FACTORING – REVIEW

Part A: Common Factoring

Ex. 1 Factor.

a) $4x^2y - 16xy^2$

b) $m(2a - 3b) - 2n(2a - 3b)$

c) $7x^3y^{-3} + 21x^4y^{-4}$

d) $3a^2(a + 4)^5 + \frac{1}{2}a(a + 4)^4$

Part B: Factor by Grouping

Ex. 1 Factor: $2y^2 - 6y + 4my - 12m$

Part C: Trinomial Factoring

Ex. 1 Factor.

a) $2m^2 - 5mn + 3n^2$

b) $14x^2 + 77x - 147$

Part D: Difference of Squares Factoring

Ex. 1 Factor.

a) $16m^2 - 81n^2$

b) $(2x - 1)^2 - 49$

c) $x^2 - 6x + 9 - y^2 - 4y - 4$

FUNCTIONS AND RELATIONS

A **relation** is a set of ordered pairs.
A **function** is a relation such that for each independent variable, there is only one dependent variable. Using the vertical line test, a relation is a function if any vertical line crosses the graph at only one point.

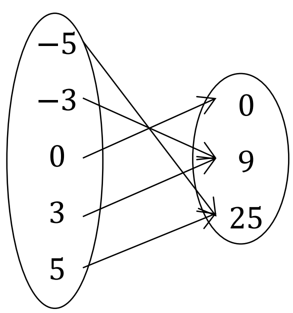
The **domain** of a relation is the set of all possible ________________.
The **range** of a relation is the set of all possible ________________.

Ex. 1 Which of the following are functions?

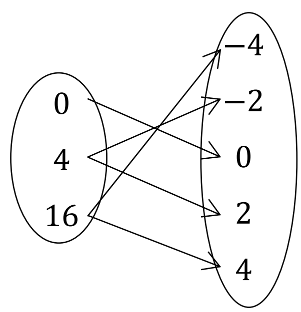
a) $\{(2,1),(3,5),(3,6),(4,-2),(5,-1)\}$

b) $\{(3,-2),(4,1),(5,1),(6,0),(7,-2)\}$

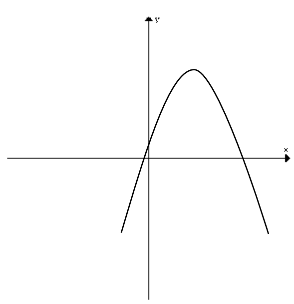
c)



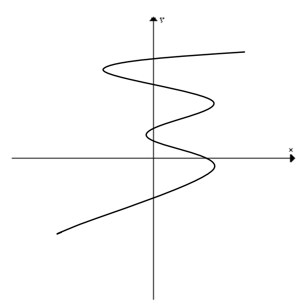
d)



e)



f)



g) $y = 3x - 7$

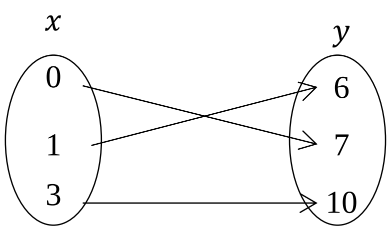
h) $x^2 + y^2 = 9$

Ex. 2 Find the domain and range.

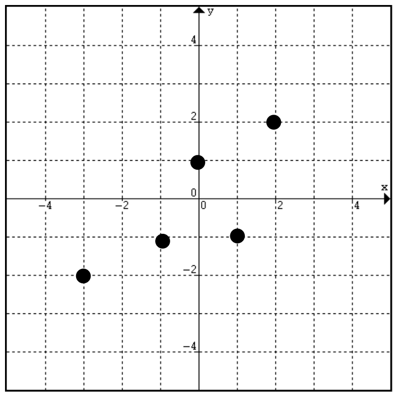
a) $\{(0,0),(1,-1),(2,4),(3,-9),(4,16)\}$

b) $\{(1,2),(2,1),(2,3),(3,0),(3,4)\}$

c)



d)



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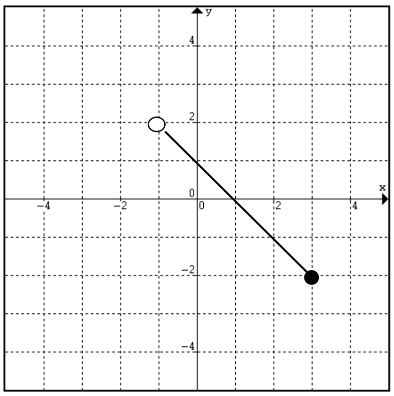

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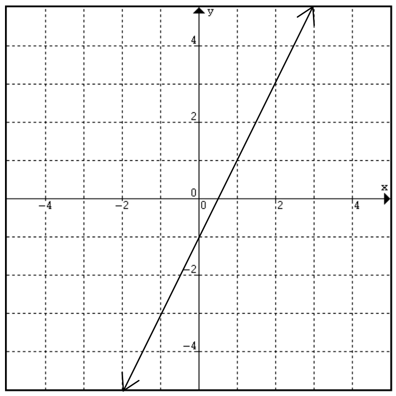
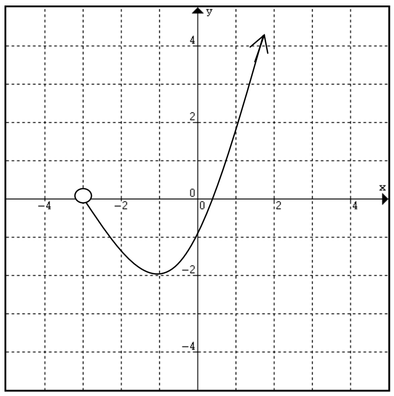
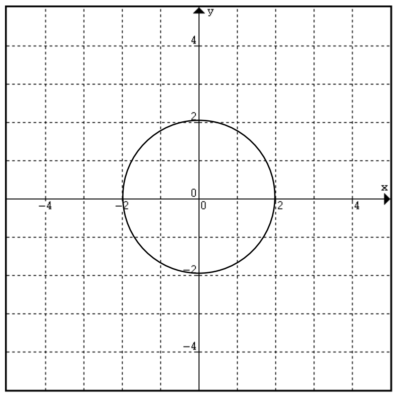


g)



h)



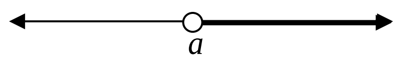
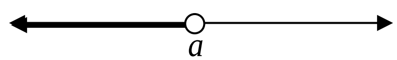
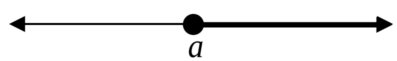
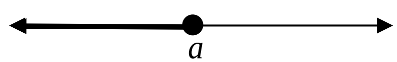
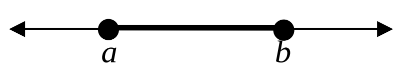
i) $y = x^2$

j) $y = \frac{1}{x}$

INTERVAL NOTATION

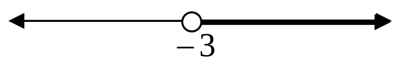
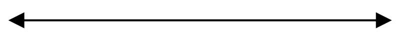
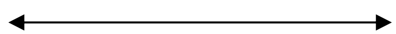
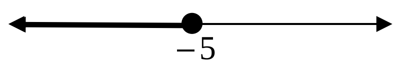
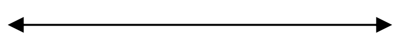
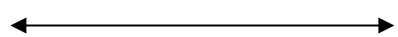
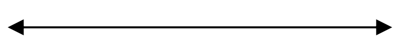
Sets of real numbers can be described in the following ways.

1. In words
2. Graphically on a number line
3. Inequality
4. Set notation
5. Interval (or bracket) notation

In Words	Number Line	Inequality	Set Notation	Interval Notation
x is greater than a		$x > a$	$\{x \in R \mid x > a\}$	(a, ∞)
x is less than a		$x < a$	$\{x \in R \mid x < a\}$	$(-\infty, a)$
x is greater than or equal to a		$x \geq a$	$\{x \in R \mid x \geq a\}$	$[a, \infty)$
x is less than or equal to a		$x \leq a$	$\{x \in R \mid x \leq a\}$	$(-\infty, a]$
x is greater than a and less than b		$a < x < b$	$\{x \in R \mid a < x < b\}$	(a, b)
x is greater than a and less than or equal to b		$a < x \leq b$	$\{x \in R \mid a < x \leq b\}$	$(a, b]$
x is greater than or equal to a and less than b		$a \leq x < b$	$\{x \in R \mid a \leq x < b\}$	$[a, b)$
x is greater than or equal to a and less than or equal to b		$a \leq x \leq b$	$\{x \in R \mid a \leq x \leq b\}$	$[a, b]$
x is an element of all real numbers		$-\infty < x < \infty$	$\{x \in R\}$	$(-\infty, \infty)$

For interval notation, square brackets indicate that the end value is included in the interval and round brackets indicate that the end value is not included. A round bracket is always used with positive or negative infinity.

Ex. 1 Complete the following chart.

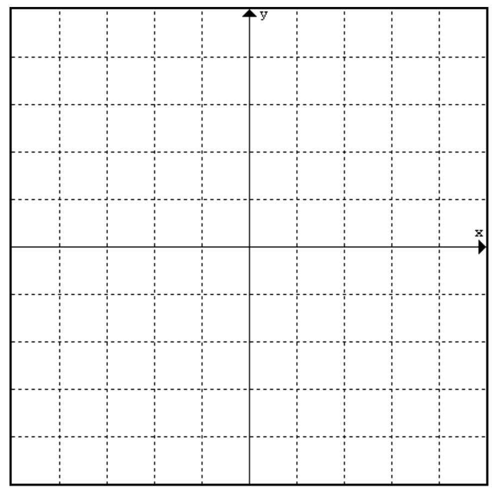
In Words	Number Line	Inequality	Set Notation	Interval Notation
			$\{x \in R \mid x > -3\}$	
		$x < 7$		$(-\infty, 7)$
			$\{x \in R \mid x \geq 2\}$	$[2, \infty)$
		$x \leq -5$		
x is greater than 3 and less than 6		$3 < x < 6$		
x is greater than -5 and less than or equal to 0			$\{x \in R \mid -5 < x \leq 0\}$	
x is greater than or equal to -1 and less than 2				$[-1, 2)$
x is greater than or equal to -8 and less than or equal to -2				

ABSOLUTE VALUE

$f(x) = |x|$ is the absolute value function. On a number line, this function describes the distance, $f(x)$, of any number x from the origin. Since distance cannot be negative, absolute value always returns a positive result.

The graph of $f(x) = |x|$ is comprised of two functions.

$$f(x) = \begin{cases} x, x \geq 0 \\ -x, x < 0 \end{cases}$$



Ex. 1 Evaluate.

a) $|-3|$

b) $-2|-3|$

c) $|-7| + |-5|$

d) $|3| - 2|2 - 5|$

e) $|3 - 5| - |7 - 11|$

f) $\frac{|-3|}{2} - \left|\frac{-5}{10}\right|$

Ex. 2 Are the following equivalent?

a) $3|x|$ and $|3x|$

b) $2|x|$ and $|-2x|$

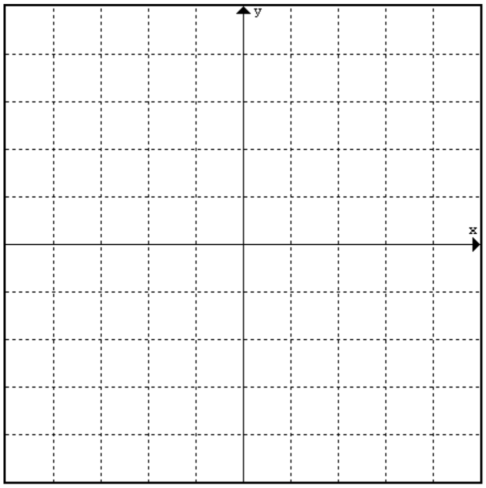
c) $|x + 3|$ and $|x| + 3$

Ex. 3 Graph $f(x) = 2|x+1| - 3$

Recall: In $y = f(x) \rightarrow y = af\big(k(x-d)\big) + c$,

$(x, y) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$

(x, y)	
$(0,0)$	
$(1,1)$	
$(-1,1)$	



Ex. 4 For each of the following,

i. Express on a number line.

ii. Express using absolute value notation.

a) $-7 < x < 7$

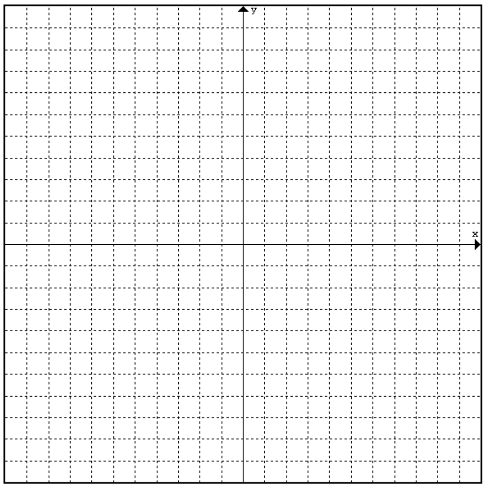
b) $x \leq -6$ or $x \geq 6$

For $|x| \leq c$, then $-c \leq x \leq c$, where c is a positive real number

For $|x| \geq c$, then $x \leq -c$ or $x \geq c$, where c is a positive real number

Ex. 5 Graph on a number line.

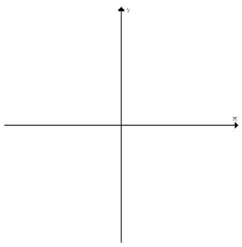
$|x+3| \leq 5$



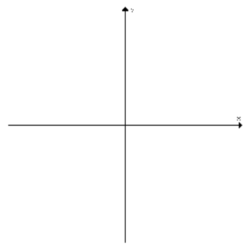
PROPERTIES OF FUNCTIONS

Functions can be described based on their appearance and may have more than one descriptor.

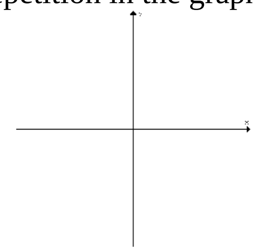
1. Increasing function
   As x increases, y increases.



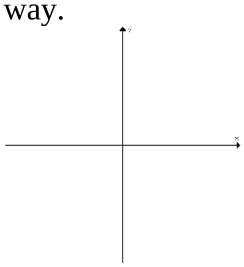
2. Decreasing function
   As x increases, y decreases.



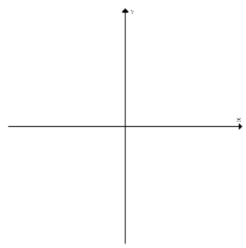
3. Periodic function
   There's a regular repetition in the graph.



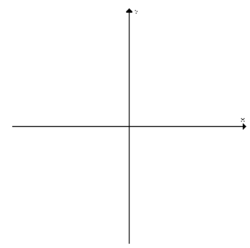
4. Step function
   Graph increases/ decreases in a step-like way.



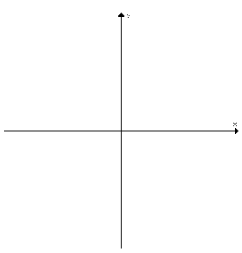
5. Discrete function
   Graph consists of separate (discrete) points.



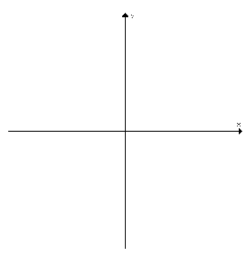
6. Continuous function
   Graph can be drawn without lifting the pencil.



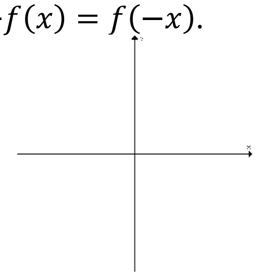
7. Discontinuous function
   Graph cannot be drawn without lifting the pencil.



8. Even function
   Function is symmetric about the y-axis. Has the property $f(x) = f(-x)$.



9. Odd function
   Function has rotational symmetry about the origin. Has the property $-f(x) = f(-x)$.



Interval of increase is the interval within a function's domain where the y-values increase as the x-values increase.
Interval of decrease is the interval within the function's domain where the y-values decrease as the x-values increase.

End behaviour refers to the y value as x approaches infinity (________) and as x approaches negative infinity (________).

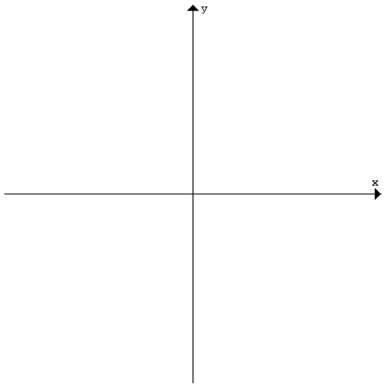
BASIC GRAPHS

1. Graph each of the following relations and complete the table.

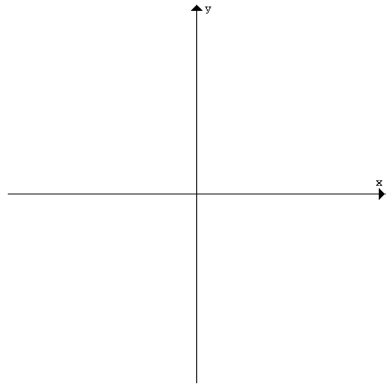
Equation	Domain & Range	Continuous / Discontinuous	Min/Max value and where it occurs	Symmetry	Interval(s) of Increase/Decrease	End Behaviour
$x = a$						
$y = a$						
$y = x$						
$y = x^2$						
$y = x^3$						
$x^2 + y^2 = r^2$						

Equation	Domain & Range	Continuous / Discontinuous	Min/Max value and where it occurs	Symmetry	Interval(s) of Increase/Decrease	End Behaviour
$y = \sqrt{x}$						
$y = \frac{1}{x}$						
$y = 2^x$						
$y = \left(\frac{1}{2}\right)^x$						
$y = \sin x$						
$y = \cos x$						
$y = \lvert x \rvert$						

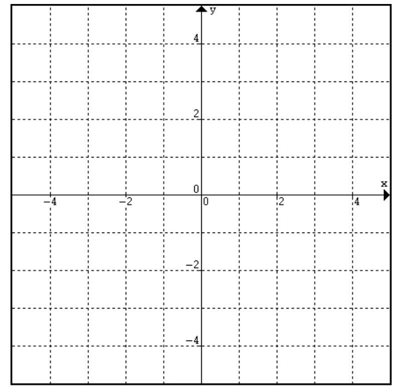
$x = a$



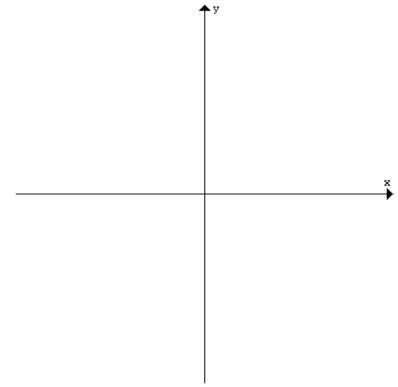
$y = a$



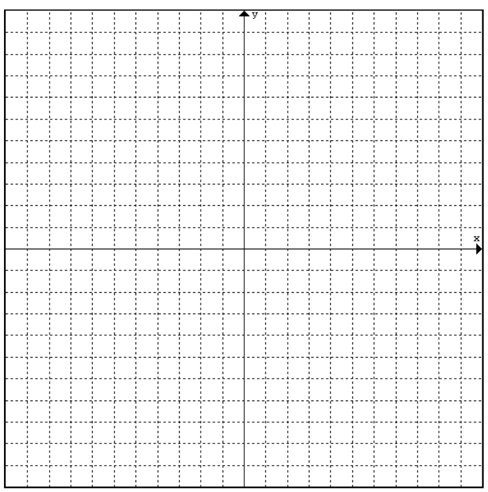
$y = x$



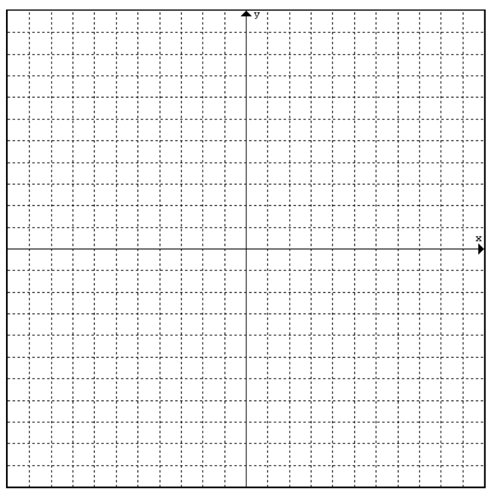
$x^2 + y^2 = r^2$



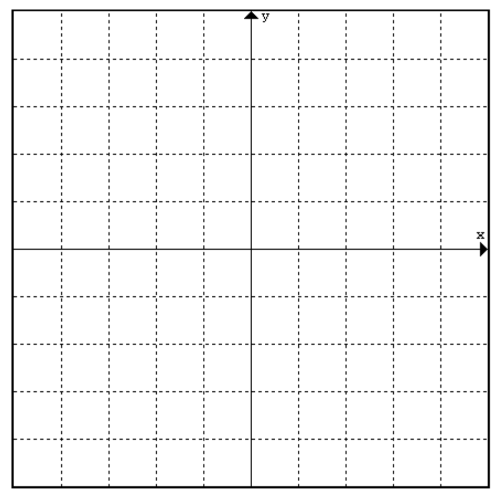
$y = x^2$



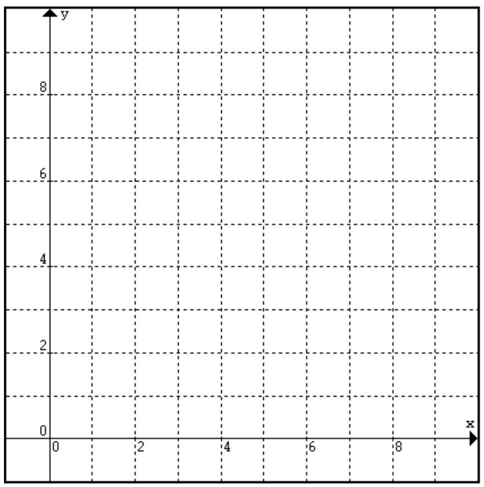
$y = x^3$



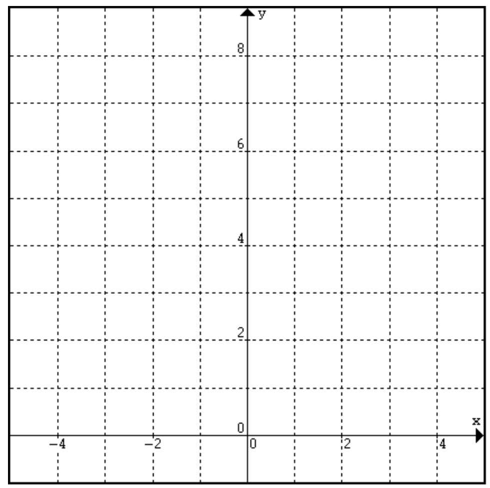
$y = \frac{1}{x}$



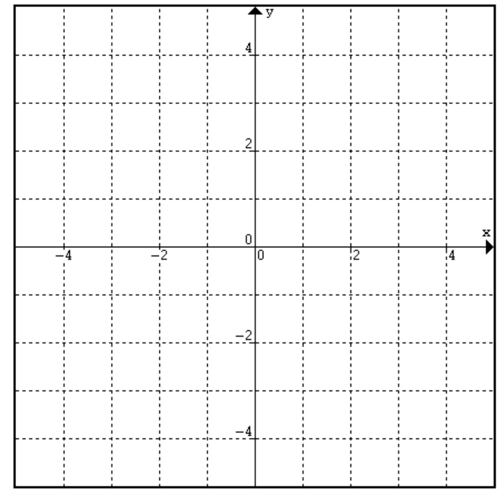
$y = \sqrt{x}$



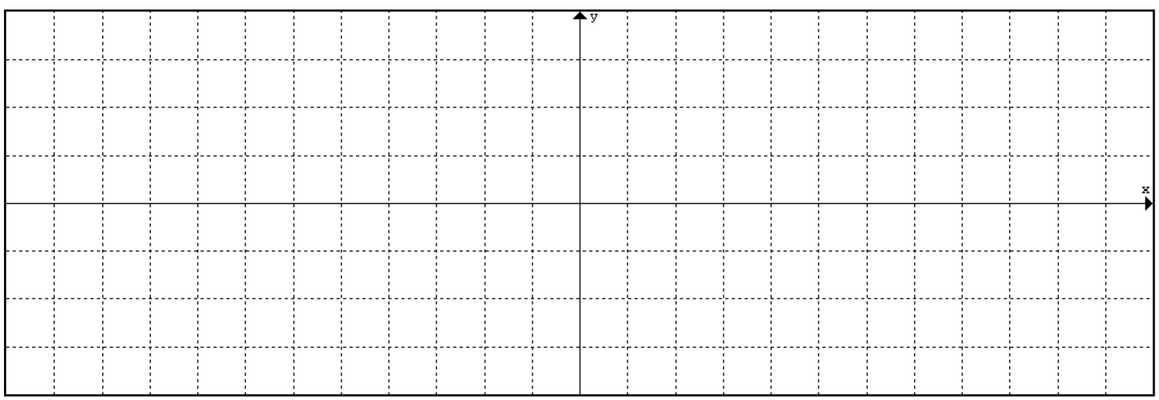
$y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$



$y = |x|$



$y = \sin x$ and $y = \cos x$

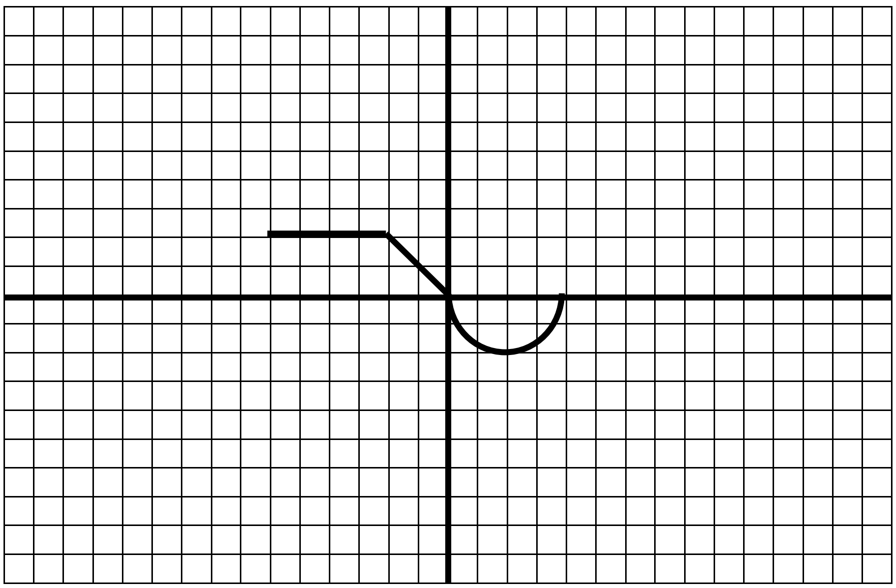


TRANSFORMATIONS – A REVIEW

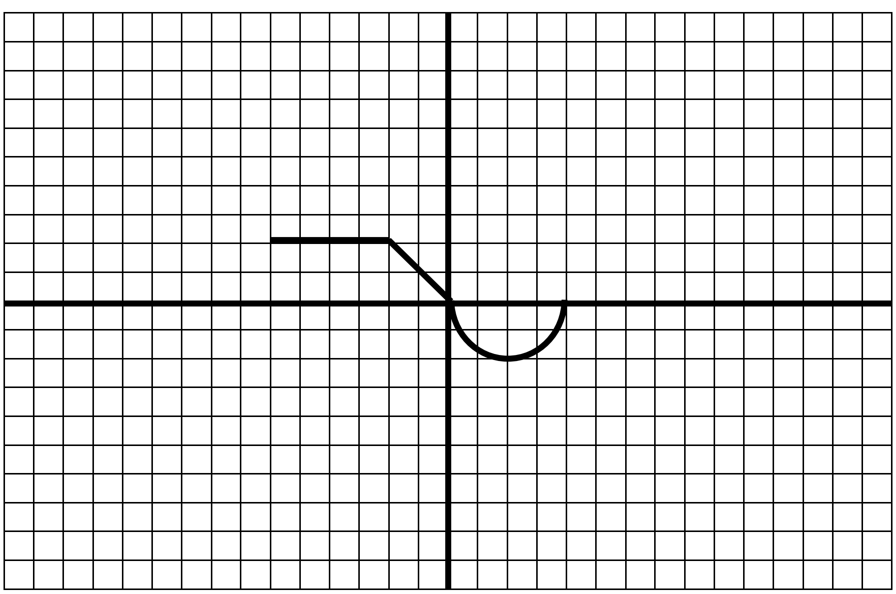
Part A: Single Transformations

Given $y = f(x)$, graph and describe each transformation.

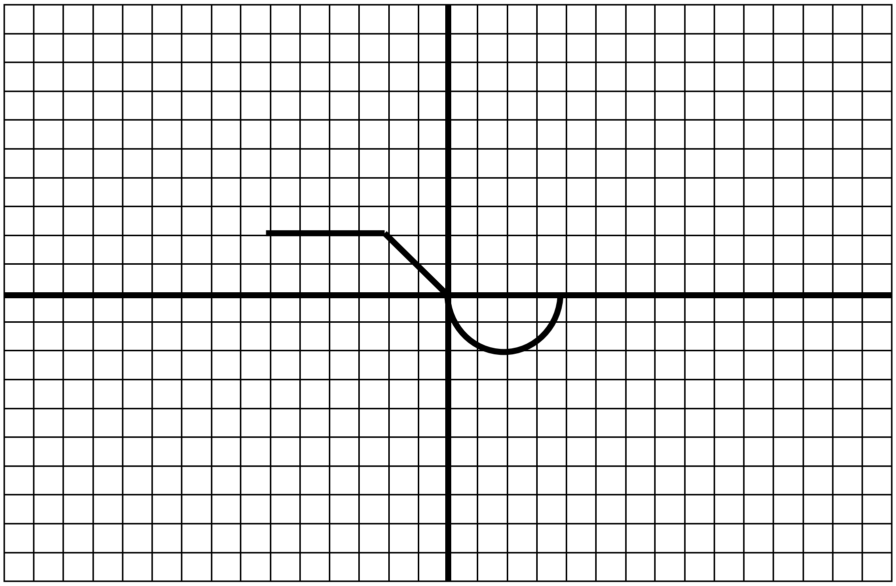
a) $y = f(x) + 3$
b) $y = f(x) - 2$
c) $y = f(x - 4)$
d) $y = f(x + 5)$



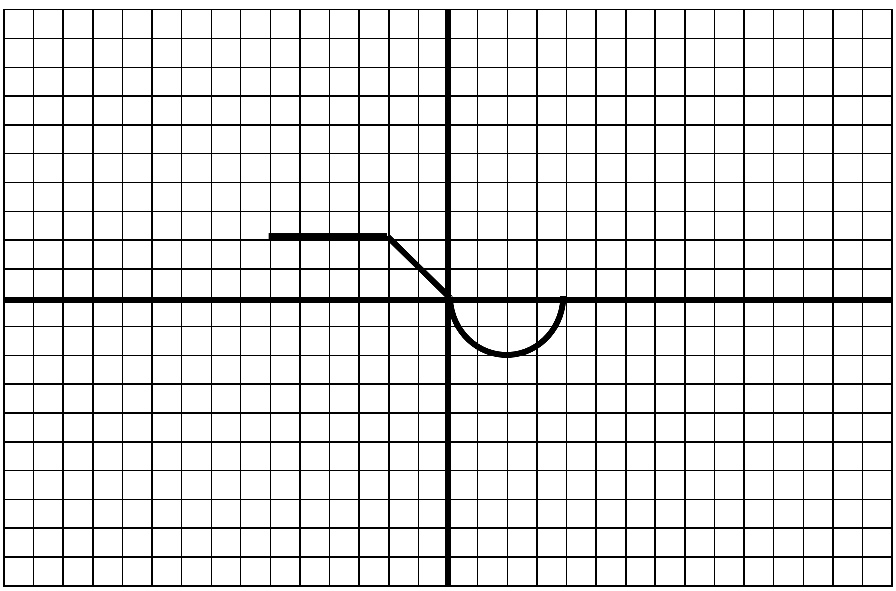
e) $y = 3f(x)$
f) $y = \frac{1}{2}f(x)$
g) $y = f(2x)$
h) $y = f\left(\frac{1}{2}x\right)$



i) $y = -f(x)$ j) $y = f(-x)$



k) $y = |f(x)|$



Part B: Combinations of Transformations

If $a < 0$, then reflect in ______________________ If $d > 0$, then translation ___________

If $|a| > 1$, then vertical ______________________ If $d < 0$, then translation ___________

If $|a| < 1$, then vertical ______________________

$$y = af\big(k(x - c)\big) + d$$

If $k < 0$, then reflect in ______________________ If $c > 0$, then translation ___________

If $|k| > 1$, then horizontal ______________________ If $c < 0$, then translation ___________

If $|k| < 1$, then horizontal ______________________

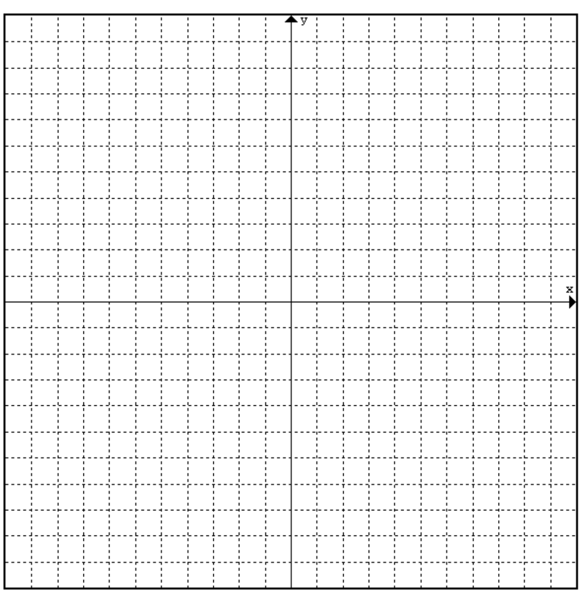
Transformations are performed in the following order:

1. Reflections
2. Stretches and compressions
3. Translations

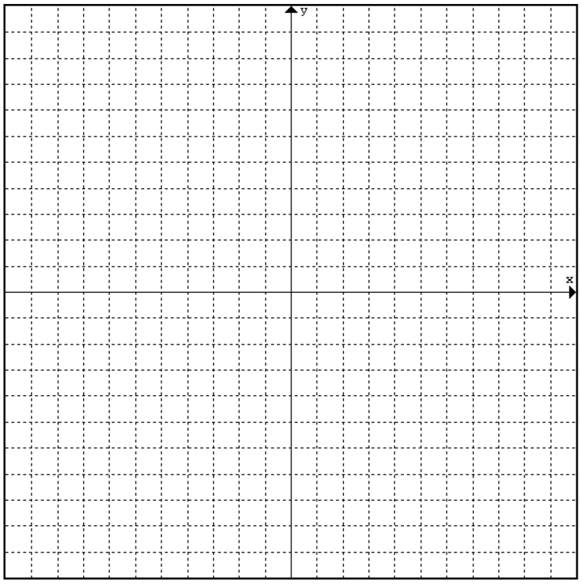
Mapping:

$$(x, y) \rightarrow \left(\frac{x}{k} + c, ay + d\right)$$

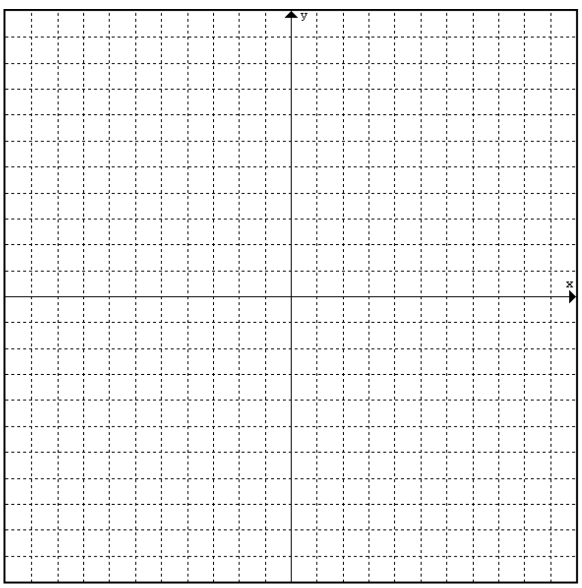
Ex. 1 Graph $y = -2(x - 3)^2 + 2$ using transformations.



Ex. 2 Graph $y = \sqrt{-x+4} - 2$ using mapping method.



Ex. 3 Sketch $y = \frac{1}{x+4} + 3$.



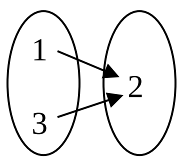
INVERSES

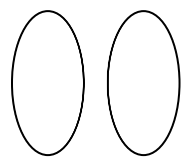
The inverse of a relation is obtained by ____________________ the components of each ordered pair. _____ becomes _____ and _____ becomes _____.

Note: $f^{-1}(x)$ does not equal $\dfrac{1}{f(x)}$

Ex. 1 Find the inverse of the following.

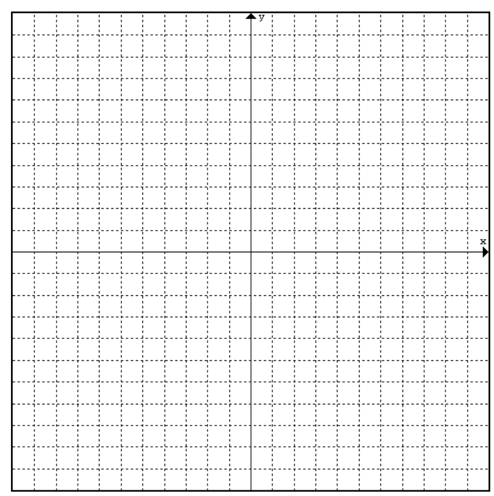
a) $f(x)$ inverse of $f(x)$





b) $f(x) = \{(2,3),(5,2),(7,2)\}$

Ex. 2 Given $f(x) = (x+2)^2$.



a) Graph the function.

b) Graph the inverse.

c) Is the inverse a function?

d) What is the relation between the function and its inverse?

e) Under what condition(s) will the inverse of this function also be a function?

f) State the domain and range of the function and its inverse. What do you notice?

g) State the equation of the inverse.

Ex. 3 Find the inverse of $f(x)=\frac{2}{5}x-3$ by applying the opposite operations in reverse order.

Ex. 4 Algebraically find the inverse of $f(x)=3\sqrt{x+1}-5$.

Ex. 5 Using f and f^{-1} from Ex. 4, find $f\left(f^{-1}\right)$ and $f^{-1}(f)$. What do you notice?

Summary:

1. The inverse of a relation is obtained by switching the x and y coordinates.
2. The original and inverse relations are reflections of each other in the line $y=x$.
3. The original and inverse relations have switched domain and range.
4. The equation of the inverse can be found by:
   a) applying the opposite operations in reverse order.
   b) switching x and y and isolating y.
5. If f and g are inverses, then $f(g)=g(f)=x$
6. The inverse of a function is not necessarily a function.
7. The inverse of a function is also a function iff the original function is one-to-one.