Polynomial Division

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1. Factor:

a) $z^3 - 27$

$(z-3)(z^2+3z+9)$

b) $y^3 + 1$

$(y+1)(y^2-y+1)$

c) $8x^3 - 64$

$(2x+4)(4x^2-8x+16)$

d) $a^3 - 8b^3$

$(a-2b)(a^2+2ab+4b^2)$

e) $8x^3 + 27y^3$

$(2x+3y)(4x^2-6xy+9y^2)$

f) $64x^3 + 1$

$(4x+1)(16x^2-4x+1)$

2. Express as a product and simplify all factors:

a) $(x+1)^3 - 1$

$((x+1)-1)((x+1)^2+(x+1)+1)$

$x(x^2+2x+1+x+2)$

$x(x^2+3x+3)$

b) $(2x)^3 + 1$

$(2x+1)((2x)^2-2x+1)$

$(2x+1)(4x^2-2x+1)$

c) $(x+2)^3 - x^3$

$((x+2)-x)((x+2)^2+x(x+2)+x^2)$

$2(x^2+4x+4+x^2+2x+x^2)$

$2(3x^2+6x+4)$

d) $(2x+1)^3 + (2y)^3$

$((2x+1)+2y)((2x+1)^2-2y(2x+1)+4y^2)$

$(2x+2y+1)(4x^2+4x+1-4xy-2y+4y^2)$

$(2x+2y+1)(4x^2+4y^2-4xy+4x-2y+1)$

e) $(a+2b)^3 - (a-2b)^3$

$((a+2b)-(a-2b))((a+2b)^2+(a+2b)(a-2b)+(a-2b)^2)$

$4b(a^2+4ab+4b^2+a^2-4b^2+a^2-4ab+4b^2)$

$4b(3a^2+4b^2)$

g) $(x+3)^3 + (x-3)^3$

$((x+3)+(x-3))((x+3)^2-(x+3)(x-3)+(x-3)^2)$

$2x(x^2+6x+9-x^2+9+x^2-6x+9)$

$2x(x^2+27)$

3. Factor:

a) $x^6 - y^6$

b) $x^6 + y^6$

c) $64a^6 - 1$

d) $1 + 64y^6$

e) $(x+y)^6 - (x-y)^6$

f) $(x+y)^6 + (x-y)^6$

4. The volume, V, of the frustrum of a right circular cone of radii a, b and height h is given by:

$$V = \frac{1}{3}\pi h\left(\frac{b^3-a^3}{b-a}\right)$$

a) Express V as a polynomial in a and b.

b) Show that when $a = b$ the formula becomes $V = \pi a^2 h$.