Practice:

1. Factor the following polynomials using the factor theorem.

(a) x^3-4x^2+x+6
(b) $x^3+8x^2+21x+18$
(c) $x^4-x^3-3x^2+x+2$

2. Factor each expression

(a) x^3-8
(b) $27x^3+1$
(c) $625x^3-40$
(d) $125-64x^3$

3. Factor fully: $abx^3+(a+b-ab)x^2+(1-a-b)x-1$ [note $P(1)=0$]

4. a) Factor $x^{12}-1$ fully.
b) List all polynomials of the form $x^4+bx^3+cx^2+dx+e$ with rational coefficients that are factors of the polynomial, $x^{12}-1$.

Answer

1. a) $(x+1)(x-2)(x-3)$ b) $(x+2)(x+3)^2$ c) $(x-2)(x-1)(x+1)^2$

2. a) $(x-2)(x^2+2x+4)$ b) $(3x+1)(9x^2-3x+1)$

c) $5(5x-2)(25x^2+10x+4)$ d) $-(4x-5)(16x^2+20x+25)$

3. $abx^3+(a+b-ab)x^2+(1-a-b)x-1=(ax+1)(bx+1)(x-1)$; note $P(1)=0$

4. a) $x^{12}-1=(x-1)(x+1)(x^2+1)(x^2+x+1)(x^2-x+1)(x^4-x^2+1)$

b) There are seven such 4*th* degree polynomial factors:

o x^4-x^2+1

o $(x^2+x+1)(x^2-x+1)=x^4+x^2+1$

o $(x-1)(x+1)(x^2+1)=x^4-1$

o $(x-1)(x+1)(x^2+x+1)=x^4+x^3-x-1$

o $(x-1)(x+1)(x^2-x+1)=x^4-x^3+x-1$

o $(x^2+1)(x^2+x+1)=x^4+x^3+2x^2+x+1$

o $(x^2+1)(x^2-x+1)=x^4-x^3+2x^2-x+1$