(a) increases (b) decreases
(c) remains the same (d) may increase or decrease.

4. Two equal positive charges are kept at points A and B. The electric potential at the points between A and B (excluding these points) is studied while moving from A to B. The potential
(a) continuously increases
(b) continuously decreases
(c) increases then decreases
(d) decreases then increases.

5. The electric field at the origin is along the positive x-axis. A small circle is drawn with the centre at the origin cutting the axes at points A, B, C and D having coordinates $(a, 0)$, $(0, a)$, $(-a, 0)$, $(0, -a)$ respectively. Out of the points on the periphery of the circle, the potential is minimum at
(a) A (b) B (c) C (d) D.

6. If a body is charged by rubbing it, its weight
(a) remains precisely constant
(b) increases slightly
(c) decreases slightly
(d) may increase slightly or may decrease slightly.

7. An electric dipole is placed in a uniform electric field. The net electric force on the dipole
(a) is always zero
(b) depends on the orientation of the dipole
(c) can never be zero
(d) depends on the strength of the dipole.

8. Consider the situation of figure (29-Q3). The work done in taking a point charge from P to A is W_A, from P to B is W_B and from P to C is W_C.
(a) $W_A < W_B < W_C$ (b) $W_A > W_B > W_C$
(c) $W_A = W_B = W_C$ (d) None of these

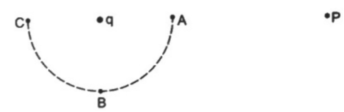

Figure 29-Q3

9. A point charge q is rotated along a circle in the electric field generated by another point charge Q. The work done by the electric field on the rotating charge in one complete revolution is
(a) zero (b) positive (c) negative
(d) zero if the charge Q is at the centre and nonzero otherwise.

OBJECTIVE II

1. Mark out the correct options.
(a) The total charge of the universe is constant.
(b) The total positive charge of the universe is constant.
(c) The total negative charge of the universe is constant.
(d) The total number of charged particles in the universe is constant.

2. A point charge is brought in an electric field. The electric field at a nearby point
(a) will increase if the charge is positive
(b) will decrease if the charge is negative
(c) may increase if the charge is positive
(d) may decrease if the charge is negative.

3. The electric field and the electric potential at a point are E and V respectively.
(a) If $E = 0$, V must be zero.
(b) If $V = 0$, E must be zero.
(c) If $E \neq 0$, V cannot be zero.
(d) If $V \neq 0$, E cannot be zero.

4. The electric potential decreases uniformly from 120 V to 80 V as one moves on the x-axis from $x = -1$ cm to $x = +1$ cm. The electric field at the origin
(a) must be equal to 20 Vcm^{-1}
(b) may be equal to 20 Vcm^{-1}
(c) may be greater than 20 Vcm^{-1}
(d) may be less than 20 Vcm^{-1}.

5. Which of the following quantities do not depend on the choice of zero potential or zero potential energy?
(a) Potential at a point
(b) Potential difference between two points
(c) Potential energy of a two-charge system
(d) Change in potential energy of a two-charge system.

6. An electric dipole is placed in an electric field generated by a point charge.
(a) The net electric force on the dipole must be zero.
(b) The net electric force on the dipole may be zero.
(c) The torque on the dipole due to the field must be zero.
(d) The torque on the dipole due to the field may be zero.

7. A proton and an electron are placed in a uniform electric field.
(a) The electric forces acting on them will be equal.
(b) The magnitudes of the forces will be equal.
(c) Their accelerations will be equal.
(d) The magnitudes of their accelerations will be equal.

8. The electric field in a region is directed outward and is proportional to the distance r from the origin. Taking the electric potential at the origin to be zero,
(a) it is uniform in the region
(b) it is proportional to r
(c) it is proportional to r^2
(d) it increases as one goes away from the origin.

EXERCISES

1. Find the dimensional formula of ε_0.
2. A charge of 1·0 C is placed at the top of your college building and another equal charge at the top of your house. Take the separation between the two charges to be 2·0 km. Find the force exerted by the charges on each other. How many times of your weight is this force ?
3. At what separation should two equal charges, 1·0 C each, be placed so that the force between them equals the weight of a 50 kg person ?
4. Two equal charges are placed at a separation of 1·0 m. What should be the magnitude of the charges so that the force between them equals the weight of a 50 kg person ?
5. Find the electric force between two protons separated by a distance of 1 fermi (1 fermi = 10^{-15} m). The protons in a nucleus remain at a separation of this order.
6. Two charges $2 \cdot 0 \times 10^{-6}$ C and $1 \cdot 0 \times 10^{-6}$ C are placed at a separation of 10 cm. Where should a third charge be placed such that it experiences no net force due to these charges ?
7. Suppose the second charge in the previous problem is $-1 \cdot 0 \times 10^{-6}$ C. Locate the position where a third charge will not experience a net force.
8. Two charged particles are placed at a distance 1·0 cm apart. What is the minimum possible magnitude of the electric force acting on each charge ?
9. Estimate the number of electrons in 100 g of water. How much is the total negative charge on these electrons ?
10. Suppose all the electrons of 100 g water are lumped together to form a negatively charged particle and all the nuclei are lumped together to form a positively charged particle. If these two particles are placed 10·0 cm away from each other, find the force of attraction between them. Compare it with your weight.
11. Consider a gold nucleus to be a sphere of radius 6·9 fermi in which protons and neutrons are distributed. Find the force of repulsion between two protons situated at largest separation. Why do these protons not fly apart under this repulsion ?
12. Two insulating small spheres are rubbed against each other and placed 1 cm apart. If they attract each other with a force of 0·1 N, how many electrons were transferred from one sphere to the other during rubbing ?
13. NaCl molecule is bound due to the electric force between the sodium and the chlorine ions when one electron of sodium is transferred to chlorine. Taking the separation between the ions to be $2 \cdot 75 \times 10^{-8}$ cm, find the force of attraction between them. State the assumptions (if any) that you have made.
14. Find the ratio of the electric and gravitational forces between two protons.
15. Suppose an attractive nuclear force acts between two protons which may be written as $F = Ce^{-kr}/r^2$. (a) Write down the dimensional formulae and appropriate SI units of C and k. (b) Suppose that $k = 1$ fermi^{-1} and that the repulsive electric force between the protons is just balanced by the attractive nuclear force when the separation is 5 fermi. Find the value of C.
16. Three equal charges, $2 \cdot 0 \times 10^{-6}$ C each, are held fixed at the three corners of an equilateral triangle of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest two.
17. Four equal charges $2 \cdot 0 \times 10^{-6}$ C each are fixed at the four corners of a square of side 5 cm. Find the Coulomb force experienced by one of the charges due to the rest three.
18. A hydrogen atom contains one proton and one electron. It may be assumed that the electron revolves in a circle of radius 0·53 angstrom (1 angstrom = 10^{-10} m and is abbreviated as Å) with the proton at the centre. The hydrogen atom is said to be in the ground state in this case. Find the magnitude of the electric force between the proton and the electron of a hydrogen atom in its ground state.
19. Find the speed of the electron in the ground state of a hydrogen atom. The description of ground state is given in the previous problem.
20. Ten positively charged particles are kept fixed on the x-axis at points $x = 10$ cm, 20 cm, 30 cm, ..., 100 cm. The first particle has a charge $1 \cdot 0 \times 10^{-8}$ C, the second 8×10^{-8} C, the third 27×10^{-8} C and so on. The tenth particle has a charge 1000×10^{-8} C. Find the magnitude of the electric force acting on a 1 C charge placed at the origin.
21. Two charged particles having charge $2 \cdot 0 \times 10^{-8}$ C each are joined by an insulating string of length 1 m and the system is kept on a smooth horizontal table. Find the tension in the string.
22. Two identical balls, each having a charge of $2 \cdot 00 \times 10^{-7}$ C and a mass of 100 g, are suspended from a common point by two insulating strings each 50 cm long. The balls are held at a separation 5·0 cm apart and then released. Find (a) the electric force on one of the charged balls (b) the components of the resultant force on it along and perpendicular to the string (c) the tension in the string (d) the acceleration of one of the balls. Answers are to be obtained only for the instant just after the release.
23. Two identical pith balls are charged by rubbing against each other. They are suspended from a horizontal rod through two strings of length 20 cm each, the separation between the suspension points being 5 cm. In equilibrium, the separation between the balls is 3 cm. Find the mass of each ball and the tension in the strings. The charge on each ball has a magnitude $2 \cdot 0 \times 10^{-8}$ C.
24. Two small spheres, each having a mass of 20 g, are suspended from a common point by two insulating strings of length 40 cm each. The spheres are identically charged and the separation between the balls at

equilibrium is found to be 4 cm. Find the charge on each sphere.

25. Two identical pith balls, each carrying a charge q, are suspended from a common point by two strings of equal length l. Find the mass of each ball if the angle between the strings is 2θ in equilibrium.

26. A particle having a charge of $2 \cdot 0 \times 10^{-4}$ C is placed directly below and at a separation of 10 cm from the bob of a simple pendulum at rest. The mass of the bob is 100 g. What charge should the bob be given so that the string becomes loose?

27. Two particles A and B having charges q and $2q$ respectively are placed on a smooth table with a separation d. A third particle C is to be clamped on the table in such a way that the particles A and B remain at rest on the table under electrical forces. What should be the charge on C and where should it be clamped?

28. Two identically charged particles are fastened to the two ends of a spring of spring constant 100 N m^{-1} and natural length 10 cm. The system rests on a smooth horizontal table. If the charge on each particle is $2 \cdot 0 \times 10^{-8}$ C, find the extension in the length of the spring. Assume that the extension is small as compared to the natural length. Justify this assumption after you solve the problem.

29. A particle A having a charge of $2 \cdot 0 \times 10^{-6}$ C is held fixed on a horizontal table. A second charged particle of mass 80 g stays in equilibrium on the table at a distance of 10 cm from the first charge. The coefficient of friction between the table and this second particle is $\mu = 0 \cdot 2$. Find the range within which the charge of this second particle may lie.

30. A particle A having a charge of $2 \cdot 0 \times 10^{-6}$ C and a mass of 100 g is placed at the bottom of a smooth inclined plane of inclination 30°. Where should another particle B, having same charge and mass, be placed on the incline so that it may remain in equilibrium?

31. Two particles A and B, each having a charge Q, are placed a distance d apart. Where should a particle of charge q be placed on the perpendicular bisector of AB so that it experiences maximum force? What is the magnitude of this maximum force?

32. Two particles A and B, each carrying a charge Q, are held fixed with a separation d between them. A particle C having mass m and charge q is kept at the middle point of the line AB. (a) If it is displaced through a distance x perpendicular to AB, what would be the electric force experienced by it. (b) Assuming $x << d$, show that this force is proportional to x. (c) Under what conditions will the particle C execute simple harmonic motion if it is released after such a small displacement? Find the time period of the oscillations if these conditions are satisfied.

33. Repeat the previous problem if the particle C is displaced through a distance x along the line AB.

34. The electric force experienced by a charge of $1 \cdot 0 \times 10^{-6}$ C is $1 \cdot 5 \times 10^{-3}$ N. Find the magnitude of the electric field at the position of the charge.

35. Two particles A and B having charges of $+2 \cdot 00 \times 10^{-6}$ C and of $-4 \cdot 00 \times 10^{-6}$ C respectively are held fixed at a separation of 20·0 cm. Locate the point(s) on the line AB where (a) the electric field is zero (b) the electric potential is zero.

36. A point charge produces an electric field of magnitude $5 \cdot 0 \text{ N C}^{-1}$ at a distance of 40 cm from it. What is the magnitude of the charge?

37. A water particle of mass 10·0 mg and having a charge of $1 \cdot 50 \times 10^{-6}$ C stays suspended in a room. What is the magnitude of electric field in the room? What is its direction?

38. Three identical charges, each having a value $1 \cdot 0 \times 10^{-8}$ C, are placed at the corners of an equilateral triangle of side 20 cm. Find the electric field and potential at the centre of the triangle.

39. Positive charge Q is distributed uniformly over a circular ring of radius R. A particle having a mass m and a negative charge q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming $x << R$, find the time period of oscillation of the particle if it is released from there.

40. A rod of length L has a total charge Q distributed uniformly along its length. It is bent in the shape of a semicircle. Find the magnitude of the electric field at the centre of curvature of the semicircle.

41. A 10-cm long rod carries a charge of +50 μC distributed uniformly along its length. Find the magnitude of the electric field at a point 10 cm from both the ends of the rod.

42. Consider a uniformly charged ring of radius R. Find the point on the axis where the electric field is maximum.

43. A wire is bent in the form of a regular hexagon and a total charge q is distributed uniformly on it. What is the electric field at the centre? You may answer this part without making any numerical calculations.

44. A circular wire-loop of radius a carries a total charge Q distributed uniformly over its length. A small length dL of the wire is cut off. Find the electric field at the centre due to the remaining wire.

45. A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

46. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} C is at rest in a uniform, horizontal electric field of 20 kVm^{-1}. Find the tension in the thread.

47. A particle of mass m and charge q is thrown at a speed u against a uniform electric field E. How much distance will it travel before coming to momentary rest?

48. A particle of mass 1 g and charge $2 \cdot 5 \times 10^{-4}$ C is released from rest in an electric field of $1 \cdot 2 \times 10^{4} \text{ N C}^{-1}$. (a) Find the electric force and the force of gravity acting on this particle. Can one of these forces be neglected in comparison with the other for approximate analysis? (b) How long will it take for the particle to travel a distance of 40 cm? (c) What will be the speed of the particle after travelling this distance? (d) How much is the work done by the electric force on the particle during this period?

49. A ball of mass 100 g and having a charge of 4.9×10^{-5} C is released from rest in a region where a horizontal electric field of 2.0×10^{4} N C^{-1} exists. (a) Find the resultant force acting on the ball. (b) What will be the path of the ball? (c) Where will the ball be at the end of 2 s?

50. The bob of a simple pendulum has a mass of 40 g and a positive charge of 4.0×10^{-6} C. It makes 20 oscillations in 45 s. A vertical electric field pointing upward and of magnitude 2.5×10^{4} N C^{-1} is switched on. How much time will it now take to complete 20 oscillations?

51. A block of mass m having a charge q is placed on a smooth horizontal table and is connected to a wall through an unstressed spring of spring constant k as shown in figure (29-E1). A horizontal electric field E parallel to the spring is switched on. Find the amplitude of the resulting SHM of the block.

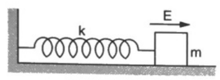

Figure 29-E1

52. A block of mass m containing a net positive charge q is placed on a smooth horizontal table which terminates in a vertical wall as shown in figure (29-E2). The distance of the block from the wall is d. A horizontal electric field E towards right is switched on. Assuming elastic collisions (if any) find the time period of the resulting oscillatory motion. Is it a simple harmonic motion?

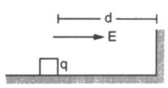

Figure 29-E2

53. A uniform electric field of 10 N C^{-1} exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50 cm.

54. 12 J of work has to be done against an existing electric field to take a charge of 0·01 C from A to B. How much is the potential difference $V_B - V_A$?

55. Two equal charges, 2.0×10^{-7} C each, are held fixed at a separation of 20 cm. A third charge of equal magnitude is placed midway between the two charges. It is now moved to a point 20 cm from both the charges. How much work is done by the electric field during the process?

56. An electric field of 20 N C^{-1} exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the points A and B are given by,
(a) $A = (0, 0)$; $B = (4\text{ m}, 2\text{ m})$
(b) $A = (4\text{ m}, 2\text{ m})$; $B = (6\text{ m}, 5\text{ m})$
(c) $A = (0, 0)$; $B = (6\text{ m}, 5\text{ m})$.
Do you find any relation between the answers of parts (a), (b) and (c)?

57. Consider the situation of the previous problem. A charge of -2.0×10^{-4} C is moved from the point A to the point B. Find the change in electrical potential energy $U_B - U_A$ for the cases (a), (b) and (c).

58. An electric field $\vec{E} = (\vec{i}\,20 + \vec{j}\,30)$ N C^{-1} exists in the space. If the potential at the origin is taken to be zero, find the potential at (2 m, 2 m).

59. An electric field $\vec{E} = \vec{i}Ax$ exists in the space, where $A = 10$ V m^{-2}. Take the potential at (10 m, 20 m) to be zero. Find the potential at the origin.

60. The electric potential existing in space is $V(x, y, z) = A(xy + yz + zx)$. (a) Write the dimensional formula of A. (b) Find the expression for the electric field. (c) If A is 10 SI units, find the magnitude of the electric field at (1 m, 1 m, 1 m).

61. Two charged particles, having equal charges of 2.0×10^{-5} C each, are brought from infinity to within a separation of 10 cm. Find the increase in the electric potential energy during the process.

62. Some equipotential surfaces are shown in figure (29-E3). What can you say about the magnitude and the direction of the electric field?

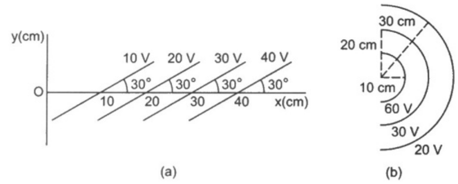

Figure 29-E3

63. Consider a circular ring of radius r, uniformly charged with linear charge density λ. Find the electric potential at a point on the axis at a distance x from the centre of the ring. Using this expression for the potential, find the electric field at this point.

64. An electric field of magnitude 1000 N C^{-1} is produced between two parallel plates having a separation of 2·0 cm as shown in figure (29-E4). (a) What is the potential difference between the plates? (b) With what minimum speed should an electron be projected from the lower plate in the direction of the field so that it may reach the upper plate? (c) Suppose the electron is projected from the lower plate with the speed calculated in part (b). The direction of projection makes an angle of 60° with the field. Find the maximum height reached by the electron.

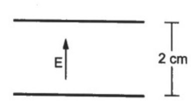

Figure 29-E4

65. A uniform field of $2{\cdot}0\ \mathrm{N\,C^{-1}}$ exists in space in x-direction. (a) Taking the potential at the origin to be zero, write an expression for the potential at a general point (x, y, z). (b) At which points, the potential is 25 V ? (c) If the potential at the origin is taken to be 100 V, what will be the expression for the potential at a general point ? (d) What will be the potential at the origin if the potential at infinity is taken to be zero ? Is it practical to choose the potential at infinity to be zero ?

66. How much work has to be done in assembling three charged particles at the vertices of an equilateral triangle as shown in figure (29-E5) ?

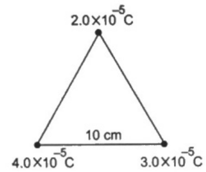

Figure 29-E5

67. The kinetic energy of a charged particle decreases by 10 J as it moves from a point at potential 100 V to a point at potential 200 V. Find the charge on the particle.

68. Two identical particles, each having a charge of $2{\cdot}0 \times 10^{-4}$ C and mass of 10 g, are kept at a separation of 10 cm and then released. What would be the speeds of the particles when the separation becomes large ?

69. Two particles have equal masses of 5·0 g each and opposite charges of $+4{\cdot}0 \times 10^{-5}$ C and $-4{\cdot}0 \times 10^{-5}$ C. They are released from rest with a separation of 1·0 m between them. Find the speeds of the particles when the separation is reduced to 50 cm.

70. A sample of HCl gas is placed in an electric field of $2{\cdot}5 \times 10^{4}\ \mathrm{N\,C^{-1}}$. The dipole moment of each HCl molecule is $3{\cdot}4 \times 10^{-30}$ Cm. Find the maximum torque that can act on a molecule.

71. Two particles A and B, having opposite charges $2{\cdot}0 \times 10^{-6}$ C and $-2{\cdot}0 \times 10^{-6}$ C, are placed at a separation of 1·0 cm. (a) Write down the electric dipole moment of this pair. (b) Calculate the electric field at a point on the axis of the dipole 1·0 m away from the centre. (c) Calculate the electric field at a point on the perpendicular bisector of the dipole and 1·0 m away from the centre.

72. Three charges are arranged on the vertices of an equilateral triangle as shown in figure (29-E6). Find the dipole moment of the combination.

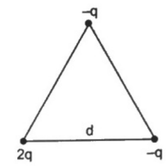

Figure 29-E6

73. Find the magnitude of the electric field at the point P in the configuration shown in figure (29-E7) for $d >> a$. Take $2qa = p$.

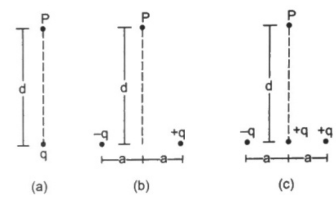

Figure 29-E7

74. Two particles, carrying charges $-q$ and $+q$ and having equal masses m each, are fixed at the ends of a light rod of length a to form a dipole. The rod is clamped at an end and is placed in a uniform electric field E with the axis of the dipole along the electric field. The rod is slightly tilted and then released. Neglecting gravity find the time period of small oscillations.

75. Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6·4 g (take the atomic weight of copper to be $64\ \mathrm{g\,mol^{-1}}$).

□

ANSWERS

OBJECTIVE I

1. (c) 2. (d) 3. (a) 4. (d) 5. (a) 6. (d)
7. (a) 8. (c) 9. (a)

OBJECTIVE II

1. (a) 2. (c), (d) 3. none
4. (b), (c) 5. (b), (d) 6. (d)
7. (b) 8. (c)

EXERCISES

1. $I^2M^{-1}L^{-3}T^4$
2. $2{\cdot}25 \times 10^3$ N
3. $4{\cdot}3 \times 10^3$ m
4. $2{\cdot}3 \times 10^{-4}$ C
5. 230 N
6. 5·9 cm from the larger charge in between the two charges
7. 34·1 cm from the larger charge on the line joining the charge in the side of the smaller charge
8. $2{\cdot}3 \times 10^{-24}$ N
9. $3{\cdot}35 \times 10^{25}$, $5{\cdot}35 \times 10^6$ C
10. $2{\cdot}56 \times 10^{25}$ N
11. 1·2 N
12. 2×10^{11}
13. $3{\cdot}05 \times 10^{-9}$ N
14. $1{\cdot}23 \times 10^{36}$
15. (a) ML^3T^{-2}, L^{-1}, $N\,m^2$, m^{-1} (b) $3{\cdot}4 \times 10^{-26}\ N\,m^2$
16. 24·9 N at 30° with the extended sides from the charge under consideration
17. 27·5 N at 45° with the extended sides of the square from the charge under consideration
18. $8{\cdot}2 \times 10^{-8}$ N
19. $2{\cdot}18 \times 10^6$ m s^{-1}
20. $4{\cdot}95 \times 10^5$ N
21. $3{\cdot}6 \times 10^{-6}$ N
22. (a) 0·144 N
    (b) zero, 0·095 N away from the other charge
    (c) 0·986 N and (d) 0·95 m s^{-2} perpendicular to the string and going away from the other charge
23. 8·2 g, $8{\cdot}2 \times 10^{-2}$ N
24. $4{\cdot}17 \times 10^{-8}$ C
25. $\dfrac{q^2 \cot\theta}{16\pi\varepsilon_0 g l^2 \sin^2\theta}$
26. $5{\cdot}4 \times 10^{-9}$ C
27. $-(6 - 4\sqrt{2})\,q$, between q and $2q$ at a distance of $(\sqrt{2} - 1)\,d$ from q
28. $3{\cdot}6 \times 10^{-6}$ m
29. between $\pm\, 8{\cdot}71 \times 10^{-8}$ C
30. 27 cm from the bottom
31. $d/2\sqrt{2}$, $3{\cdot}08 \dfrac{Qq}{4\pi\varepsilon_0 d^2}$
32. (a) $\dfrac{Qqx}{2\pi\varepsilon_0\left(x^2 + \dfrac{d^2}{4}\right)^{3/2}}$ (c) $\left[\dfrac{m\pi^3\varepsilon_0 d^3}{Qq}\right]^{\frac{1}{2}}$
33. time period $= \left[\dfrac{\pi^3\varepsilon_0 m d^3}{2Qq}\right]^{\frac{1}{2}}$
34. $1{\cdot}5 \times 10^3$ N C^{-1}
35. (a) 48·3 cm from A along BA
    (b) 20 cm from A along BA and $\dfrac{20}{3}$ cm from A along AB
36. $8{\cdot}9 \times 10^{-11}$ C
37. 65·3 N C^{-1}, upward
38. zero, $2{\cdot}3 \times 10^3$ V
39. $\left[\dfrac{16\pi^3\varepsilon_0 m R^3}{Qq}\right]^{1/2}$
40. $\dfrac{Q}{2\varepsilon_0 L^2}$
41. $5{\cdot}2 \times 10^7$ N C^{-1}
42. $R/\sqrt{2}$
43. zero
44. $\dfrac{QdL}{8\pi^2\varepsilon_0 a^3}$
45. $\dfrac{q}{4\pi\varepsilon_0 d^2}$ towards the charge q
46. $8{\cdot}8 \times 10^{-4}$ N
47. $\dfrac{mu^2}{2qE}$
48. (a) 3·0 N, $9{\cdot}8 \times 10^{-3}$ N (b) $1{\cdot}63 \times 10^{-2}$ s
    (c) 49·0 m s^{-1} (d) 1·20 J
49. (a) 1·4 N making an angle of 45° with $\vec{g}$ and $\vec{E}$
    (b) straight line along the resultant force
    (c) 28 m from the starting point on the line of motion
50. 52 s
51. qE/k
52. $\sqrt{\dfrac{8\,md}{qE}}$
53. 5 V
54. 1200 volts
55. $3{\cdot}6 \times 10^{-3}$ J
56. (a) −80 V (b) −40 V (c) −120 V
57. 0·016 J, 0·008 J, 0·024 J
58. −100 V
59. 500 V
60. (a) $MT^{-3}I^{-1}$ (b) $-A\,\{\vec{i}(y+z) + \vec{j}(z+x) + \vec{k}(x+y)\}$
    (c) 35 N C^{-1}
61. 36 J
62. (a) 200 V m^{-1} making an angle 120° with the x-axis
    (b) radially outward, decreasing with distance as $E = \dfrac{6\text{ Vm}}{r^2}$.
63. $\dfrac{r\lambda}{2\varepsilon_0(r^2 + x^2)^{1/2}}$, $\dfrac{r\lambda x}{2\varepsilon_0(r^2 + x^2)^{3/2}}$

64. (a) 20 V (b) $2{\cdot}65 \times 10^{6}$ m s^{-1} (c) 0·50 cm

65. (a) $-(2{\cdot}0 \text{ V m}^{-1})\, x$

(b) points on the plane $x = -12{\cdot}5$ m

(c) $100 \text{ V} - (2{\cdot}0 \text{ V m}^{-1})\, x$

(d) infinity

66. 234 J

67. 0·1 C

68. 600 m s^{-1}

69. 54 m s^{-1} for each particle

70. $8{\cdot}5 \times 10^{-26}$ Nm

71. (a) $2{\cdot}0 \times 10^{-8}$ Cm (b) 360 N C^{-1} (c) 180 N C^{-1}

72. $qd\sqrt{3}$, along the bisector of the angle at $2q$, away from the triangle

73. (a) $\frac{q}{4\pi\varepsilon_0 d^2}$ (b) $\frac{p}{4\pi\varepsilon_0 d^3}$ (c) $\frac{1}{4\pi\varepsilon_0 d^3}\sqrt{q^2 d^2 + p^2}$

74. $2\pi\sqrt{\frac{ma}{qE}}$

75. 6×10^{22}

□

CHAPTER 30

GAUSS'S LAW

Gauss's law is one of the fundamental laws of physics. It relates the electric field to the charge distribution which has produced this field. In section (30.1) we define the flux of an electric field and in the next section we discuss the concept of a solid angle. These will be needed to state and understand Gauss's law.

30.1 FLUX OF AN ELECTRIC FIELD THROUGH A SURFACE

Consider a hypothetical plane surface of area ΔS and suppose a uniform electric field $\vec{E}$ exists in the space (figure 30.1). Draw a line perpendicular to the surface and call one side of it, the positive normal to the surface. Suppose, the electric field $\vec{E}$ makes an angle θ with the positive normal.

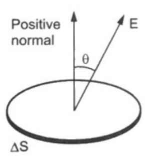

Figure 30.1

The quantity

$$\Delta\Phi = E\,\Delta S\cos\theta$$

is called *the flux of the electric field through the chosen surface*. If we draw a vector of magnitude ΔS along the positive normal, it is called the *area-vector* $\Delta\vec{S}$ corresponding to the area ΔS. One can then write

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S}.$$

Remember, the direction of the area-vector is always along the normal to the corresponding surface. If the field $\vec{E}$ is perpendicular to the surface, it is parallel to the area-vector. If $\vec{E}$ is along the positive normal, $\theta = 0$, and $\Delta\Phi = E\,\Delta S$. If it is opposite to the positive normal, $\theta = \pi$, and $\Delta\Phi = -E\,\Delta S$. If the electric field is parallel to the surface, $\theta = \pi/2$, and $\Delta\Phi = 0$.

Flux is a scalar quantity and may be added using the rules of scalar addition. Thus, if the surface ΔS has two parts ΔS_1 and ΔS_2, the flux through ΔS equals the flux through ΔS_1 plus the flux through ΔS_2. This gives us a clue to define the flux through surfaces which are not plane, as well as the flux when the field is not uniform. We divide the given surface into smaller parts so that each part is approximately plane and the variation of electric field over each part can be neglected. We calculate the flux through each part separately, using the relation $\Delta\Phi = \vec{E}\cdot\Delta\vec{S}$ and then add the flux through all the parts. Using the techniques of integration, the flux is

$$\Phi = \int \vec{E}\cdot d\vec{S}$$

where integration has to be performed over the entire surface through which the flux is required.

The surface under consideration may be a closed one, enclosing a volume, such as a spherical surface. A hemispherical surface is an open surface. A cylindrical surface is also open. A cylindrical surface plus two plane surfaces perpendicular to the axis enclose a volume and these three taken together form a closed surface. When flux through a closed surface is required, we use a small circular sign on the integration symbol;

$$\Phi = \oint \vec{E}\cdot d\vec{S}.$$

It is customary to take the outward normal as positive in this case.

Example 30.1

A square frame of edge 10 cm *is placed with its positive normal making an angle of* 60° *with a uniform electric field of* $20\ \mathrm{V\,m^{-1}}$. *Find the flux of the electric field through the surface bounded by the frame.*

Solution : The surface considered is plane and the electric field is uniform (figure 30.2). Hence, the flux is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S}$$
$$= E\,\Delta S\cos 60^\circ$$

$$= (20 \text{ V m}^{-1})(0{\cdot}01 \text{ m}^2)\left(\frac{1}{2}\right) = 0{\cdot}1 \text{ Vm}.$$

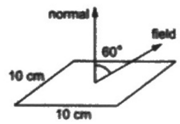

Figure 30.2

Example 30.2

A charge q is placed at the centre of a sphere. Taking outward normal as positive, find the flux of the electric field through the surface of the sphere due to the enclosed charge.

Solution :

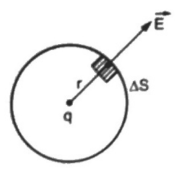

Figure 30.3

Let us take a small element ΔS on the surface of the sphere (Figure 30.3). The electric field here is radially outward and has the magnitude

$$\frac{q}{4\pi\varepsilon_0 r^2},$$

where r is the radius of the sphere. As the positive normal is also outward, $\theta = 0$ and the flux through this part is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = \frac{q}{4\pi\varepsilon_0 r^2}\Delta S.$$

Summing over all the parts of the spherical surface,

$$\Phi = \sum \Delta\Phi = \frac{q}{4\pi\varepsilon_0 r^2}\sum \Delta S = \frac{q}{4\pi\varepsilon_0 r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}.$$

Example 30.3

A uniform electric field exists in space. Find the flux of this field through a cylindrical surface with the axis parallel to the field.

Solution :

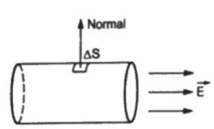

Figure 30.4

Consider figure (30.4) and take a small area ΔS on the cylindrical surface. The normal to this area will be perpendicular to the axis of the cylinder. But the electric field is parallel to the axis and hence

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S \cos(\pi/2) = 0.$$

This is true for each small part of the cylindrical surface. Summing over the entire surface, the total flux is zero.

30.2 SOLID ANGLE

Solid angle is a generalisation of the plane angle. In figure (30.5a) we show a plane curve AB. The end points A and B are joined to the point O. We say that the curve AB subtends an *angle* or a *plane angle* at O. An angle is formed at O by the two lines OA and OB passing through O.

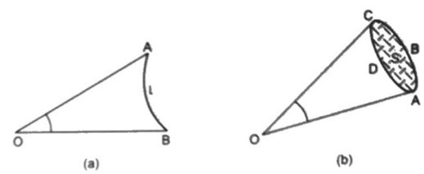

Figure 30.5

To construct a solid angle, we start with a surface S (figure 30.5b) and join all the points on the periphery such as A, B, C, D, etc., with the given point O. We then say that a *solid angle* is formed at O and that the surface S has subtended the solid angle. The solid angle is formed by the lines joining the points on the periphery with O. The whole figure looks like a cone. As a typical example, think of the paper containers used by Moongfaliwalas.

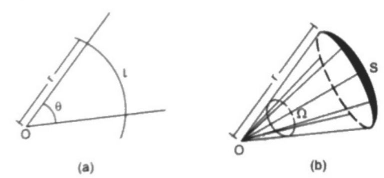

Figure 30.6

How do we measure a solid angle ? Let us consider how do we measure a plane angle. See figure (30.6a). We draw a circle of any radius r with the centre at O and measure the length l of the arc intercepted by the angle. The angle θ is then defined as $\theta = l/r$. In order to measure a solid angle at the point O (figure 30.6b), we draw a sphere of any radius r with O as the centre and measure the area S of the part of the sphere intercepted by the cone. The solid angle Ω is then defined as

$$\Omega = S/r^2.$$

Note that this definition makes the solid angle a dimensionless quantity. It is independent of the radius of the sphere drawn.

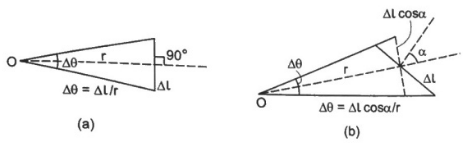

Figure 30.7

Next, consider a plane angle subtended at a point O by a small line segment Δl (figure 30.7a). Suppose, the line joining O to the middle point of Δl is perpendicular to Δl. As the segment is small, we can approximately write

$$\Delta\theta = \frac{\Delta l}{r}.$$

As Δl gets smaller, the approximation becomes better. Now suppose, the line joining O to Δl is not perpendicluar to Δl (figure 30.7b). Suppose, this line makes an angle α with the perpendicular to Δl. The angle subtended by Δl at O is

$$\Delta\theta = \frac{\Delta l \cos\alpha}{r}.$$

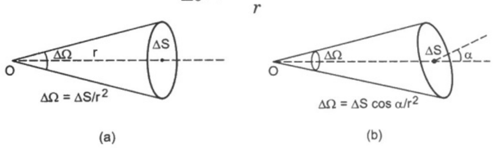

Figure 30.8

Similarly, if a small plane area ΔS (figure 30.8a) subtends a solid angle $\Delta\Omega$ at O in such a way that the line joining O to ΔS is normal to ΔS, we can write $\Delta\Omega = \Delta S / r^2$. But if the line joining O to ΔS makes an angle α with the normal to ΔS (figure 30.8b), we should write

$$\Delta\Omega = \frac{\Delta S \cos\alpha}{r^2}.$$

A complete circle subtends an angle

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = 2\pi$$

at the centre. In fact, any closed curve subtends an angle 2π at any of the internal points. Similarly, a complete sphere subtends a solid angle

$$\Omega = \frac{S}{r^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

at the centre. Also, any closed surface subtends a solid angle 4π at any internal point.

How much is the angle subtended by a closed plane curve at an external point ?

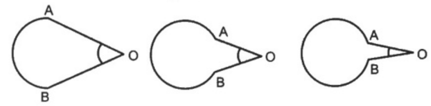

Figure 30.9

See figure (30.9). As we gradually close the curve, the angle finally diminishes to zero. A closed curve subtends zero angle at an external point. Similarly, a closed surface subtends zero solid angle at an external point.

30.3 GAUSS'S LAW AND ITS DERIVATION FROM COULOMB'S LAW

The statement of the Gauss's law may be written as follows:

The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by ε_0. In symbols,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} \qquad \text{... (30.1)}$$

where q_{in} is the net charge enclosed by the surface through which the flux is calculated.

It should be carefully noted that the electric field on the left-hand side of equation (30.1) is the resultant electric field due to all the charges existing in the space, whereas, the charge appearing on the right-hand side includes only those which are inside the closed surface.

Gauss's law is taken as a fundamental law of nature, a law whose validity is shown by experiments. However, historically Coulomb's law was discovered before Gauss's law and it is possible to derive Gauss's law from Coulomb's law.

Proof of Gauss's Law (Assuming Coulomb's Law)

Flux due to an internal charge

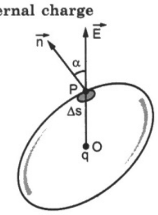

Figure 30.10

Suppose a charge q is placed at a point O inside a "closed" surface (figure 30.10). Take a point P on the surface and consider a small area ΔS on the surface around P. Let $OP = r$. The electric field at P due to the charge q is

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

along the line OP. Suppose this line OP makes an angle α with the outward normal to ΔS. The flux of the electric field through ΔS is

$$\Delta\Phi = \vec{E}\cdot\Delta\vec{S} = E\,\Delta S\cos\alpha$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}\Delta S\cos\alpha$$

$$= \frac{q}{4\pi\varepsilon_0}\Delta\Omega$$

where $\Delta\Omega = \frac{\Delta S\cos\alpha}{r^2}$ is the solid angle subtended by ΔS at O. The flux through the entire surface is

$$\Phi = \Sigma\,\frac{q}{4\pi\varepsilon_0}\,\Delta\Omega = \frac{q}{4\pi\varepsilon_0}\,\Sigma\,\Delta\Omega.$$

The sum over $\Delta\Omega$ is the total solid angle subtended by the closed surface at the internal point O and hence is equal to 4π.

The total flux of the electric field due to the internal charge q through the closed surface is, therefore,

$$\Phi = \frac{q}{4\pi\varepsilon_0}\,4\pi = \frac{q}{\varepsilon_0}\cdot \qquad \ldots \text{(i)}$$

Flux due to an external charge

Now, suppose a charge q is placed at a point O outside the closed surface. The flux of the electric field due to q through the small area ΔS is again

$$\Delta\Phi = \frac{q}{4\pi\varepsilon_0}\,\frac{\Delta S\cos\alpha}{r^2} = \frac{q}{4\pi\varepsilon_0}\,\Delta\Omega.$$

When we sum over all the small area elements of the closed surface we get $\Sigma\Delta\Omega = 0$ as this is the total solid angle subtended by the closed surface at an external point. Hence,

$$\Phi = 0. \qquad \ldots \text{(ii)}$$

Flux due to a combination of charges

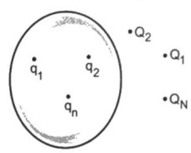

Figure 30.11

Finally, consider a general situation (figure 30.11) where charges q_1, q_2, ..., q_n are inside a closed surface and charges Q_1, Q_2, ..., Q_N are outside it. The resultant electric field at any point is

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \ldots + \vec{E}_n + \vec{E'}_1 + \vec{E'}_2 + \ldots + \vec{E'}_N$$

where E_i and E'_i are the fields due to q_i and Q_i respectively. Thus, the flux of the resultant electric field through the closed surface is

$$\Phi = \oint \vec{E}\cdot d\vec{S} = \oint \vec{E}_1\cdot d\vec{S} + \oint \vec{E}_2\cdot d\vec{S} + \ldots + \oint \vec{E}_n\cdot d\vec{S}$$

$$+ \oint \vec{E'}_1\cdot d\vec{S} + \oint \vec{E'}_2\cdot d\vec{S} + \ldots + \oint \vec{E'}_N\cdot d\vec{S}. \qquad \ldots \text{(iii)}$$

Now, $\oint \vec{E}_i\cdot d\vec{S}$ is the flux of the electric field due to the charge q_i only. As this charge is inside the closed surface, from (i), it is equal to q_i/ε_0. Also, $\oint \vec{E'}_i\cdot d\vec{S}$ is the flux of the electric field due to the charge Q_i which is outside the closed surface. This flux is, therefore, zero from (ii). Using these results in (iii),

$$\Phi = \frac{q_1}{\varepsilon_0} + \frac{q_2}{\varepsilon_0} + \ldots + \frac{q_n}{\varepsilon_0} + 0 + \ldots + 0$$

or, $$\Phi = \frac{1}{\varepsilon_0}\,\Sigma\,q_i$$

or, $$\oint \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}.$$

This completes the derivation of Gauss's law (equation 30.1).

We once again emphasise that the electric field appearing in the Gauss's law is the resultant electric field due to all the charges present inside as well as outside the given closed surface. On the other hand, the charge q_{in} appearing in the law is only the charge contained within the closed surface. The contribution of the charges outside the closed surface in producing the flux is zero. A surface on which Gauss's law is applied, is sometimes called the *Gaussian surface.*

Example 30.4

A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring (figure 30.12). Find the flux of the electric field through the surface of the sphere.

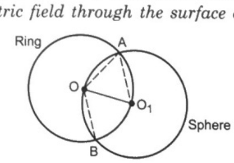

Figure 30.12

Solution : From the geometry of the figure, $OA = OO_1$ and $O_1A = O_1O$. Thus, OAO_1 is an equilateral triangle. Hence $\angle AOO_1 = 60°$ or $\angle AOB = 120°$.

The arc AO_1B of the ring subtends an angle 120° at the centre O. Thus, one third of the ring is inside the sphere.