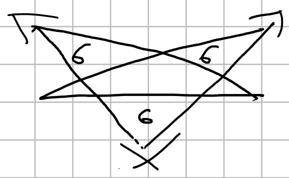
ESERCIZI CON TONY



1) $\int \pi \, dx = \pi x + c$

2) $\int \frac{x^3}{2} \, dx = \frac{x^4}{2 \cdot 4} = \frac{x^4}{8} + c$

3) $\int \left(x^5 + 5x^4 + \frac{x^3}{3} + 3x^2 + 1 \right) dx$

POTERE DELL'ADDITIVITÀ INTEGRALE

$\int x^5 + \int 5x^4 + \int \frac{x^3}{3} + \int 3x^2 + \int 1 \, dx =$

$= \frac{x^6}{6} + x^5 + \frac{x^4}{12} + x^3 + x = \frac{x^6}{6} + x^5 + \frac{x^4}{12} + x^3 + c$

4) $\int (3e^x + 1) \, dx = 3e^x + x + c$

5) $\int \left(x^{1/4} + x/2 \right) dx = \int x^{1/4} + \int x/2 \, dx = \frac{4x^{5/4}}{5} + \frac{x^2}{4} + c$

6) $\int 2 \sin(x) \, dx = 2 \int \sin(x) dx = -2\cos x + c$

7) $\int \frac{2}{x} dx = 2 \int \frac{1}{x} \, dx = 2 \log|x| + c$

8) $\int \frac{1+x}{x} dx = \int \frac{1}{x} + \int 1 \, dx = \log|x| + x + c$

9) $\int \left(3^x + \cos(x) + \frac{x^6}{7} \right) dx =$ $\quad \int a^x = \frac{a^x}{\log a} + c$

$= \int 3^x + \int \cos x + \int \frac{x^6}{7} \, dx =$

$= \frac{3^x}{\log 3} + \sin x + \frac{x^7}{49} + c$

10) $\int \frac{2}{1 + x^2} \, dx = 2 \int \frac{1}{1+x^2} \, dx = 2 \arctan(x) + c$

11) $\int \frac{3}{\sqrt{1-x^2}} \, dx = 3 \int \frac{1}{\sqrt{1-x^2}} \, dx = -3 \arctan(x) + c$

12) $\int \frac{x^6 + 8x^7}{\sqrt[3]{x^2}} \, dx = \int \frac{x^6 + 8x^7}{x^{2/3}} \, dx = \int \left(x^6 \cdot \frac{1}{x^{2/3}} \right) + 8 \int \left(x^7 \cdot \frac{1}{x^{2/3}} \right) dx =$

$\frac{18}{3} - 2/3 = \frac{16}{3} \qquad \frac{21}{3} - \frac{2}{3} = \frac{19}{3}$

$\int x^{16/3} \, dx = \frac{x^{19/3}}{19/3} = \frac{3x^{19/3}}{19} \qquad \int x^{24/3} dx = \frac{x^{24/3}}{24/3} = \frac{3x^8}{24}$

$= \frac{3x^{19/3}}{19} + \frac{24x^8}{24} + c = \frac{3x^{19/3}}{19} + x^8 + c$

13) $\int 2e^{2x} dx = 2\int e^{2x} dx = 2 \cdot \frac{e^{2x}}{2} = e^{2x} + c$

$\int e^{f(x)} dx = e^{f(x)} \cdot f'(x)$ $\quad \int \left(e^{2x} \cdot \frac{2}{2}\right) dx = \frac{1}{2} \int \left(e^{2x} \cdot 2\right) dx$

$\Downarrow$

e^{2x}

14) $\int \cos(2x)\, dx$

$\int \cos(2x) \cdot \frac{2}{2}\, dx = \frac{1}{2} \int 2\cos(2x)\, dx$

Pongo $y = 2x$

$dy = D(2x)dx = 2dx$

$\frac{1}{2} \int \cos(y)\, dy = \frac{\sin(2x)}{2} + c$

15) $\int \frac{e^{\ln(x)}}{x} dx = \int e^{\ln(x)} \cdot \frac{1}{x} dx$ $\quad$ Pongo $y = \ln(x)$ $\quad dy = \frac{dx}{x}$

$= \int e^y\, dy = e^{\ln(x)} + c$

16) $\int \cos\left(\frac{3x}{2}\right) dx = \int \cos\left(\frac{3x}{2}\right) \cdot \underbrace{\frac{3}{2} \cdot \frac{2}{3}}_{=1} dx = \frac{2}{3} \int \cos(f(x)) \cdot f'(x)\, dx$

Pongo $y = \frac{3x}{2}$ $\quad dy = \frac{3}{2} dx$

$= \frac{2}{3} \int \cos(y)\, dy = \frac{2\sin(y)}{3} + c = \frac{2\sin\left(\frac{3x}{2}\right)}{3} + c$

17) $\int \frac{1}{4\sin^2(x)} dx = \frac{1}{4} \int \frac{1}{\sin^2(x)} dx = \frac{-\cot(x)}{4} + c$

18) $\int \left(\frac{2^{5x}}{3} + \sin\left(\frac{x}{2}\right)\right) dx = \underset{A}{\int \frac{2^{5x}}{3}} + \underset{B}{\int \sin\left(\frac{x}{2}\right) dx}$

B -> $\int \sin\left(\frac{x}{2}\right) \cdot \frac{2}{2} dx = 2\int \sin\left(\frac{x}{2}\right) \cdot \frac{1}{2} dx =$ $\quad$ Pongo $y = x/2$ $\quad dy = \frac{dx}{2}$

$= 2\int \sin y\, dy = -2\cos(y) + c = -2\cos\left(\frac{x}{2}\right) + c$

A -> $\frac{1}{3} \int 2^{5x} dx = \frac{1}{3} \int 2^{5x} \cdot \frac{5}{5} dx = \frac{1}{15} \int 5 \cdot 2^{5x} dx$

Pongo $y = 5x$ $\quad dy = 5dx$

$\frac{1}{15} \int 2^y dy = \frac{1}{15}\left(\frac{2^y}{\log 2}\right) + c$ $\quad \frac{2^{5x}}{15\log 2} + c$

I) $\int x \ln(x)dx$

II) $\int xe^x dx$

III) $\int (4x-1)sin(x)dx$

IV) $\int \log^2(x)dx$

V) $\int \arctan(x)dx$

VI) $\int x^2 \arctan(x)dx$

VII) $\int \log(x^2+3)dx$

VIII) $\int e^x \cos(x)dx$

IX) $\int \sqrt{1-x^2}dx$

X) $\int \sin(2x)e^{-x}dx$

XI) $\int \frac{\log(x)}{x^4}dx$

XII) $\int \sqrt{2-x^2}dx$

XIII) $\int x\sin(x)\cos(x)dx$

XIV) $\int x\frac{\cos(x)}{\sin^3(x)}dx$

XV) $\int xe^x\cos(x)dx$

XVI) $\int x^2e^xdx$

XVII) $\int x^2\log(x+1)dx$

Integrazione per parti per integrali indefiniti

Applicando la precedente formula al caso di un intervallo $[a,x] \subseteq [a,b]$ abbiamo subito la formula di integrazione per parti per gli integrali indefiniti

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx + c$$

dove c è una costante arbitraria.

1) $\int x \ln(x)\,dx =$

$f(x) = \ln(x)$ $\quad f'(x) = \frac{1}{x}$

$g'(x) = x$ $\quad g(x) = \frac{x^2}{2}$

$\ln(x)\frac{x^2}{2} - \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx =$

$= \ln(x)\frac{x^2}{2} - \int \frac{x}{2}\,dx = \frac{\ln(x)x^2}{2} - \frac{x^2}{4} = \frac{2(\ln(x)x^2) - x^2}{4} =$

$= \frac{2\ln(x)x^2 - x^2}{4}$

2) $\int xe^x\,dx =$

$f(x) = x$ $\quad f'(x) = 1$

$g'(x) = e^x$ $\quad g(x) = e^x$

$= xe^x - \int e^x\,dx = xe^x - e^x + c$

3) $\int (4x-1)\sin(x)\,dx$

$f(x) = 4x-1$ $\quad f'(x) = 4$

$g'(x) = \sin(x)$ $\quad g(x) = -\cos(x)$

$(4x-1)(-\cos(x)) - \int 4(-\cos(x))\,dx =$

$= -4\cos(x)x + \cos(x) + 4\int \cos(x)\,dx =$

$= -4\cos(x)x + \cos(x) + 4\sin(x) + c$

4) $\int \log^2(x)\,dx = \int \log(x)\cdot\log(x) =$

$f(x) = \log(x)$ $\quad f'(x) = 1/x$

$g'(x) = \log(x)$ $\quad g(x) = x\log(x) - x$

$x\log^2(x) - x\log(x) - \int \frac{x\log(x) - x}{x}\,dx = \underbrace{x(\log^2(x) - \log(x))}_{A} - \int \log(x) - 1\,dx$

$= A - \left(\int \log(x)\,dx - \int 1\,dx\right) = A - (x\log(x) - x - x)$

$= x(\log^2(x) - \log(x) - \log(x) + 2)$

$= x(\log^2(x) - 2\log(x) + 2)$

5) $\int \arctan(x)\,dx = \int 1\cdot\arctan(x)\,dx =$

$f(x) = \arctan(x)$

$f'(x) = \frac{1}{1+x^2}$

$g'(x) = 1$

$g(x) = x$

$= x\arctan(x) - \int \frac{x}{1+x^2}\,dx =$

$= x\arctan(x) - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx =$

$\int \frac{h'(x)}{h(x)}\,dx = \log(|h'(x)|)$

$= x\arctan(x) - \frac{1}{2}\log(1+x^2)$

$1 + x^2 \geq 0$

7) $\int \log(x^2+3)\,dx =$

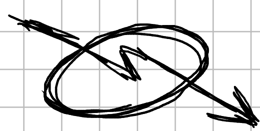
$f(x) = \log(x^2+3) \qquad f'(x) = \frac{2x}{x^2+3}$

$g'(x) = 1 \qquad g(x) = x$

$= \underbrace{x\log(x^2+3)}_{A} - \int \frac{2x^2}{x^2+3}\,dx =$

$= A - 2\int\left(\frac{x^2+3}{x^2+3} - \frac{3}{x^2+3}\right)dx = A - 2\left(\int 1\,dx - 3\int \frac{1}{x^2+3}\,dx\right) =$

$= A - 2\left(x - 3\left(\frac{1}{\sqrt3}\arctan\left(\frac{x}{\sqrt3}\right)\right)\right) =$



$= x\log(x^2+3) - 2x + \frac{6}{\sqrt3}\arctan\left(\frac{x}{\sqrt3}\right)$

9) $\int\sqrt{1-x^2}\,dx = \int \frac{1-x^2}{\sqrt{1-x^2}}\,dx =$

$g'(x) = \frac{1}{\sqrt{1-x^2}} \qquad g(x) = \arcsin x$

$f(x) = 1 - x^2 \qquad f'(x) = -2x$

$= \underbrace{\arcsin(x) - x^2\arcsin(x)}_{A} - \int -2x\arcsin(x)\,dx =$ MERDA

10) $\int \sin(2x)\,e^{-x}\,dx =$

$\int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)\,dx$

$= \underbrace{-\frac{e^{-x}\cos(2x)}{2}}_{A} - \int \frac{e^{-x}\cos(2x)}{2}\,dx =$

$f(x) = e^{-x} \qquad f'(x) = -e^{-x}$

$g'(x) = \sin(2x) \qquad g(x) = -\frac{\cos(2x)}{2}$

$h(x) = e^{-x} \qquad h'(x) = -e^{-x}$

$s'(x) = \cos(2x) \qquad s(x) = \frac{\sin(2x)}{2}$

$= A - \frac{1}{2}\int e^{-x}\cos(2x)\,dx =$

$= A - \frac{1}{2}\left(\underbrace{\frac{e^{-x}\sin(2x)}{2}}_{B} - \int -\frac{e^{-x}\sin(2x)}{2}\,dx\right) = A - \frac{1}{2}\left[B + \frac{1}{2}\int e^{-x}\sin(2x)\,dx\right] =$

PONGO I ALLA PROPOSIZIONE INIZIALE

$I = -\frac{e^{-x}\cos(2x)}{2} - \frac{1}{2}\left(\frac{e^{-x}\sin(2x)}{2} + \frac{I}{2}\right) =$

$= -\frac{e^{-x}\cos(2x)}{2} - \frac{e^{-x}\sin(2x)}{4} - \frac{I}{4}$

$\Rightarrow \frac{5I}{4} = -\frac{e^{-x}\cos(2x)}{2} - \frac{e^{-x}\sin(2x)}{4}$

$5I = -2e^{-x}\cos(2x) - e^{-x}\sin(2x)$

$I = -\frac{2e^{-x}\cos(2x)}{5} - \frac{e^{-x}\sin(2x)}{5}$

$+ C$