be integral multiples of e. If an object contains n_1 protons and n_2 electrons, the net charge on the object is

$$n_1(e) + n_2(-e) = (n_1 - n_2)e.$$

Indeed, there are elementary particles other than protons and electrons, which carry charge. However, they all carry charges which are integral multiples of e. Thus, the charge on any object is always an integral multiple of e and can be changed only in steps of e, i.e., charge is quantized.

The step size e is usually so small that we can easily neglect the quantization. If we rub a glass rod with a silk cloth, typically charges of the order of a microcoulomb appear on the rubbed objects. Now, 1 μC contains n units of basic charge e where

$$n = \frac{1\ \mu\text{C}}{1{\cdot}6 \times 10^{-19}\ \text{C}} \approx 6 \times 10^{12}.$$

The step size is thus very small as compared to the charges usually found and in many cases we can assume a continuous charge variation.

Charge is Conserved

The charge of an isolated system is conserved. It is possible to create or destroy charged particles but it is not possible to create or destroy *net charge*. In a beta decay process, a neutron converts itself into a proton and a fresh electron is created. The charge however, remains zero before and after the event.

Frictional Electricity : Induction

The simplest way to experience electric charges is to rub certain solid bodies against each other. Long ago, around 600 BC, the Greeks knew that when amber is rubbed with wool, it acquires the property of attracting light objects such as small pieces of paper. This is because amber becomes electrically charged. If we pass a comb through dry hair, the comb becomes electrically charged and can attract small pieces of paper. An automobile becomes charged when it travels through the air. A paper sheet becomes charged when it passes through a printing machine. A gramophone record becomes charged when cleaned with a dry cloth.

The explanation of appearance of electric charge on rubbing is simple. All material bodies contain large number of electrons and equal number of protons in their normal state. When rubbed against each other, some electrons from one body may pass on to the other body. The body that receives the extra electrons, becomes negatively charged. The body that donates the electrons, becomes positively charged because it has more protons than electrons. Thus, when a glass rod is rubbed with a silk cloth, electrons are transferred from the glass rod to the silk cloth. The glass rod becomes positively charged and the silk cloth becomes negatively charged.

If we take a positively charged glass rod near small pieces of paper, the rod attracts the pieces. Why does the rod attract paper pieces which are uncharged ? This is because the positively charged rod attracts the electrons of a paper piece towards itself. Some of the electrons accumulate at that edge of the paper piece which is closer to the rod. At the farther end of the piece there is a deficiency of electrons and hence positive charge appears there. Such a redistribution of charge in a material, due to the presence of a nearby charged body, is called *induction*. The rod exerts larger attraction on the negative charges of the paper piece as compared to the repulsion on the positive charges. This is because the negative charges are closer to the rod. Hence, there is a net attraction between the rod and the paper piece.

29.2 COULOMB'S LAW

The experiments of Coulomb and others established that the force exerted by a charged particle on the other is given by

$$F = \frac{kq_1q_2}{r^2}, \qquad \ldots \text{(29.1)}$$

where q_1 and q_2 are the charges on the particles, r is the separation between them and k is a constant. The force is attractive if the charges are of opposite signs and is repulsive if they are of the same sign. We can write Coulomb's law as

$$\vec{F} = \frac{kq_1q_2\,\vec{r}}{r^3},$$

where $\vec{r}$ is the position vector of the force-experiencing particle with respect to the force-exerting particle. In this form, the equation includes the direction of the force.

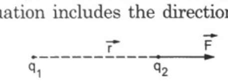

Figure 29.1

As F, q_1, q_2 and r are all independently defined quantities, the constant k can be measured experimentally. In SI units, the constant k is measured to be $8{\cdot}98755 \times 10^{9}\ \text{N m}^2\ \text{C}^{-2}$.

The constant k is often written as $\frac{1}{4\pi\varepsilon_0}$ so that equation (29.1) becomes

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}. \qquad \ldots \text{(29.2)}$$

The constant ε_0 is called the *permittivity of free space* and its value is

$$\varepsilon_0 = \frac{1}{4\pi k} = 8{\cdot}85419 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}.$$

29.3 ELECTRIC FIELD

We have already discussed in the chapter on gravitation that a particle cannot directly interact with another particle kept at a distance. A particle creates a gravitational field around it and this field exerts force on another particle placed in it. The electric force between two charged particles is also seen as a two-step process. A charge produces something called an *electric field* in the space around it and this electric field exerts a force on any charge (except the source charge itself) placed in it. The electric field has its own existence and is present even if there is no additional charge to experience the force. The field takes finite time to propagate. Thus, if a charge is displaced from its position, the field at a distance r will change after a time $t = r/c$, where c is the speed of light. We define the *intensity of electric field* at a point as follows:

Bring a charge q at the given point without disturbing any other charge that has produced the field. If the charge q experiences an electric force $\vec{F}$, we define the intensity of electric field at the given point as

$$\vec{E} = \frac{\vec{F}}{q} \cdot \qquad \dots (29.3)$$

The charge q used to define $\vec{E}$ is called a *test charge*.

One way to ensure that the test charge q does not disturb other charges is to keep its magnitude very small. If this magnitude is not small, the positions of the other charges may change. Equation (29.3) then gives the electric field due to the charges in the changed positions. The intensity of electric field is often abbreviated as *electric field*.

The electric field at a point is a vector quantity. Suppose, $\vec{E}_1$ is the field at a point due to a charge Q_1 and $\vec{E}_2$ is the field at the same point due to a charge Q_2. The resultant field when both the charges are present, is $\vec{E} = \vec{E}_1 + \vec{E}_2$.

Electric Field due to a Point Charge

Consider a point charge Q placed at a point A (figure 29.2). We are interested in the electric field $\vec{E}$ at a point P at a distance r from Q. Let us imagine a test charge q placed at P. The charge Q creates a field $\vec{E}$ at P and this field exerts a force $\vec{F} = q\vec{E}$ on the charge q. But, from Coulomb's law the force on the charge q in the given situation is

$$F = \frac{Qq}{4\pi\varepsilon_0 r^2}$$

along AP. The electric field at P is, therefore,

$$E = \frac{F}{q} = \frac{Q}{4\pi\varepsilon_0 r^2} \qquad \dots (29.4)$$

along AP.

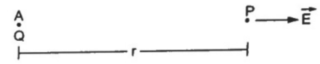

Figure 29.2

The electric field due to a set of charges may be obtained by finding the fields due to each individual charge and then adding these fields according to the rules of vector addition.

Example 29.1

Two charges $10\ \mu C$ *and* $-10\ \mu C$ *are placed at points A and B separated by a distance of* 10 cm. *Find the electric field at a point P on the perpendicular bisector of AB at a distance of* 12 cm *from its middle point.*

Solution :

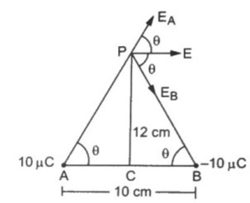

Figure 29.3

The situation is shown in figure (29.3). The distance $AP = BP = \sqrt{(5\text{ cm})^2 + (12\text{ cm})^2} = 13$ cm.

The field at the point P due to the charge $10\ \mu C$ is

$$E_A = \frac{10\ \mu C}{4\pi\varepsilon_0 (13\text{ cm})^2} = \frac{(10 \times 10^{-6}\text{ C}) \times (9 \times 10^9\text{ N m}^2\text{ C}^{-2})}{169 \times 10^{-4}\text{ m}^2}$$

$$= 5{\cdot}3 \times 10^6\text{ N C}^{-1}.$$

This field is along AP. The field due to $-10\ \mu C$ at P is $E_B = 5{\cdot}3 \times 10^6\text{ N C}^{-1}$ along PB. As E_A and E_B are equal in magnitude, the resultant will bisect the angle between the two. The geometry of the figure shows that this resultant is parallel to the base AB. The magnitude of the resultant field is

$$E = E_A \cos\theta + E_B \cos\theta$$

$$= 2 \times (5{\cdot}3 \times 10^6\text{ N C}^{-1}) \times \frac{5}{13}$$

$$= 4{\cdot}1 \times 10^6\text{ N C}^{-1}.$$

If a given charge distribution is continuous, we can use the technique of integration to find the resultant electric field at a point. A small element dQ is chosen in the distribution and the field $d\vec{E}$ due to dQ is

calculated. The resultant field is then calculated by integrating the components of $d\vec{E}$ under proper limits.

Example 29.2

A ring of radius a contains a charge q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance x from the centre.

Solution :

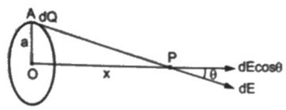

Figure 29.4

Figure (29.4) shows the situation. Let us consider a small element of the ring at the point A having a charge dQ. The field at P due to this element is

$$dE = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}.$$

By symmetry, the field at P will be along the axis OP. The component of dE along this direction is

$$dE\cos\theta = \frac{dQ}{4\pi\varepsilon_0 (AP)^2}\left(\frac{OP}{AP}\right)$$

$$= \frac{x\,dQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}.$$

The net field at P is

$$E = \int dE\cos\theta = \int \frac{x\,dQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}$$

$$= \frac{x}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}\int dQ = \frac{xQ}{4\pi\varepsilon_0 (a^2 + x^2)^{3/2}}.$$

29.4 LINES OF ELECTRIC FORCE

The electric field in a region can be graphically represented by drawing certain curves known as *lines of electric force* or *electric field lines*. Lines of force are drawn in such a way that the tangent to a line of force gives the direction of the resultant electric field there. Thus, the electric field due to a positive point charge is represented by straight lines originating from the charge (figure 29.5a). The electric field due to a negative point charge is represented by straight lines terminating at the charge (figure 29.5b). If we draw the lines isotropically (the lines are drawn uniformly in all directions, originating from the point charge), we can compare the intensities of the field at two points by just looking at the distribution of the lines of force.

Consider two points P_1 and P_2 in figure (29.5). Draw equal small areas through P_1 and P_2 perpendicular to the lines. More number of lines pass through the area at P_1 and less number of lines pass through the area at P_2. Also, the intensity of electric field is more at P_1 than at P_2. In fact, the electric field is proportional to the lines per unit area if the lines originate isotropically from the charge.

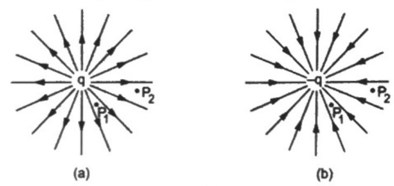

Figure 29.5

We can draw the lines of force for a charge distribution containing more than one charge. From each charge we can draw the lines isotropically. The lines may not be straight as one moves away from a charge. Figure (29.6) shows the shapes of these lines for some charge distributions.

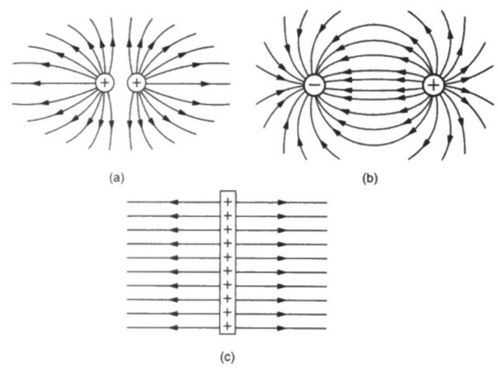

Figure 29.6

The lines of force are purely a geometrical construction which help us to visualise the nature of electric field in a region. They have no physical existence.

29.5 ELECTRIC POTENTIAL ENERGY

Consider a system of charges. The charges of the system exert electric forces on each other. If the position of one or more charges is changed, work may be done by these electric forces. We define *change in electric potential energy* of the system as negative of the work done by the electric forces as the configuration of the system changes.

Consider a system of two charges q_1 and q_2. Suppose, the charge q_1 is fixed at a point A and the charge q_2 is taken from a point B to a point C along the line ABC (figure 29.7).

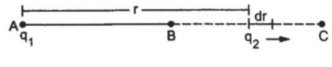

Figure 29.7

Let the distance $AB = r_1$ and the distance $AC = r_2$.

Consider a small displacement of the charge q_2 in which its distance from q_1 changes from r to $r + dr$. The electric force on the charge q_2 is

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \text{ towards } \vec{AB}.$$

The work done by this force in the small displacement dr is

$$dW = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr.$$

The total work done as the charge q_2 moves from B to C is

$$W = \int_{r_1}^{r_2} \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} dr = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

No work is done by the electric force on the charge q_1 as it is kept fixed. The change in potential energy $U(r_2) - U(r_1)$ is, therefore,

$$U(r_2) - U(r_1) = -W = \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \quad \ldots (29.5)$$

We choose the potential energy of the two-charge system to be zero when they have infinite separation (that means when they are widely separated). This means $U(\infty) = 0$. The potential energy when the separation is r is

$$U(r) = U(r) - U(\infty)$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 r}. \quad \ldots (29.6)$$

The above equation is derived by assuming that one of the charges is fixed and the other is displaced. However, the potential energy depends essentially on the separation between the charges and is independent of the spatial location of the charged particles. Equations (29.5) and (29.6) are, therefore, general.

Equation (29.6) gives the electric potential energy of a pair of charges. If there are three charges q_1, q_2 and q_3, there are three pairs. Similarly for an N-particle system, the potential energy of the system is equal to the sum of the potential energies of the N pairs of charged particles.

Example 29.3

Three particles, each having a charge of 10 μC, *are placed at the vertices of an equilateral triangle of side* 10 cm. *Find the work done by a person in pulling them apart to infinite separations.*

Solution : The potential energy of the system in the initial condition is

$$U = \frac{3 \times (10\,\mu\text{C}) \times (10\,\mu\text{C})}{4\pi\varepsilon_0 (10\text{ cm})}$$

$$= \frac{(3 \times 10^{-10}\text{ C}^2) \times (9 \times 10^9\text{ N m}^2\text{ C}^{-2})}{0{\cdot}1\text{ m}} = 27\text{ J}.$$

When the charges are infinitely separated, the potential energy is reduced to zero. If we assume that the charges do not get kinetic energy in the process, the total mechanical energy of the system decreases by 27 J. Thus, the work done by the person on the system is −27 J.

29.6 ELECTRIC POTENTIAL

The electric field in a region of space is described by assigning a vector quantity $\vec{E}$ at each point. The same field can also be described by assigning a scalar quantity V at each point. We now define this scalar quantity known as *electric potential.*

Suppose, a test charge q is moved in an electric field from a point A to a point B while all the other charges in question remain fixed. If the electric potential energy changes by $U_B - U_A$ due to this displacement, we define the *potential difference* between the point A and the point B as

$$V_B - V_A = \frac{U_B - U_A}{q}. \quad \ldots (29.7)$$

Conversely, if a charge q is taken through a potential difference $V_B - V_A$, the electric potential energy is increased by $U_B - U_A = q(V_B - V_A)$. This equation defines potential difference between any two points in an electric field. We can define absolute electric potential at any point by choosing a reference point P and saying that the potential at this point is zero. The electric potential at a point A is then given by (equation 29.7)

$$V_A = V_A - V_P = \frac{U_A - U_P}{q}. \quad \ldots (29.8)$$

So, *the potential at a point A is equal to the change in electric potential energy per unit test charge when it is moved from the reference point to the point A.*

Suppose, the test charge is moved in an electric field without changing its kinetic energy. The total work done on the charge should be zero from the work–energy theorem. If W_{ext} and W_{el} be the work done by the external agent and by the electric field as the charge moves, we have,

$$W_{ext} + W_{el} = 0$$

or, $$W_{ext} = -W_{el} = \Delta U,$$

where ΔU is the change in electric potential energy. Using this equation and equation (29.8), the potential at a point A may also be defined as follows:

The potential at a point A is equal to the work done per unit test charge by an external agent in moving the test charge from the reference point to the point A (without changing its kinetic energy).

The choice of reference point is purely ours. Generally, a point widely separated from all charges in question is taken as the reference point. Such a point is assumed to be at infinity.

As potential energy is a scalar quantity, potential is also a scalar quantity. Thus, if V_1 is the potential at a given point due to a charge q_1 and V_2 is the potential at the same point due to a charge q_2, the potential due to both the charges is $V_1 + V_2$.

29.7 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

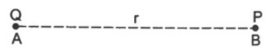

Figure 29.8

Consider a point charge Q placed at a point A (figure 29.8). We have to find the electric potential at a point P where $AP = r$. Let us take the reference point at $r = \infty$. Suppose, a test charge q is moved from $r = \infty$ to the point P. The change in electric potential energy of the system is, from equation (29.6),

$$U_P - U_\infty = \frac{Qq}{4\pi\varepsilon_0 r}.$$

The potential at P is, from equation (29.8),

$$V_P = \frac{U_P - U_\infty}{q} = \frac{Q}{4\pi\varepsilon_0 r}. \qquad \dots (29.9)$$

The electric potential due to a system of charges may be obtained by finding potentials due to the individual charges using equation (29.9) and then adding them. Thus,

$$V = \frac{1}{4\pi\varepsilon_0}\sum \frac{Q_i}{r_i}.$$

Example 29.4

Two charges $+10\,\mu C$ and $+20\,\mu C$ are placed at a separation of 2 cm. Find the electric potential due to the pair at the middle point of the line joining the two charges.

Solution : Using the equation $V = \frac{Q}{4\pi\varepsilon_0 r}$, the potential due to $+10\,\mu C$ is

$$V_1 = \frac{(10\times 10^{-6}\,\mathrm{C})\times(9\times 10^{9}\,\mathrm{N\,m^2C^{-2}})}{1\times 10^{-2}\,\mathrm{m}} = 9\,\mathrm{MV}.$$

The potential due to $+20\,\mu C$ is

$$V_2 = \frac{(20\times 10^{-6}\,\mathrm{C})\times(9\times 10^{9}\,\mathrm{N\,m^2C^{-2}})}{1\times 10^{-2}\,\mathrm{m}} = 18\,\mathrm{MV}.$$

The net potential at the given point is

$$9\,\mathrm{MV} + 18\,\mathrm{MV} = 27\,\mathrm{MV}.$$

If the charge distribution is continuous, we may use the technique of integration to find the electric potential.

29.8 RELATION BETWEEN ELECTRIC FIELD AND POTENTIAL

Suppose, the electric field at a point $\vec{r}$ due to a charge distribution is $\vec{E}$ and the electric potential at the same point is V. Suppose, a point charge q is displaced slightly from the point $\vec{r}$ to $\vec{r} + \vec{dr}$. The force on the charge is

$$\vec{F} = q\vec{E}$$

and the work done by the electric field during the displacement is

$$dW = \vec{F}\cdot\vec{dr} = q\vec{E}\cdot\vec{dr}.$$

The change in potential energy is

$$dU = -\,dW = -\,q\vec{E}\cdot\vec{dr}.$$

The change in potential is

$$dV = \frac{dU}{q}$$

or,

$$dV = -\,\vec{E}\cdot\vec{dr}. \qquad \dots (29.10)$$

Integrating between the points $\vec{r_1}$ and $\vec{r_2}$, we get

$$V_2 - V_1 = -\int_{\vec{r_1}}^{\vec{r_2}} \vec{E}\cdot\vec{dr} \qquad \dots (29.11)$$

where V_1 and V_2 are the potentials at $\vec{r_1}$ and $\vec{r_2}$ respectively. If we choose $\vec{r_1}$, at the reference point (say at infinity) and $\vec{r_2}$ at $\vec{r}$, equation (29.11) becomes

$$V(\vec{r}) = -\int_{\infty}^{\vec{r}} \vec{E}\cdot\vec{dr}. \qquad \dots (29.12)$$

Example 29.5

Figure (29.9) shows two metallic plates A and B placed parallel to each other at a separation d. A uniform electric field E exists between the plates in the direction from plate B to plate A. Find the potential difference between the plates.

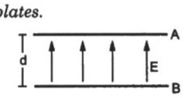

Figure 29.9

Solution : Let us take the origin at plate A and x-axis along the direction from plate A to plate B. We have

$$V_B - V_A = -\int_{\vec{r}_A}^{\vec{r}_B} \vec{E} \cdot d\vec{r} = -\int_0^d - E\, dx = Ed.$$

If we work in Cartesian coordinate system

$$E = E_x \vec{i} + E_y \vec{j} + E_z \vec{k}$$

and $$d\vec{r} = dx\, \vec{i} + dy\, \vec{j} + dz\, \vec{k}.$$

Thus, from (29.10)

$$dV = -E_x\, dx - E_y\, dy - E_z\, dz. \qquad \text{... (i)}$$

If we change x to $x + dx$ keeping y and z constant, $dy = dz = 0$ and from (i),

$$E_x = -\frac{\partial V}{\partial x}.$$

Similarly, $$E_y = -\frac{\partial V}{\partial y} \qquad \text{... (29.13)}$$

and $$E_z = -\frac{\partial V}{\partial z}.$$

The symbols $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}$, etc., are used to indicate that while differentiating with respect to one coordinate, the others are kept constant.

If we know the electric field in a region, we can find the electric potential using equation (29.12) and if we know the electric potential in a region, we can find the electric field using (29.13).

Equation (29.10) may also be written as

$$dV = -E\, dr \cos\theta$$

where θ is the angle between the field $\vec{E}$ and the small displacement $d\vec{r}$. Thus,

$$-\frac{dV}{dr} = E \cos\theta. \qquad \text{... (29.14)}$$

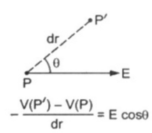

Figure 29.10

We see that, $-\frac{dV}{dr}$ gives the component of the electric field in the direction of displacement $d\vec{r}$. In figure (29.10), we show a small displacement $PP' = dr$. The electric field is E making an angle θ with PP'. We have

$$dV = V(P') - V(P)$$

so that $$\frac{V(P) - V(P')}{dr} = E\cos\theta.$$

This gives us a method to get the component of the electric field in any given direction if we know the potential. Move a small distance dr in the given direction and see the change dV in the potential. The component of electric field along that direction is $-\frac{dV}{dr}$.

If we move a distance dr in the direction of the field, θ is zero and $-\frac{dV}{dr} = E$ is maximum. Thus, *the electric field is along the direction in which the potential decreases at the maximum rate.*

If a small displacement $d\vec{r}$ perpendicular to the electric field is considered, $\theta = 90°$ and $dV = -\vec{E} \cdot d\vec{r} = 0$. The potential does not vary in a direction perpendicular to the electric field.

Equipotential Surfaces

If we draw a surface in such a way that the electric potential is the same at all the points of the surface, it is called an *equipotential surface*. The component of electric field parallel to an equipotential surface is zero, as the potential does not change in this direction. Thus, the electric field is perpendicular to the equipotential surface at each point of the surface. For a point charge, the electric field is radial and the equipotential surfaces are concentric spheres with centres at the charge (figure 29.11).

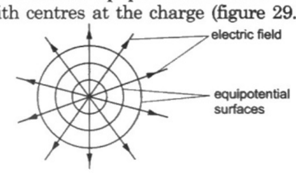

Figure 29.11

29.9 ELECTRIC DIPOLE

A combination of two charges $+q$ and $-q$ separated by a small distance d constitutes an *electric dipole*. The *electric dipole moment* of this combination is defined as a vector

$$\vec{p} = q\vec{d}, \qquad \text{... (29.15)}$$

where $\vec{d}$ is the vector joining the negative charge to the positive charge. The line along the direction of the dipole moment is called the *axis of the dipole.*

Electric Potential due to a Dipole

Suppose, the negative charge $-q$ is placed at a point A and the positive charge q is placed at a point

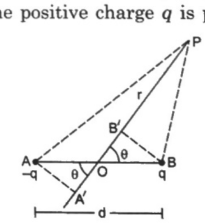

Figure 29.12

B (figure 29.12), the separation $AB = d$. The middle point of AB is O. The potential is to be evaluated at a point P where $OP = r$ and $\angle POB = \theta$. Also, let $r >> d$.

Let AA' be the perpendicular from A to PO and BB' be the perpendicular from B to PO. As d is very small compared to r,

$$AP \approx A'P = OP + OA'$$

$$= OP + AO\cos\theta = r + \frac{d}{2}\cos\theta.$$

Similarly, $BP \approx B'P = OP - OB'$

$$= r - \frac{d}{2}\cos\theta.$$

The potential at P due to the charge $-q$ is

$$V_1 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{AP} \approx -\frac{1}{4\pi\varepsilon_0}\frac{q}{r + \frac{d}{2}\cos\theta}$$

and that due to the charge $+q$ is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP} \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{r - \frac{d}{2}\cos\theta}.$$

The net potential at P due to the dipole is

$$V = V_1 + V_2$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r - \frac{d}{2}\cos\theta} - \frac{q}{r + \frac{d}{2}\cos\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q\,d\cos\theta}{r^2 - \frac{d^2}{4}\cos^2\theta}$$

$$\approx \frac{1}{4\pi\varepsilon_0}\frac{q\,d\cos\theta}{r^2}$$

or, $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}. \qquad \ldots (29.16)$$

Generalised Definition of Electric Dipole

The potential at a distance r from a point charge q is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}.$$

It is inversely proportional to r and is independent of direction. The potential due to a dipole is inversely proportional to r^2 and depends on direction as shown by the term $\cos\theta$ in equation (29.16). In general, any charge distribution that produces electric potential given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

is called an electric dipole. The constant p is called its dipole moment and the direction from which the angle θ is measured to get the above equation is called the direction of the dipole moment.

Electric Field due to a Dipole

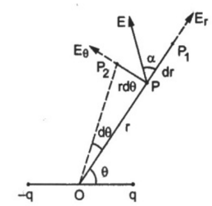

Figure 29.13

We can find the electric field due to an electric dipole using the expression (29.16) for the electric potential. In figure (29.13), PP_1 is a small displacement in the direction of OP and PP_2 is a small displacement perpendicular to OP. Thus, PP_1 is in radial direction and PP_2 is in transverse direction. In going from P to P_1, the angle θ does not change and the distance OP changes from r to $r + dr$. Thus, $PP_1 = dr$. In going from P to P_2, the angle θ changes from θ to $\theta + d\theta$ while the distance r remains almost constant. Thus, $PP_2 = r\,d\theta$. From equation (29.14), the component of the electric field at P in the radial direction PP_1 is

$$E_r = -\frac{dV}{PP_1} = -\frac{dV}{dr} = -\frac{\partial V}{\partial r}. \qquad \ldots (i)$$

The symbol ∂ specifies that θ should be treated as constant while differentiating with respect to r.

Similarly, the component of the electric field at P in the transverse direction PP_2 is

$$E_\theta = -\frac{dV}{PP_2} = -\frac{dV}{rd\theta} = -\frac{1}{r}\frac{\partial V}{\partial\theta}. \qquad \ldots (ii)$$

As $$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2},$$

$$E_r = -\frac{\partial V}{\partial r} = -\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}(p\cos\theta)\frac{d}{dr}\left(\frac{1}{r^2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \qquad \ldots (iii)$$

and $$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = -\frac{1}{r}\frac{1}{4\pi\varepsilon_0}\frac{\partial}{\partial\theta}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= -\frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\frac{d}{d\theta}(\cos\theta)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}. \qquad \ldots (iv)$$

The resultant electric field at P (figure 29.13) is

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\sqrt{\left(\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{p\sin\theta}{r^3}\right)^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2\theta + 1}. \quad \ldots (29.17)$$

If the resultant field makes an angle α with the radial direction OP, we have

$$\tan\alpha = \frac{E_\theta}{E_r} = \frac{p\sin\theta/r^3}{2p\cos\theta/r^3} = \frac{1}{2}\tan\theta$$

or, $$\alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right). \quad \ldots (29.18)$$

Special Cases

(a) $\theta = 0$

In this case, the point P is on the axis of the dipole. From equation (29.16), the electric potential is $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$.

The field at such a point is, from equation (29.17), $E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$ along the axis. Such a position of the point P is called an *end-on position.*

(b) $\theta = 90°$

In this case the point P is on the perpendicular bisector of the dipole. The potential here is zero while the field is, from equation (29.17), $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$.

The angle α is given by

$$\tan\alpha = \frac{\tan\theta}{2} = \infty$$

or, $$\alpha = 90°.$$

The field is antiparallel to the dipole axis. Such a position of the point P is called a *broadside-on position.*

29.10 TORQUE ON AN ELECTRIC DIPOLE PLACED IN AN ELECTRIC FIELD

Consider an electric dipole placed in a uniform electric field $\vec{E}$. The dipole consists of charges $-q$ placed at A and $+q$ placed at B (figure 29.14). The mid-point of AB is O and the length $AB = d$. Suppose the axis of the dipole AB makes an angle θ with the electric field at a certain instant.

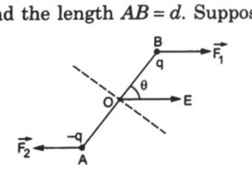

Figure 29.14

The force on the charge $+q$ is $\vec{F}_1 = q\vec{E}$ and the force on the charge $-q$ is $\vec{F}_2 = -q\vec{E}$. Let us calculate the torques $(\vec{r} \times \vec{F})$ of these forces about O.

The torque of $\vec{F}_1$ about O is

$$\vec{\Gamma}_1 = \overrightarrow{OB} \times \vec{F}_1 = q(\overrightarrow{OB} \times \vec{E})$$

and the torque of $\vec{F}_2$ about O is

$$\vec{\Gamma}_2 = \overrightarrow{OA} \times \vec{F}_2 = -q(\overrightarrow{OA} \times \vec{E}) = q(\overrightarrow{AO} \times \vec{E}).$$

The net torque acting on the dipole is

$$\vec{\Gamma} = \vec{\Gamma}_1 + \vec{\Gamma}_2$$
$$= q(\overrightarrow{OB} \times \vec{E}) + q(\overrightarrow{AO} \times \vec{E})$$
$$= q(\overrightarrow{OB} + \overrightarrow{AO}) \times \vec{E}$$
$$= q\,\overrightarrow{AB} \times \vec{E} = \vec{p} \times \vec{E}. \quad \ldots (29.19)$$

The direction of the torque is perpendicular to the plane containing the dipole axis and the electric field. In figure (29.14), this is perpendicular to the plane of paper and is going into the page. The magnitude is $\Gamma = |\vec{\Gamma}| = pE\sin\theta$.

29.11 POTENTIAL ENERGY OF A DIPOLE PLACED IN A UNIFORM ELECTRIC FIELD

When an electric dipole is placed in an electric field $\vec{E}$, a torque $\vec{\Gamma} = \vec{p} \times \vec{E}$ acts on it (figure 29.14). If we rotate the dipole through a small angle $d\theta$, the work done by the torque is

$$dW = \Gamma d\theta$$
$$= -pE\sin\theta\, d\theta.$$

The work is negative as the rotation $d\theta$ is opposite to the torque.

The change in electric potential energy of the dipole is, therefore,

$$dU = -\,dW = pE\sin\theta\, d\theta.$$

If the angle θ is changed from 90° to θ, the change in potential energy is

$$U(\theta) - U(90°) = \int_{90°}^{\theta} pE\sin\theta\, d\theta$$
$$= pE\,[-\cos\theta]_{90°}^{\theta}$$
$$= -pE\cos\theta = -\vec{p}\cdot\vec{E}.$$

If we choose the potential energy of the dipole to be zero when $\theta = 90°$ (dipole axis is perpendicular to the field), $U(90°) = 0$ and the above equation becomes

$$U(\theta) = -\vec{p}\cdot\vec{E}. \quad \ldots (29.20)$$

29.12 CONDUCTORS, INSULATORS AND SEMICONDUCTORS

Any piece of matter of moderate size contains millions and millions of atoms or molecules. Each atom contains a positively charged nucleus and several electrons going round it.

In gases, the atoms or molecules almost do not interact with each other. In solids and liquids, the interaction is comparatively stronger. It turns out that the materials may be broadly divided into three categories according to their behaviour when they are placed in an electric field.

In some materials, the outer electrons of each atom or molecule are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called *free electrons*. They are also known as *conduction electrons*. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called *conductors*.

Another class of materials is called *insulators* in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no *free electrons*. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can't leave their parent atoms or molecules and hence can't move through long distances. Such materials are also called *dielectrics*.

In *semiconductors*, the behaviour is like an insulator at the temperature 0 K. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behaviour is in between a conductor and an insulator and hence, the name semiconductor. A freed electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

We shall learn more about conductivity in later chapters. At the moment we accept the simple approximate model described above. The conductors have large number of free electrons everywhere in the material whereas the insulators have none. The discussion of semiconductors is deferred to a separate chapter.

Roughly speaking, the metals are conductors and the nonmetals are insulators. The above discussion may be extended to liquids and gases. Some of the liquids, such as mercury, and ionized gases are conductors.

29.13 THE ELECTRIC FIELD INSIDE A CONDUCTOR

Consider a conducting plate placed in a region. Initially, there is no electric field and the conduction electrons are almost uniformly distributed within the plate (shown by dots in figure 29.15a). In any small volume (which contains several thousand molecules) the number of electrons is equal to the number of protons in the nuclei. The net charge in the volume is zero.

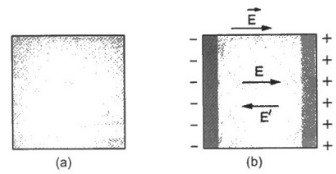

Figure 29.15

Now, suppose an electric field $\vec{E}$ is created in the direction left to right (figure 29.15b). This field exerts force on the free electrons from right to left. The electrons move towards left, the number of electrons on the left face increases and the number on the right face decreases. The left face becomes negatively charged and the right face becomes positively charged. These extra charges produce an extra electric field E' inside the plate from right to left. The electrons continue to drift and the internal field $\vec{E'}$ becomes stronger and stronger. A situation comes when the field $\vec{E'}$ due to the redistribution of free electrons becomes equal in magnitude to $\vec{E}$. The net electric field inside the plate is then zero. The free electrons there do not experience any net force and the process of further drifting stops. Thus, a steady state is reached in which some positive and negative charges appear at the surface of the plate and there is no electric field inside the plate.

Whenever a conductor is placed in an electric field some of the free electrons redistribute themselves on the surface of the conductor. The redistribution takes place in such a way that the electric field is zero at all the points inside the conductor. The redistribution takes a time which is, in general, less than a millisecond. Thus, *there can be no electric field inside a conductor in electrostatics.*

Worked Out Examples

1. *Charges* $5{\cdot}0\times10^{-7}$ C, $-2{\cdot}5\times10^{-7}$ C *and* $1{\cdot}0\times10^{-7}$ C *are held fixed at the three corners A, B, C of an equilateral triangle of side* 5·0 cm. *Find the electric force on the charge at C due to the rest two.*

Solution :

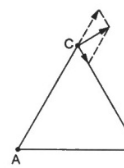

Figure 29-W1

The force on C due to A

$$=\frac{1}{4\pi\varepsilon_0}\frac{(5\times10^{-7}\text{ C})\,(1\times10^{-7}\text{ C})}{(0{\cdot}05\text{ m})^2}$$

$$=9\times10^9\text{ Nm}^2\text{C}^{-2}\times\frac{5\times10^{-14}\text{ C}^2}{25\times10^{-4}\text{ m}^2}=0{\cdot}18\text{ N}.$$

This force acts along AC. The force on C due to B

$$=\frac{1}{4\pi\varepsilon_0}\frac{(2{\cdot}5\times10^{-7}\text{ C})\,(1\times10^{-7}\text{ C})}{(0{\cdot}05\text{ m})^2}=0{\cdot}09\text{ N}.$$

This attractive force acts along CB. As the triangle is equilateral, the angle between these two forces is 120°. The resultant electric force on C is

$$[(0{\cdot}18\text{ N})^2+(0{\cdot}09\text{ N})^2+2(0{\cdot}18\text{ N})\,(0{\cdot}09\text{ N})\,(\cos120°)]^{1/2}$$

$$=0{\cdot}16\text{ N}.$$

The angle made by this resultant with CB is

$$\tan^{-1}\frac{0{\cdot}18\sin120°}{0{\cdot}09+0{\cdot}18\cos120°}=90°.$$

2. *Two particles A and B having charges* $8{\cdot}0\times10^{-6}$ C *and* $-2{\cdot}0\times10^{-6}$ C *respectively are held fixed with a separation of* 20 cm. *Where should a third charged particle be placed so that it does not experience a net electric force?*

Solution : As the net electric force on C should be equal to zero, the force due to A and B must be opposite in direction. Hence, the particle should be placed on the line AB. As A and B have charges of opposite signs, C cannot be between A and B. Also, A has larger magnitude of charge than B. Hence, C should be placed closer to B than A. The situation is shown in figure (29-W2).

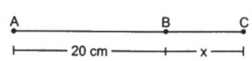

Figure 29-W2

Suppose $BC=x$ and the charge on C is Q.

The force due to $A=\dfrac{(8{\cdot}0\times10^{-6}\text{ C})Q}{4\pi\varepsilon_0(20\text{ cm}+x)^2}$.

The force due to $B=\dfrac{(2{\cdot}0\times10^{-6}\text{ C})Q}{4\pi\varepsilon_0 x^2}$.

They are oppositely directed and to have a zero resultant, they should be equal in magnitude. Thus,

$$\frac{8}{(20\text{ cm}+x)^2}=\frac{2}{x^2}$$

or, $$\frac{20\text{ cm}+x}{x}=2,\text{ giving }x=20\text{ cm}.$$

3. *Three equal charges, each having a magnitude of* $2{\cdot}0\times10^{-6}$ C, *are placed at the three corners of a right-angled triangle of sides* 3 cm, 4 cm *and* 5 cm. *Find the force on the charge at the right-angle corner.*

Solution :

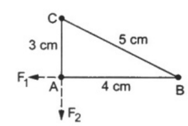

Figure 29-W3

The situation is shown in figure (29-W3). The force on A due to B is

$$F_1=\frac{(2{\cdot}0\times10^{-6}\text{ C})\,(2{\cdot}0\times10^{-6}\text{ C})}{4\pi\varepsilon_0\,(4\text{ cm})^2}$$

$$=9\times10^9\text{ N m}^2\text{ C}^{-2}\times4\times10^{-12}\text{ C}^2\times\frac{1}{16\times10^{-4}\text{ m}^2}$$

$$=22{\cdot}5\text{ N}.$$

This force acts along BA. Similarly, the force on A due to C is $F_2=40$ N in the direction of CA. Thus, the net electric force on A is

$$F=\sqrt{F_1^2+F_2^2}$$

$$=\sqrt{(22{\cdot}5\text{ N})^2+(40\text{ N})^2}=45{\cdot}9\text{ N}.$$

This resultant makes an angle θ with BA where

$$\tan\theta=\frac{40}{22{\cdot}5}=\frac{16}{9}.$$

4. *Two small iron particles, each of mass* 280 mg, *are placed at a distance* 10 cm *apart. If* 0·01% *of the electrons of one particle are transferred to the other, find the electric force between them. Atomic weight of iron is* 56 g mol^{-1} *and there are 26 electrons in each atom of iron.*

Solution : The atomic weight of iron is 56 g mol^{-1}. Thus, 56 g of iron contains 6×10^{23} atoms and each atom contains 26 electrons. Hence, 280 mg of iron contains

$$\frac{280 \text{ mg} \times 6 \times 10^{23} \times 26}{56 \text{ g}} = 7{\cdot}8 \times 10^{22} \text{ electrons.}$$

The number of electrons transferred from one particle to another

$$= \frac{0{\cdot}01}{100} \times 7{\cdot}8 \times 10^{22} = 7{\cdot}8 \times 10^{18}.$$

The charge transferred is, therefore,

$$1{\cdot}6 \times 10^{-19} \text{ C} \times 7{\cdot}8 \times 10^{18} = 1{\cdot}2 \text{ C}.$$

The electric force between the particles is

$$(9 \times 10^{9} \text{ N m}^{2} \text{ C}^{-2}) \frac{(1{\cdot}2 \text{ C})^{2}}{(10 \times 10^{-2} \text{ m})^{2}}$$

$$= 1{\cdot}3 \times 10^{12} \text{ N}.$$

This equals the load of approximately 2000 million grown-up persons !

5. *A charge Q is to be divided on two objects. What should be the values of the charges on the objects so that the force between the objects can be maximum ?*

Solution : Suppose one object receives a charge q and the other $Q - q$. The force between the objects is

$$F = \frac{q(Q-q)}{4\pi\varepsilon_0 d^{2}},$$

where d is the separation between them. For F to be maximum, the quantity

$$y = q(Q-q) = Qq - q^{2}$$

should be maximum. This is the case when,

$$\frac{dy}{dq} = 0 \text{ or, } Q - 2q = 0 \text{ or, } q = Q/2.$$

Thus, the charge should be divided equally on the two objects.

6. *Two particles, each having a mass of* 5 g *and charge* $1{\cdot}0 \times 10^{-7}$ C, *stay in limiting equilibrium on a horizontal table with a separation of* 10 cm *between them. The coefficient of friction between each particle and the table is the same. Find the value of this coefficient.*

Solution : The electric force on one of the particles due to the other is

$$F = (9 \times 10^{9} \text{ N m}^{2} \text{ C}^{-2}) \times (1{\cdot}0 \times 10^{-7} \text{C})^{2} \times \frac{1}{(0{\cdot}10 \text{ m})^{2}}$$

$$= 0{\cdot}009 \text{ N}.$$

The frictional force in limiting equilibrium

$$f = \mu \times (5 \times 10^{-3} \text{ kg}) \times 9{\cdot}8 \text{ m s}^{-2}$$

$$= (0{\cdot}049 \, \mu) \text{ N}.$$

As these two forces balance each other,

$$0{\cdot}049 \, \mu = 0{\cdot}009$$

or, $\mu = 0{\cdot}18.$

7. *A vertical electric field of magnitude* $4{\cdot}00 \times 10^{5}$ N C^{-1} *just prevents a water droplet of mass* $1{\cdot}00 \times 10^{-4}$ kg *from falling. Find the charge on the droplet.*

Solution : The forces acting on the droplet are

(i) the electric force $q\vec{E}$ and

(ii) the force of gravity $m\vec{g}$.

To just prevent from falling, these two forces should be equal and opposite. Thus,

$$q(4{\cdot}00 \times 10^{5} \text{ N C}^{-1}) = (1{\cdot}00 \times 10^{-4} \text{ kg}) \times (9{\cdot}8 \text{ m s}^{-2})$$

or, $q = 2{\cdot}45 \times 10^{-9}$ C.

8. *Three charges, each equal to q, are placed at the three corners of a square of side a. Find the electric field at the fourth corner.*

Solution :

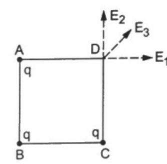

Figure 29-W4

Let the charges be placed at the corners A, B and C (figure 29-W4). We shall calculate the electric field at the fourth corner D. The field E_1 due to the charge at A will have the magnitude $\frac{q}{4\pi\varepsilon_0 a^{2}}$ and will be along AD. The field E_2 due to the charge at C will have the same magnitude and will be along CD.

The field E_3 due to the charge at B will have the magnitude $\frac{q}{4\pi\varepsilon_0 (\sqrt{2}a)^{2}}$ and will be along BD. As E_1 and E_2 are equal in magnitude, their resultant will be along the bisector of the angle between E_1, E_2 and hence along E_3. The magnitude of this resultant is $\sqrt{E_1^{2} + E_2^{2}}$ as the angle between E_1 and E_2 is $\pi/2$. The resultant electric field at D is, therefore, along E_3 and has magnitude

$$\sqrt{E_1^{2} + E_2^{2}} + E_3$$

$$= \sqrt{\left(\frac{q}{4\pi\varepsilon_0 a^{2}}\right)^{2} + \left(\frac{q}{4\pi\varepsilon_0 a^{2}}\right)^{2}} + \frac{q}{4\pi\varepsilon_0 (\sqrt{2}a)^{2}}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\sqrt{2}}{a^{2}} + \frac{1}{2a^{2}}\right] = (2\sqrt{2} + 1)\frac{q}{8\pi\varepsilon_0 a^{2}}.$$

9. *A charged particle of mass* 1·0 g *is suspended through a silk thread of length* 40 cm *in a horizontal electric field of* $4{\cdot}0 \times 10^{4}$ N C^{-1}. *If the particle stays at a distance of* 24 cm *from the wall in equilibrium, find the charge on the particle.*