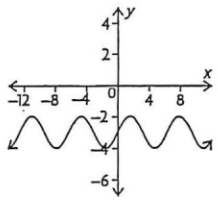
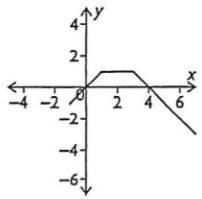
CHECK Your Understanding

1. State the domain and range of each relation. Then determine whether the relation is a function, and justify your answer.

a)



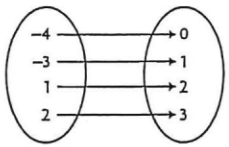
b)



c) $\{(1, 4), (1, 9), (2, 7), (3, -5), (4, 11)\}$

d) $y = 3x - 5$

e)



f) $y = -5x^2$

2. State the domain and range of each relation. Then determine whether the relation is a function, and justify your answer.

a) $y = -2(x + 1)^2 - 3$

b) $y = \dfrac{1}{x + 3}$

c) $y = 2^{-x}$

d) $y = \cos x + 1$

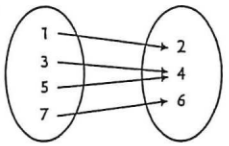
e) $x^2 + y^2 = 9$

f) $y = 2 \sin x$

PRACTISING

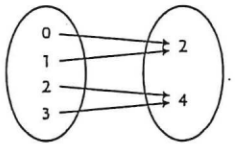
3. Determine whether each relation is a function, and state its domain and range.

a)



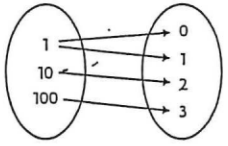
b) $\{(2, 3), (1, 3), (5, 6), (0, -1)\}$

c)



d) $\{(2, 5), (6, 1), (2, 7), (8, 3)\}$

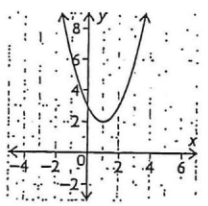
e)



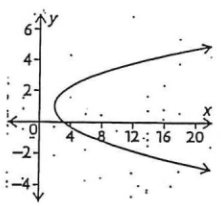
f) $\{(1, 2), (2, 1), (3, 4), (4, 3)\}$

4. Determine whether each relation is a function, and state its domain and range. K

a)



b)



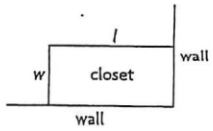
c) $x^2 = 2y + 1$

d) $x = y^2$

e) $y = \dfrac{3}{x}$

f) $f(x) = 3x + 1$

5. Determine the equations that describe the following function rules:
   a) The input is 3 less than the output.
   b) The output is 5 less than the input multiplied by 2.
   c) Subtract 2 from the input and then multiply by 3 to find the output.
   d) The sum of the input and output is 5.



6. Martin wants to build an additional closet in a corner of his bedroom. Because the closet will be in a corner, only two new walls need to be built. The total length of the two new walls must be 12 m. Martin wants the length of the closet to be twice as long as the width, as shown in the diagram.
   a) Explain why $l = 2w$.
   b) Let the function $f(l)$ be the sum of the length and the width. Find the equation for $f(l)$.
   c) Graph $y = f(l)$.
   d) Find the desired length and width.

7. The following table gives Tina's height above the ground while riding a Ferris wheel, in relation to the time she was riding it. A

Time (s)	0	20	40	60	80	100	120	140	160	180	200	220	240
Height (m)	5	10	5	0	5	10	5	0	5	10	5	0	5

   a) Draw a graph of the relation, using time as the independent variable and height as the dependent variable.
   b) What is the domain?
   c) What is the range?
   d) Is this relation a function? Justify your answer.
   e) Another student sketched a graph, but used height as the independent variable. What does this graph look like?
   f) Is the relation in part e) a function? Justify your answer.

8. Consider what happens to a relation when the coordinates of all its ordered pairs are switched.
   a) Give an example of a function that is still a function when its coordinates are switched.
   b) Give an example of a function that is no longer a function when its coordinates are switched.
   c) Give an example of a relation that is not a function, but becomes a function when its coordinates are switched.
9. Explain why a relation that fails the vertical line test is not a function.
10. Consider the relation between x and y that consists of all points (x, y) such that the distance from (x, y) to the origin is 5.
    a) Is $(4, 3)$ in the relation? Explain.
    b) Is $(1, 5)$ in the relation? Explain.
    c) Is the relation a function? Explain.
11. The table below lists all the ordered pairs that belong to the function $g(x)$.

x	0	1	2	3	4	5
$g(x)$	3	4	7	12	19	28

    a) Determine an equation for $g(x)$.
    b) Does $g(3) - g(2) = g(3 - 2)$? Explain.
12. The factors of 4 are 1, 2, and 4. The sum of the factors is $1 + 2 + 4 = 7$. The sum of the factors is called the sigma function. Therefore, $f(4) = 7$.
    a) Find $f(6)$, $f(7)$, and $f(8)$.
    b) Is $f(15) = f(3) \times f(5)$?
    c) Is $f(12) = f(3) \times f(4)$?
    d) Are there others that will work?
13. Make a concept map to show what you have learned about functions. Put "FUNCTION" in the centre of your concept map, and include the following words:

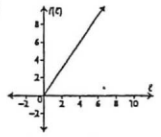
algebraic model, dependent variable, domain, function notation, graphical model, independent variable, mapping model, numerical model, range, vertical line test

Extending

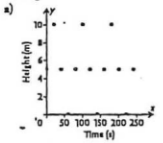
14. Consider the relations $x^2 + y^2 = 25$ and $y = \sqrt{25 - x^2}$. Draw the graphs of these relations, and determine whether each relation is a function. State the domain and range of each relation.
15. You already know that y is a function of x if and only if the graph passes the vertical line test. When is x a function of y? Explain.

Lesson 1.1, pp. 11–15

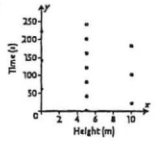
1. a) $D = \{x \in R\}$; $R = \{y \in R \mid -4 \le y \le -2\}$; This is a function because it passes the vertical line test.
   b) $D = \{x \in R \mid -1 \le x \le 7\}$; $R = \{y \in R \mid -3 \le y \le 1\}$; This is a function because it passes the vertical line test.
   c) $D = \{1, 2, 3, 4\}$; $R = \{-5, 4, 7, 9, 11\}$; This is not a function because 1 is sent to more than one element in the range.
   d) $D = \{x \in R\}$; $R = \{y \in R\}$; This is a function because every element in the domain produces exactly one element in the range.
   e) $D = \{-4, -3, 1, 2\}$; $R = \{0, 1, 2, 3\}$; This is a function because every element of the domain is sent to exactly one element in the range.
   f) $D = \{x \in R\}$; $R = \{y \in R \mid y \le 0\}$; This is a function because every element in the domain produces exactly one element in the range.
2. a) $D = \{x \in R\}$; $R = \{y \in R \mid y \le -3\}$; This is a function because every element in the domain produces exactly one element in the range.
   b) $D = \{x \in R \mid x \ne -3\}$; $R = \{y \in R \mid y \ne 0\}$; This is a function because every element in the domain produces exactly one element in the range.
   c) $D = \{x \in R\}$; $R = \{y \in R \mid y > 0\}$; This is a function because every element in the domain produces exactly one element in the range.
   d) $D = \{x \in R\}$; $R = \{y \in R \mid 0 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.
   e) $D = \{x \in R \mid -3 \le x \le 3\}$; $R = \{y \in R \mid -3 \le y \le 3\}$; This is not a function because (0, 3) and (0, −3) are both in the relation.
   f) $D = \{x \in R\}$; $R = \{y \in R \mid -2 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.
3. a) function; $D = \{1, 3, 5, 7\}$; $R = \{2, 4, 6\}$
   b) function; $D = \{0, 1, 2, 5\}$; $R = \{-1, 3, 6\}$
   c) function; $D = \{0, 1, 2, 3\}$; $R = \{2, 4\}$
   d) not a function; $D = \{2, 6, 8\}$; $R = \{1, 3, 5, 7\}$
   e) not a function; $D = \{1, 10, 100\}$; $R = \{0, 1, 2, 3\}$
   f) function; $D = \{1, 2, 3, 4\}$; $R = \{1, 2, 3, 4\}$
4. a) function; $D = \{x \in R\}$; $R = \{y \in R \mid y \ge 2\}$.
   b) not a function; $D = \{x \in R \mid x \ge 2\}$; $R = \{y \in R\}$
   c) function; $D = \{x \in R\}$; $R = \{y \in R \mid y \ge -0.5\}$
   d) not a function; $D = \{x \in R \mid x \ge 0\}$; $R = \{y \in R\}$
   e) function; $D = \{x \in R \mid x \ne 0\}$; $R = \{y \in R \mid y \ne 0\}$
   f) function; $D = \{x \in R\}$; $R = \{y \in R\}$
5. a) $y = x + 3$
   b) $y = 2x - 5$
   c) $y = 3(x - 2)$
   d) $y = -x + 5$
6. a) The length is twice the width.
   b) $f(l) = \frac{3}{2}l$
   c)



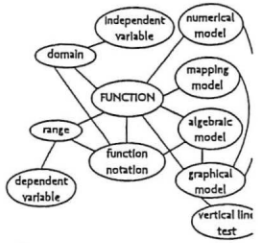
   d) length = 8 m; width = 4 m
7. a)



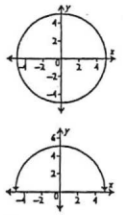
   b) $D = \{0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240\}$
   c) $R = \{0, 5, 10\}$
   d) It is a function because it passes the vertical line test.
   e)



   f) It is not a function because (5, 0) and (5, 40) are both in the relation.
8. a) {(1, 2), (3, 4), (5, 6)}
   b) {(1, 2), (3, 2), (5, 6)}
   c) {(2, 1), (2, 3), (5, 6)}
9. If a vertical line passes through a function and hits two points, those two points have identical x-coordinates and different y-coordinates. This means that one x-coordinate is sent to two different elements in the range, violating the definition of *function*.
10. a) Yes, because the distance from (4, 3) to (0, 0) is 5.
    b) No, because the distance from (1, 5) to (0, 0) is not 5.
    c) No, because (4, 3) and (4, −3) are both in the relation.
11. a) $g(x) = x^2 + 3$
    b) $g(3) - g(2) = 12 - 7$
    $= 5$
    $g(3 - 2) = g(1)$
    $= 4$
    So, $g(3) - g(2) \ne g(3 - 2)$.
12. a) $f(6) = 12$; $f(7) = 8$; $f(8) = 15$
    b) Yes, $f(15) = f(3) \times f(5)$
    c) Yes, $f(12) = f(3) \times f(4)$
    d) Yes, there are others that will work. $f(a) \times f(b) = f(a \times b)$ whenever a and b have no common factors other than 1.
13. Answers may vary. For example:



14.



    The first is not a function because it fails the vertical line test:
    $D = \{x \in R \mid -5 \le x \le 5\}$;
    $R = \{y \in R \mid -5 \le y \le 5\}$.
    The second is a function because it passes the vertical line test:
    $D = \{x \in R \mid -5 \le x \le 5\}$;
    $R = \{y \in R \mid 0 \le y \le 5\}$.
15. x is a function of y if the graph passes the horizontal line test. This occurs when any horizontal line hits the graph at most once.