

# Team notebook

greedy is good

December 23, 2019

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## 1 Basics

### 1.1 Dos and Donts

---

```
/* INSTRUCTIONS
1. Focus on the problem, Not on the Scoreboard (Specially Lad)
2. Review code before submitting. 2 min review < 20 min penalty
3. Watch out for overflows, out of bound errors
4. Stay Calm. Good Luck. Have Fun :) */

// Compiler Settings : alias g++=g++ -g -O2 -std=gnu++14 -Wall
```

---

## 1.2 Templates

### 1.2.1 Akshat

---

```
// #pragma GCC optimize("Ofast")
// #pragma GCC optimize ("unroll-loops")
// #pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,abm,mmx,avx,tune=native")
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
using namespace std;
#define ll long long int
#define ld unsigned long long int
#define pi pair<ll,ll>
#define pb push_back
#define pf push_front
#define pu push
#define po pop
#define fi first
#define se second
#define mk make_pair
#define ve vector
#define lr(n) for(ll i=0;i<n;i++)
#define all(x) x.begin(),x.end()
#define be begin
#define sz(a) (ll)a.size()
#define INF 1e18
```

---

### 1.2.2 Lad

---

```
#include <bits/stdc++.h>
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
using namespace std;
typedef long long int ll;
typedef unsigned long long int ull;
typedef long double ld;
typedef pair <ll, ll> pll;
typedef pair <int, int> pii;
typedef tree <ll, null_type, less <ll>, rb_tree_tag, tree_order_statistics_node_update>
ordered_set;
// order_of_key(val): returns the number of values less than val
// find_by_order(k): returns an iterator to the kth largest element (0-based)
#define pb push_back
#define mp make_pair
#define ff first
#define ss second
#define all(a) a.begin(), a.end()
```

---

```
#define sz(a) (ll)(a.size())
#define endl "\n"
template <class Ch, class Tr, class Container>
basic_ostream <Ch, Tr> & operator << (basic_ostream <Ch, Tr> & os, Container const& x)
{
    os << "{ ";
    for(auto& y : x)
    {
        os << y << " ";
    }
    return os << "}";
}
template <class X, class Y>
ostream & operator << (ostream & os, pair <X, Y> const& p)
{
    return os << "[" << p.ff << ", " << p.ss << "]";
}
ll gcd(ll a, ll b)
{
    if(b==0)
    {
        return a;
    }
    return gcd(b, a%b);
}
ll modexp(ll a, ll b, ll c)
{
    a%=c;
    ll ans = 1;
    while(b)
    {
        if(b&1)
        {
            ans = (ans*a)%c;
        }

        a = (a*a)%c;
        b >>= 1;
    }
    return ans;
}
const ll L = 1e5+5;
int main()
{
    ios_base::sync_with_stdio(false);
    cin.tie(NULL); cout.tie(NULL);
}
```

---

## 2 DP

### 2.1 Convex Hull DP2

---

```

struct Line { // gives max value of x
    ll k, m;
    mutable ll p;
    bool operator<(const Line& o) const {
        return k < o.k;
    }
    bool operator<(const ll &x) const{
        return p < x;
    }
};
struct LineContainer : multiset<Line, less<>> {
    const ll inf = LLONG_MAX;
    ll div(ll a, ll b){
        return a / b - ((a ^ b) < 0 && a % b);
    }
    bool isect(iterator x, iterator y) {
        if (y == end()) { x->p = inf; return false; }
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

LineContainer lc;

```

---

### 2.2 LIS Using Segment Tree

---

```

int compare(pair<int, int> p1, pair<int, int> p2){
    if (p1.first == p2.first)
        return p1.second > p2.second;
    return p1.first < p2.first;
}
void buildTree(int* tree, int pos, int low, int high, int index, int value) {

```

```

    if (index < low || index > high)
        return;
    if (low == high) {
        tree[pos] = value;
        return;
    }
    int mid = (high + low) / 2;
    buildTree(tree, 2 * pos + 1, low, mid, index, value);
    buildTree(tree, 2 * pos + 2, mid + 1, high, index, value);
    tree[pos] = max(tree[2 * pos + 1], tree[2 * pos + 2]);
}
int findMax(int* tree, int pos, int low, int high, int start, int end) {
    if (low >= start && high <= end)
        return tree[pos];
    if (start > high || end < low)
        return 0;
    int mid = (high + low) / 2;
    return max(findMax(tree, 2 * pos + 1, low, mid, start, end),
               findMax(tree, 2 * pos + 2, mid + 1, high, start, end));
}
int findLIS(int arr[], int n) {
    pair<int, int> p[n];
    for (int i = 0; i < n; i++) {
        p[i].first = arr[i];
        p[i].second = i;
    }
    sort(p, p + n, compare);
    int len = pow(2, (int)(ceil(sqrt(n))) + 1) - 1;
    int tree[len];
    memset(tree, 0, sizeof(tree));
    for (int i = 0; i < n; i++) {
        buildTree(tree, 0, 0, n - 1, p[i].second,
                  findMax(tree, 0, 0, n - 1, 0, p[i].second) + 1);
    }
    return tree[0];
}

```

---

### 2.3 SOS DP

---

```

ll N = 1000;
void SumOverSubsets(ll a[], ll n) {
    ll sos[1 << n] = {0};
    ll dp[N][N];
    for (ll x = 0; x < (1 << n); x++) {
        for (ll i = 0; i < n; i++) {
            if (x & (1 << i)) {
                if (i == 0)
                    dp[x][i] = a[x] + a[x ^ (1 << i)];
                else // dp recurrence
                    dp[x][i] = dp[x][i - 1] +
                        dp[x ^ (1 << i)][i - 1];
            }
        }
    }
}

```

```

    }
    else{ // i-th bit is not set
        if (i == 0)
            dp[x][i] = a[x]; // base condition
        else
            dp[x][i] = dp[x][i - 1]; // dp recurrence
    }
    }
    sos[x] = dp[x][n - 1];
}
for (ll i = 0; i < (1 << n); i++)
    cout << sos[i] << " ";
}

int main(){
    ll n; cin >> n;
    ll a[n];
    for (int i = 0; i < n; ++i){
        cin >> a[i];
    }
    SumOverSubsets(a, (1ll)log2(n));
    return 0;
}

```

### 3 Data Structures

#### 3.1 BIT

```

// 1-based indexing
/* Problem Statement:
Given a sequence of n numbers a1, a2,..., an and a number of k-queries.
A k-query is a triple (i, j, k) (1<=i<=j<=n). For each k-query
(i, j, k), you have to return the number of elements greater than k in
the subsequence ai, ai+1, ..., aj. */
struct M{
    ll key, key2, key3, key4;
};
bool cmp(struct M a, struct M b){
    if(a.key==b.key) return b.key4<=a.key4;
    return (a.key > b.key);
}
bool cmp2(struct M a, struct M b){
    return a.key4<b.key4;
}
ll bit[30002];
ll update(ll idx, ll n){
    while(idx<=n){
        bit[idx]++;
        idx=idx+(idx&(-idx));
    }
}

```

```

ll query(ll idx){
    ll sum=0;
    while(idx>0){
        sum+=bit[idx];
        idx=idx-(idx&(-idx));
    }
    return sum;
}

struct M Ssp[230000];
int main(){
    ll n; cin >> n; ll q;
    for (int i = 0; i < n; ++i){
        ll a; cin >> a;
        Ssp[i].key=a;
        Ssp[i].key2=Ssp[i].key3=0;
        Ssp[i].key4=i;
    }
    cin >> q;
    for (int i = 0; i < q; ++i){
        ll l,r,k; cin >> l >> r >> k;
        Ssp[i+n].key=k;
        Ssp[i+n].key2=l;
        Ssp[i+n].key3=r;
        Ssp[i+n].key4=i+n;
    }
    sort(Ssp, Ssp+n+q, cmp);
    for (int i = 0; i < n+q; ++i){
        if(!Ssp[i].key2)
            update(Ssp[i].key4+1,n);
        else
            Ssp[i].key=query(Ssp[i].key3)-query(Ssp[i].key2-1);
    }
    sort(Ssp, Ssp+n+q, cmp2);
    for (int i = 0; i < n+q; ++i){
        if(Ssp[i].key2)
            printf("%lld\n", Ssp[i].key);
    }
}

```

#### 3.2 Centroid Decomposition

```

// E. Xenia and Tree, Codeforces
#define ln 20
#define N 100001
#define INF 1e9
ll n; vector<vector<ll>>ar(N);
ll lev[N]; ll pa[N][ln];
ll centroidMarked[N]={0};
ll sub[N]; ll par[N]; ll ans[N];
// dist(u,v)
void dfs(ll u, ll p, ll l){

```

```

    pa[u][0]=p;
    lev[u]=1;
    for(auto i:ar[u]){
        if(i!=p)
            dfs(i,u,1+1);
    }
}
ll lca(ll u,ll v){
    if(lev[u]<lev[v]) swap(u,v);
    ll log;
    for(log=1;(1<<log)<=lev[u];log++);
    log--;
    for(ll i=log;i>=0;i--){
        if(lev[u]-(1<<i)>=lev[v])
            u=pa[u][i];
    }
    if(u==v) return u;
    for(ll i=log;i>=0;i--){
        if(pa[u][i]!=-1 && pa[u][i]!=pa[v][i])
            u=pa[u][i],v=pa[v][i];
    }
    return pa[u][0];
}
ll dist(ll u,ll v){
    return lev[u]+lev[v]-2*lev[lca(u,v)];
}
// decompose
ll nn;
void dfs1(ll u,ll p){
    nn++;
    sub[u]=1;
    for(auto i:ar[u]){
        if(i!=p && !centroidMarked[i]){
            dfs1(i,u);
            sub[u]+=sub[i];
        }
    }
}
ll dfs2(ll u,ll p){
    for(auto i:ar[u]){
        if(i!=p && !centroidMarked[i] && sub[i]>nn/2)
            return dfs2(i,u);
    }
    return u;
}
void decompose(ll u,ll p){
    nn=0;
    dfs1(u,p);
    ll centroid=dfs2(u,p);
    centroidMarked[centroid]=1;
    par[centroid]=p;
    for(auto i:ar[centroid]){
        if(!centroidMarked[i]){

```

```

            decompose(i,centroid);
        }
    }
}
// query
void update(ll u){
    ll x=u;
    while(x!=-1){
        ans[x]=min(ans[x],dist(u,x));
        x=par[x];
    }
}
ll query(ll u){
    ll x=u;
    ll an=INF;
    while(x!=-1){
        an=min(an,ans[x]+dist(u,x));
        x=par[x];
    }
    return an;
}
int main(){
    ll m;
    cin>>n>>m;
    for(ll i=1,u,v;i<n;i++){
        cin>>u>>v;
        ar[u].pb(v);
        ar[v].pb(u);
    }
    for(ll i=0;i<n;i++){
        for(ll j=0;j<n;j++){
            pa[i][j]=-1;
        }
    }
    dfs(1,-1,0);
    for(ll i=1;i<n;i++){
        for(ll j=1;j<n;j++){
            if(pa[j][i-1]!=-1)
                pa[j][i]=pa[pa[j][i-1]][i-1];
        }
    }
    decompose(1,-1);
    for(ll i=0;i<n;i++){
        ans[i]=INF;
    }
    update(1);
    while(m--){
        ll t,v;
        cin>>t;
        if(t==2){
            cin>>v;
            cout << query(v) << "\n";
        }
        else{
            cin>>v;

```

```

        update(v);
    }
}

```

### 3.3 Merge Sort Tree

```

// Merge Sort Tree to calculate kth smallest number in a range
// Works for online queries // Problem Codeforces 1262D2
bool cmp(pll a, pll b){
    if(a.ff == b.ff){
        return a.ss < b.ss;
    }
    return a.ff > b.ff;
}
ll kd[30][L], a[L], pos[L], Real[L];
void init(ll d, ll b, ll e){
    if(b == e){
        kd[d][b] = pos[b];
        return;
    }
    ll m = (b + e) >> 1;
    init(d + 1, b, m);
    init(d + 1, m + 1, e);
    ll i = b, j = m + 1;
    ll ptr = 0;
    while(i <= m && j <= e){
        if(kd[d + 1][i] < kd[d + 1][j]){
            kd[d][b + (ptr++)] = kd[d + 1][i++];
        }else{
            kd[d][b + (ptr++)] = kd[d + 1][j++];
        }
    }
    while(i <= m) kd[d][b + (ptr++)] = kd[d + 1][i++];
    while(j <= e) kd[d][b + (ptr++)] = kd[d + 1][j++];
}
inline ll find(ll d, ll b, ll e, ll x1, ll x2){
    return upper_bound(kd[d] + b, kd[d] + e + 1, x2) - lower_bound(kd[d] + b, kd[d] + e + 1, x1);
}
ll get(ll n, ll x1, ll x2, ll k){
    ll d = 0, b = 1, e = n;
    while(b != e){
        ll l11 = find(d + 1, b, (b + e) / 2, x1, x2);
        ll mm = ((b + e) >> 1LL);
        if(l11 >= k){
            e = mm;
        }else{
            b = mm + 1;
            k -= l11;
        }
    }
}

```

```

        ++d;
    }
    return b;
}
ll copy_it[L];
int main(){
    ll n;
    cin >> n;
    vector<ll> a(n, 0);
    vector<pll> pq;
    for(ll i=0; i<n; i++){
        ll t;
        cin >> t;
        copy_it[i] = t;
        pq.pb(mp(t, i));
    }
    sort(all(pq), cmp);
    vector<ll> vals;
    for(ll i=1; i<=n; i++){
        a[i] = pq[i-1].ss;
        vals.pb(a[i]);
    }
    sort(all(vals));
    for(ll i=1; i<=n; i++){
        ll old = a[i];
        a[i] = lower_bound(all(vals), a[i]) - vals.begin() + 1;
        pos[a[i]] = i;
        Real[a[i]] = old;
    }
    init(0, 1, n);
    ll m;
    cin >> m;
    while(m--){
        ll k, which;
        cin >> k >> which;
        cout << copy_it[Real[get(n, 1, k, which)]] << endl;
    }
}

```

### 3.4 Mo's Algorithm

```

void remove(idx); // TODO: remove value at idx from data structure
void add(idx); // TODO: add value at idx from data structure
int get_answer(); // TODO: extract the current answer of the data structure
int block_size;
struct Query {
    int l, r, idx;
    bool operator<(Query other) const
    {
        return make_pair(l / block_size, r) <
            make_pair(other.l / block_size, other.r);
    }
}

```

```

    }
};
vector<int> mo_s_algorithm(vector<Query> queries) {
    vector<int> answers(queries.size());
    sort(queries.begin(), queries.end());
    // TODO: initialize data structure
    int cur_l = 0;
    int cur_r = -1;
    // invariant: data structure will always reflect the range [cur_l, cur_r]
    for (Query q : queries) {
        while (cur_l > q.l) {
            cur_l--;
            add(cur_l);
        }
        while (cur_r < q.r) {
            cur_r++;
            add(cur_r);
        }
        while (cur_l < q.l) {
            remove(cur_l);
            cur_l++;
        }
        while (cur_r > q.r) {
            remove(cur_r);
            cur_r--;
        }
        answers[q.idx] = get_answer();
    }
    return answers;
}

```

### 3.5 Persistent Segment Tree

```

struct node{
    ll val;
    node *l, *r;
    node(){
        l=r=NULL;
    }
    node(node *left, node *right, ll v){
        l=left;
        r=right;
        val=v;
    }
};
struct psegtree{
    void build(vector<ll>&ar, node *root, ll l, ll r){
        if(l==r){
            root->val=ar[l];
            return;
        }
    }
}

```

```

    ll b=(l+r)/2;
    root->l=new node(NULL, NULL, 0);
    root->r=new node(NULL, NULL, 0);
    build(ar,root->l, l, b);
    build(ar,root->r, b+1, r);
    root->val=root->l->val+root->r->val;
}
void upgrade(node *pre,node *cur,ll l,ll r,ll idx,ll val){
    if(l==r){
        cur->val=val;
        return;
    }
    ll b=(l+r)/2;
    if(idx<=b){
        cur->r = pre->r;
        cur->l = new node(NULL, NULL, 0);
        upgrade(pre->l,cur->l,l,b,idx,val);
    }
    else{
        cur->l=pre->l;
        cur->r=new node(NULL, NULL, 0);
        upgrade(pre->r,cur->r,b+1,r,idx,val);
    }
    cur->val=cur->l->val+cur->r->val;
}
ll get(node *root,ll l,ll r,ll st,ll en){
    if(l>r || en<l || st>r){
        return 0;
    }
    if(l>=st && r<=en){
        return root->val;
    }
    ll b=(l+r)/2;
    return get(root->l,l,b,st,en)+get(root->r,b+1,r,st,en);
}
};

```

### 3.6 SQRT Decomposition

```

int build(int ary[],int sto[],int n){
    int a=sqrt(n);
    for (int i = 0; i < n; ++i)
        sto[i/a]+=ary[i];
    for (int i = 0; i < ceil(sqrt(n)); ++i)
        cout << sto[i]<<" ";
    cout << endl;
}
int main(){
    int n; cin >> n;
    int ary[n];
    for (int i = 0; i < n; ++i) cin >> ary[i];
}

```

```

int a=sqrt(n);
int sto[a+1];
for (int i = 0; i < a+1; ++i)sto[i]=0;
build(ary,sto,n);
int q;
cin >> q;
while(q--){
    int type;
    cin >> type;
    if(type==1){ //update
        int ind,val;
        cin >> ind >> val;
        sto[ind/a]+=(val-ary[ind]);
        ary[ind]=val;
    }
    else{
        int l,r;
        cin >> l >> r;
        int ans=0;
        for (int i = l; i <=r;){
            if(i%a==0&&r-i>=a){
                ans+=sto[i/a];
                i+=a;
            }
            else{
                ans+=ary[i];
                i++;
            }
        }
        cout << ans << endl;
    }
}
}

```

---

### 3.7 Segment Tree with Lazy Propagation

```

// SPOJ CNTPRIME // 1-based indexing
ll a[L]; ll seg[4*L]; ll lazy[4*L];
ll merge(ll a, ll b){
    return (a+b);
}
void build(ll pos, ll tl, ll tr){
    if(tl == tr){
        if(isPrime[a[tl]])
            seg[pos] = 1;
        return;
    }
    ll mid = tl + (tr-tl)/2;
    build(2*pos, tl, mid);
    build(2*pos+1, mid+1, tr);
    seg[pos] = merge(seg[2*pos], seg[2*pos+1]);
}

```

```

}
void update(ll pos, ll tl, ll tr, ll l, ll r, ll val){
    if(lazy[pos] != 0){
        if(isPrime[lazy[pos]])
            seg[pos] = tr-tl+1;
        else
            seg[pos] = 0;
        if(tl != tr){
            lazy[2*pos] = lazy[pos];
            lazy[2*pos+1] = lazy[pos];
        }
        lazy[pos] = 0;
    }
    if(tl > r || tr < l)
        return;
    if(tl >= l && tr <= r){
        if(isPrime[val])
            seg[pos] = tr-tl+1;
        else
            seg[pos] = 0;
        if(tl != tr){
            lazy[2*pos] = val;
            lazy[2*pos+1] = val;
        }
        lazy[pos] = 0;
        return;
    }
    ll mid = tl + (tr-tl)/2;
    update(2*pos, tl, mid, l, r, val);
    update(2*pos+1, mid+1, tr, l, r, val);
    seg[pos] = merge(seg[2*pos], seg[2*pos+1]);
}
ll query(ll pos, ll tl, ll tr, ll l, ll r){
    if(lazy[pos] != 0)
        // same as update
    if(l > tr || r < tl)
        return 0;
    if(tl >= l && tr <= r)
        return seg[pos];
    ll mid = tl + (tr-tl)/2;
    return merge(query(2*pos, tl, mid, l, r), query(2*pos+1, mid+1, tr, l, r));
}

```

---

### 3.8 Segment Tree

```

// SPOJ GSS3 // 1-based indexing
typedef struct node{
    ll ans, pref, suff, sum;
} node;
ll a[L];
node seg[4*L];

```



```

node merge(node a, node b){
    node x;
    // Merge Function
    return x;
}
void build(ll pos, ll tl, ll tr){
    if(tl == tr){
        seg[pos] = a[tl]; // Leaf Node
        return;
    }
    ll mid = tl + (tr-tl)/2;
    build(2*pos, tl, mid);
    build(2*pos+1, mid+1, tr);
    seg[pos] = merge(seg[2*pos], seg[2*pos+1]);
}
void update(ll pos, ll tl, ll tr, ll idx, ll val){
    if(tl == tr){
        seg[pos] = val; // Assign updated Value
        return;
    }
    ll mid = tl + (tr - tl)/2;
    if(tl <= idx && idx <= mid){
        update(2*pos, tl, mid, idx, val);
    }
    else{
        update(2*pos+1, mid+1, tr, idx, val);
    }
    seg[pos] = merge(seg[2*pos], seg[2*pos+1]);
}

```

### 3.9 Trie

```

struct node{
    vector<ll>val;
    vector<node*>pt;
    node(){
    }
    node(ll c){
        val.resize(c,0);
        pt.resize(c,NULL);
    }
};
struct trie{
    ll chr;
    trie(ll c){
        chr=c;
    }
    void add(node *root, string &s){
        node *cur=root;
        for(auto x:s){
            if(cur->val[x-'a']==0){
                cur->val[x-'a']=1;

```

```

                cur->pt[x-'a']=new node(chr);
            }
            cur=cur->pt[x-'a'];
        }
    }
    ll find(node *root, string &s, ll x){
        if(s[x]=='\0')
            return 1;
        if(root->val[s[x]-'a']==0){
            return 0;
        }
        else{
            return find(root->pt[s[x]-'a'],s,x+1);
        }
    }
};
int main(){
    trie obj(26);
    node *root=new node(26);
    ll q;
    cin>>q;
    while(q--){
        ll a;
        cin>>a;
        if(a==1){
            string s;
            cin>>s;
            cout << obj.find(root,s,0) << "\n";
        }
        else{
            string s;
            cin>>s;
            obj.add(root,s);
        }
    }
}

```

### 3.10 Wavelet Tree

```

ll MAX=1e6;
struct wavelet_tree{
    ll lo,hi;
    wavelet_tree *l,*r;
    vector<ll>b;
    wavelet_tree(ll *from,ll *to,ll x,ll y){
        lo = x,hi = y;
        if(lo == hi || from >= to)return;
        ll mid = (lo+hi)/2;
        auto f = [mid](ll x){
            return x <= mid;
        };

```

```

    b.reserve(to-from+1);
    b.push_back(0);
    for(auto it = from; it!=to; it++){
        b.push_back(b.back() + f(*it));
    }

    auto pivot = stable_partition(from, to, f);
    l = new wavelet_tree(from, pivot, lo, mid);
    r = new wavelet_tree(pivot, to, mid + 1, hi);
}
// kth smallest element in [l, r]
ll kth(ll le, ll ri, ll k){
    if(le > ri) return 0;
    if(lo == hi) return lo;
    ll inLeft = b[ri] - b[le-1];
    ll lb = b[le-1]; //amt of nos in first (l-1) nos that go in left
    ll rb = b[ri]; //amt of nos in first (r) nos that go in left
    if(k <= inLeft) return this->l->kth(lb+1, rb, k);
    return this->r->kth(le-lb, ri-rb, k-inLeft);
}
// count of nos in [l, r] less than or equal to k
ll LTE(ll le, ll ri, ll k){
    if(le>ri || k < this->lo) return 0;
    if(this->hi <= k) return ri-le+1;
    ll lb = b[le-1], rb = b[ri];
    return this->l->LTE(lb+1, rb, k) + this->r->LTE(le-lb, ri-rb, k);
}
//count of nos in [l, r] equal to k
int count(ll le, ll ri, ll k) {
    if(le > ri || k < lo || k > hi) return 0;
    if(lo == hi) return ri - le + 1;
    int lb = b[le-1], rb = b[ri], mid = (lo+hi)/2;
    if(k <= mid) return this->l->count(lb+1, rb, k);
    return this->r->count(le-lb, ri-rb, k);
}
};
int main(){
    ll n; cin>>n;
    ll ar[n+1];
    wavelet_tree obj(ar+1, ar+n+1, 1, MAX);
}

```

## 4 Graphs

### 4.1 Basic Graph Algorithms

```

vector<ll>path(N, INF); // Dijkstra's
vector<ll>visit(N, 0);
void dijk(auto &ar, ll x){
    priority_queue<pair<ll, ll>, vector<pair<ll, ll>>, greater<pair<ll, ll>>> pq;
    pq.push(make_pair(x, 0));

```

```

    path[x] = 0;
    while(!pq.empty()){
        auto p=pq.top(); pq.pop();
        if(visit[p.first] == 1) continue;
        visit[p.first] = 1;
        for(auto i:ar[p.first]){
            if(visit[i.first] == 1){
                continue;
            }
            if(path[i.first] > path[p.first] + i.second){
                path[i.first] = path[p.first] + i.second;
                pq.push(make_pair(i.first, path[i.first]));
            }
        }
    }
}
struct edge{ // Bellman Ford
    ll u,v,w;
};
vector<ll>path(N, INF);
vector<ll>par(N, 0);
ll n;
ll bellman_ford(auto &ar, ll x){
    ll m = sz(ar);
    path[x] = 0;
    for(ll i=1; i < n; i++){
        for(ll j = 0; j < m; j++){
            if(path[ar[j].v] > path[ar[j].u] + ar[j].w){
                path[ar[j].v] = path[ar[j].u] + ar[j].w;
                par[ar[j].v] = ar[j].u;
            }
        }
    }
    for(ll i = 0; i < m; i++){
        if(ar[i].v > ar[i].u + ar[i].w)
            return 0;
    }
    return 1;
}
ll graph[N][N]; // Floyd Warshall
ll n;
void floydWarshal(){
    for(ll k = 1; k <= n; k++){
        for(ll i = 1; i <= n; i++){
            for(ll j = 1; j <= n; j++){
                if(graph[i][j] > graph[i][k] + graph[k][j]){
                    graph[i][j] = graph[i][k] + graph[k][j];
                }
            }
        }
    }
}
vector<ll>visit(N, 0); // Shortest Path in DAG

```

```

stack<ll>st;
void st_dfs(auto &ar, ll x){
    visit[x] = 1;
    for(auto i:ar[x]){
        if(visit[i.first] == 0){
            st_dfs(ar, i.first);
        }
    }
    st.push(x);
}
void toposort(auto &ar){
    ll n = sz(ar)-1;
    for(ll i=1; i <= n; i++){
        if(visit[i] == 0)
            st_dfs(ar, i);
    }
}
vector<ll>path(N, INF);
void shortpathDAG(auto &ar, ll x){
    toposort(ar);
    path[x] = 0;
    while(!st.empty()){
        auto t = st.top(); st.pop();
        if(t == x){
            st.push(x);
            break;
        }
    }
    while(!st.empty()){
        auto t = st.top(); st.pop();
        for(auto i:ar[t]){
            if(path[i.first] > path[t] + i.second){
                path[i.first] = path[t] + i.second;
            }
        }
    }
}
}

```

## 4.2 Dinics $EV^2$

```

const ll N=1e4+5,inf=1e10;
struct edge{
    int a,b;
    ll c,f ;
    edge(int u,int v,ll cap):a(u),b(v),c(cap),f(0){}
};
struct flows{
    const static ll inf = 1e18 ;
    int level[N], Dptr[N], s, t ;
    queue<int> Q; vector<edge> E,E2; vll ad[N] ;
    void add(int a,int b,int c){

```

```

        if(a==b)return ;
        ad[a].pb(E.size()),E.pb(edge(a,b,c)) ;
        ad[b].pb(E.size()),E.pb(edge(b,a,0)) ;
    }
    bool bfs(void){
        memset(level,0,sizeof(level));
        Q.push(s);
        level[s]=1;
        while(!Q.empty()){
            int sz=Q.size(),v ;
            while(sz--){
                v = Q.front();Q.pop() ;
                for(auto &e:ad[v]) {
                    if(!level[E[e].b]&&E[e].f<E[e].c){
                        level[E[e].b]=level[v]+1;
                        Q.push(E[e].b);
                    }
                }
            }
            return level[t]>0 ;
        }
    }
    ll dfs(int x,ll flow){
        if(!flow) return 0;
        if(x==t) return flow ;
        for(int &pt=Dptr[x];pt<ad[x].size();++pt){
            int e=ad[x][pt];
            if(level[E[e].b]==level[x]+1){
                if(ll pushed=dfs(E[e].b,min(flow,E[e].c-E[e].f))){
                    E[e].f+=pushed ;
                    E[e^1].f -= pushed;
                    return pushed ;
                }
            }
        }
        return 0 ;
    }
    ll dinic(void){
        ll flow=0 ;
        while(bfs()){
            memset(Dptr,0,sizeof(Dptr));
            while(ll pushed=dfs(s,inf)) flow+=pushed;
        }
        return flow ;
    }
    void reset(void){
        for(auto &e:E)e.f=0;
    }
};
int main(){
    ll n,m;
    cin >> n >> m;
    flows F;

```

```

lp(i,0,m){
    ll a,b,w;
    cin >> a >> b >> w;
    F.add(a,b,w);
}
cin >> F.s>> F.t;
cout<<F.dinic()<<endl;
return 0;
}

```

### 4.3 Dinics Push Relabel EV<sup>2</sup>

```

/* Push Relabel O(n^3) implementation using FIFO method to chose push vertex.
This uses gapRelabel heuristic to fasten the process even further. If only
the maxFlow value is required then the algo can be stopped as soon as the
gap relabel method is called. However, to get the actual flow values in the
edges, we need to let the algo terminate itself.
This implementation assumes zero based vertex indexing. Edges to the graph
can be added using the addEdge method only. capacity for residual edges is
set to be zero. To get the actual flow values iterate through the edges and
check for flow for an edge with cap > 0.
This implimentaion is superior over dinic's for graphs where graph is dense
locally at some places and mostly sparse. For randomly generated graphs, this
implimentation gives results within seconds for n = 10000 nodes, m = 1000000
edges. */
typedef ll fType;
struct edge{
    ll from, to;
    fType cap, flow;
    edge(ll from, ll to, fType cap, fType flow = 0) : from(from), to(to), cap(cap),
        flow(flow) {}
};
struct PushRelabel{
    ll N; vector<edge> edges;
    vector<vector<ll>> > G; vector<ll> h, inQ, count;
    vector<fType> excess; queue<ll> Q;
    PushRelabel(ll N) : N(N), count(N<<1), G(N), h(N), inQ(N), excess(N) {}
    void addEdge(ll from, ll to, ll cap) {
        G[from].push_back(edges.size());
        edges.push_back(edge(from, to, cap));
        G[to].push_back(edges.size());
        edges.push_back(edge(to, from, 0));
    }
    void enqueue(ll u) {
        if(!inQ[u] && excess[u] > 0) Q.push(u), inQ[u] = true;
    }
    void Push(ll edgeIdx) {
        edge & e = edges[edgeIdx];
        ll toPush = min<fType>(e.cap - e.flow, excess[e.from]);
        if(toPush > 0 && h[e.from] > h[e.to]) {
            e.flow += toPush;

```

```

            excess[e.to] += toPush;
            excess[e.from] -= toPush;
            edges[edgeIdx^1].flow -= toPush;
            enqueue(e.to);
        }
    }
    void Relabel(ll u) {
        count[h[u]] -= 1; h[u] = 2*N-2;
        for (ll i = 0; i < G[u].size(); ++i) {
            edge & e = edges[G[u][i]];
            if(e.cap > e.flow) h[u] = min(h[u], h[e.to]);
        }
        count[+h[u]] += 1;
    }
    void gapRelabel(ll height) {
        for (ll u = 0; u < N; ++u) if(h[u] >= height && h[u] < N) {
            count[h[u]] -= 1;
            count[h[u] = N] += 1;
            enqueue(u);
        }
    }
    void Discharge(ll u) {
        for (ll i = 0; excess[u] > 0 && i < G[u].size(); ++i) {
            Push(G[u][i]);
        }
        if(excess[u] > 0) {
            if(h[u] < N && count[h[u]] < 2) gapRelabel(h[u]);
            else Relabel(u);
        }
        else if(!Q.empty()) { // dequeue
            Q.pop();
            inQ[u] = false;
        }
    }
    fType getFlow(ll src, ll snk) {
        h[src] = N; inQ[src] = inQ[snk] = true;
        count[0] = N - (count[N] = 1);
        for (ll i = 0; i < G[src].size(); ++i) {
            excess[src] += edges[G[src][i]].cap;
            Push(G[src][i]);
        }
        while (!Q.empty()) {
            Discharge(Q.front());
        }
        return excess[snk];
    }
};
int main(){
    ll n, m;
    cin >> n >> m;
    PushRelabel df(n);
    while(m--) {
        ll x, y, c;

```

```

        cin >> x >> y >> c;
        --x, --y;
        if (x != y) {
            df.addEdge(x, y, c);
            df.addEdge(y, x, c);
        }
    }
    cout << df.getFlow(0, n-1) << "\n";
}

```

#### 4.4 Dinics with Binary Search

```

class Dinics {
public:
    typedef int flowType; // can use float/double
    static const flowType INF = 1e9; // maximum capacity
    static const flowType EPS = 0; // minimum capacity/flow change
private:
    int nodes, src, dest;
    vector<int> dist, q, work;
    struct Edge {
        int to, rev;
        flowType f, cap;
    };
    vector< vector<Edge> > g;
    bool dinic_bfs() {
        fill(dist.begin(), dist.end(), -1);
        dist[src] = 0;
        int qt = 0;
        q[qt++] = src;
        for (int qh = 0; qh < qt; qh++) {
            int u = q[qh];
            for (int j = 0; j < (int) g[u].size(); j++) {
                Edge &e = g[u][j];
                int v = e.to;
                if (dist[v] < 0 && e.f < e.cap) {
                    dist[v] = dist[u] + 1;
                    q[qt++] = v;
                }
            }
        }
        return dist[dest] >= 0;
    }
    int dinic_dfs(int u, int f) {
        if (u == dest)
            return f;
        for (int &i = work[u]; i < (int) g[u].size(); i++) {
            Edge &e = g[u][i];
            if (e.cap <= e.f) continue;
            int v = e.to;
            if (dist[v] == dist[u] + 1) {

```

```

                flowType df = dinic_dfs(v, min(f, e.cap - e.f));
                if (df > 0) {
                    e.f += df;
                    g[v][e.rev].f -= df;
                    return df;
                }
            }
        }
        return 0;
    }
public:
    Dinics(int n): dist(n, 0), q(n, 0),
        work(n, 0), g(n), nodes(n) {}
    // s->t (cap); t->s (rcap)
    void addEdge(int s, int t, flowType cap, flowType rcap = 0) {
        g[s].push_back({t, (int) g[t].size(), 0, cap});
        g[t].push_back({s, (int) g[s].size() - 1, 0, rcap});
    }
    flowType maxFlow(int _src, int _dest) {
        src = _src;
        dest = _dest;
        flowType result = 0;
        while (dinic_bfs()) {
            fill(work.begin(), work.end(), 0);
            flowType delta;
            while ((delta = dinic_dfs(src, INF)) > EPS)
                result += delta;
        }
        return result;
    }
};
vector<pair<ll, ll>> g[100];
int main() {
    ll n, m, x;
    cin >> n >> m >> x;
    for (ll i = 1; i <= m; i++) {
        ll u, v, c;
        cin >> u >> v >> c;
        g[u].push_back({v, c});
        // g[v].push_back({u, c});
    }
    double lb = 0, ub = 100000000, mid = (lb + ub) / 2;
    double ans = 0;
    int cnt = 100;
    while (cnt) {
        cnt--;
        mid = (lb + ub) / 2;
        Dinics d(n);
        for (int i = 1; i < n + 1; ++i) {
            for (auto j : g[i]) {
                if (j.second / mid > 1e7)

```

```

        d.addEdge(i-1, j.first-1, x);
    }
    else
        d.addEdge(i-1, j.first-1, floor((j.second)/mid));
}
if(d.maxFlow(0, n-1)>=x)
    lb=mid;
else
    ub=mid;
ans=mid;
}
cout <<fixed<<setprecision(10)<< ans*x;
return 0;
}

```

#### 4.5 Ford Fulkerson

```

const ll inf=1e10,N=1005;
ll flow[N][N],cap[N][N],p[N],timer,ans,vis[N];
vll G[N];
bool bfs(ll st,ll end){
    queue<ll> q;
    q.push(st);
    while(!q.empty()){
        ll a=q.front();
        q.pop();
        if(a==end)
            return true;
        lp(i,0,G[a].size()){
            ll u=G[a][i];
            if(vis[u]!=timer && cap[a][u] > flow[a][u]){
                p[u] = a;
                vis[u]=timer;
                q.push(u);
            }
        }
    }
    return false;
}
int main(){
    ll n,m;
    cin >> n>> m;
    lp(i,0,m){
        ll a,b,w;
        cin >> a >> b >> w;
        G[a].pb(b);
        G[b].pb(a);
        cap[a][b]=w;
    }
    ll st,end;
    cin >> st >> end;
}

```

```

ll x=inf;
timer++;
while(bfs(st,end)){
    cout<<endl;
    timer++;
    ll mn=inf;
    ll i=end;
    while(i!=st){
        cout<<i<<" ";
        mn=min(mn,cap[p[i]][i]-flow[p[i]][i]);
        i=p[i];
    }
    cout<<endl;
    i=end;
    while(i!=st){
        flow[p[i]][i]+=mn;
        flow[i][p[i]]-=mn;
        i=p[i];
    }
    cout<<mn<<endl;
    ans+=mn;
    memset(p,0,sizeof p);
}
cout<<ans<<endl;
}

```

#### 4.6 Heavy Light Decomposition

```

// QTREE SPOJ
struct node{
    ll depth,par,size,chain,posInBase;
};
#define ln 16
#define N 100001
ll n,chainNo,ptr;
vector<vector<pair<ll,pair<ll,ll>>>>ar(N);
node nd[N];
ll chainHead[N],otherEnd[N];
vector<ll> baseArray(N);
ll pa[N][ln];
struct segtree{
    struct node{
        ll sum;
    };
    vector<node> seg;
    segtree(){
        segtree(ll n){
            seg.resize(4*n+4,{0});
        }
        segtree(ll n, vector<ll> &ar){
            seg.resize(4*n+4);
        }
    }
}

```

```

    build(ar, 1, 1, n);
}
node merge(node a, node b){
    node k;
    k.sum=max(a.sum,b.sum);
    return k;
}
// build segtree
node get(ll pos,ll l,ll r,ll st,ll en){
    if(l>en || r<st || l>r){
        node k={-1};
        return k;
    }
    if(st<=l && en>=r){
        return seg[pos];
    }
    ll b=(l+r)/2;
    return merge(get(2*pos,l,b,st,en),get(2*pos+1,b+1,r,st,en));
}
// update segtree
};
ll query(segtree &obj,ll u,ll v){
    if(u==v) return 0;
    ll uchain,vchain=nd[v].chain,cost=0;
    while(1){
        uchain=nd[u].chain;
        if(uchain==vchain){
            if(u==v) return cost;
            return max(cost,obj.get(1,1,n-1,nd[v].posInBase+1,nd[u].posInBase).sum);
        }
        cost=max(cost,obj.get(1,1,n-1,nd[chainHead[uchain]].posInBase,nd[u].posInBase).sum);
        u=nd[chainHead[uchain]].par;
    }
}
ll lca(ll u,ll v){
    if(nd[u].depth < nd[v].depth) swap(u,v);
    ll log;
    for(log=1;(1<<log)<=nd[u].depth;log++);
    log--;
    for(ll i=log;i>=0;i--){
        if(nd[u].depth-(1<<i)>=nd[v].depth){
            u=pa[u][i];
        }
    }
    if(u==v) return v;
    for(ll i=log;i>=0;i--){
        if(pa[u][i]!=-1 && pa[u][i]!=pa[v][i]){
            u=pa[u][i],v=pa[v][i];
        }
    }
    return pa[u][0];
}
void hld(ll cur,ll cost,ll pre){
    if(chainHead[chainNo]==-1){

```

```

        chainHead[chainNo]=cur;
    }
    nd[cur].chain=chainNo;
    nd[cur].posInBase=ptr;
    baseArray[ptr++]=cost;

    ll sc=-1,ncost;
    for(auto i:ar[cur]){
        if(i.first==pre) continue;
        if(sc==-1 || nd[sc].size<nd[i.first].size){
            sc=i.first;
            ncost=i.second.first;
        }
    }
    if(sc!=-1){
        hld(sc,ncost,cur);
    }
    for(auto i:ar[cur]){
        if(i.first==pre) continue;
        if(sc!=i.first){
            chainNo++;
            hld(i.first,i.second.first,cur);
        }
    }
}
void dfs(ll x,ll p,ll d){
    nd[x].depth=d;
    nd[x].par=p;
    nd[x].size=1;
    for(auto i:ar[x]){
        if(i.first==p) continue;
        otherEnd[i.second.second]=i.first;
        dfs(i.first,x,d+1);
        nd[x].size+=nd[i.first].size;
    }
}
int main(){
    ll t; cin>>t;
    while(t--){
        cin>>n;
        chainNo=0,ptr=0;
        for(ll i=0;i<=n;i++){
            ar[i].clear();
            chainHead[i]=-1;
            for(ll j=0;j<ln;j++){
                pa[i][j]=-1;
            }
        }
        for(ll i=1,u,v,w;i<=n;i++){
            cin>>u>>v>>w;
            ar[u].push_back(mk(v,mk(w,i)));
            ar[v].push_back(mk(u,mk(w,i)));
        }
    }
}

```

```

    dfs(1,0,-1);
    hld(1,-1,-1);
    segtree obj(n-1,baseArray);
    for(ll i=1;i<=n;i++)
        pa[i][0]=nd[i].par;
    for(ll i=1;i<ln;i++)
        for(ll j=1;j<=n;j++)
            if(pa[j][i-1]!=-1)
                pa[j][i]=pa[pa[j][i-1]][i-1];
    while(1){
        string s;
        cin>>s;
        if(s[0]=='D') break;
        ll a,b;
        cin>>a>>b;
        if(s[0]=='Q')
            cout << max(query(obj,a,lca(a,b)),query(obj,b,lca(a,b))) << "\n";
        else{
            obj.update(1,1,n-1,nd[otherEnd[a]].posInBase,b);
        }
    }
}
}
}

```

## 4.7 Kruskal's Algorithm

```

ll find(ll s){
    if(parent[s]==s){
        return s;
    }
    return parent[s]=find(parent[s]);
}
void unionSet(ll x, ll y){
    ll a = find(x);
    ll b = find(y);
    if(unionSize[a] > unionSize[b]){
        swap(x, y);
    }
    parent[a] = b;
    unionSize[b] += unionSize[a];
}
ll kruskals(ll M){
    ll ans = 0;
    for(ll i=0; i<M; i++){
        ll u = weights[i].ss.ff;
        ll v = weights[i].ss.ss;
        ll w = weights[i].ff;

        if(find(u)!=find(v))
        {
            ans+=w;

```

```

        unionSet(u, v);
    }
}
return ans;
}
int main(){
    ll N, M;
    cin >> N >> M;
    for(ll i=0; i<L; i++)
    {
        parent[i] = i;
        unionSize[i] = 1;
    }
    for(ll i=0; i<M; i++)
    {
        ll u, v, w;
        cin >> u >> v >> w;

        adj[u].pb(mp(v, w));
        adj[v].pb(mp(u, w));

        weights.pb(mp(w, mp(u, v)));
    }
    sort(weights.begin(), weights.end());
    cout << kruskals(M) << endl;
}

```

## 4.8 LCA

```

struct LCA {
    vector<ll> height, euler, first, segtree;
    vector<bool> visited;
    ll n;
    LCA(vector<vector<ll>> &adj, ll root = 0) {
        n = adj.size();
        height.resize(n);
        first.resize(n);
        euler.reserve(n * 2);
        visited.assign(n, false);
        dfs(adj, root);
        ll m = euler.size();
        segtree.resize(m * 4);
        build(1, 0, m - 1);
    }
    void dfs(vector<vector<ll>> &adj, ll node, ll h = 0) {
        visited[node] = true;
        height[node] = h;
        first[node] = euler.size();
        euler.push_back(node);
        for (auto to : adj[node]) {
            if (!visited[to]) {

```



```

        dfs(adj, to, h + 1);
        euler.push_back(node);
    }
}

void build(ll node, ll b, ll e) {
    if (b == e) {
        segtree[node] = euler[b];
    } else {
        ll mid = (b + e) / 2;
        build(node << 1, b, mid);
        build(node << 1 | 1, mid + 1, e);
        ll l = segtree[node << 1], r = segtree[node << 1 | 1];
        segtree[node] = (height[l] < height[r]) ? l : r;
    }
}

ll query(ll node, ll b, ll e, ll L, ll R) {
    if (b > R || e < L)
        return -1;
    if (b >= L && e <= R)
        return segtree[node];
    ll mid = (b + e) >> 1;

    ll left = query(node << 1, b, mid, L, R);
    ll right = query(node << 1 | 1, mid + 1, e, L, R);
    if (left == -1) return right;
    if (right == -1) return left;
    return height[left] < height[right] ? left : right;
}

ll lca(ll u, ll v) {
    ll left = first[u], right = first[v];
    if (left > right)
        swap(left, right);
    return query(1, 0, euler.size() - 1, left, right);
}

};
vector<vector<ll>>>ar;
LCA obj(ar);

```

## 4.9 Min-Cost Max-Flow

```

struct Edge{
    int from, to, capacity, cost;
};
vector<vector<int>>> adj, cost, capacity;
const int INF = 1e9;
void shortest_paths(int n, int v0, vector<int>& d, vector<int>& p) {
    d.assign(n, INF);
    d[v0] = 0;
    vector<bool> inq(n, false);
    queue<int> q;

```

```

        q.push(v0);
        p.assign(n, -1);
        while (!q.empty()) {
            int u = q.front();
            q.pop();
            inq[u] = false;
            for (int v : adj[u]) {
                if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
                    d[v] = d[u] + cost[u][v];
                    p[v] = u;
                    if (!inq[v]) {
                        inq[v] = true;
                        q.push(v);
                    }
                }
            }
        }
    }

int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t) {
    adj.assign(N, vector<int>());
    cost.assign(N, vector<int>(N, 0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to);
        adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost;
        cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity;
    }

    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p);
        if (d[t] == INF)
            break;

        // find max flow on that path
        int f = K - flow;
        int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]);
            cur = p[cur];
        }

        // apply flow
        flow += f;
        cost += f * d[t];
        cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f;
            capacity[cur][p[cur]] += f;
            cur = p[cur];
        }
    }
}

```

```

    }
}
if (flow < K)
    return -1;
else
    return cost;
}

```

---

## 5 Math and Number Theory

### 5.1 Extended Euclidean

```

ll x, y;
ll extendedeuc(ll a, ll b){
    if (b==0){
        x=1;
        y=0;
    }
    else{
        extendedeuc(b, a%b);
        ll t=x;
        x=y;
        y=t-y*(a/b);
    }
}
int main(){
    ll a, b, c;
    cin >> a >> b >> c;
    if (c%gcd(a, b)!=0){
        cout << "-1";
        return 0;
    }
    extendedeuc(a, b);
    cout << -x*(c)/gcd(a,b) << " "<<-y*c/gcd(a, b);
    return 0;
}

```

---

### 5.2 FFT

```

typedef complex<double> cd;
const double PI = acos(-1);
void fft(vector<cd> &a, bool invert){
    ll n=a.size();
    for(ll i=1,j=0; i<n; i++){
        ll bit=n>>1;
        for(; j<bit; bit>>=1)
            j ^= bit;
    }
}

```

---

```

        j ^= bit;
        if(i < j)
            swap(a[i], a[j]);
    }
    for(ll len=2; len<=n; len <= 1){
        double ang=2*PI/len*(invert ? -1 : 1);
        cd wlen(cos(ang), sin(ang));
        for(ll i=0; i<n; i+=len){
            cd w(1);
            for(ll j=0; j<len/2; j++){
                cd u = a[i+j], v = a[i+j+len/2]*w;
                a[i+j] = u+v;
                a[i+j+len/2] = u-v;
                w *= wlen;
            }
        }
    }
    if(invert){
        for(cd & x : a)
            x /= n;
    }
}
vector<ll> multiply(vector<ll> const &a, vector<ll> const &b){
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    ll n=1;
    while(n < a.size()+b.size())
        n <= 1;
    fa.resize(n,0);
    fb.resize(n,0);
    fft(fa, false);
    fft(fb, false);
    for(ll i=0; i<n; i++)
        fa[i] *= fb[i];
    fft(fa, true);
    vector<ll> result(n);
    for(ll i=0; i<n; i++)
        result[i] = llround(fa[i].real());
    return result;
}
int main(){
    ll t;
    cin>>t;
    while(t--){
        ll n;
        cin>>n;
        vector<ll>a(n+1), b(n+1);
        for(ll i=0;i<n;i++){
            cin>>a[n-i];
        }
        for(ll i=0;i<n;i++){
            cin>>b[n-i];
        }
        auto c = multiply(a, b);
    }
}

```

```

    for (ll i=2*n; i>=0; i--){
        cout << c[i] << " ";
    }
    cout << "\n";
}

```

### 5.3 Gauss

```

// ----- Gauss Jordan -----
const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or a big number
int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }
    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

```

```

//Gauss Jordan For Mod
int gauss (vector < bitset<N> > a, int n, int m, bitset<N> & ans) {
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        for (int i=row; i<n; ++i)
            if (a[i][col]) {
                swap (a[i], a[row]);
                break;
            }
        if (! a[row][col])
            continue;
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row && a[i][col])
                a[i] ^= a[row];
        ++row;
    }
    // The rest of implementation is the same as above
}

```

### 5.4 Matrix Exponentiation

```

typedef vector<vector<ll> > matrix;
matrix mul(matrix A, matrix B){
    matrix C(K, vector<ll>(K));
    lp(i,0, K) lp(j,0, K) lp(k,0, K)
        C[i][j] = (C[i][j] + A[i][k] * B[k][j]) % mod;
    return C;
}
// Only Square Matrices
matrix pow(matrix A, ll p){
    if (p == 1)
        return A;
    if (p % 2)
        return mul(A, pow(A, p-1));
    matrix X = pow(A, p/2);
    return mul(X, X);
}

```

### 5.5 NTT and Some Other Transformations

```

#define INF 1e16
//Fast WalshHadamard transform (XOR)
#define poly vector<ll>
poly FWHT(poly P, bool inverse) {
    for (len = 1; 2 * len <= degree(P); len <= 1) {
        for (i = 0; i < degree(P); i += 2 * len) {

```

```

        for (j = 0; j < len; j++) {
            u = P[i + j];
            v = P[i + len + j];
            P[i + j] = u + v;
            P[i + len + j] = u - v;
        }
    }
    if (inverse) {
        for (i = 0; i < degree(P); i++)
            P[i] = P[i] / degree(P);
    }
    return P;
}
// & operator
poly transform(poly P, bool inverse) {
    for (len = 1; 2 * len <= degree(P); len <= 1) {
        for (i = 0; i < degree(P); i += 2 * len) {
            for (j = 0; j < len; j++) {
                u = P[i + j];
                v = P[i + len + j];

                if (!inverse) {
                    P[i + j] = v;
                    P[i + len + j] = u + v;
                } else {
                    P[i + j] = -u + v;
                    P[i + len + j] = u;
                }
            }
        }
    }
    return P;
}
// NTT
//
//          k          g
// 5767169    19    3
// 7340033    20    3
// 23068673   21    3
// 104857601  22    3
// 167772161  25    3
// 469762049  26    3
// 998244353  23    3
// 1004535809 21    3
// 2013265921 27   31
// 2281701377 27    3
const ll mod = 998244353;
ll inverse(ll x, ll y){
    ll rem = 1;
    while(y != 0){
        if(y % 2 == 1){
            rem=(rem * x) % mod;
        }
    }

```

```

        x=(x * x) % mod;
        y /= 2;
    }
    return rem;
}
const ll root = 3;
const ll root_1 = inverse(root, mod - 2);
const ll root_pw = 1 << 23;
void ntt(vector<ll> &a, bool invert){
    ll n = a.size();
    for(ll i = 1, j = 0; i < n; i++){
        ll bit = n >> 1;
        for(; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
        if(i < j)
            swap(a[i], a[j]);
    }

    for(ll len = 2; len <= n; len <= 1){
        ll wlen = invert ? root_1 : root;
        for(ll i = len; i < root_pw; i <= 1)
            wlen = wlen * wlen % mod;

        for(ll i = 0; i < n; i += len){
            ll w = 1;
            for(ll j = 0; j < len / 2; j++){
                ll u = a[i + j], v = a[i + j + len / 2] * w % mod;
                a[i + j] = u + v < mod ? u + v : u + v - mod;
                a[i + j + len / 2] = u - v >= 0 ? u - v : u - v + mod;
                w = w * wlen % mod;
            }
        }
    }

    if(invert){
        ll n_1 = inverse(n, mod - 2);
        for(ll &x:a)
            x = x * n_1 % mod;
    }
}
vector<ll> multiply(vector<ll> const &a, vector<ll> const &b){
    vector<ll> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    ll n = 1;
    while(n < a.size() + b.size())
        n <= 1;
    fa.resize(n, 0);
    fb.resize(n, 0);
    ntt(fa, false);
    ntt(fb, false);
    for(ll i = 0; i < n; i++)
        fa[i] = fa[i] * fb[i] % mod;
    ntt(fa, true);
}

```

```

        return fa;
    }
    int main(){
        ll t; cin>>t;
        while(t--){
            ll n;
            cin>>n;
            vector<ll>a(n+1), b(n+1);
            for(ll i=0;i<=n;i++){
                cin>>a[n-i];
            }
            for(ll i=0;i<=n;i++){
                cin>>b[n-i];
            }
            auto c = multiply(a, b);
            for(ll i=2*n;i>=0;i--){
                cout << c[i] << " ";
            }
            cout << "\n";
        }
    }

```

## 5.6 Shoelace Formula

```

// Used to calculate area of convex polygon, given
// its coordinates in the x and y plane
// (X[i], Y[i]) are coordinates of i'th point.
double polygonArea(double X[], double Y[], int n) {
    double area = 0.0;
    int j = n - 1;
    for (int i = 0; i < n; i++){
        area += (X[j] + X[i]) * (Y[j] - Y[i]);
        j = i; // j is previous vertex to i
    }
    return abs(area / 2.0);
}

```

## 5.7 Union of Rectangles

```

/*primes*/
//ll p1=1e6+3, p2=1616161, p3=3959297, p4=7393931;
int n; const int N=1e6;
struct rect{
    int x1, y1, x2, y2;
};
struct event_x{
    int typ, x, idx;
    event_x(int x, int t, int idx):x(x), typ(t), idx(idx){}
}

```

```

};
struct event_y{
    int typ, y, idx;
    event_y(int y, int t, int idx):y(y), typ(t), idx(idx){}
};
vector<rect> vec;
vector<event_x> Sx;
vector<pii> tree;
vi lazy;
void init(){
    vec.resize(n);
    tree.resize(4*N, mp(0, 0));
    lazy.resize(4*N, 0);
}
bool comp_x(event_x e1, event_x e2){
    if(e1.x!=e2.x) return e1.x<e2.x;
    return e1.typ<e2.typ;
}
void update(int start, int end, int node, int l, int r, int delta){
    int len=end-start+1;
    if(start>r || end<l) return ;

    if(start>=l && end<=r){
        tree[node].ss+=delta;
        if(tree[node].ss==0) tree[node].ff=tree[2*node].ff+tree[2*node+1].ff;
        else tree[node].ff=len;
        return ;
    }

    int mid=(start+end)/2;
    update(start, mid, 2*node, l, r, delta);
    update(mid+1, end, 2*node+1, l, r, delta);
    if(tree[node].ss==0) tree[node].ff=tree[2*node].ff+tree[2*node+1].ff;
    return ;
}
int query(int start, int end, int node, int l, int r){
    if(start>r || end<l) return 0;
    if(start>=l && end<=r){
        return tree[node].ff;
    }
    int mid=(start+end)/2;
    return query(start, mid, 2*node, l, r)+query(mid+1, end, 2*node+1, l, r);
}
int main(){
    cin>>n;
    init();
    fr(i, n){
        cin>>vec[i].x1>>vec[i].y1>>vec[i].x2>>vec[i].y2;
        Sx.pb(event_x(vec[i].x1, 0, i));
        Sx.pb(event_x(vec[i].x2, 1, i));
    }
    sort(all(Sx), comp_x);
    ll ans=0;

```

```

11 px=Sx[0].x, dy, dx, cnt, py;
for(auto i:Sx){
    dx=i.x-px;
    dy=query(0, N, 1, 0, N);
    ans+=dx*dy;
    px=i.x;
    if(i.typ==0){
        update(0, N, 1, vec[i.idx].y1, vec[i.idx].y2-1, 1);
        continue;
    }
    update(0, N, 1, vec[i.idx].y1, vec[i.idx].y2-1, -1);
}
cout<<ans<<endl;
}

```

## 6 Misc

### 6.1 Big Integer

```

const int bignumlen=2200;
const int Blen=8;
const int64 base=1000000000;
struct bignum{
    int len;
    int64 data[bignumlen];
    int64 &operator [](int x){ return(data[x]);}
    const int64 &operator [](int x)const { return(data[x]);}
    bignum (){
        memset(data,0,sizeof(data));
        len=0;
    }
    void clear(){
        for(int i=len;i>=1;--i)data[i]=0;
        len=0;
    }
    int check (const bignum &a,const bignum &b){
        if(a.len>b.len)return(0);
        if(b.len>a.len)return(1);
        for(int i=a.len;i>=1;--i){
            if(a.data[i]<b.data[i])return(1);
            if(b.data[i]<a.data[i])return(0);
        }
        return 2;
    }
    bool operator < (const bignum &b){ return(check(*this,b)==1);}
    bool operator > (const bignum &b){ return(check(*this,b)==0);}
    bool operator <=(const bignum &b){ return(check(*this,b)>=1);}
    bool operator >=(const bignum &b){ return(check(*this,b)%2==0);}
    bool operator !=(const bignum &b){ return(check(*this,b)!=2);}
    bool operator ==(const bignum &b){ return(check(*this,b)==2);}
}

```

```

bignum operator=(const bignum &x){
    for(int i=x.len+1;i<=len;++i)data[i]=0;
    for(int i=1;i<=x.len;++i)data[i]=x.data[i];
    len=x.len;
    return *this;
}
bignum operator=(int64 x){
    for(int i=len;i>=0;--i)data[i]=0;
    len=0;
    while(x){
        data[++len]=x%base;
        x/=base;
    }
    return *this;
}
bignum(int64 x){
    memset(data,0,sizeof(data));
    len=0;
    (*this)=x;
}
bignum operator *(const bignum &b){
    int i,j;
    bignum tmp;
    for(i=1;i<=len;++i)if(data[i]!=0){
        for(j=1;j<=b.len;++j)if(b.data[j]!=0){
            tmp.data[i+j-1]+=data[i]*b.data[j];
            tmp.data[i+j]+=tmp.data[i+j-1]/base;
            tmp.data[i+j-1]%=base;
        }
        tmp.len=len+b.len-1;
        while(tmp.data[tmp.len+1])tmp.len++;
        return tmp;
    }
    bignum operator *(int64 x){
        int i;
        bignum tmp;
        for(i=1;i<=len;++i)tmp[i]=data[i]*x;
        tmp.len=len;
        for(i=1;i<=len;++i){
            tmp[i+1]+=tmp[i]/base,tmp[i]%=base;
            if(tmp[i+1]&&tmp[i+1]>tmp.len)tmp.len++;
        }
        return tmp;
    }
    bignum operator /(int64 x){
        int i;
        bignum tmp;
        int64 y=0;
        for(i=len;i>=1;--i){
            y=y*base+data[i];
            tmp[i]=y/x;
            y%=x;
        }
    }
}

```

```

    tmp.len=len;
    while(tmp[tmp.len]==0&&tmp.len>1)tmp.len--;
    return tmp;
}
bignum operator /(const bignum &b){
    if(b.len<=1 && b[1]==0){
        printf("error!  0  !");
        for(;;);
    }
    int i,l1=(len-1)*Blen,l2=(b.len-1)*Blen;
    int64 x=data[len],y=b[b.len];
    while(x)/=10,l1++;
    while(y)/=10,l2++;
    bignum tmp,chu,B;
    chu=*this; B=b;

    for(i=1;i*Blen<=l1-l2;++i)B*=base;
    for(i=1;i<=(l1-l2)%Blen;++i)B*=10;
    for(i=l1-l2;i>=0;--i){
        x=0;
        while(chu>=B)chu-=B,x++;
        tmp[i/Blen+1]=tmp[i/Blen+1]*10+x;
        B/=10;
    }
    tmp.len=(l1-l2)/Blen+1;
    while(tmp.len>=1 && !tmp[tmp.len])tmp.len--;
    return tmp;
}
bignum operator +(const bignum &b){
    bignum tmp;
    int i,l=max(len,b.len);
    for(i=1;i<=l;++i)tmp[i]=data[i]+b[i];
    for(i=1;i<=l;++i)tmp[i+1]+=tmp[i]/base,tmp[i]%=base;
    tmp.len=l;
    if(tmp[tmp.len+1])tmp.len++;
    return tmp;
}
bignum operator +(int64 x){
    bignum tmp; tmp=*this;
    tmp[1]+=x;
    for(int i=1;i<=len&&tmp[i]>=base;++i)tmp[i+1]+=tmp[i]/base,tmp[i]%=base;
    while(tmp[tmp.len+1])tmp.len++;
    return tmp;
}
bignum operator -(const bignum &b){
    int i;
    bignum tmp;
    for(i=1;i<=len;++i)tmp.data[i]=data[i]-b.data[i];
    for(i=1;i<=len;++i){
        if(tmp[i]<0)tmp.data[i]+=base,tmp.data[i+1]--;
    }
    tmp.len=len;
    while(tmp[tmp.len]==0&&tmp.len>1)tmp.len--;

```

```

        return tmp;
    }
    bignum operator -(int64 x){
        bignum tmp; tmp=*this;
        tmp[1]-=x;
        for(int i=1;i<=len&&tmp[i]<0;++i){
            tmp[i+1]+=(tmp[i]+1)/base-1;
            tmp[i]=(tmp[i]+1)%base+base-1;
        }
        while(!tmp[tmp.len]&&tmp.len>1)tmp.len--;
        return tmp;
    }
    int64 operator %(int64 x){
        int i;
        int64 y=0;
        for(i=len;i>=1;--i)y=(y*base+data[i])%x;
        return y;
    }
    bignum operator %(const bignum &b){
        if(b.len<=1 && b[1]==0){
            printf("error!  0  mod!");
            for(;;);
        }
        int i,l1=(len-1)*Blen,l2=(b.len-1)*Blen;
        int64 x=data[len],y=b[b.len];
        while(x)/=10,l1++;
        while(y)/=10,l2++;
        bignum chu,B;
        chu=*this; B=b;

        for(i=1;i*Blen<=l1-l2;++i)B*=base;
        for(i=1;i<=(l1-l2)%Blen;++i)B*=10;
        for(i=l1-l2;i>=0;--i){
            while(chu>=B)chu-=B;
            B/=10;
        }
        return chu;
    }
    bignum operator +=(const bignum &b){return *this=(*this+b);}
    bignum operator *=(const bignum &b){return *this=(*this*b);}
    bignum operator -=(const bignum &b){return *this=(*this-b);}
    bignum operator /=(const bignum &b){return *this=(*this/b);}
    bignum operator %=(const bignum &b){return *this=(*this%b);}
    bignum operator *=(int64 x) {return (*this=(*this *x));}
    bignum operator +=(int64 x) {return (*this=(*this +x));}
    bignum operator -=(int64 x) {return (*this=(*this -x));}
    bignum operator /=(int64 x) {return (*this=(*this /x));}
    void read(){
        char c[bignumlen*Blen+10];
        scanf("%s",c+1);
        int l=strlen(c+1);
        (*this).clear();
        int64 x;

```

```

    for(int i=1;i<=(l-1)/Blen+1;++i){
        x=0;
        for(int j=1-Blen*i+1;j<=1-Blen*i+Blen;++j)if(j>=1)x=x*10+c[j]-48;
        data[++len]=x;
    }
}
void write(){
    printf("%I64d",data[len]);
    for(int i=len-1;i>=1;--i)printf("%0*I64d",Blen,data[i]);
}
}p,q,pp,qq;
bignum gcd(const bignum &A,const bignum &B){
    bignum a=A,b=B,res=1;
    while(!(a[1]&1) && !(b[1]&1))a/=2,b/=2,res*=2;
    for(;;){
        if(a.len==1 && a[1]==0)return b*res;
        if(b.len==1 && b[1]==0)return a*res;
        while(!(a[1]&1))a/=2;
        while(!(b[1]&1))b/=2;
        if(a>b)a-=b;
        else b-=a;
    }
}
}

```

## 6.2 Closest Pair

```

const ll N=1e5+5,inf=1e18;
pll pnts [N];
int compare(pll a, pll b){
    return a.px<b.px;
}
double closest_pair(int n){
    sort(pnts,pnts+n,compare);
    double best=inf;
    set<pll> box;
    box.insert(pnts[0]);
    int left = 0;
    for (int i=1;i<n;++i){
        while (left<i && pnts[i].px-pnts[left].px > best)
            box.erase(pnts[left++]);
        ll cnt=0;
        cout<<pnts[i].px<<" "<<pnts[i].py<<endl;
        for(auto it=box.lower_bound(make_pair(pnts[i].py-best,
            pnts[i].px-best));it!=box.end() && pnts[i].py+best>=it->py;it++){
            cnt++;
            best = min(best, sqrt(pow(pnts[i].py - it->py, 2.0)+pow(pnts[i].px - it->px,
                2.0)));
        }
        box.insert(pnts[i]);
    }
    return best;
}

```

```

}
int main(){
    ll n;
    cin >> n;
    lp(i,0,n){
        ll a,b;
        cin >> a >> b;
        pnts[i].px=a;
        pnts[i].py=b;
    }
    cout<<closest_pair(n)<<endl;
}

```

## 6.3 Hare Tortoise Method

```

// UVA 11053
ll a, b, N;
ll f(ll x){
    return ((a*x)%N*x)%N + b)%N;
}
int main(){
    cin >> N >> a >> b;
    ll tortoise = f(0);
    ll hare = f(f(0));
    while(tortoise != hare){
        tortoise = f(tortoise);
        hare = f(f(hare));
    }
    ll die = 1;
    tortoise = f(tortoise);
    while(tortoise != hare){
        tortoise = f(tortoise);
        die++;
    }
    cout << N - die << endl;
}

```

## 7 String Algorithms

### 7.1 KMP

```

int main(){
    string c,t;
    cin>>c>>t;
    ll l=t.length();
    vector<ll>p(l);
    p[0]=0;
}

```



```

for(ll i = 1, j = 0; i < l; i++){
    while(j > 0 && t[i] != t[j]){
        j = p[j-1];
    }
    if(t[i] == t[j])
        j++;
    p[i] = j;
}
ll n = c.length(), ans=0;
for(ll i = 0, j = 0; i < n; i++){
    if(c[i] == t[j]){
        if(j == l-1){
            ans++;
            j = p[j];
            continue;
        }
        j++;
    }
    else if(j > 0){
        j = p[j-1];
        i--;
    }
}
}

```

## 8 Theory

---

```

/* Total number of spanning trees in a complete graph
with n vertices is given by  $n^{(n-2)}$  */

```

```

/* Sprague-Grundy Theorem: The losing states are exactly
those with Grundy number equal to 0.
Grundy number of the current state is the smallest whole
number which is not the Grundy number of any state that
can be reached in the next step.
Mathematically, if s1, s2 ... sk are the game states directly
reachable from s,
Grundy(s)=min({0,1,...} - {Grundy(s1),Grundy(s2),...,Grundy(sk)}) */

```

```

/* Sums of Games:
1. Player chooses a game and makes a move in it. Grundy number
of a position is xor of grundy numbers of positions in summed games.
2. Player chooses a non-empty subset of games (possibly, all) and
makes moves in all of them. A position is losing iff each game
is in a losing position.
3. Player chooses a proper subset of games (not empty and not all),
and makes moves in all chosen ones. A position is losing iff grundy
numbers of all games are equal.
4. Player must move in all games, and loses if cant move in some game.
A position is losing if any of the games is in a losing position. */

```

```

/* Misere Nim. A position with pile sizes a1, a2,..., an >= 1, not all
equal to 1, is losing iff a1 xor a2 ... xor an = 0 (like in normal nim.)
A position with n piles of size 1 is losing iff n is odd. */

```

---