LAB 1 – Stochastic Processes

Part A (Week 1):

1. Generate 3 random sequences (i.e., '**white noise**' WN) each comprising 200'000 samples. Set the standard deviation of these sequences to 2 [units]. For the sake of simplicity, assume that the generated sequences are sampled at 1 Hz.
   Hint: in Matlab, the function "randn" generates white noise with variance equaling unity; but you may use also Python ("numpy.random.randn") for your assignment.

2. Use these sequences to generate three realizations of accumulated white noise, also known as **'random walk' (**RW).

3. Use the previously generated random sequences to generate three realizations each of 1st order **Gauss-Markov** (GM) process (i.e. in your notes this process is called 'exponentially correlated random variable') for two correlation times (e.g. T_1=2000 and T_2=500 samples). *Hint*: in Matalb use 'exp()'.

By now, you should have 12 different sequences. Save all the realizations in a text file or as a mat file (MATLAB). The data should be stored in a column format with 8 digits after 0.

Part B (Week 2):

4. Compute the noise characteristics for each sequence by a: **autocorrelation** (AC) function; b: **power-spectral-density** (PSD); c: **Allan Variance**

5. Examine each noise characteristic obtained above **graphically** (suggestion: make a common plot for each type of noise while toggling the colors between the different realizations)

6. **Optional:** Compute the parameters of stochastic processes and compare your findings with the values determined via GMWM[1]. It is sufficient to upload one sequence for each noise.

<u>Report content:</u>

1. Figure 1:
   a. Subfigure: 3 WN realization with different color and legends
   b. Subfigure: 3 RW realization with different color and legends
   c. Subfigure: 3 GM (T=2000) realization with different color and legends
   d. Subfigure: 3 GM (T=500) realization with different color and legends
2. Repeat the same figure structure for the AC (Figure 2) and PSD (Figure 3) characteristics
3. Figure 4 (4 subplots): Plot Allan Variance for **one** realization only
   a) **Optional:** Figure 5 (4 subplots): Plot Wavelet variance for **one** realization only

[1] https://smac-group.github.io/gui4gmwm/

4. Answers to the following questions[2]:
   a. How does the shape of the empirically determined autocorrelation function **correspond** to the theoretical ones in all cases? Explain differences.
   b. How do the *empirically* determined values of **standard deviation** (i.e., calculated from all realizations) and **correlation length** (derived from the plot) **deviate** from the *true* values (i.e., those used in simulation)? **Make a quantitative comparison using a table.**

Stochastic Process	Parameter	True	Empirical
WN1	std		
WN2	std		
WN3	std		
RW1	std		
…			
GM1 T1	Std T		
…			
GM1 T2	Std T		
…			

   c. Which noise characteristic best identified the underlying stochastic process and the stochastic parameters?

5. Submit your report in addition to the MATLAB/Python code via Moodle. Please make sure that your code is well commented and runs!

Lab weight: 3% + **1%** (for optional exercise)
Distributed: Week 1
Deadline without penalty: 2 weeks after distribution (i.e., lecture of week 3)

HINT on useful Matlab (in Signal Processing Toolbox) and Python functions:

Autocorrelation:
xcorr.m (MATLAB)
numpy.correlate (Python)

Power Spectral Density (PSD):
pwelch.m (MATLAB)
scipy.signal.welch (Python)

[2]The answers and the comments should be relevant, short and consistent as would be expected during an oral exam. In other words, large number of pages does not prove that you well understood the subject.