EXERCISES 3-6 Factoring the Sum and Difference of Cubes

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1. Factor:
   a) $z^3 - 27$ b) $y^3 + 1$ c) $8x^3 - 64$
   d) $a^3 - 8b^3$ e) $8x^3 + 27y^3$ f) $64x^3 + 1$

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2. Express as a product and simplify all factors:
   a) $(x + 1)^3 - 1$ b) $(2x)^3 + 1$ c) $(x + 2)^3 - x^3$
   d) $(2x + 1)^3 + (2y)^3$ e) $(a + 2b)^3 - (a - 2b)^3$
   f) $(x + y)^3 + (x - y)^3$ g) $(x + 3)^3 + (x - 3)^3$
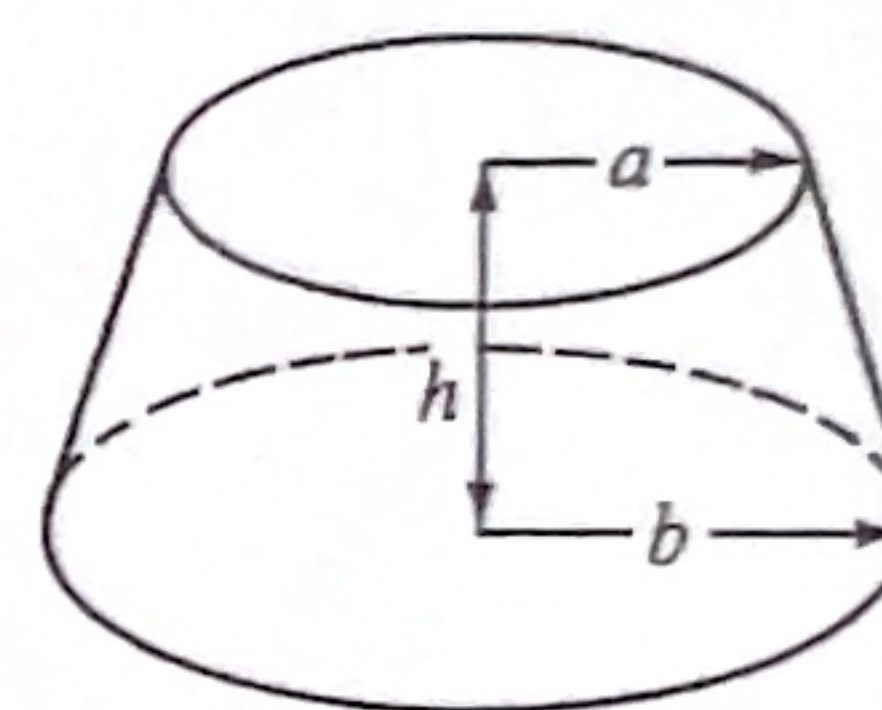
3. Factor:
   a) $x^6 - y^6$ b) $x^6 + y^6$ c) $64a^6 - 1$
   d) $1 + 64y^6$ e) $(x + y)^6 - (x - y)^6$ f) $(x + y)^6 + (x - y)^6$

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4. The volume, V, of the frustrum of a right circular cone of radii a, b and height h is given by:

$$V = \frac{1}{3}\pi h\left(\frac{b^3 - a^3}{b - a}\right).$$

   a) Express V as a polynomial in a and b.
   b) Show that when $a = b$ the formula becomes: $V = \pi a^2 h$.

Answers:

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1. a) $(z - 3)(z^2 + 3z + 9)$ b) $(y + 1)(y^2 - y + 1)$
   c) $8(x - 2)(x^2 + 2x + 4)$
   d) $(a - 2b)(a^2 + 2ab + 4b^2)$
   e) $(2x + 3y)(4x^2 - 6xy + 9y^2)$
   f) $(4x + 1)(16x^2 - 4x + 1)$
2. a) $x(x^2 + 3x + 3)$ b) $(2x + 1)(4x^2 - 2x + 1)$
   c) $2(3x^2 + 6x + 4)$
   d) $(2x + 2y + 1)(4x^2 + 4y^2 - 4xy + 4x - 2y + 1)$
   e) $4b(3a^2 + 4b^2)$ f) $2x(x^2 + 3y^2)$
   g) $2x(x^2 + 27)$
3. a) $(x + y)(x - y)(x^4 + x^2y^2 + y^4)$
   b) $(x^2 + y^2)(x^4 - x^2y^2 + y^4)$
   c) $(2a - 1)(2a + 1)(16a^4 + 4a^2 + 1)$
   c) $(1 + 4y^2)(1 - 4y^2 + 16y^4)$
   e) $4xy(x^2 + 3y^2)(y^2 + 3x^2)$
   f) $2(x^2 + y^2)(x^4 + 14x^2y^2 + y^4)$
4. a) $V = \frac{1}{3}\pi h(a^2 + ab + b^2)$

   b) When $b = a$, $V = \frac{1}{3}\pi h(a^2 + a^2 + a^2)$, or $\pi a^2 h$