École Polytechnique Fédérale de Lausanne

Lab2 - Inertial Navigation in 2D Errorless Signal

ENV-548: Sensor Orientation

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1 Task 1: Nominal measurements

1.1 Uniform circular motion

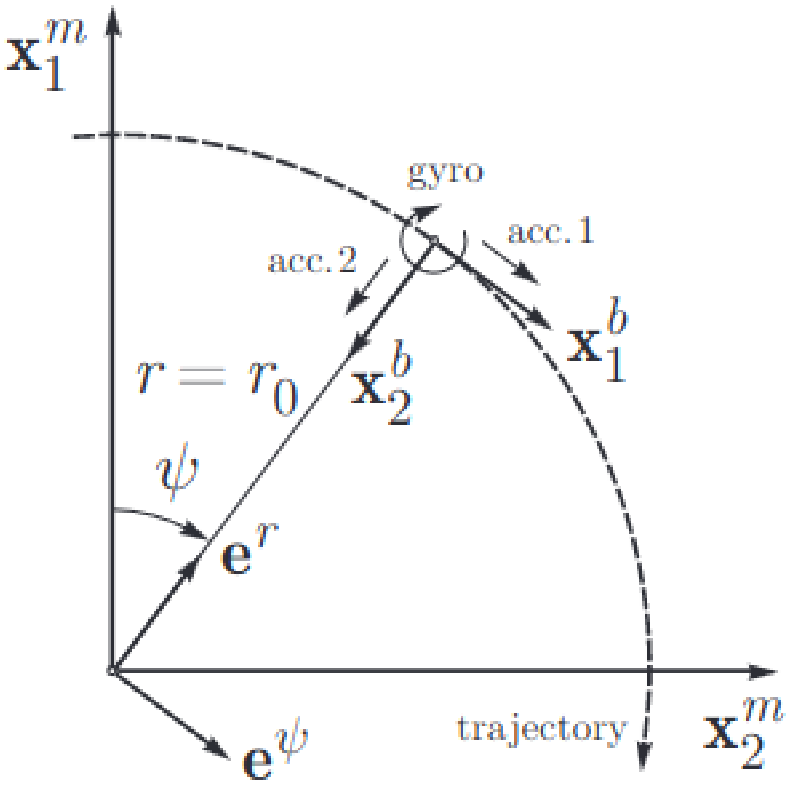


Figure 1: Axes correspondences : $X = x_1^b$, $Y = x_2^m$, $N = x_1^m$ and $E = x_2^m$

1.2 Nominal measurements

For this classic case of 'Uniform Circular Motion', the acceleration and angular velocities of the body are known to be constant and defined as :

$$gyro = \omega$$
$$a_x = 0$$
$$a_y = \omega^2 * R$$

with ω the angular velocity of the body around the origin of the frame N-E (x_1^m-x_2^m)
Therefore the nominal measurements are exactly the values cited above

2 Task 2 : True trajectory (PVA)

Given the initial condition and the nominal measurements, we can determine the trajectory variables (attitude α, position, velocity) with the integration method (dt infinitesimal) :

- Initial conditions:

$$\begin{aligned}
\alpha[t=0] &= \pi/2 \\
v_N[t=0] &= 0 \\
v_E[t=0] &= \omega * R \\
p_N[t=0] &= R \\
p_E[t=0] &= 0
\end{aligned}$$

- States in N-E frame:

$$\begin{aligned}
\alpha[t=T] &= \alpha[t=0] + \int_{t=0}^{t=T} gyro * dt \\
&= \alpha[t=0] + gyro * t\Big|_{t=0}^{t=T} \\
a_N[t=T] &= a_x * cos(\alpha[t=T]) - a_y * sin(\alpha[t=T]) \\
a_E[t=T] &= a_x * sin(\alpha[t=T]) + a_y * cos(\alpha[t=T]) \\
v_N[t=T] &= v_N[t=0] + \int_{t=0}^{t=T} a_N[t]dt \\
&= v_N[t=0] + (a_x * sin(\alpha[t]) + a_y * cos(\alpha[t]))/gyro\Big|_{t=0}^{t=T} \\
v_E[t=T] &= v_E[t=0] + \int_{t=0}^{t=T} a_E[t]dt \\
&= v_E[t=0] + (-a_x * cos(\alpha[t]) + a_y * sin(\alpha[t]))/gyro\Big|_{t=0}^{t=T} \\
p_N[t=T] &= p_N[t=0] + \int_{t=0}^{t=T} (\int_{t=0}^{t=T} a_N[t]dt)dt \\
&= p_N[t=0] + (-a_x * cos(\alpha[t]) + a_y * sin(\alpha[t]))/gyro^2\Big|_{t=0}^{t=T} \\
p_E[t=T] &= p_E[t=0] + \int_{t=0}^{t=T} (\int_{t=0}^{t=T} a_E[t]dt)dt \\
&= p_E[t=0] + (-a_x * sin(\alpha[t]) - a_y * cos(\alpha[t]))/gyro^2\Big|_{t=0}^{t=T}
\end{aligned}$$

3 Task 3 : Trajectories

In order to solve numerically the navigation-differential equations using integration methods, the acceleration at each step is first computed as followed:

$$a_E = -a_y * sin(\theta) + a_x * cos(\theta)$$
$$a_N = -a_y * cos(\theta) - a_x * sin(\theta)$$

Then, the velocity and position on axis E and N are computed using integration methods of different orders:

- order 1:

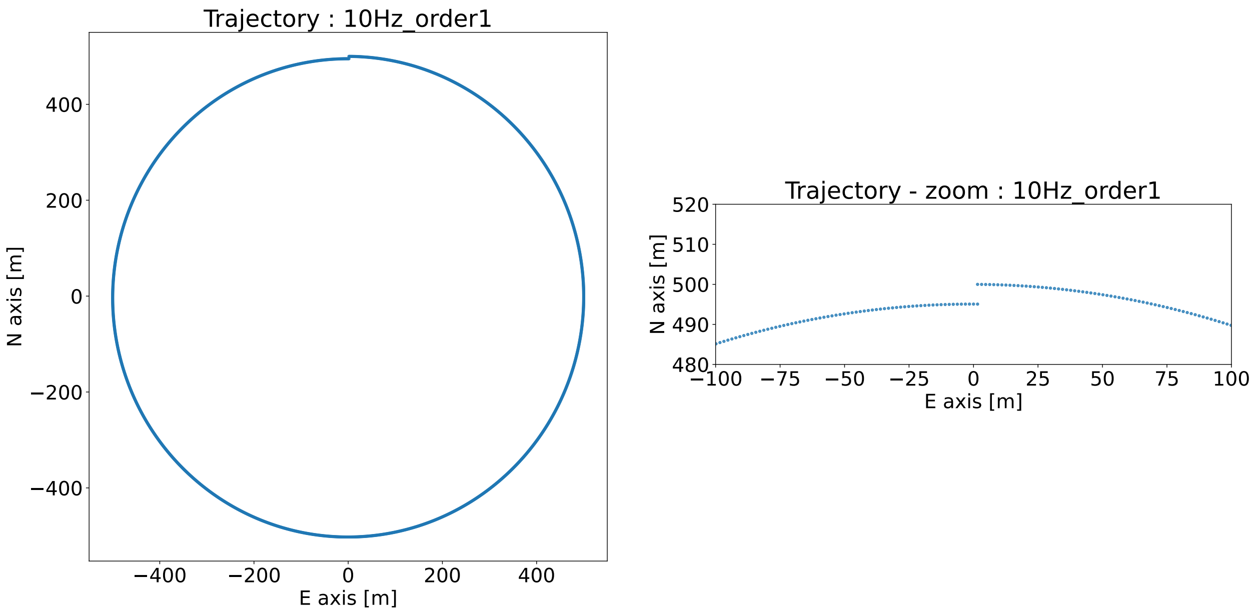
$$v_E(t_k) = v_E(t_{k-1}) + a_E(t_k) \cdot \Delta t$$
$$p_E(t_k) = p_E(t_{k-1}) + v_E(t_k) \cdot \Delta t$$

- order 2:

$$v_E(t_k) = v_E(t_{k-1}) + \frac{1}{2}[a_E(t_k) + a_E(t_{k-1})] \cdot \Delta t$$
$$p_E(t_k) = p_E(t_{k-1}) + \frac{1}{2}[v_E(t_k) + v_E(t_{k-1})] \cdot \Delta t$$

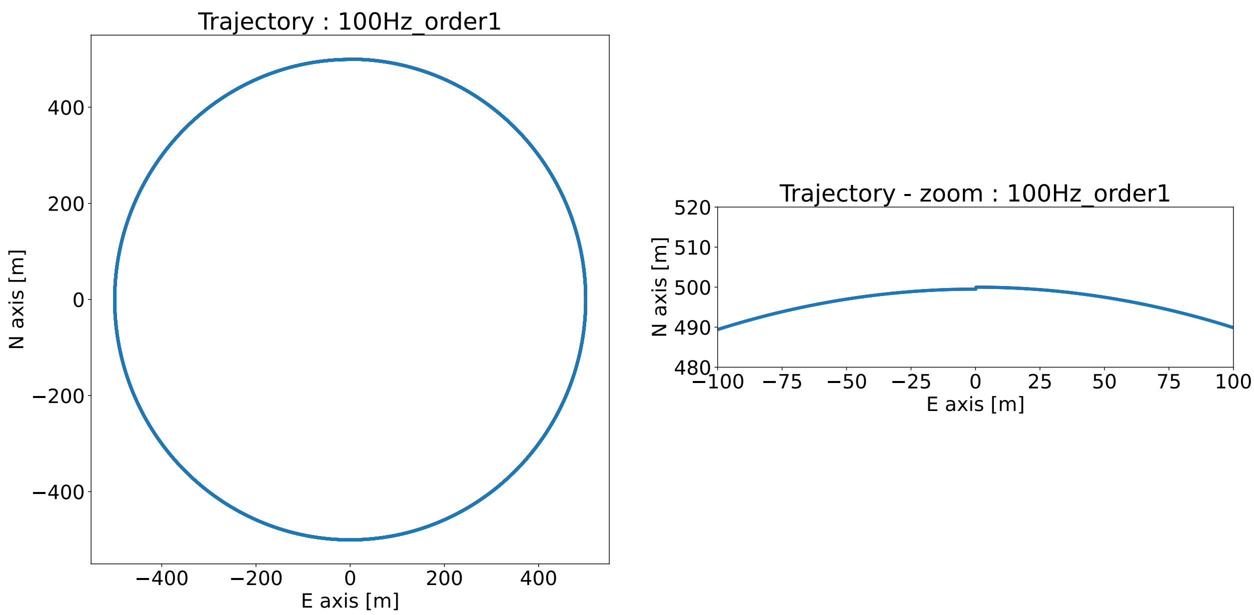
and respectively for v_N and p_N.

Figures 2, 3, 4 and 5 show the simulations of a full revolution trajectory using the two different order method at two different frequency of sampling.



(a) Full revolution (b) Zoom on revolution completion point

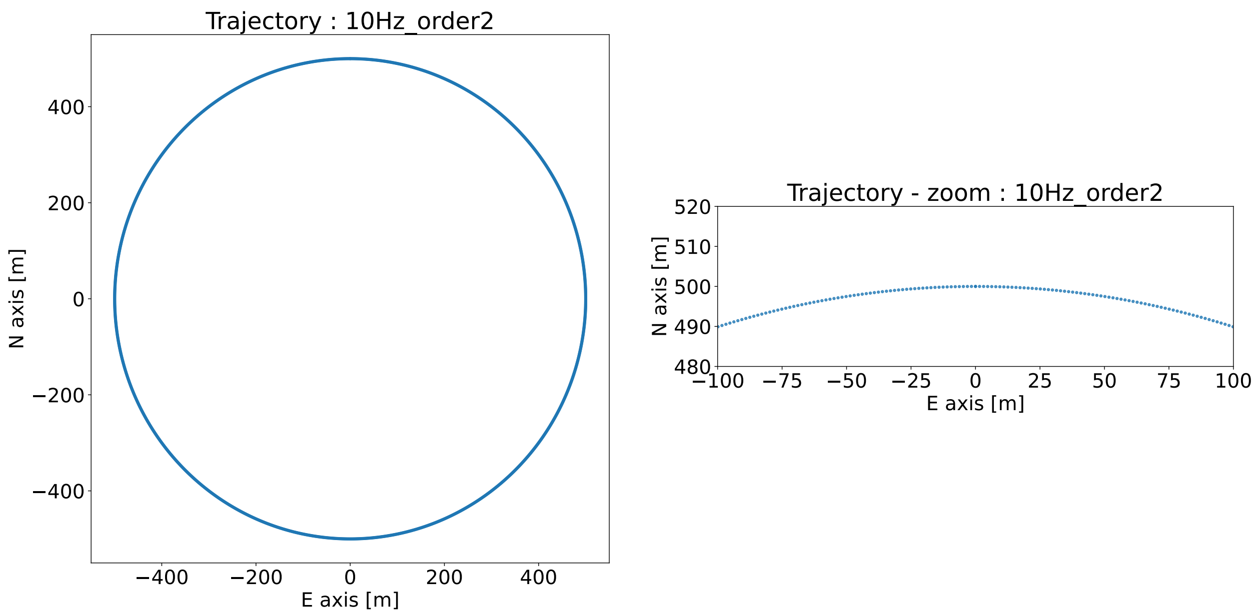
Figure 2: Trajectory during one revolution computed by integration of order 1 with a sampling at 10Hz



(a) Full revolution

(b) Zoom on revolution completion point

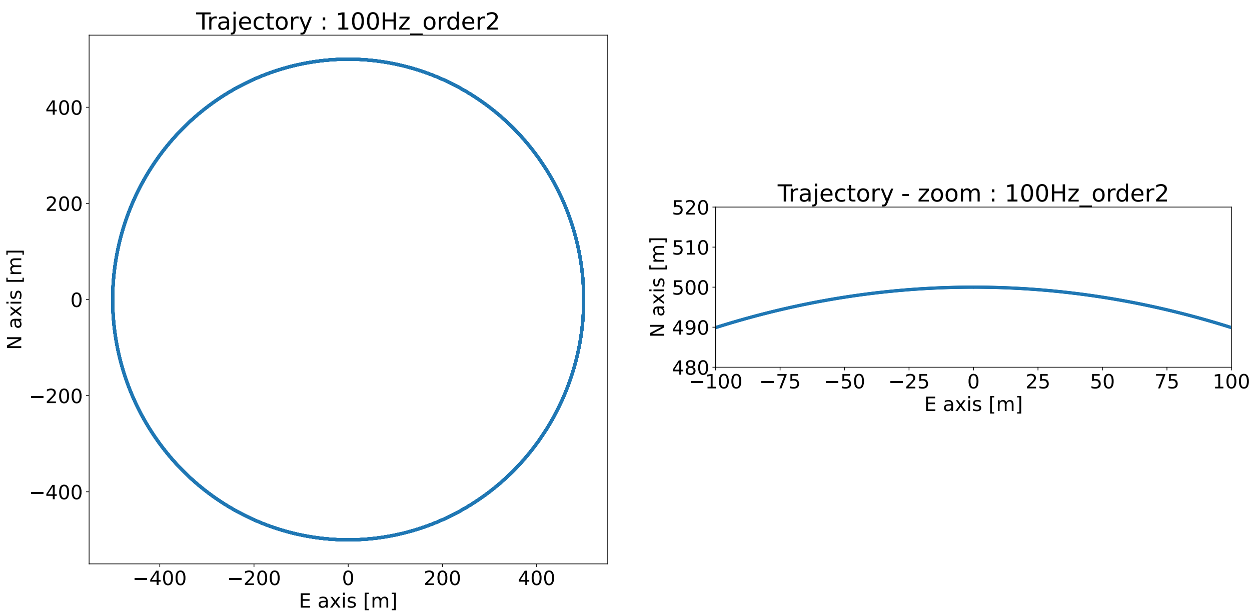
Figure 3: Trajectory during one revolution computed by integration of order 1 with a sampling at 100Hz



(a) Full revolution

(b) Zoom on revolution completion point

Figure 4: Trajectory during one revolution computed by integration of order 2 with a sampling at 10Hz



(a) Full revolution

(b) Zoom on revolution completion point

Figure 5: Trajectory during one revolution computed by integration of order 2 with a sampling at 100Hz

4 Task 4 : Trajectory errors (PVA)

After completing task 3, the results for the azimuth, position and velocity of each realization is compared to the *true value* by computing the difference at each time step. The results can be seen at section below

4.1 Frequency = 10Hz, Order = 1

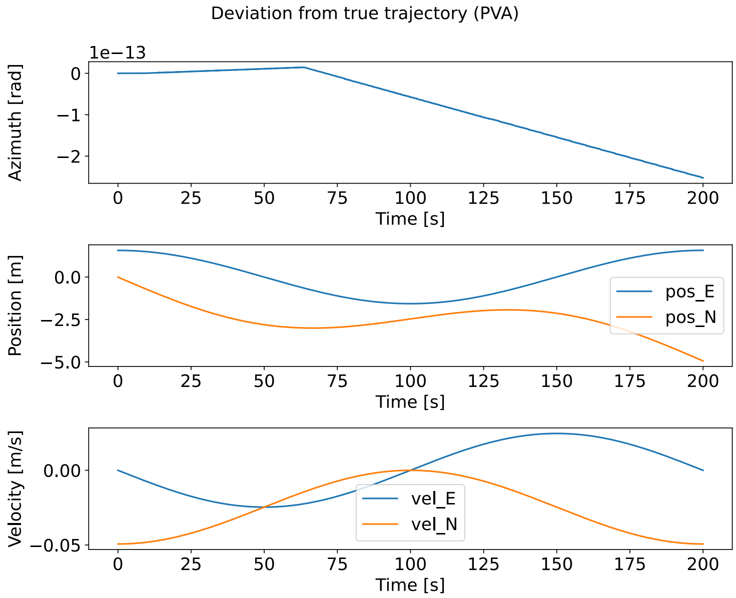


Figure 6: Error of the azimuth, position and velocity on integration of order 1 with a sampling at 10Hz

4.2 Frequency = 100Hz, Order = 1

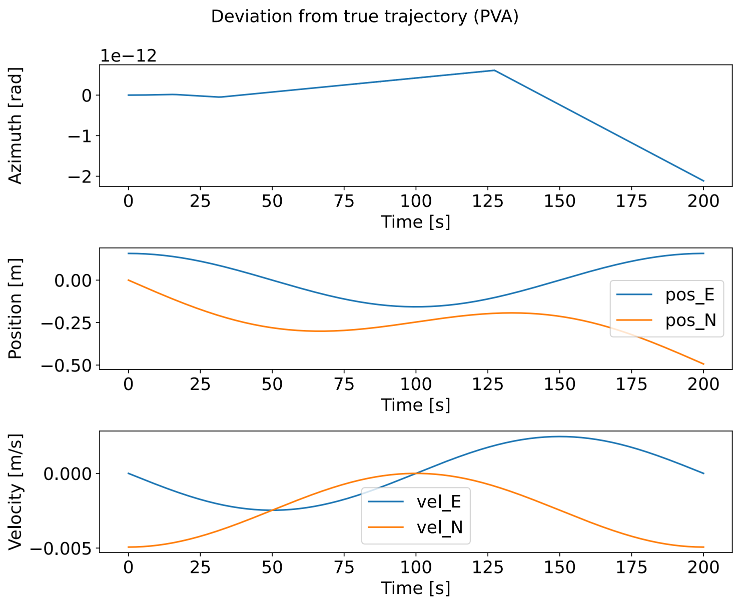


Figure 7: Error of the azimuth, position and velocity on integration of order 1 with a sampling at 100Hz

4.3 Frequency = 10Hz, Order = 2

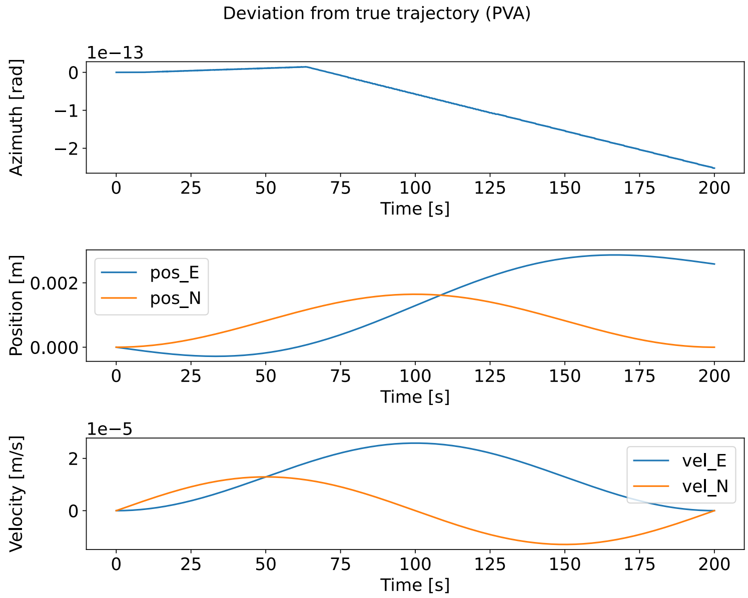


Figure 8: Error of the azimuth, position and velocity on integration of order 2 with a sampling at 10Hz

4.4 Frequency = 100Hz, Order = 2

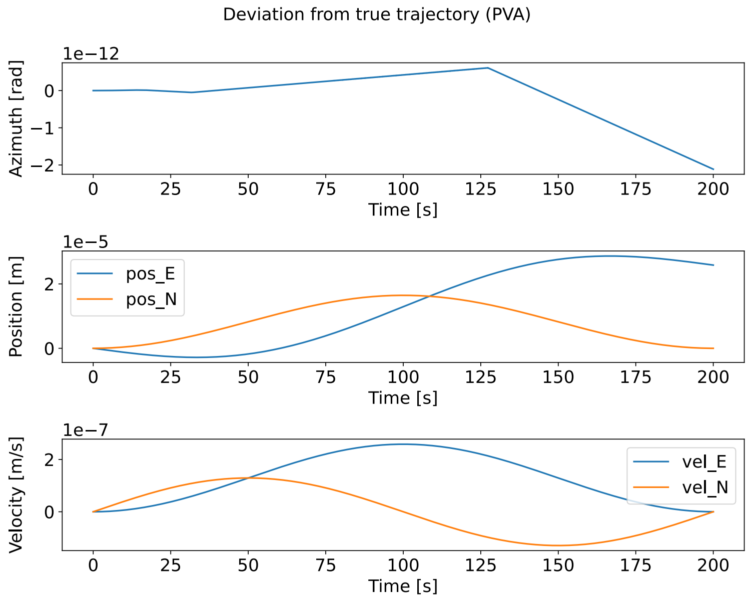


Figure 9: Error of the azimuth, position and velocity on integration of order 2 with a sampling at 100Hz

5 Questions

5.1 Question A

What are the maximum committed errors (in x and y) in PVA after one revolution for each case ?

Table 1: Maximum Committed Errors

Cases		**(P, V)**		**A**
Units		**([m], [m/s]))**		**[rad]**
Freq	**Integ. order**	**North**	**East**	
10	1	(4.940, 0.049)	(1.573, 0.025)	2.522e-13
100	1	(0.494, 0.005)	(0.157, 0.002)	2.114e-12
10	2	(0.002, 1.292e-5)	(0.003, 2.583e-5)	2.522e-13
100	2	(1.645e-5, 1.292e-7)	(2.866e-5, 2.583e-7)	2.114e-12

5.2 Question B

Which integration method would you recommend using and why ? Looking at the table 1, it can be seen that the integration method of order 2 gives far better results than the method of order 1 with a very small extra-computational cost. Moreover, the difference between computing it with a sampling of 100Hz reduce the errors by a factor of magnitude 100. However, the computation is heavier.

Therefore, we would recommend the method of order 2 in almost any cases and the sampling should be adapted to the need of the simulation at hand.