DIVIDING A POLYNOMIAL BY A POLYNOMIAL

Consider:

$$23\overline{)668}$$

Ex. 1 Divide $3x^2 - 5x - 18$ by $x + 2$ and check.

Ex. 2 Divide $-6x^3 + 5x^2 + 2x + 7$ by $3x - 1$.

Ex. 3 Divide $x^4 - x^2 - 3x + 8$ by $x^2 - x + 3$.

Ex. 4 Divide $4x - 5x^2 + 3x^4 - 2$ by $x - 1$.

Ex. 5 Rewrite $\frac{2x^3 - 7x^2 + 7}{2x - 3}$ in the form $Q(x) + \frac{R(x)}{D(x)}$.

REMAINDER THEOREM

Consider:

Determine the remainder when $f\left(x\right)=x^3-4x^2-5x-1$ is divided by $x-2$.

Determine the value of $f\left(2\right)$.

Consider:

Determine the remainder when $f\left(x\right)=-3x^3+x^2-4x$ is divided by $3x+2$.

Determine the value of $f\left(-\frac{2}{3}\right)$.

thm REMAINDER THEOREM

If $f(x)$ is divided by $x-b$, then the remainder is $f(b)$.

If $f(x)$ is divided by $ax-b$, then the remainder is $f\left(\frac{b}{a}\right)$.

Ex. 1 Find the remainder when $5x^3-3x^2+2x-8$ is divided by

a) $x+3$

b) $x-5$

c) $2x+1$

Ex. 2 When x^3-2x^2-kx+6 is divided by $x+5$, the remainder is 11. Find the value of k.

FACTOR THEOREM

thm FACTOR THEOREM

$x-b$ is a factor of $f(x)$ iff $f(b)=0$

$ax-b$ is a factor of $f(x)$ iff $f\left(\frac{b}{a}\right)=0$

Ex. 1 Determine which binomials are factors of $x^3-6x^2+3x+10$.

a) $x-5$

b) $x-1$

Ex. 2 Factor.

a) x^3+2x^2-5x-6

b) $x^3-5x^2-4x+20$

c) $x^4+5x^3+5x^2-5x-6$

d) $8x^3+12x^2-2x-3$

Ex. 3 If x^3-kx^2-x+30 is divisible by $x-5$, determine the value of k.

SUM AND DIFFERENCE OF CUBES

form SUM AND DIFFERENCE OF CUBES

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Ex. 1 Factor.

a) $x^3 - 1000$

b) $27y^3 + 64$

c) $250x^3 - 16y^3$

d) $(x-1)^3 - 8y^3$

e) $(x+2)^3 + (x-2)^3$

f) $x^6 - 1$

POWER FUNCTIONS

def POLYNOMIAL FUNCTION

A function in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

where $a_i \in R$, $n \in W$

Ex. $y = 4x^3 - 3x^2 + 1$, $y = -2x^4 - 0.5x^2 + x$, $y = 8x^9 - 3x^7 + x^5 - x + 5$, $y = -9x^3$

def POWER FUNCTION

A polynomial function in the form

$$f(x) = ax^n$$

where $a \in R$, $n \in W$

Ex. $y = -9x^3$, $y = x^2$, $y = 5x^7$, $y = -0.48x$

Consider:

1. $f(x) = x$
3. $f(x) = x^3$
5. $f(x) = x^5$

2. $f(x) = x^2$
4. $f(x) = x^4$
6. $f(x) = x^6$

Symmetry:

def LINE SYMMETRY
A relation exhibits line symmetry if it is symmetrical in a __________.

def POINT SYMMETRY
A relation exhibits point symmetry if part of the graph can be rotated _____ and coincide with another part of the graph.

If n is odd, then $\exists$ point symmetry about the _______________.

If n is even, then $\exists$ line symmetry in the _______________.

End Behaviour:

End behaviour refers to the y value as x approaches infinity (________) and as x approaches negative infinity (________).

If $a > 0$,

	n	
	odd	even
as $x \to \infty$		
as $x \to -\infty$		

If $a < 0$,

	n	
	odd	even
as $x \to \infty$		
as $x \to -\infty$		

Domain:

Range:

If n is odd,

If n is even and

$a > 0$,

$a < 0$,

TRANSFORMATIONS OF $f(x) = x^n$

Ex. 1 Given $f(x) = -32\left[\frac{1}{2}(x-6)\right]^4 + 10$

a) Describe the transformations.

b) State the coordinates of the vertex or point of inflection if it exists.

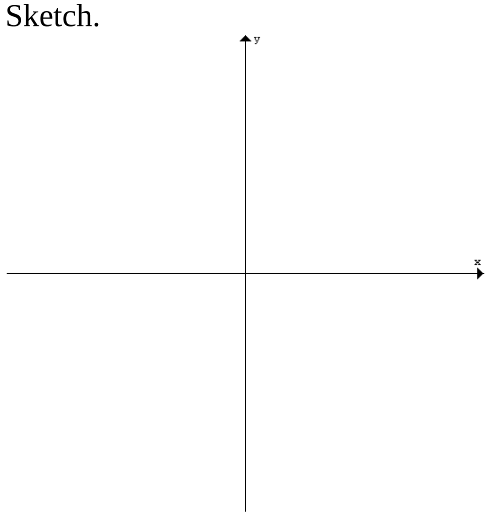
c) Determine the equation of the axis of symmetry if it exists.

d) State the domain and range.

e) Find the y-intercept.

f) Find the x-intercept(s).

g) Sketch.



Ex. 2 Given the function $f(x)=x^3$ and $y=-f(2x+2)-5$.

a) State the coordinates of the vertex or point of inflection if it exists.

b) Determine the equation of the axis of symmetry if it exists.

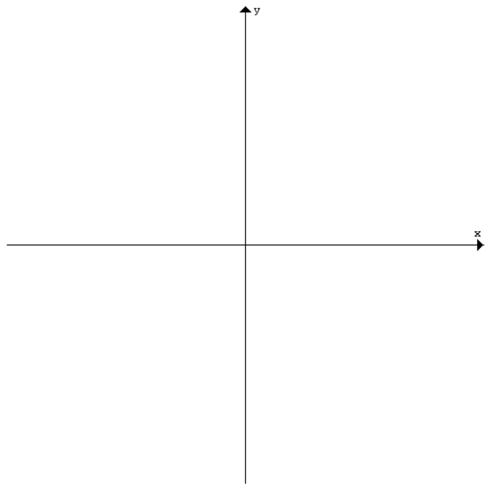
c) State the domain and range.

d) State the equation of the cubic under the given transformations.

e) Find the y-intercept.

f) Find the x-intercept(s).

g) Sketch.



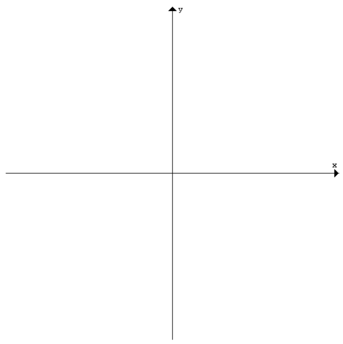
Ex. 3 What is the equation of a quintic polynomial function with the following transformations: reflection in the y-axis, vertical compression by a factor of $\frac{2}{3}$, translation right by 2 units, translation up by 4 units

CUBIC FUNCTIONS

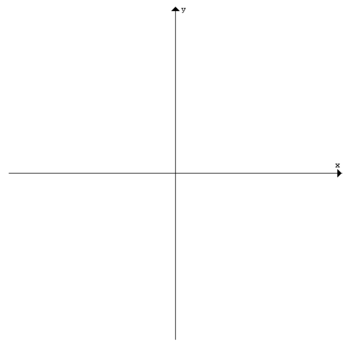
The general cubic function is in the form $f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ where $a_i \in R$.

1. Sketch the following. Get the basic shape, show intercept(s) and show end behaviour.

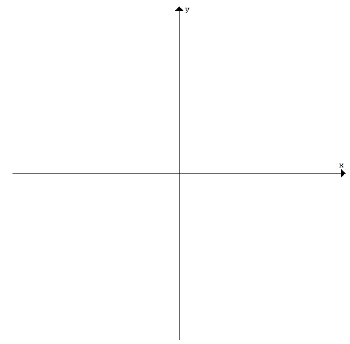
a) $f(x) = x^3$



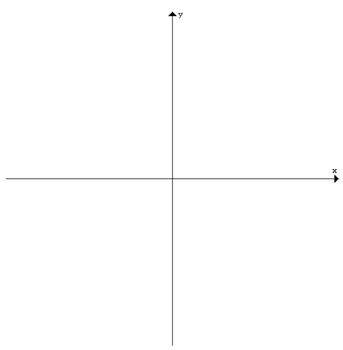
b) $f(x) = x^3 - 2x^2 - x + 2$



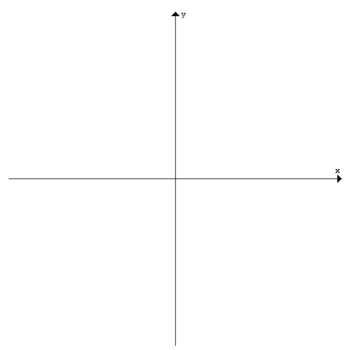
c) $f(x) = x^3 - 3x - 2$



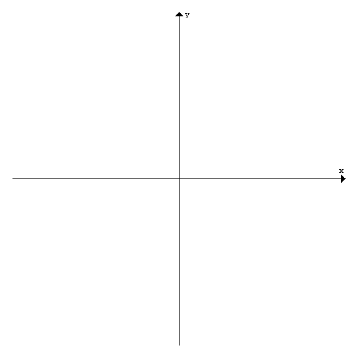
d) $f(x) = 2x^3 - 6x^2 + 8$



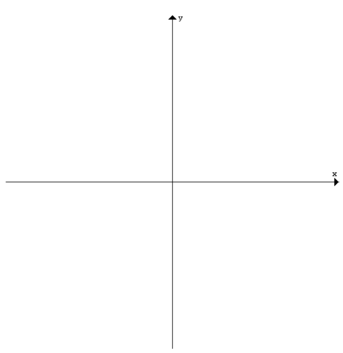
e) $f(x) = 5x^3 + 6x^2 + 2x + 1$



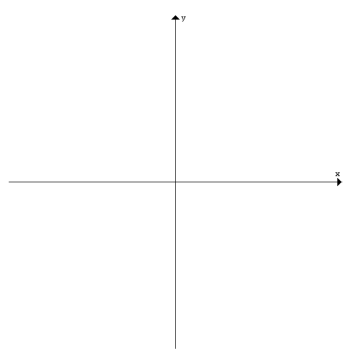
f) $f(x) = x^3 + x^2 + x - 3$



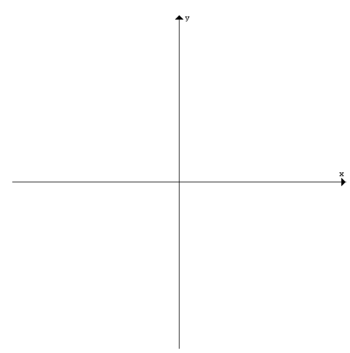
g) $f(x) = -x^3 - 5x^2 - 7x - 3$



h) $f(x) = -0.5x^3 + 2x^2$



i) $f(x) = -x^3 - 3x^2 - 3x - 1$



2. Describe the end behaviour of graphs with a positive leading coefficient. (Graphs a to f.)

3. Describe the end behaviour of graphs with a negative leading coefficient. (Graphs g to i.)

4. How many x-intercepts can a cubic function have?

5. How many absolute maximum/minimum points can a cubic function have?

6. How many local maximum/minimum points can a cubic have?

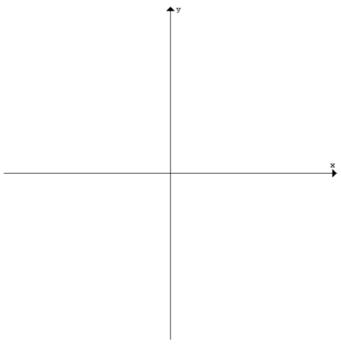
7. For the general cubic function in standard form, $f(x)=ax^3+bx^2+cx+d$, what does the value of d represent?

QUARTIC FUNCTIONS

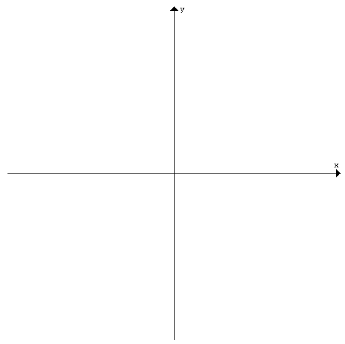
The general quartic function is in the form $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$ where $a_i \in R$.

8. Sketch the following. Get the basic shape, show intercept(s) and show end behaviour.

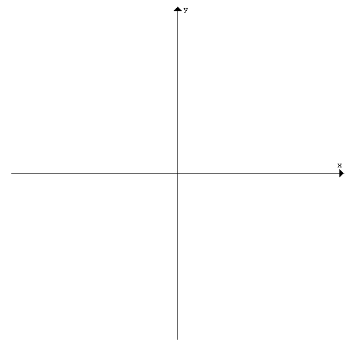
a) $f(x) = x^4$



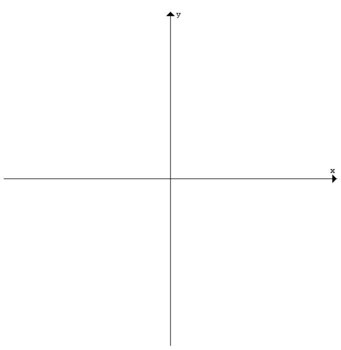
b) $f(x) = x^4 - 5x^2 + 4$



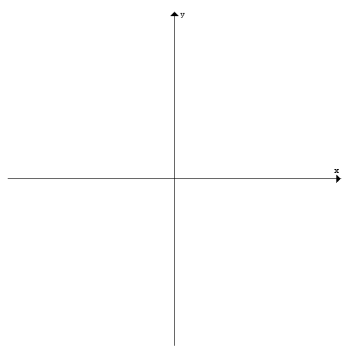
c) $f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$



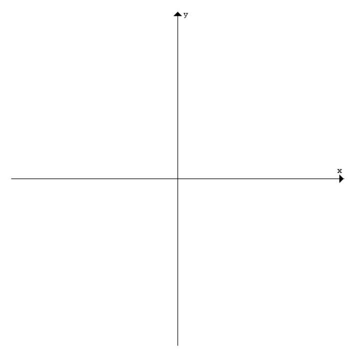
d) $f(x) = -2x^4 + 6x^2 - 4x$



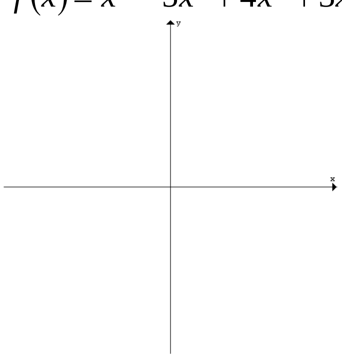
e) $f(x) = x^4 - 1$



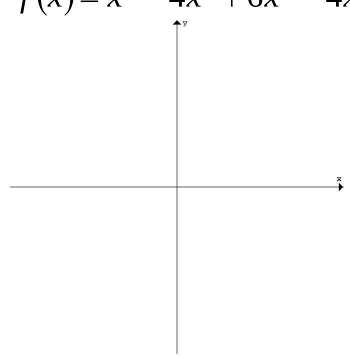
f) $f(x) = -x^4 + 3x^2 + 4$



g) $f(x) = x^4 - 5x^3 + 4x^2 + 3x + 9$



h) $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$



9. Describe the end behaviour of graphs with a positive leading coefficient.

10. Describe the end behaviour of graphs with a negative leading coefficient.

11. How many x-intercepts can a quartic function have?

12. How many absolute maximum/minimum points can a quartic function have?

13. How many local maximum/minimum points can a quartic have?

14. For the general quartic function in standard form, $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, what does the value of e represent?

POLYNOMIAL FUNCTIONS – SUMMARY

def POLYNOMIAL FUNCTION
A function in the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$$

where $a_i \in R$, $n \in W$

<u>Domain:</u>

<u>Range:</u>

If n is odd,

If n is even and

$a > 0$,

$a < 0$,

<u>End Behaviour:</u>

If n is odd,

	$a > 0$	$a < 0$
as $x \to \infty$		
as $x \to -\infty$		

If n is even,

	$a > 0$	$a < 0$
as $x \to \infty$		
as $x \to -\infty$		

<u>Zeros:</u>

If n is odd, then the function may have _____ to _____ x-intercepts.

If n is even, then the function may have _____ to _____ x-intercepts.

Order of Zeros:

If the order is one, then the graph ________________ the x-axis.

If the order is odd and greater than 1, then the graph ________________ the x-axis and there is a

____.

If the order is even, then the graph ________________ but does not ________________ the x-axis.

Absolute Minimum / Maximum:

def ABSOLUTE MINIMUM / MAXIMUM
The smallest / largest y-value in the ________________ graph.

If n is odd, then an absolute min ______________________.

If n is even and

$a > 0$, then an absolute min ______________________.

$a < 0$, then an absolute min ______________________.

If n is odd, then an absolute max ______________________.

If n is even and

$a > 0$, then an absolute max ______________________.

$a < 0$, then an absolute max ______________________.

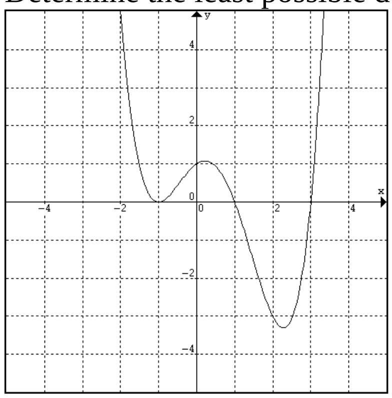
Local Minimum / Maximum:

def LOCAL MINIMUM / MAXIMUM
The smallest / largest y-value within an ________________.

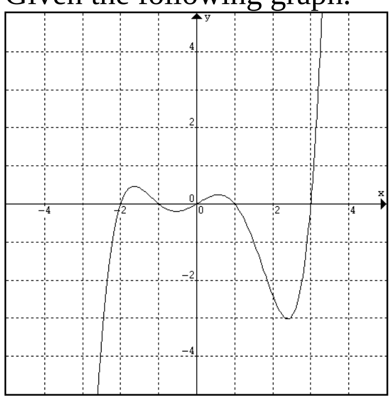
For a polynomial of degree $n > 1$, then the function may have a total of _____ to _____ local min / max.

POLYNOMIAL FUNCTION PROBLEMS

Ex. 1 Determine the least possible degree of the following function.



Ex. 2 Given the following graph.



a) Determine the zeros.

b) Determine the least possible degree of this function.

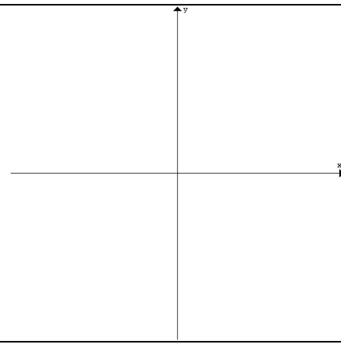
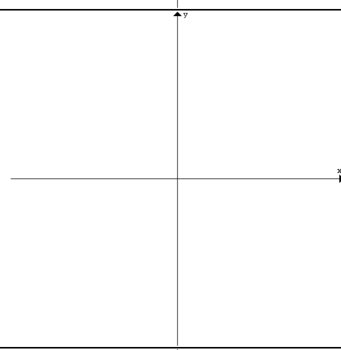
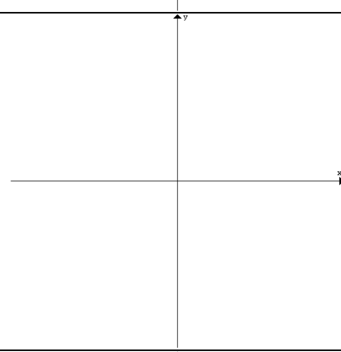
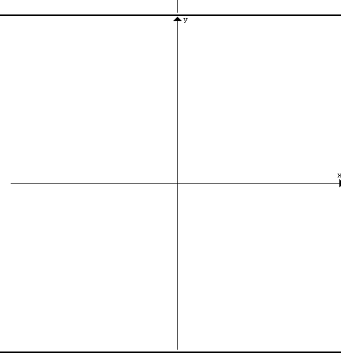
c) What is the sign of the leading coefficient?

d) Describe the end behaviour.

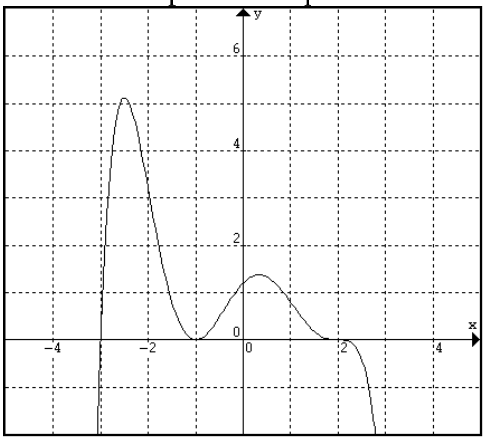
e) State the number of local min/max.

f) State the number of absolute min/max.

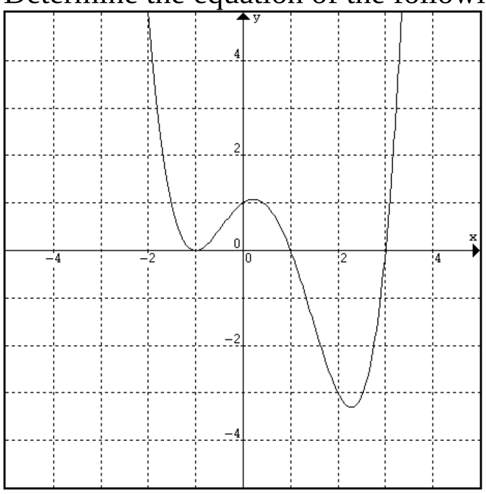
POLYNOMIAL FUNCTIONS – FACTORED FORM

Equation	Sketch	Degree	Sign of Leading Coeff.	Zero(s) & Order
$y=(x-1)(x+1)(x+3)^3$				
$y=-(x+3)(x-2)^3$				
$y=-2x(x-1)^2(x+2)$				
$y=x(x-4)^4$				

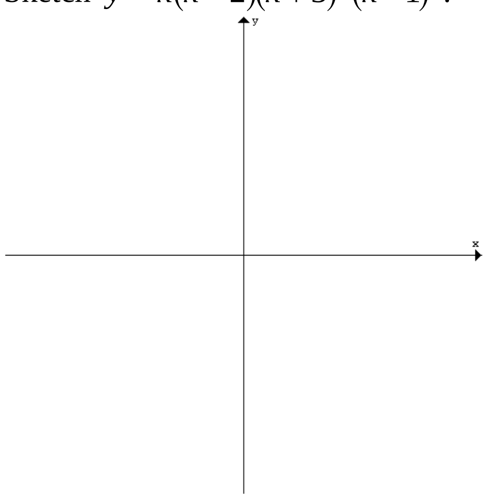
Ex. 1 Determine a possible equation for the following.



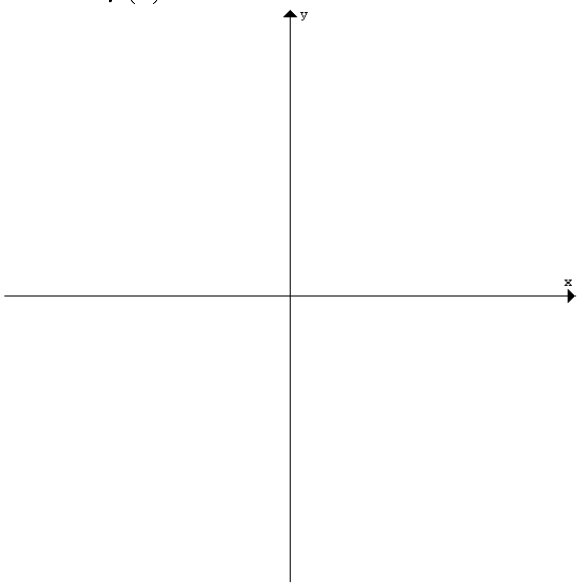
Ex. 2 Determine the equation of the following.



Ex. 3 Sketch $y = x(x-2)(x+3)^2(x-1)^3$.



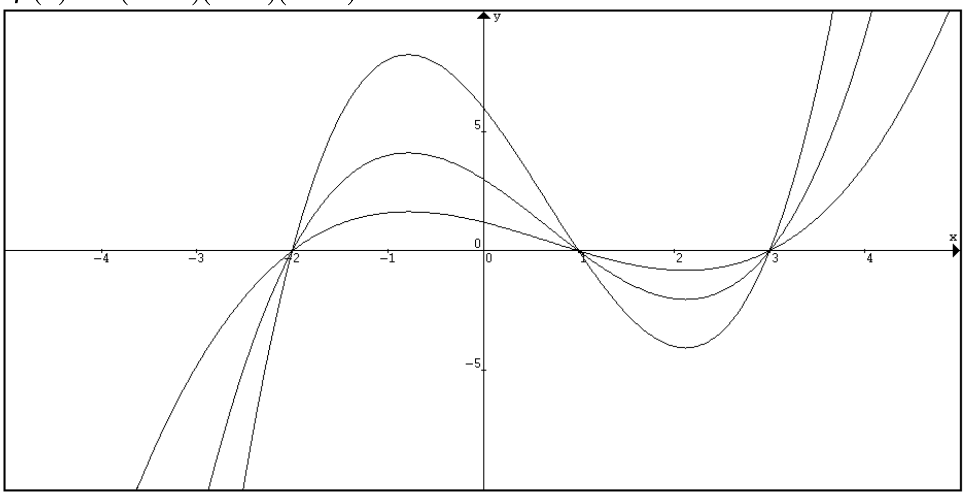
Ex. 4 Sketch $f(x) = 3x^4 + 6x^3 - 9x^2 - 24x - 12$.



FAMILY OF POLYNOMIAL FUNCTIONS

Polynomial functions that have the same zeros and are vertical dilations of each other belong to the same family and have equations of the form $y = af(x)$. To determine the equation of a specific member of the family, a point (not a zero) on the function must be known.

For example, the family of cubics with zeros at -2, 1, and 3 has the equation $f(x) = a(x+2)(x-1)(x-3)$.



Ex. 1 Find the equation of the family of quartics with zeros -4, 2 (order 2), and 6.

Ex. 2 Find an equation of a quintic with zeros -2 (order 2), -4 (order 2), and 7.

Ex. 3 Determine the equation of the cubic with zeros -5 (order 2) and $\frac{1}{2}$ (order 1) and y-intercept -50.

POLYNOMIAL FUNCTIONS – FINITE DIFFERENCES

Consider:

1. $f(x) = 2x^2 + x - 1$

x	y		
-2			
-1			
0			
1			
2			

Calculate $an!$

2. $f(x) = 5x^3 + x$

x	y			
-2				
-1				
0				
1				
2				

Calculate $an!$

3. $f(x) = -x^4 - x + 1$

x	y				
-2					
-1					
0					
1					
2					
3					

Calculate $an!$

The n^{th} differences $= an!$, where a is the leading coefficient and n is the degree of the polynomial function.

EVEN / ODD FUNCTIONS AND SYMMETRY

def EVEN FUNCTION

A function is even if

$$f(-x) = f(x)$$

An even function is symmetrical in the _______________.

def ODD FUNCTION

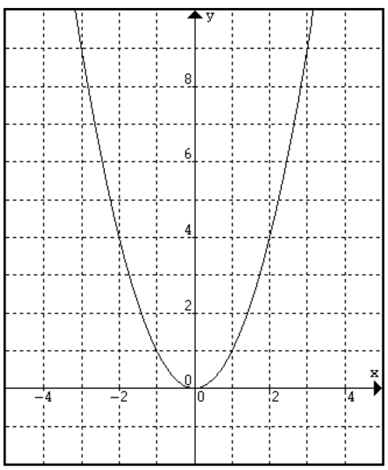
A function is odd if

$$f(-x) = -f(x)$$

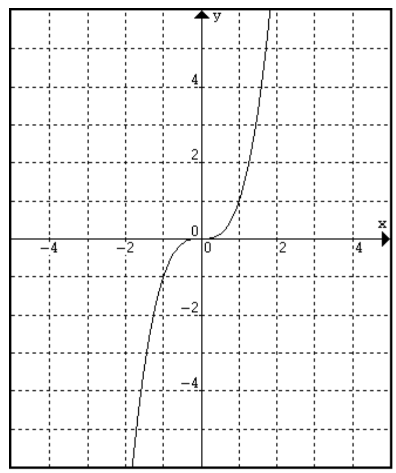
An odd function is symmetrical about the _______________.

Ex. 1 State whether the functions whose graphs are given are even, odd or neither.

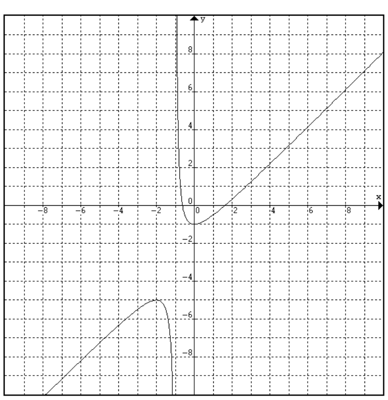
a)



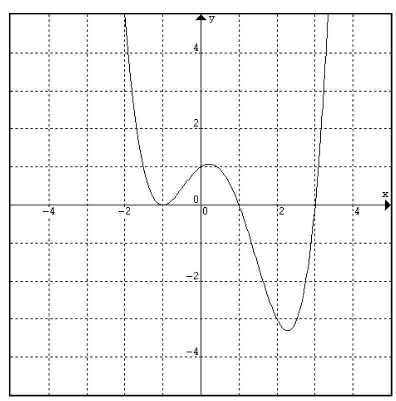
c)



b)



d)



Ex. 2 Determine if $f(x) = 2x^3 - x + 1$ is odd or not.

Ex. 3 Determine if $f(x) = \frac{1}{x}$ has a line of symmetry in the y-axis, point of symmetry about the origin or neither.

Ex. 4 Is $f(x) = 2(x-1)(x+2)(x-2)(x+1)$ an even function?

SOLVING POLYNOMIAL EQUATIONS

Ex. 1 Solve: $(x+1)(x-3)(2x-3)=0$

Ex. 2 Solve: $3x^4-4x^3-5x^2+2x=0$

Ex. 3 Solve: $x^3-27=0$

a) if $x \in R$

b) if $x \in C$

Ex. 4 A 10 cm by 8 cm rectangular piece of paper is to be made into a box by cutting out squares from each corner and folding up the sides. The box's volume is 48 cm^3. What is the length of sides of each square that is cut out?

SOLVING LINEAR INEQUALITIES

Isolate for the unknown variable the same way as in an equation. But, when both sides are multiplied or divided by a negative value, the sign is reversed.

Ex. 1 Solve: $3(x+2)-x>7$

a) Algebraically

b) Graphically

Ex. 2 Solve: $4(x-5)-7(x-1)\leq 2$

a) Algebraically

b) Graphically

Ex. 3 Solve: $30\leq 3(2x+4)-2(x+1)\leq 46$

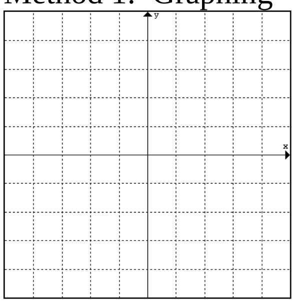
a) Algebraically

b) Graphically

SOLVING POLYNOMIAL INEQUALITIES

Ex. 1 Solve: $3x > 6$

Method 1: Graphing



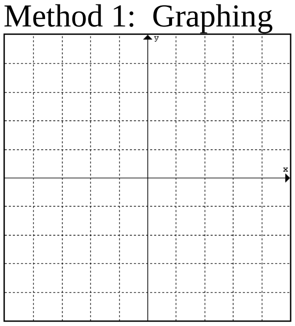
Method 2: Interval Table

x	
$f(x)$	

Ex. 2 Solve:

a) $-2x^2 - 2x \leq 2x - 6$

Method 1: Graphing



Method 2: Interval Table

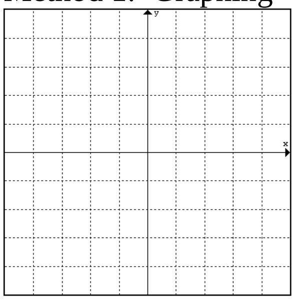
x	
$f(x)$	

b) $-2x^2 - 2x > 2x - 6$

Ex. 3 Solve:

a) $x^2 - 4x + 4 < 0$

Method 1: Graphing



Method 2: Interval Table

x	
$f(x)$	

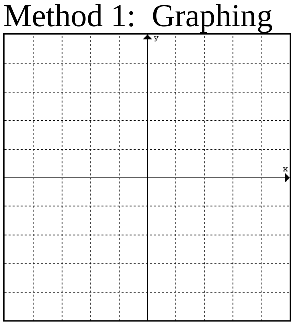
b) $x^2 - 4x + 4 \le 0$

c) $x^2 - 4x + 4 > 0$

d) $x^2 - 4x + 4 \ge 0$

Ex. 4 Solve: $x^3 - 4x^2 + 3 < -3(x-1)$

Method 1: Graphing

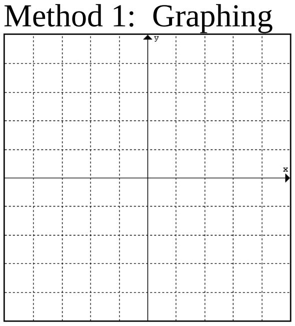


Method 2: Interval Table

x	
$f(x)$	

Ex. 5 Solve: $-x^4 - 10x^3 - 33x^2 - 40x - 16 \le 0$

Method 1: Graphing



Method 2: Interval Table

x	
$f(x)$	