

Sum and Difference of Cubes

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1. Factor:

- a)** $z^3 - 27$

$(z - 3)(z^2 + 3z + 9)$
- b)** $y^3 + 1$

$(y + 1)(y^2 - y + 1)$
- c)** $8x^3 - 64$

$(2x + 4)(4x^2 - 8x + 16)$
- d)** $a^3 - 8b^3$

$(a - 2b)(a^2 + 2ab + 4b^2)$
- e)** $8x^3 + 27y^3$

$(2x + 3y)(4x^2 - 6xy + 9y^2)$
- f)** $64x^3 + 1$

$(4x + 1)(16x^2 - 4x + 1)$

2. Express as a product and simplify all factors:

- a)** $(x + 1)^3 - 1$

$((x + 1) - 1)((x + 1)^2 + (x + 1) + 1)$

$x(x^2 + 2x + 1 + x + 2)$

$x(x^2 + 3x + 3)$
- b)** $(2x)^3 + 1$

$(2x + 1)((2x)^2 - 2x + 1)$

$(2x + 1)(4x^2 - 2x + 1)$
- c)** $(x + 2)^3 - x^3$

$((x + 2) - x)((x + 2)^2 + x(x + 2) + x^2)$

$2(x^2 + 4x + 4 + x^2 + 2x + x^2)$

$2(3x^2 + 6x + 4)$
- d)** $(2x + 1)^3 + (2y)^3$

$((2x + 1) + 2y)((2x + 1)^2 - 2y(2x + 1) + 4y^2)$

$(2x + 2y + 1)(4x^2 + 4x + 1 - 4xy - 2y + 4y^2)$

$(2x + 2y + 1)(4x^2 + 4y^2 - 4xy + 4x - 2y + 1)$
- e)** $(a + 2b)^3 - (a - 2b)^3$

$((a + 2b) - (a - 2b))((a + 2b)^2 + (a + 2b)(a - 2b) + (a - 2b)^2)$

$4b(a^2 + 4ab + 4b^2 + a^2 - 4b^2 + a^2 - 4ab + 4b^2)$

$4b(3a^2 + 4b^2)$
- g)** $(x + 3)^3 + (x - 3)^3$

$((x + 3) + (x - 3))((x + 3)^2 - (x + 3)(x - 3) + (x - 3)^2)$

$2x(x^2 + 6x + 9 - x^2 + 9 + x^2 - 6x + 9)$

$2x(x^2 + 27)$

3. Factor:

- a)** $x^6 - y^6$
- b)** $x^6 + y^6$
- c)** $64a^6 - 1$
- d)** $1 + 64y^6$
- e)** $(x + y)^6 - (x - y)^6$
- f)** $(x + y)^6 + (x - y)^6$

4. The volume, V , of the frustrum of a right circular cone of radii a , b and height h is given by:

$$V = \frac{1}{3}\pi h(\frac{b^2+a^2}{b-a})$$

- a)** Express V as a polynomial in a and b .
- b)** Show that when $a = b$ the formula becomes $V = \pi a^2 h$.