Polynomial Division

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1.

a) Divide $x^4 - 16x^3 + 4x^2 + 10x - 11$ by each of the following binomials.

i) $x - 2$

$$\begin{array}{l} \qquad\quad x^3 - 14x^2 - 24x - 38 \\ x - 2\overline{)x^4 - 16x^3 + 4x^2 + 10x - 11} \\ \underline{-(x^4 - 2x^3)} \\ \quad -14x^3 + 4x^2 \\ \underline{-(-14x^3 + 28x^2)} \\ \qquad -24x^2 + 10x \\ \underline{-(-24x^2 + 48x)} \\ \qquad\quad -38x - 11 \\ \underline{-(-38x + 76)} \\ \qquad\qquad -87 \end{array}$$

ii) $x + 4$

$$\begin{array}{l} \qquad\quad x^3 - 20x^2 + 84x - 326 \\ x + 4\overline{)x^4 - 16x^3 + 4x^2 + 10x - 11} \\ \underline{-(x^4 + 4x^3)} \\ \quad -20x^3 + 4x^2 \\ \underline{-(-20x^3 - 80x^2)} \\ \qquad 84x^2 + 10x \\ \underline{-(84x^2 + 336x)} \\ \qquad\quad -326x - 11 \\ \underline{-(-326x - 1304)} \\ \qquad\qquad 1293 \end{array}$$

iii) $x - 1$

$$x - 1\overline{)x^4 - 16x^3 + 4x^2 + 10x - 11}$$

b) Are any of the binomials in part a) factors of $x^4 - 16x^3 + 4x^2 + 10x - 11$? Explain.

No, this is because there is a remainder when you try to divide it with any of the above binomials, meaning it doesn't factor fully.

2. State the degree of the quotient for each of the following division statements, if possible.

a) $(x^4 - 15x^3 + 2x^2 + 12x - 10) \div (x^2 - 4)$

b) $(5x^3 - 4x^2 + 3x - 4) \div (x + 3)$

c) $(x^4 - 7x^3 + 2x^2 + 9x) \div (x^3 - x^2 + 2x + 1)$

d) $(2x^2 + 5x - 4) \div (x^4 + 3x^3 - 5x^2 + 4x - 2)$

3. Complete the divisions in question 2, if possible.

a) $(x^4 - 15x^3 + 2x^2 + 12x - 10) \div (x^2 - 4)$

b) $(5x^3 - 4x^2 + 3x - 4) \div (x + 3)$

c) $(x^4 - 7x^3 + 2x^2 + 9x) \div (x^3 - x^2 + 2x + 1)$

d) $(2x^2 + 5x - 4) \div (x^4 + 3x^3 - 5x^2 + 4x - 2)$

4. Complete the following table.

Dividend	Divisor	Quotient	Remainder
$2x^3 - 5x^2 + 8x + 4$	$x + 3$	$2x^2 - 11x + 41$	-199
$6x^4 + 12x^3 - 10x^2 - 4x + 32$	$2x + 4$	$3x^3 - 5x + 8$	-3
$6x^4 + 2x^3 + 3x^2 - 11x - 9$	$3x + 1$	$2x^3 + x - 4$	-5
$3x^3 + x^2 - 6x + 16$	$x + 2$	$3x^2 - 5x + 4$	8

5. Calculate each of the following using long division.

a) $(x^3 - 2x + 1) \div (x - 4)$

b) $(x^3 + 2x^2 - 6x + 1) \div (x + 2)$

c) $(2x^3 + 5x^2 - 4x - 5) \div (2x + 1)$

d) $(x^4 + 3x^3 - 2x^2 + 5x - 1) \div (x^2 + 7)$

e) $(x^4 + 6x^2 - 8x + 12) \div (x^3 - x^2 - x + 1)$

f) $(x^5 + 4x^4 + 9x + 8) \div (x^4 + x^3 + x^2 + x - 2)$