

**More Challenging Questions**

- When  $x^3 - kx^2 - 10kx + 25$  is divided by  $x - 2$ , the remainder is 9. Find the value of  $k$ .
- If  $2x^3 - 9x^2 + 13x + k$  is divisible by  $2x - 1$ , then it is also divisible by  
(A)  $x - 2$  (B)  $x - 1$  (C)  $x + \frac{1}{2}$  (D)  $x - \frac{1}{2}$  (E)  $2x + 1$
- If one root of the equation  $x^3 - 5x^2 + 5x - 1 = 0$  is  $2 - \sqrt{3}$ , then find the sum of the other two roots.
- If  $m$ ,  $n$ , and 1 are non-zero roots of the equation  $x^3 - mx^2 + nx - 1 = 0$ , then find the sum of the roots.
- The remainder when  $f(x) = x^5 - 2x^4 + ax^3 - x^2 + bx - 2$  is divided by  $x + 1$  is  $-7$ . When  $f(x)$  is divided by  $x - 2$ , the remainder is 32. Determine the remainder when  $f(x)$  is divided by  $x - 1$ .
- (a) For the polynomial  $f(x) = ax^3 + bx^2 + cx + d$ , the sum of the coefficients is equal to zero. (i.e.  $a + b + c + d = 0$ ). Show that the polynomial is divisible by  $x - 1$ .  
(b) Solve  $2x^3 - 3x^2 + 1 = 0$ .  
(c) Find the sum of the coefficients in the expansion of  $g(x) = (x + 2)(x^2 + 2x + 1)^2$
- Find  $a$  and  $b$  so that the quartic function  $f(x) = a^2x^4 + 3x^3 + b^2x^2 + 4abx + 4ab$  leaves a remainder of 10 on division by  $x + 1$  and a remainder of 20 on division by  $x$ .
- An unknown polynomial  $f(x)$  of degree 37 yields a remainder of 1 when divided by  $x - 1$ , a remainder of 3 when divided by  $x - 3$ , a remainder of 21 when divided by  $x - 5$ . Find the remainder when  $f(x)$  is divided by  $(x - 1)(x - 3)(x - 5)$ .
- If  $ax^3 + bx + c$ , with  $a \neq 0$ ,  $c \neq 0$ , has a factor of the form  $x^2 + px + 1$ , show that  $a^2 - c^2 = ab$ .
- Given that the cubic equation  $x^3 - 3x^2 + ax + b = 0$  has rational coefficients and has the root  $-1 + i\sqrt{3}$ , determine the values of  $a$  and  $b$ .

Answers: 1) 1 2) D 3)  $3 + \sqrt{3}$  4)  $-1$  5)  $-3$  6) b) 1, 1,  $-\frac{1}{2}$  c) 48 7) 4 possible solutions 8)  $2x^2 - 7x + 6$   
 10)  $a = -6$ ,  $b = -20$