Functions & Interval Notation

Jacob Zante

September 9th, 2024

1. State the domain and range

a)

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | -4 \leq y \leq -2\}$

b)

D: $\{x \in \mathbb{R} | -1 \leq x \leq 7\}$

R: $\{y \in \mathbb{R} | -3 \leq y \leq -1\}$

c)

D: $\{1, 2, 3, 4\}$

R: $\{-5, 4, 7, 9, 11\}$

d)

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

e)

D: $\{-4, -3, 1, 2\}$

R: $\{0, 1, 2, 3\}$

f)

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | y \leq 0\}$

2. State the domain and range, then deterine whether the relation is a function, and justify your answer.

a) $y = -2(x+1)^2 - 3$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | y \leq -3\}$

This relation is a function, as it passes the vertical line test.

b) $y = \frac{1}{x+3}$

D: $\{x \in \mathbb{R} | x \neq -3\}$

R: $\{y \in \mathbb{R} | y \neq 0\}$

This relation is a function, as it passes the vertical line test.

c) $y = 2^{-x}$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | y > 0\}$

This relation is a function, as it passes the vertical line test.

d) $y = \cos x + 1$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | -1 \leq y \leq 1\}$

This relation is a function, as it passes the vertical line test.

e) $x^2 + y^2 = 9$

D: $\{x \in \mathbb{R} | -3 \leq x \leq 3\}$

R: $\{y \in \mathbb{R} | -3 \leq y \leq 3\}$

This relation is not a function, as it does not pass the vertical line test.

f) $y = 2\sin x$

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | -2 \leq y \leq 2\}$

This relation is a function, as it passes the vertical line test.

3. Determine whether each relation is a function, and state its domain and range.

a)

This relation is a function, because each x value only cooresponds to one y value.

D: $\{1, 3, 5, 7\}$

R: $\{2, 4, 6\}$

b)

This relation is a function, because each x value only cooresponds to one y value.

D: $\{0, 1, 2, 5\}$

R: $\{-1, 3, 6\}$

c)

This relation is a function, because each x value only cooresponds to one y value.

D: $\{0, 1, 2, 3\}$

R: $\{2, 4\}$

d)

This relation is not a function, because there is a value of x that cooresponds to more tha

D: $\{2, 6, 8\}$

R: $\{1, 3, 5, 7\}$

e)

This relation is not a function, because there is a value of x that cooresponds to more tha

D: $\{1, 10, 100\}$

R: $\{0, 1, 2, 3\}$

f)

This relation is a function, because each x value only cooresponds to one y value.

D: $\{1, 2, 3, 4\}$

R: $\{1, 2, 3, 4\}$

4. Determine whether each relation is a function, and state its domain and range.

a)

This relation is a function, as it passes the vertical line test.

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | y \geq 2\}$

b)

This relation is not a function, as it does not pass the vertical line test.

D: $\{x \in \mathbb{R} | x \geq 2\}$

R: $\{y \in \mathbb{R}\}$

c) $x^2 = 2y + 1$

This relation is a function, as when you isloate for y, $y = 2(x^2 - 1)$ it becomes clear that t

D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} | y \geq -2\}$

d) $x = y^2$

This relation is not a function, as when you isolate for y, $y = \pm\sqrt{x}$ it becomes clear that

D: $\{x \in \mathbb{R} | x \geq 0\}$

R: $\{y \in \mathbb{R}\}$

e) $y = \frac{3}{x}$

This relation is a function, as there is only one possible y value for every x value.

D: $\{x \in \mathbb{R} | x \neq 0\}$

R: $\{y \in \mathbb{R} | y \neq 0\}$

f) $f(x) = 3x + 1$

This relation is a function, as there is only one possible y value for every x value.

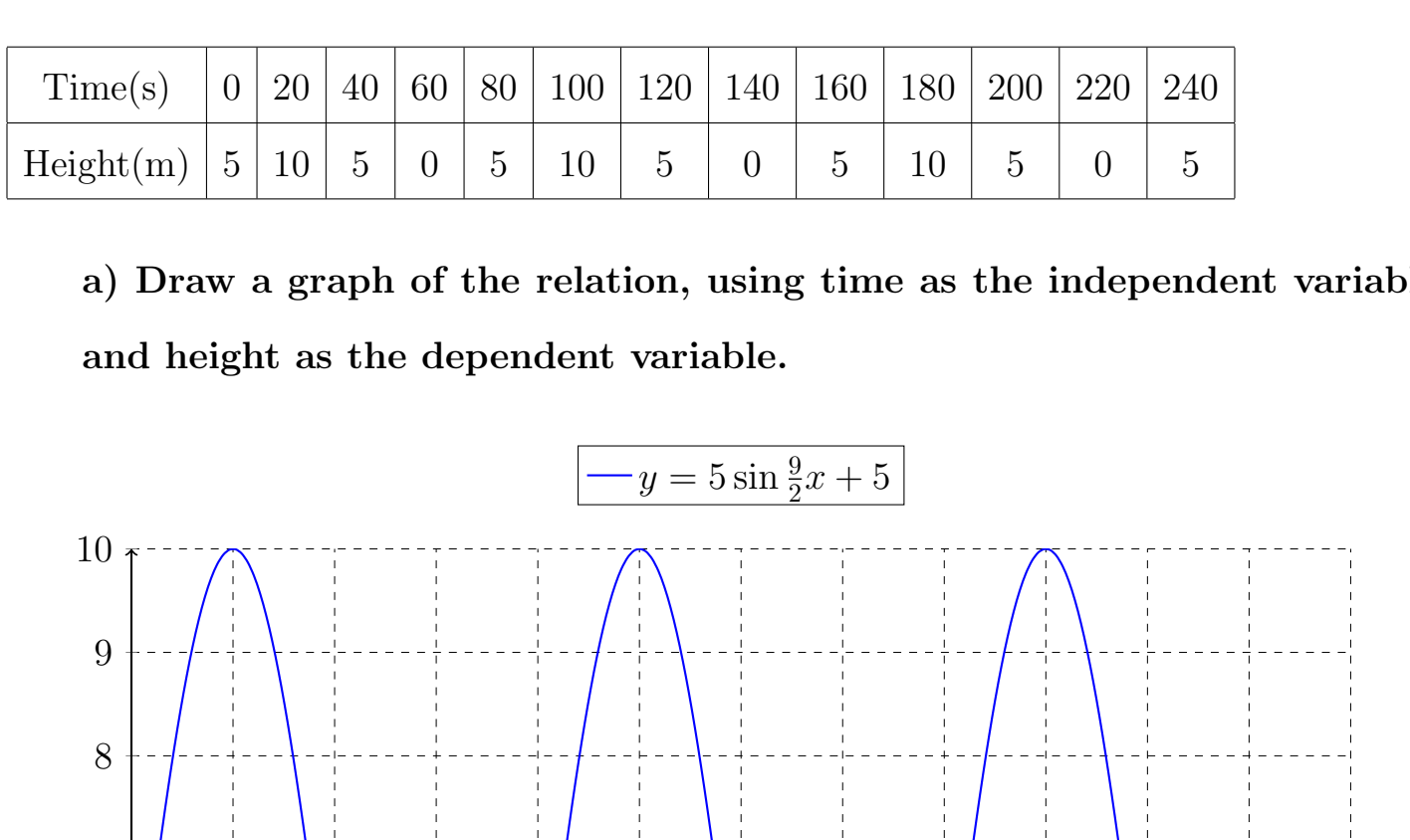
D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

7. The following table give's Tina's height above the ground while riding a Ferris wheel, in relation to the time she was riding it.

Time(s)	0	20	40	60	80	100	120	140	160	180	200	220	240
Height(m)	5	10	5	0	5	10	5	0	5	10	5	0	5

a) Draw a graph of the relation, using time as the independent variable and height as the dependent variable.



b, c) What is the domain? What is the range?

D: $\{x \in \mathbb{R} | 0 \leq x \leq 240\}$

R: $\{y \in \mathbb{R} | 0 \leq y \leq 10\}$

d) Is this relation a function? Justify your answer.

This relation is a function, as it passes the vertical line test.

e) Another student sketched a graph, but used height as the independent variable. What does this graph look like?

$x = 5\sin\frac{9}{2}y + 5$

$x - 5 = 5\sin\frac{9}{2}y$

$\frac{x-5}{5} = \sin\frac{9}{2}y$

$\frac{x-5}{5} \cdot \frac{2}{9} = \sin y$

$\frac{2(x-5)}{45} = \sin y$

$\arcsin(\frac{2(x-5)}{45}) = y$

$y = \arcsin(\frac{2(x-5)}{45})$

$x = 5\sin\frac{9}{2}y + 5$

$x = 5(\sin\frac{9}{2}y + 1)$

$\frac{x}{5} = \sin\frac{9}{2}y + 1$

$\frac{x}{5} - 1 = \sin\frac{9}{2}y$

$\arcsin(\frac{x}{5} - 1) = \frac{9}{2}y$

$\frac{2}{9}\arcsin(\frac{x}{5} - 1) = y$

$y = \frac{2}{9}\arcsin(\frac{x}{5} - 1)$

$x = 5\sin\frac{9}{2}y + 5$

$x - 5 = 5\sin\frac{9}{2}y$

$\frac{x-5}{5} = \sin\frac{9}{2}y$

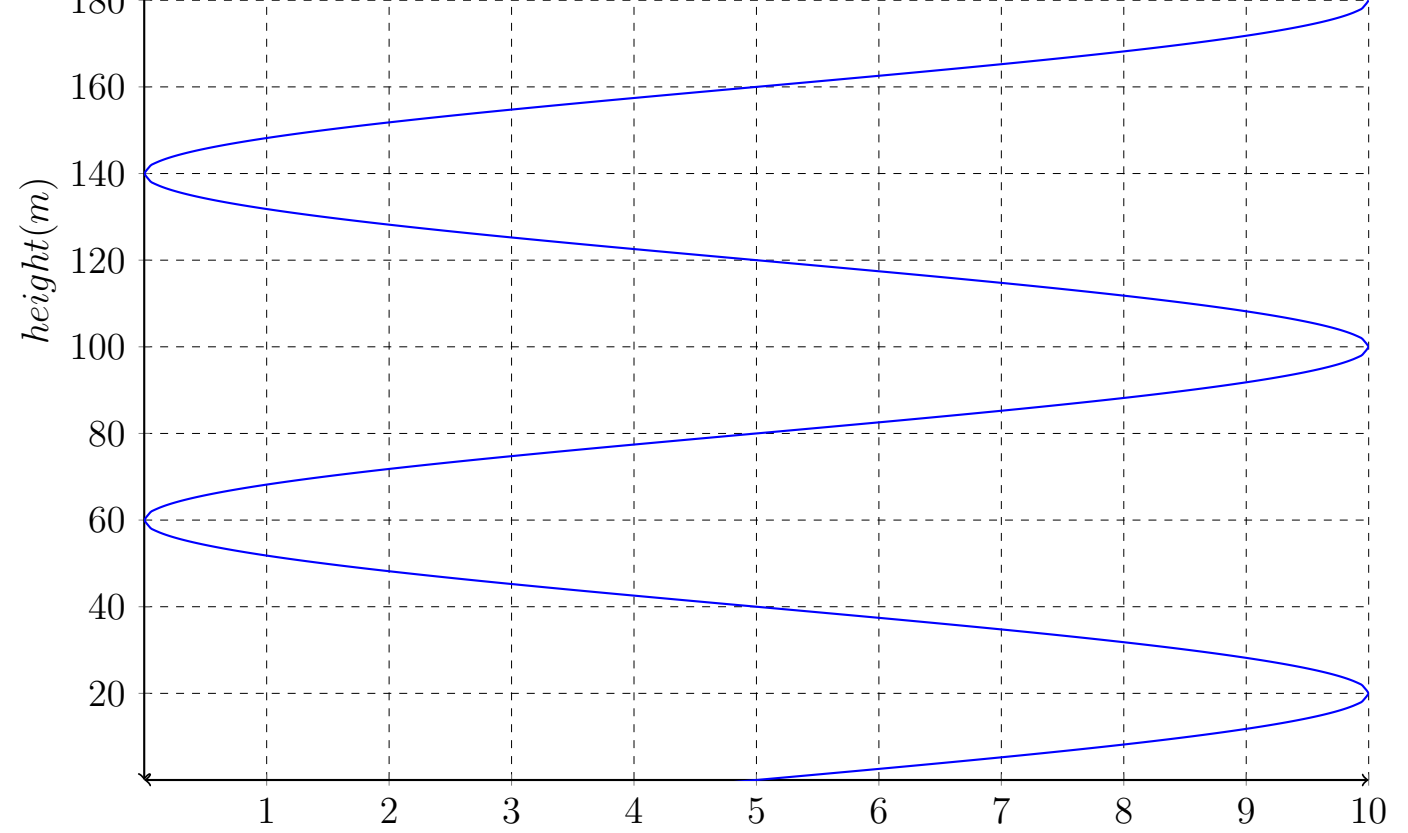
$\frac{x}{5} - \frac{5}{5} = \sin\frac{9}{2}y$

$\frac{x}{5} - 1 = \sin\frac{9}{2}y$

$\arcsin(\frac{x}{5} - 1) = \frac{9}{2}y$

$\frac{2}{9}\arcsin(\frac{x}{5} - 1) = y$

$y = \frac{2}{9}\arcsin(\frac{x}{5} - 1)$



f) Is the relation in part e) a function? Justify your answer.

No, the relation in part e) is not a function, as it does not pass the vertical line test.

8. Consider what happens to a relation when the coordinatges of all its ordered pairs are switched.

a) Give an example of a function that is still a function when its coordinates are switched.

$\{(0,0), (1,1), (4,2)\}$

b) Give an example of a function that is no longer a function when its coordinates are switched.

$y = x^2$

c) Give an example of a relation that is not a function, but becomes a function when its coordinates are switched.

$y = \pm\sqrt{x}$

9. Explain why a relation that fails the vertical line test is not a function.

a function is an equation or set of ordered pairs where every x-value only cooresponds to one y-value, and when a relation fails the vertical line test, it means that a vertical line can be drawn that intersects the graph at more than one point, which means that there is an x value that cooresponds to more than one y-value.

11. The table below lists all the ordered pairs that belong to the function $g(x)$

x	0	1	2	3	4	5
$g(x)$	3	4	7	12	19	28

a) Determine an equation for $g(x)$

$g(x) = x^2 + 3$

b) Does $g(3) - g(2) = g(3-2)$? Explain.

$g(3) - g(2) = (3^2 + 3) - (2^2 + 3)$

$g(3) - g(2) = (12) - (7)$

$g(3) - g(2) = 5$

$g(3-2) = (3-2)^2 + 3$

$g(3-2) = (1)^2 + 3$

$g(3-2) = 4$

$\therefore g(3) - g(2) \neq g(3-2)$