Computing the turbulent kinetic energy dissipation rate from lidar quasi-vertical stares

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The stabilization table was rolling with the ship, so it sampled some component of the mean horizontal wind.

1 The ship coordinate system

Define the ship coordinate to be "northward" toward the bow and "eastward" to starboard. The direction angle of the wind is from is 0° for wind from the north, and 90° for wind from the east. The x coordinate points forward (north), and the y coordinate points to starboard (east). z is downward for a right-handed north, east, down (NED) coordinate system.

Pitch θ (y-axis) and roll ϕ (x axis) rotations are defined in this coordinate system. Positive roll angle is a rotation of the port upward; positive pitch angle is a rotation of the bow upward.

2 Winds

The vertical wind is w. The ship-relative wind is W. Ship-relative horizontal wind is (U, V) = (-speed cos(dir), -speed sin(dir)). The Doppler lidar measures the radial velocity away from the lidar $\hat{w}$ and

$$\hat{w}^2 = (-U \sin\theta)^2 + (V \cos\theta \sin\phi)^2 + (W \cos\theta \cos\phi)^2.$$

We solve this for the air velocity,

$$W^2 = \frac{\hat{w}^2 - (U \sin\theta)^2 - (V \cos\theta \sin\phi)^2}{(\cos\theta \cos\phi)^2}$$

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3 Vectors among the lidar, and two target volumes

The radial vector to the target volume is

$$\hat{\mathbf{r}} = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \text{range} \begin{pmatrix} -\sin\theta \\ \cos\theta\sin\phi \\ \cos\phi\cos\theta \end{pmatrix}$$

The displacement between two *simultaneous* target volumes is

$$\hat{\mathbf{r}}_1 - \hat{\mathbf{r}}_2 = \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} = \text{range} \begin{pmatrix} -\sin\theta_1 - (-\sin\theta_2) \\ \cos\theta_2\sin\phi_1 - \cos\theta_2\sin\phi_2 \\ \cos\theta_1\cos\phi_1 - \cos\theta_2\cos\phi_2 \end{pmatrix}$$

In the time τ the ship moves relative to the air a horizontal displacement of $(X, Y) = \tau(U, V)$. The total displacement of the sample volumes is

$$\mathbf{r}_2 - \mathbf{r}_1 = \begin{pmatrix} \hat{x} + X \\ \hat{y} + Y \\ \hat{z} \end{pmatrix}$$

Expanding, the distance between the sample volumes is

$$r = |\mathbf{r}_2 - \mathbf{r}_1| = \left| \begin{pmatrix} \tau U - \text{range}(\sin\theta_1 + \sin\theta_2) \\ \tau V + \text{range}(\cos\theta_1\sin\phi_1 - \cos\theta_2\sin\phi_2) \\ \text{range}(\cos\theta_1\cos\phi_1 - \cos\theta_2\cos\phi_2) \end{pmatrix} \right|$$

This is the displacement argument of the (second order) structure function

$$D(r) = \overline{(w_2 - w_1)^2},$$

which behaves as $D = N + Ar^{2/3}$. The structure function is averaged as a function of r. We will retain sample pairs of (D, r) and bin average in r, perhaps with bins equally populated by sample pairs.

4 Ship motion fluctuations

The ship heave, pitch, and roll also induce fluctuating velocities in the transmitter that must be added. (The separation of scale between the mean ship-relative velocity and these fast fluctuations breaks down in maneuvers where the ship changes direction quickly.) Heave, pitch, and roll is either measured at the lidar on the stabilization table, where it can be added directly to the radial Doppler $\hat{w}$, measured in the ship orientation at the location of the lidar, or at a reference location on the ship (see survey).

Presently, I read a Notre Dame VectorNav file for measuring and correcting the motion of the table, but I don't know the quantities and units. The ship's POSMV IMU is also available from a reference location. If we use the POSMV, we need to calculate the velocities and moments from the heave and angular rates.

Assuming the ship is rigid, the heave is the same everywhere. Angular rates induce motion in proportion to the moment $\mathbf{L} = (L_x, L_y, L_z)$ be the displacement vector from the reference location (of the POSMV) to the lidar. At the lidar the ship moves by

$$\dot{x} = -\dot{\theta} L_z \dot{y} = \dot{\phi} L_z \dot{z} = \dot{\theta} L_x - \dot{\phi} L_y.$$

Varying yaw is ignored.

5 The structure function for arbitrary displacements

We sample the vertical component of the Doppler (radial) velocity along arbitrary displacements **r**, which may be either longitudinally oriented along the resolved velocity component, or longitudinal to it. The Kolmogorov similarity for second order structure functions is in general

$$D_{ij}(\mathbf{r}) = C_2 \epsilon^{2/3} r^{2/3} (4\delta_{ij} - r_i r_j / r^2)/3$$

where C_2 is a universal constant. For sampling only the vertical coordinate $i = j = 1$,

$$D_{11}(\mathbf{r}) = C_2 \epsilon^{2/3} r^{2/3} (4 - r_1^2 / r^2)/3.$$

where r_1 is the vertical (longitudinal) displacement and $r = |\mathbf{r}|$ is the scalar distance between samples.