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Extended Kalman Filter
Lab 6 – INS as model (4)

General non-linear perturbation with random noise

$$\Delta\dot{\mathbf{x}} = \underbrace{\left[\frac{\partial f()}{\partial \mathbf{x}}\right]_{\mathbf{x}=\mathbf{x}^*}}_{\mathbf{F}} \Delta\mathbf{x} + \mathbf{u}(t)$$

(2) perturbation of 2D INS → **F :**

$$\begin{aligned} \delta\dot{\alpha} &= \delta\omega^b_{mb} \\ \delta\dot{\mathbf{v}}^m &= \mathbf{R}^m_b \boldsymbol{\Omega}^b_{mb} \mathbf{f}^b + \mathbf{R}^m_b \delta\mathbf{f}^b \\ \delta\dot{\mathbf{p}}^m &= \delta\mathbf{v}^m \end{aligned}$$

2D IMU perturbation with random noise

- **F** + **noise** - together per element
- **case** : all errors in 'deltas' are modeled as a white noise e.g. $\delta\dot{\alpha} = \delta\omega^b_{mb} + w_g$

$$\begin{bmatrix} \delta\dot{\alpha} \\ \delta\dot{v}_n \\ \delta\dot{v}_e \\ \delta\dot{p}_n \\ \delta\dot{p}_e \end{bmatrix} = \underbrace{\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ -f^m_2 & \cdot & \cdot & \cdot & \cdot \\ -f^m_1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot \end{bmatrix}}_{\mathbf{F}_{11}} \begin{bmatrix} \delta\alpha \\ \delta v_n \\ \delta v_e \\ \delta p_n \\ \delta p_e \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & \cdot & \cdot \\ \cdot & \cos\alpha & -\sin\alpha \\ \cdot & \sin\alpha & \cos\alpha \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}}_{\mathbf{G}_{11}} \begin{bmatrix} w_g \\ w_{a_1} \\ w_{a_2} \end{bmatrix}$$

Extended Kalman Filter
Lab 6 – INS as model (7) details

Refer to Lab 6 help and/or black–board $c\alpha \rightarrow \cos\alpha$, $s\alpha \rightarrow \sin\alpha$

$$\underbrace{\begin{bmatrix} \delta\dot{\alpha} \\ \delta\dot{v}_n \\ \delta\dot{v}_e \\ \delta\dot{p}_n \\ \delta\dot{p}_e \\ \delta\dot{b}_c \\ \delta\dot{b}_g \\ \delta\dot{b}_{a_1} \\ \delta\dot{b}_{a_2} \end{bmatrix}}_{\delta\dot{\mathbf{x}}} = \underbrace{\left[\begin{array}{c|cccc} & 1 & 1 & \cdot & \cdot \\ & \cdot & \cdot & c\alpha & -s\alpha \\ \mathbf{F}_{11}^{5\times5} & \cdot & \cdot & s\alpha & c\alpha \\ & \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot \\ \hline & \cdot & \cdot & \cdot & \cdot \\ & \cdot & -\beta_g & \cdot & \cdot \\ \mathbf{0}^{4\times5} & \cdot & \cdot & -\beta_{a_1} & \cdot \\ & \cdot & \cdot & \cdot & -\beta_{a_2} \end{array}\right]}_{\mathbf{F}} \underbrace{\begin{bmatrix} \delta\alpha \\ \delta v_n \\ \delta v_e \\ \delta p_n \\ \delta p_e \\ \delta b_c \\ \delta b_g \\ \delta b_{a_1} \\ \delta b_{a_2} \end{bmatrix}}_{\delta\mathbf{x}} + \underbrace{\left[\begin{array}{c|ccc} & & & \\ \mathbf{G}_{11}^{5\times3} & & \mathbf{0}^{5\times3} & \\ & & & \\ \hline & \cdot & \cdot & \cdot \\ \mathbf{0}^{4\times3} & 1 & \cdot & \cdot \\ & \cdot & 1 & \cdot \\ & \cdot & \cdot & 1 \end{array}\right]}_{\mathbf{G}} \underbrace{\begin{bmatrix} w_g \\ w_{a_1} \\ w_{a_2} \\ q_b \\ q_{a_1} \\ q_{a_2} \end{bmatrix}}_{\mathbf{w}}$$

Extended Kalman Filter

Lab 6 – INS as model (9) - flowchart