The equivalent capacitance between A and B is easily found to be $C + \frac{CC_1}{C + C_1}\cdot$ But being equivalent to the original ladder, the equivalent capacitance is also C_1.

Thus, $$C_1 = C + \frac{CC_1}{C + C_1}$$

or, $$C_1C + C_1^2 = C^2 + 2\,CC_1$$

or, $$C_1^2 - CC_1 - C^2 = 0$$

giving $$C_1 = \frac{C + \sqrt{C^2 + 4\,C^2}}{2} = \frac{1 + \sqrt{5}}{2}\,C.$$

Negative value of C_1 is rejected.

14. *Find the energy stored in the electric field produced by a metal sphere of radius R containing a charge Q.*

Solution : Consider a thin spherical shell of radius x ($> R$) and thickness dx concentric with the given metal sphere. The energy density in the shell is

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 x^2}\right)^2.$$

The volume of the shell is $4\pi x^2 dx$. The energy contained in the shell is, therefore,

$$dU = \frac{1}{2}\varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 x^2}\right)^2 \times 4\pi x^2 dx = \frac{Q^2 dx}{8\pi\varepsilon_0 x^2}\cdot$$

The energy contained in the whole space outside the sphere is

$$U = \int_R^\infty \frac{Q^2 dx}{8\pi\varepsilon_0 x^2} = \frac{Q^2}{8\pi\varepsilon_0 R}\cdot$$

As the field inside the sphere is zero, this is also the total energy stored in the field.

Alternative Method

Considering a concentric spherical shell at infinity, we have a spherical capacitor. The capacitance is $C = 4\pi\varepsilon_0 R$. The energy stored in this capacitor is the energy stored in the entire electric field. This energy is

$$U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\varepsilon_0 R}.$$

15. *A capacitor of capacitance C is charged by connecting it to a battery of emf $\mathcal{E}$. The capacitor is now disconnected and reconnected to the battery with the polarity reversed. Calculate the heat developed in the connecting wires.*

Solution : When the capacitor is connected to the battery, a charge $Q = C\mathcal{E}$ appears on one plate and $-Q$ on the other. When the polarity is reversed, a charge $-Q$ appears on the first plate and $+Q$ on the second. A charge $2Q$, therefore, passes through the battery from the negative to the positive terminal. The battery does a work

$$W = (2Q)\mathcal{E} = 2C\mathcal{E}^2$$

in the process. The energy stored in the capacitor is the same in the two cases. Thus, the work done by the battery appears as heat in the connecting wires. The heat produced is, therefore, $2C\mathcal{E}^2$.

16. *An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.*

Solution : Suppose the capacitance of the capacitor is C and the emf of the battery is V. The charge given to the capacitor is $Q = CV$. The work done by the battery is

$$W = QV.$$

The battery supplies this energy. The energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV.$$

The remaining energy $QV - \frac{1}{2}QV = \frac{1}{2}QV$ is lost as heat. Thus, half the energy supplied by the battery is lost as heat.

17. *A parallel-plate capacitor having plate area* 100 cm^2 *and separation* 1·0 mm *holds a charge of* 0·12 μC *when connected to a* 120 V *battery. Find the dielectric constant of the material filling the gap.*

Solution :

The capacitance of the capacitor is

$$\frac{0{\cdot}12\ \mu\text{C}}{120\ \text{V}} = 1{\cdot}0 \times 10^{-9}\ \text{F}.$$

If K be the dielectric constant, the capacitance is also given by $\frac{K\varepsilon_0 A}{d}$. Thus,

$$\frac{K \times 8{\cdot}85 \times 10^{-12}\ \text{F m}^{-1} \times 100 \times 10^{-4}\ \text{m}^2}{1{\cdot}0 \times 10^{-3}\ \text{m}} = 1{\cdot}0 \times 10^{-9}\ \text{F}$$

or, $$K = 11{\cdot}3.$$

18. *A parallel-plate capacitor is formed by two plates, each of area* 100 cm^2, *separated by a distance of* 1 mm. *A dielectric of dielectric constant* 5·0 *and dielectric strength* $1{\cdot}9 \times 10^7\ \text{V m}^{-1}$ *is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.*

Solution : If Q be the charge on the capacitor, the surface charge density is $\sigma = Q/A$ and the electric field is $\frac{Q}{KA\varepsilon_0}$. This should not exceed the dielectric strength $1{\cdot}9 \times 10^7\ \text{V m}^{-1}$. Thus, the maximum charge is given by

$$\frac{Q}{KA\varepsilon_0} = 1{\cdot}9 \times 10^7\ \text{V m}^{-1}$$

or, $$Q = KA\varepsilon_0 \times 1{\cdot}9 \times 10^7\ \text{V m}^{-1}$$

$$= (5{\cdot}0)\,(10^{-2}\ \text{m}^2)\,(8{\cdot}85 \times 10^{-12}\ \text{F m}^{-1}) \times (1{\cdot}9 \times 10^7\ \text{V m}^{-1})$$

$$= 8{\cdot}4 \times 10^{-6}\ \text{C}.$$

19. *The space between the plates of a parallel-plate capacitor of capacitance C is filled with three dielectric slabs of identical size as shown in figure (31-W13). If the dielectric constants of the three slabs are K_1, K_2 and K_3, find the new capacitance.*

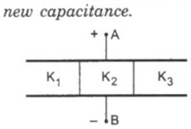

Figure 31-W13

Solution : Consider each one third of the assembly as a separate capacitor. The three positive plates are connected together at point A and the three negative plates are connected together at point B. Thus, the three capacitors are joined in parallel. As the plate area is one third of the original for each part, the capacitances of these parts will be $K_1C/3$, $K_2C/3$ and $K_3C/3$. The equivalent capacitance is, therefore,

$$C_{eq} = (K_1 + K_2 + K_3)\frac{C}{3}.$$

20. *Figure (31-W14a) shows a parallel-plate capacitor having square plates of edge a and plate-separation d. The gap between the plates is filled with a dielectric of dielectric constant K which varies parallel to an edge as*

$$K = K_0 + \alpha x$$

Figure 31-W14

where K and α are constants and x is the distance from the left end. Calculate the capacitance.

Solution :

Consider a small strip of width dx at a separation x from the left end (figure 31-W14b). This strip forms a small capacitor of plate area adx. Its capacitance is

$$dC = \frac{(K_0 + \alpha x)\varepsilon_0 a\,dx}{d}.$$

The given capacitor may be divided into such strips with x varying from 0 to a. All these strips are connected in parallel. The capacitance of the given capacitor is,

$$C = \int_0^a \frac{(K_0 + \alpha x)\varepsilon_0 a dx}{d}$$

$$= \frac{\varepsilon_0 a^2}{d}\left(K_0 + \frac{a\alpha}{2}\right).$$

21. *A parallel-plate capacitor of capacitance 100 μF is connected to a power supply of 200 V. A dielectric slab of dielectric constant 5 is now inserted into the gap between the plates. (a) Find the extra charge flown through the power supply and the work done by the supply. (b) Find the change in the electrostatic energy of the electric field in the capacitor.*

Solution : (a) The original capacitance was 100 μF. The charge on the capacitor before the insertion of the dielectric was, therefore,

$$Q_1 = 100\ \mu\text{F} \times 200\ \text{V} = 20\ \text{mC}.$$

After the dielectric slab is introduced, the capacitance is increased to 500 μF. The new charge on the capacitor is, therefore, 500 μF × 200 V = 100 mC. The charge flown through the power supply is, therefore, 100 mC − 20 mC = 80 mC. The work done by the power supply is 200 V × 80 mC = 16 J.

(b) The electrostatic field energy of the capacitor without the dielectric slab is

$$U_1 = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times (100\ \mu\text{F}) \times (200\ \text{V})^2 = 2\ \text{J}$$

and that after the slab is inserted is

$$U_2 = \frac{1}{2} \times (500\ \mu\text{F}) \times (200\ \text{V})^2 = 10\ \text{J}.$$

Thus, the energy is increased by 8 J.

22. *Figure (31-W15) shows a parallel-plate capacitor with plates of width b and length l. The separation between the plates is d. The plates are rigidly clamped and connected to a battery of emf V. A dielectric slab of thickness d and dielectric constant K is slowly inserted between the plates. (a) Calculate the energy of the system when a length x of the slab is introduced into the capacitor. (b) What force should be applied on the slab to ensure that it goes slowly into the capacitor? Neglect any effect of friction or gravity.*

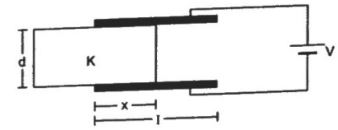

Figure 31-W15

Solution : (a) The plate area of the part with the dielectric is bx. Its capacitance is

$$C_1 = \frac{K\varepsilon_0 bx}{d}.$$

Similarly, the capacitance of the part without the dielectric is

$$C_2 = \frac{\varepsilon_0 b(l - x)}{d}.$$

These two parts are connected in parallel. The capacitance of the system is, therefore,

$$C = C_1 + C_2$$

$$= \frac{\varepsilon_0 b}{d}[l + x(K-1)]. \quad \ldots \text{(i)}$$

The energy of the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{\varepsilon_0 bV^2}{2d}[l + x(K-1)].$$

(b) Suppose, the electric field attracts the dielectric slab with a force F. An external force of equal magnitude F should be applied in opposite direction so that the plate moves slowly (no acceleration).

Consider the part of motion in which the dielectric moves a distance dx further inside the capacitor. The capacitance increases to $C + dC$. As the potential difference remains constant at V, the battery has to supply a further charge

$$dQ = (dC)V$$

to the capacitor. The work done by the battery is, therefore,

$$dW_b = V\,dQ = (dC)V^2.$$

The external force F does a work

$$dW_e = (-F\,dx)$$

during the displacement. The total work done on the capacitor is

$$dW_b + dW_e = (dC)V^2 - Fdx.$$

This should be equal to the increase dU in the stored energy. Thus,

$$\frac{1}{2}(dC)V^2 = (dC)V^2 - Fdx$$

or, $$F = \frac{1}{2}V^2\frac{dC}{dx}.$$

OBJECTIVE I

1. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the capacitor is
(a) $\frac{CV}{\varepsilon_0}$ (b) $\frac{2\,CV}{\varepsilon_0}$ (c) $\frac{CV}{2\varepsilon_0}$ (d) zero.

2. Two capacitors each having capacitance C and breakdown voltage V are joined in series. The capacitance and the breakdown voltage of the combination will be
(a) $2\,C$ and $2\,V$ (b) $C/2$ and $V/2$
(c) $2\,C$ and $V/2$ (d) $C/2$ and $2\,V$.

3. If the capacitors in the previous question are joined in parallel, the capacitance and the breakdown voltage of the combination will be
(a) $2\,C$ and $2\,V$ (b) C and $2\,V$
(c) $2\,C$ and V (d) C and V.

4. The equivalent capacitance of the combination shown in figure (31-Q1) is
(a) C (b) $2\,C$ (c) $C/2$ (d) none of these.

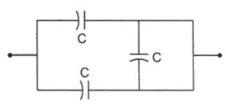

Figure 31-Q1

5. A dielectric slab is inserted between the plates of an isolated capacitor. The force between the plates will
(a) increase (b) decrease
(c) remain unchanged (d) become zero.

6. The energy density in the electric field created by a point charge falls off with the distance from the point charge as
(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$.

7. A parallel-plate capacitor has plates of unequal area. The larger plate is connected to the positive terminal of the battery and the smaller plate to its negative terminal. Let Q_+ and Q_- be the charges appearing on the positive and negative plates respectively.
(a) $Q_+ > Q_-$ (b) $Q_+ = Q_-$ (c) $Q_+ < Q_-$
(d) The information is not sufficient to decide the relation between Q_+ and Q_-.

8. A thin metal plate P is inserted between the plates of a parallel-plate capacitor of capacitance C in such a way that its edges touch the two plates (figure 31-Q2). The capacitance now becomes
(a) $C/2$ (b) $2\,C$ (c) 0 (d) indeterminate.

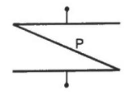

Figure 31-Q2

9. Figure (31-Q3) shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors.
(a) $C_1 > C_2$ (b) $C_1 = C_2$ (c) $C_1 < C_2$
(d) The information is not sufficient to decide the relation between C_1 and C_2.

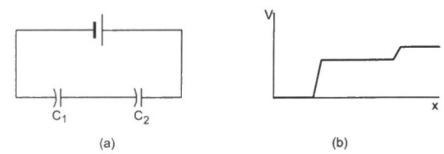

Figure 31-Q3

10. Two metal plates having charges Q, $-Q$ face each other at some separation and are dipped into an oil tank. If the oil is pumped out, the electric field between the plates will
(a) increase (b) decrease
(c) remain the same (d) become zero.

11. Two metal spheres of capacitances C_1 and C_2 carry some charges. They are put in contact and then separated. The final charges Q_1 and Q_2 on them will satisfy
(a) $\frac{Q_1}{Q_2} < \frac{C_1}{C_2}$ (b) $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$ (c) $\frac{Q_1}{Q_2} > \frac{C_1}{C_2}$ (d) $\frac{Q_1}{Q_2} = \frac{C_2}{C_1}$.

12. Three capacitors of capacitances 6 μF each are available. The minimum and maximum capacitances, which may be obtained are
(a) 6 μF, 18 μF (b) 3 μF, 12 μF
(c) 2 μF, 12 μF (d) 2 μF, 18 μF.

OBJECTIVE II

1. The capacitance of a capacitor does not depend on
(a) the shape of the plates
(b) the size of the plates
(c) the charges on the plates
(d) the separation between the plates.

2. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain the same?
(a) The electric field in the capacitor
(b) The charge on the capacitor

(c) The potential difference between the plates
(d) The stored energy in the capacitor

3. A dielectric slab is inserted between the plates of a capacitor. The charge on the capacitor is Q and the magnitude of the induced charge on each surface of the dielectric is Q'.
(a) Q' may be larger than Q.
(b) Q' must be larger than Q.
(c) Q' must be equal to Q.
(d) Q' must be smaller than Q.

4. Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now,
(a) the facing surfaces of the capacitor have equal and opposite charges
(b) the two plates of the capacitor have equal and opposite charges
(c) the battery supplies equal and opposite charges to the two plates
(d) the outer surfaces of the plates have equal charges

5. The separation between the plates of a charged parallel-plate capacitor is increased. Which of the following quantities will change ?
(a) Charge on the capacitor
(b) Potential difference across the capacitor
(c) Energy of the capacitor
(d) Energy density between the plates

6. A parallel-plate capacitor is connected to a battery. A metal sheet of negligible thickness is placed between the plates. The sheet remains parallel to the plates of the capacitor.
(a) The battery will supply more charge.
(b) The capacitance will increase.
(c) The potential difference between the plates will increase.
(d) Equal and opposite charges will appear on the two faces of the metal plate.

7. Following operations can be performed on a capacitor:
X – connect the capacitor to a battery of emf $\mathcal{E}$.
Y – disconnect the battery.
Z – reconnect the battery with polarity reversed.
W – insert a dielectric slab in the capacitor.
(a) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed.
(b) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.
(c) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.
(d) The electric field in the capacitor after the action XW is the same as that after WX.

EXERCISES

1. When $1{\cdot}0 \times 10^{12}$ electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Calculate the capacitance of the two-conductor system.

2. The plates of a parallel-plate capacitor are made of circular discs of radii 5·0 cm each. If the separation between the plates is 1·0 mm, what is the capacitance ?

3. Suppose, one wishes to construct a 1·0 farad capacitor using circular discs. If the separation between the discs be kept at 1·0 mm, what would be the radius of the discs ?

4. A parallel-plate capacitor having plate area 25 cm^2 and separation 1·00 mm is connected to a battery of 6·0 V. Calculate the charge flown through the battery. How much work has been done by the battery during the process ?

5. A parallel-plate capacitor has plate area 25·0 cm^2 and a separation of 2·00 mm between the plates. The capacitor is connected to a battery of 12·0 V. (a) Find the charge on the capacitor. (b) The plate separation is decreased to 1·00 mm. Find the extra charge given by the battery to the positive plate.

6. Find the charges on the three capacitors connected to a battery as shown in figure (31-E1). Take $C_1 = 2{\cdot}0\ \mu F$, $C_2 = 4{\cdot}0\ \mu F$, $C_3 = 6{\cdot}0\ \mu F$ and $V = 12$ volts.

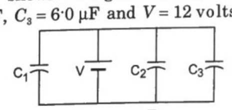

Figure 31-E1

7. Three capacitors having capacitances 20 μF, 30 μF and 40 μF are connected in series with a 12 V battery. Find the charge on each of the capacitors. How much work has been done by the battery in charging the capacitors ?

8. Find the charge appearing on each of the three capacitors shown in figure (31-E2).

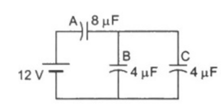

Figure 31-E2

9. Take $C_1 = 4{\cdot}0\ \mu F$ and $C_2 = 6{\cdot}0\ \mu F$ in figure (31-E3). Calculate the equivalent capacitance of the combination between the points indicated.

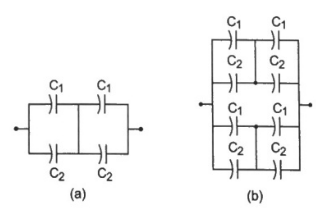

Figure 31-E3

10. Find the charge supplied by the battery in the arrangement shown in figure (31-E4).

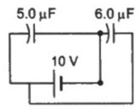

Figure 31-E4

11. The outer cylinders of two cylindrical capacitors of capacitance 2·2 μF each, are kept in contact and the inner cylinders are connected through a wire. A battery of emf 10 V is connected as shown in figure (31-E5). Find the total charge supplied by the battery to the inner cylinders.

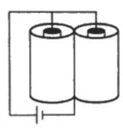

Figure 31-E5

12. Two conducting spheres of radii R_1 and R_2 are kept widely separated from each other. What are their individual capacitances ? If the spheres are connected by a metal wire, what will be the capacitance of the combination ? Think in terms of series–parallel connections.

13. Each of the capacitors shown in figure (31-E6) has a capacitance of 2 μF. Find the equivalent capacitance of the assembly between the points A and B. Suppose, a battery of emf 60 volts is connected between A and B. Find the potential difference appearing on the individual capacitors.

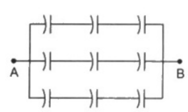

Figure 31-E6

14. It is required to construct a 10 μF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 μF are available but they can withstand only 50 V. Design a combination which can yield the desired result.

15. Take the potential of the point B in figure (31-E7) to be zero. (a) Find the potentials at the points C and D. (b) If a capacitor is connected between C and D, what charge will appear on this capacitor ?

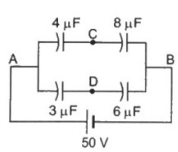

Figure 31-E7

16. Find the equivalent capacitance of the system shown in figure (31-E8) between the points a and b.

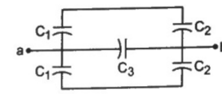

Figure 31-E8

17. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure (31-E9). The width of each stair is a and the height is b. Find the capacitance of the assembly.

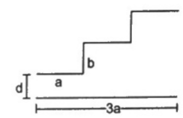

Figure 31-E9

18. A cylindrical capacitor is constructed using two coaxial cylinders of the same length 10 cm and of radii 2 mm and 4 mm. (a) Calculate the capacitance. (b) Another capacitor of the same length is constructed with cylinders of radii 4 mm and 8 mm. Calculate the capacitance.

19. A 100 pF capacitor is charged to a potential difference of 24 V. It is connected to an uncharged capacitor of capacitance 20 pF. What will be the new potential difference across the 100 pF capacitor ?

20. Each capacitor shown in figure (31-E10) has a capacitance of 5·0 μF. The emf of the battery is 50 V. How much charge will flow through AB if the switch S is closed ?

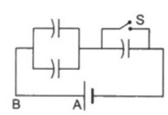

Figure 31-E10

21. The particle P shown in figure (31-E11) has a mass of 10 mg and a charge of − 0·01 μC. Each plate has a surface area 100 cm^2 on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium ?

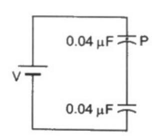

Figure 31-E11

22. Both the capacitors shown in figure (31-E12) are made of square plates of edge a. The separations between the plates of the capacitors are d_1 and d_2 as shown in the

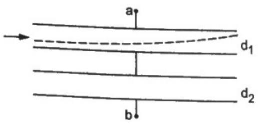

Figure 31-E12

figure. A potential difference V is applied between the points a and b. An electron is projected between the plates of the upper capacitor along the central line. With what minimum speed should the electron be projected so that it does not collide with any plate? Consider only the electric forces.

23. The plates of a capacitor are 2·00 cm apart. An electron–proton pair is released somewhere in the gap between the plates and it is found that the proton reaches the negative plate at the same time as the electron reaches the positive plate. At what distance from the negative plate was the pair released?

24. Convince yourself that parts (a), (b) and (c) of figure (31-E13) are identical. Find the capacitance between the points A and B of the assembly.

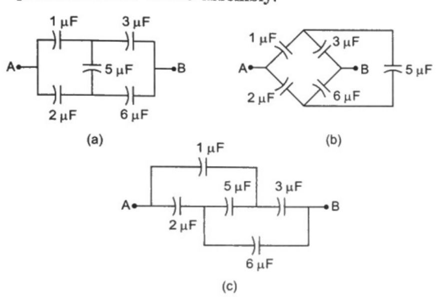

Figure 31-E13

25. Find the potential difference $V_a - V_b$ between the points a and b shown in each part of the figure (31-E14).

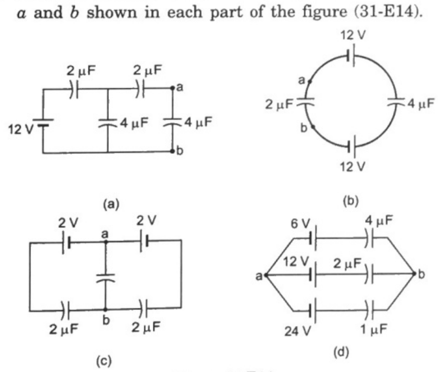

Figure 31-E14

26. Find the equivalent capacitances of the combinations shown in figure (31-E15) between the indicated points.

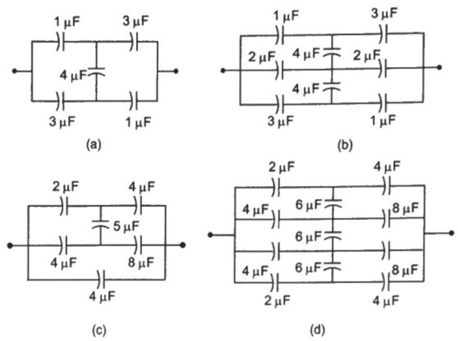

Figure 31-E15

27. Find the capacitance of the combination shown in figure (31-E16) between A and B.

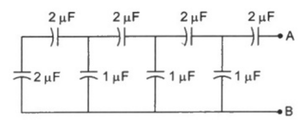

Figure 31-E16

28. Find the equivalent capacitance of the infinite ladder shown in figure (31-E17) between the points A and B.

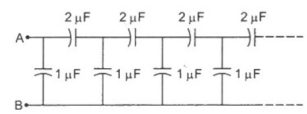

Figure 31-E17

29. A finite ladder is constructed by connecting several sections of 2 μF, 4 μF capacitor combinations as shown in figure (31-E18). It is terminated by a capacitor of capacitance C. What value should be chosen for C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between?

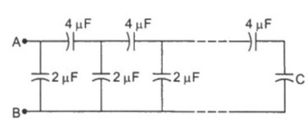

Figure 31-E18

30. A charge of $+ 2{\cdot}0 \times 10^{-8}$ C is placed on the positive plate and a charge of $- 1{\cdot}0 \times 10^{-8}$ C on the negative plate of a parallel-plate capacitor of capacitance $1{\cdot}2 \times 10^{-3}$ μF.

Calculate the potential difference developed between the plates.

31. A charge of 20 μC is placed on the positive plate of an isolated parallel-plate capacitor of capacitance 10 μF. Calculate the potential difference developed between the plates.

32. A charge of 1 μC is given to one plate of a parallel-plate capacitor of capacitance 0·1 μF and a charge of 2 μC is given to the other plate. Find the potential difference developed between the plates.

33. Each of the plates shown in figure (31-E19) has surface area $(96/\varepsilon_0) \times 10^{-12}$ Fm on one side and the separation between the consecutive plates is 4·0 mm. The emf of the battery connected is 10 volts. Find the magnitude of the charge supplied by the battery to each of the plates connected to it.

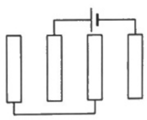

Figure 31-E19

34. The capacitance between the adjacent plates shown in figure (31-E20) is 50 nF. A charge of 1·0 μC is placed on the middle plate. (a) What will be the charge on the outer surface of the upper plate ? (b) Find the potential difference developed between the upper and the middle plates.

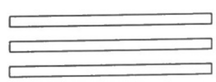

Figure 31-E20

35. Consider the situation of the previous problem. If 1·0 μC is placed on the upper plate instead of the middle, what will be the potential difference between (a) the upper and the middle plates and (b) the middle and the lower plates ?

36. Two capacitors of capacitances 20·0 pF and 50·0 pF are connected in series with a 6·00 V battery. Find (a) the potential difference across each capacitor and (b) the energy stored in each capacitor.

37. Two capacitors of capacitances 4·0 μF and 6·0 μF are connected in series with a battery of 20 V. Find the energy supplied by the battery.

38. Each capacitor in figure (31-E21) has a capacitance of 10 μF. The emf of the battery is 100 V. Find the energy stored in each of the four capacitors.

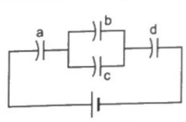

Figure 31-E21

39. A capacitor with stored energy 4·0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored in the two capacitors.

40. A capacitor of capacitance 2·0 μF is charged to a potential difference of 12 V. It is then connected to an uncharged capacitor of capacitance 4·0 μF as shown in figure (31-E22). Find (a) the charge on each of the two capacitors after the connection, (b) the electrostatic energy stored in each of the two capacitors and (c) the heat produced during the charge transfer from one capacitor to the other.

Figure 31-E22

41. A point charge Q is placed at the origin. Find the electrostatic energy stored outside the sphere of radius R centred at the origin.

42. A metal sphere of radius R is charged to a potential V. (a) Find the electrostatic energy stored in the electric field within a concentric sphere of radius $2R$. (b) Show that the electrostatic field energy stored outside the sphere of radius $2R$ equals that stored within it.

43. A large conducting plane has a surface charge density $1{\cdot}0 \times 10^{-4}$ C m^{-2}. Find the electrostatic energy stored in a cubical volume of edge 1·0 cm in front of the plane.

44. A parallel-plate capacitor having plate area 20 cm^2 and separation between the plates 1·00 mm is connected to a battery of 12·0 V. The plates are pulled apart to increase the separation to 2·0 mm. (a) Calculate the charge flown through the circuit during the process. (b) How much energy is absorbed by the battery during the process ? (c) Calculate the stored energy in the electric field before and after the process. (d) Using the expression for the force between the plates, find the work done by the person pulling the plates apart. (e) Show and justify that no heat is produced during this transfer of charge as the separation is increased.

45. A capacitor having a capacitance of 100 μF is charged to a potential difference of 24 V. The charging battery is disconnected and the capacitor is connected to another battery of emf 12 V with the positive plate of the capacitor joined with the positive terminal of the battery. (a) Find the charges on the capacitor before and after the reconnection. (b) Find the charge flown through the 12 V battery. (c) Is work done by the battery or is it done on the battery ? Find its magnitude. (d) Find the decrease in electrostatic field energy. (e) Find the heat developed during the flow of charge after reconnection.

46. Consider the situation shown in figure (31-E23). The switch S is open for a long time and then closed. (a) Find the charge flown through the battery when the switch S is closed. (b) Find the work done by the battery.

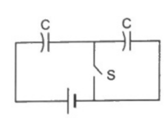

Figure 31-E23

(c) Find the change in energy stored in the capacitors.
(d) Find the heat developed in the system.

47. A capacitor of capacitance 5·00 μF is charged to 24·0 V and another capacitor of capacitance 6·0 μF is charged to 12·0 V. (a) Find the energy stored in each capacitor. (b) The positive plate of the first capacitor is now connected to the negative plate of the second and vice versa. Find the new charges on the capacitors. (c) Find the loss of electrostatic energy during the process. (d) Where does this energy go?

48. A 5·0 μF capacitor is charged to 12 V. The positive plate of this capacitor is now connected to the negative terminal of a 12 V battery and vice versa. Calculate the heat developed in the connecting wires.

49. The two square faces of a rectangular dielectric slab (dielectric constant 4·0) of dimensions 20 cm × 20 cm × 1·0 mm are metal-coated. Find the capacitance between the coated surfaces.

50. If the above capacitor is connected across a 6·0 V battery, find (a) the charge supplied by the battery, (b) the induced charge on the dielectric and (c) the net charge appearing on one of the coated surfaces.

51. The separation between the plates of a parallel-plate capacitor is 0·500 cm and its plate area is 100 cm^2. A 0·400 cm thick metal plate is inserted into the gap with its faces parallel to the plates. Show that the capacitance of the assembly is independent of the position of the metal plate within the gap and find its value.

52. A capacitor stores 50 μC charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 100 μC flows through the battery. Find the dielectric constant of the material inserted.

53. A parallel-plate capacitor of capacitance 5 μF is connected to a battery of emf 6 V. The separation between the plates is 2 mm. (a) Find the charge on the positive plate. (b) Find the electric field between the plates. (c) A dielectric slab of thickness 1 mm and dielectric constant 5 is inserted into the gap to occupy the lower half of it. Find the capacitance of the new combination. (d) How much charge has flown through the battery after the slab is inserted?

54. A parallel-plate capacitor has plate area 100 cm^2 and plate separation 1·0 cm. A glass plate (dielectric constant 6·0) of thickness 6·0 mm and an ebonite plate (dielectric constant 4·0) are inserted one over the other to fill the space between the plates of the capacitor. Find the new capacitance.

55. A parallel-plate capacitor having plate area 400 cm^2 and separation between the plates 1·0 mm is connected to a power supply of 100 V. A dielectric slab of thickness 1·0 mm and dielectric constant 5·0 is inserted into the gap. (a) Find the increase in electrostatic energy. (b) If the power supply is now disconnected and the dielectric slab is taken out, find the further increase in energy. (c) Why does the energy increase in inserting the slab as well as in taking it out?

56. Find the capacitances of the capacitors shown in figure (31-E24). The plate area is A and the separation between the plates is d. Different dielectric slabs in a particular part of the figure are of the same thickness and the entire gap between the plates is filled with the dielectric slabs.

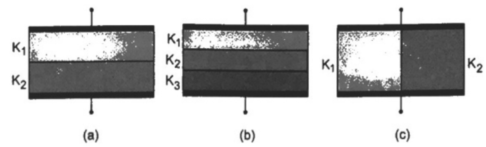

Figure 31-E24

57. A capacitor is formed by two square metal-plates of edge a, separated by a distance d. Dielectrics of dielectric constants K_1 and K_2 are filled in the gap as shown in figure (31-E25). Find the capacitance.

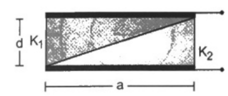

Figure 31-E25

58. Figure (31-E26) shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.

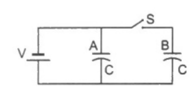

Figure 31-E26

59. A parallel-plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. Find the work done on the system in the process of inserting the slab.

60. A capacitor having a capacitance of 100 μF is charged to a potential difference of 50 V. (a) What is the magnitude of the charge on each plate? (b) The charging battery is disconnected and a dielectric of dielectric constant 2·5 is inserted. Calculate the new potential difference between the plates. (c) What charge would have produced this potential difference in absence of the dielectric slab. (d) Find the charge induced at a surface of the dielectric slab.

61. A sphercial capacitor is made of two conducting spherical shells of radii a and b. The space between the shells is filled with a dielectric of dielectric constant K

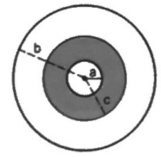

Figure 31-E27

up to a radius c as shown in figure (31-E27). Calculate the capacitance.

62. Consider an assembly of three conducting concentric spherical shells of radii a, b and c as shown in figure (31-E28). Find the capacitance of the assembly between the points A and B.

63. Suppose the space between the two inner shells of the previous problem is filled with a dielectric of dielectric constant K. Find the capacitance of the system between A and B.

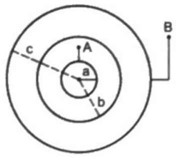

Figure 31-E28

64. An air-filled parallel-plate capacitor is to be constructed which can store 12 μC of charge when operated at 1200 V. What can be the minimum plate area of the capacitor? The dielectric strength of air is $3 \times 10^{6}\ \mathrm{V\,m^{-1}}$.

65. A parallel-plate capacitor with the plate area 100 cm^2 and the separation between the plates 1·0 cm is connected across a battery of emf 24 volts. Find the force of attraction between the plates.

66. Consider the situation shown in figure (31-E29). The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf $\mathcal{E}$. All surfaces are frictionless. Calculate the value of M for which the dielectric slab will stay in equilibrium.

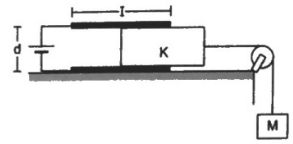

Figure 31-E29

67. Figure (31-E30) shows two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is the same for the two capacitors. The plates are rectangular in shape with width b and lengths l_1 and l_2. The left half of the dielectric slab has a dielectric constant K_1 and the right half K_2. Neglecting any friction, find the ratio of the emf of the left battery to that of the right battery for which the dielectric slab may remain in equilibrium.

68. Consider the situation shown in figure (31-E31). The plates of the capacitor have plate area A and are clamped in the laboratory. The dielectric slab is released from rest with a length a inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period.

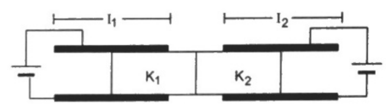

Figure 31-E30

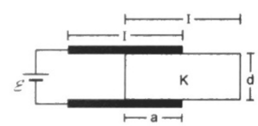

Figure 31-E31

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ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (c)	4. (b)	5. (c)	6. (d)
7. (b)	8. (d)	9. (c)	10. (a)	11. (b)	12. (d)

OBJECTIVE II

1. (c)	2. (b)	3. (d)	4. (a), (c), (d)	5. (b), (c)
6. (d)	7. (b), (c), (d)			

EXERCISES

1. $1·6 \times 10^{-8}$ F
2. $6·95 \times 10^{-5}$ μF
3. 6 km
4. $1·33 \times 10^{-10}$ C, $8·0 \times 10^{-10}$ J
5. (a) $1·33 \times 10^{-10}$ C (b) $1·33 \times 10^{-10}$ C

6. 24 μC, 48 μC, 72 μC
7. 110 μC on each, $1{\cdot}33 \times 10^{-3}$ J
8. 48 μC on the 8 μF capacitor and 24 μC on each of the 4 μF capacitors
9. (a) 5 μF (b) 10 μF
10. 110 μC
11. 44 μC
12. $4\pi\varepsilon_0 R_1$, $4\pi\varepsilon_0 R_2$; $4\pi\varepsilon_0 (R_1 + R_2)$
13. 2 μF, 20 V
15. (a) 50/3 μV at each point (b) zero
16. $C_3 + \dfrac{2\,C_1C_2}{C_1 + C_2}$
17. $\dfrac{\varepsilon_0 A(3\,d^2 + 6\,bd + 2\,b^2)}{3\,d(d+b)\,(d+2\,b)}$
18. (a) 8 pF (b) same as in (a)
19. 20 V
20. $3{\cdot}3 \times 10^{-4}$ C
21. 43 mV
22. $\left(\dfrac{Vea^2}{md_1(d_1+d_2)}\right)^{1/2}$
23. $1{\cdot}08 \times 10^{-3}$ cm
24. 2·25 μF
25. (a) $\dfrac{12}{11}$ V (b) − 8 V (c) zero (d) − 10·3 V
26. (a) $\dfrac{11}{6}$ μF (b) $\dfrac{11}{4}$ μF (c) 8 μF (d) 8 μF
27. 1 μF
28. 2 μF
29. 4 μF
30. 12·5 V
31. 1 V
32. 5 V
33. 0·16 μC
34. (a) 0·50 μC (b) 10 V
35. (a) 10 V (b) 10 V
36. (a) 1·71 V, 4·29 V (b) 184 pJ, 73·5 pJ
37. 960 μJ
38. 8 mJ in (a) and (d), 2 mJ in (b) and (c)
39. 2·0 J
40. (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ
41. $\dfrac{Q^2}{8\pi\varepsilon_0 R}$
42. (a) $\pi\varepsilon_0 RV^2$
43. $5{\cdot}6 \times 10^{-4}$ J
44. (a) $1{\cdot}06 \times 10^{-10}$ C (b) $12{\cdot}7 \times 10^{-10}$ J
 (c) $12{\cdot}7 \times 10^{-10}$ J, $6{\cdot}35 \times 10^{-10}$ J (d) $6{\cdot}35 \times 10^{-10}$ J
45. (a) 2400 μC, 1200 μC (b) 1200 μC (c) 14·4 mJ
 (d) 21·6 mJ (e) 7·2 mJ
46. (a) $C\mathcal{E}/2$, (b) $C\mathcal{E}^2/2$ (c) $C\mathcal{E}^2/4$ (d) $C\mathcal{E}^2/4$
47. (a) 1·44 mJ, 0·432 mJ (b) 21·8 μC, 26·2 μC, (c) 1·77 mJ
48. 1·44 mJ
49. 1·42 nF
50. (a) 8·5 nC (b) 6·4 nC (c) 2·1 nC
51. 88 pF
52. 3
53. (a) 30 μC (b) 3×10^3 V m^{-1} (c) 8·3 μF (d) 20 μC
54. 44 pF
55. (a) 1·18 μJ (b) 1·97 μJ
56. (a) $\dfrac{2\,K_1K_2\varepsilon_0 A}{d(K_1+K_2)}$ (b) $\dfrac{3\varepsilon_0 A\,K_1K_2K_3}{d(K_1K_2+K_2K_3+K_3K_1)}$
 (c) $\dfrac{\varepsilon_0 A}{2\,d}(K_1+K_2)$
57. $\dfrac{\varepsilon_0 K_1K_2 a^2 \ln\dfrac{K_1}{K_2}}{(K_1-K_2)d}$
58. 3 : 5
59. $\dfrac{\varepsilon_0 AV^2}{2\,d}\left(\dfrac{1}{K}-1\right)$
60. (a) 5 mC (b) 20 V (c) 2 mC (d) 3 mC
61. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(b-c)+b(c-a)}$
62. $\dfrac{4\pi\varepsilon_0 ac}{c-a}$
63. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(c-b)+c(b-a)}$
64. 0·45 m^2
65. $2{\cdot}5 \times 10^{-7}$ N
66. $\dfrac{\varepsilon_0 b\,\mathcal{E}^2(K-1)}{2\,dg}$
67. $\sqrt{\dfrac{K_2-1}{K_1-1}}$
68. $8\sqrt{\dfrac{(l-a)\,lmd}{\varepsilon_0 A\mathcal{E}^2(K-1)}}$

□