RT1 - Theory question

CS-341 : Introduction to Computer Graphics

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1 Intersection between a ray and a cylinder

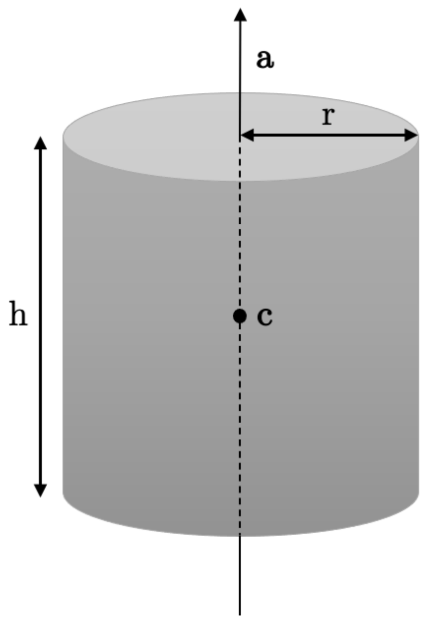

Figure 1: Cylinder and the corresponding naming of variables

In order to find the intersection points between a ray and a cylinder, we start with the implicit equation of an infinite cylinder, given its center, axis and radius, using the following formula:

$$|\vec{x} - \vec{c}|^2 = r^2 + [(\vec{x} - \vec{c}) \cdot \vec{a}]^2 \tag{1}$$

The vector $\vec{x}$ is then replaced by the explicit equation of the ray : $O + t \cdot \vec{d}$ and the equation 1 can be written as a quadratic formula with form $At^2 + Bt + C = 0$, such that:

$$\begin{aligned} A &= \vec{d}^2 - (\vec{d} \cdot \vec{a})^2 \\ B &= 2\vec{d} \cdot (\vec{O} - \vec{c}) - 2(\vec{d} \cdot \vec{a})(\vec{O} - \vec{c}) \cdot \vec{v} \\ C &= (\vec{O} - \vec{c})^2 - r^2 - [(\vec{O} - \vec{c}) \cdot \vec{a}]^2 \end{aligned} \tag{2}$$

Where square of a vector is the dot product of this vector with its transpose (e.g. $\vec{d}^2 = \vec{d} \cdot \vec{d}^T$).

Then, if the ray collide at one ore two points with the cylinder, a projection is done from the point of intersection to the axis of the cylinder. This is done by taking the dot product of the vector going from the center of the cylinder to the intersection point and the normalise axis vector of the cylinder. If this projection is half of the length of the height h of the cylinder, it means that a collision occurs between the ray and cylinder. If two intersections occur, the closest one from the origin O of the ray is kept.

Finally, the length t between the ray and the closest intersection point and the normal vector $\vec{n}$ from the intersection point in direction of the point of view are returned.
This last one is calculated as following:

$$\vec{n} = -(\vec{I} - \vec{CP})/r * sign(\vec{d} \cdot \vec{CP}) \tag{3}$$

where $\vec{I}$ is the intersection point and $\vec{CP}$ is the point along the axis where the intersection point is projected.