# On the moisture balance of the trade cumulus layer

[./ATOMIC/ATOMIC\_GOES/julia/RHB/sonde/massflux.docx]

## Introduction

The marine trade cumulus cloud regime covers a large fraction of the world oceans. Where trade cumulus predominates, cloud albedo is variable with an average of ~0.1. Climate sensitivity depends strongly on the low cloud feedback, determined in large part by the response of low trade cumulus clouds to warming (Bony, Sherwood et al. 2014). Models that generate more trade cumulus clouds in a warmer climate have mean climates that warm less; and those in which the circulation in a warmer climate dries the clouds warm more. The moisture of the cloud layer, and the resulting cloud fraction, is a competition between 1) the drying by large-scale advection and subsidence and 2) the moistening by the updrafts of the clouds themselves mixing up subcloud water vapor from near the ocean surface.

Here we present a simple model based on moisture balance between the large-scale drying and cloud moistening effects. The model is solved for the mean thermodynamic sounding, surface evaporation and precipitation, and vertical distribution of clouds observed during the ATOMIC/EUREC4A field experiment in January-February 2020. The model diagnoses the cloud moisture flux and mass fluxes required to balance the large scale drying in the present trade cumulus regime. Sensitivity experiments predict what happens to the cloud mass and moisture flux if saturation humidity increases, mean humidity decreases, and large-scale subsidence decrease, respectively.

## Model

We calculate a simple vertical water budget for the trade cumulus cloud layer. Trade cumulus clouds are intermittent shallow convection in widespread regions of mean subsidence balancing deeper convection elsewhere. The updraft area fraction is calculated over large (1000 km)2 regions, assuming updraft cores are cloudy with saturated specific humidity. The horizontal-time ensemble mean trade cumulus water (vapor + liquid) balance is written for these areas as the sum of mean subsidence drying and the divergence of the cumulus eddy flux ,

~~where is the large-scale moisture source, which is negative from subsidence and advective drying in the trade winds. All the terms are averages over an ensemble of shallow convection in an area with weak mean humidity and temperature gradients. Deep convection affects the solution remotely through the large-scale subsidence.~~

The gist of the model is that the eddy flux does what it must do to retain a steady climatological moisture profile. Updrafts, including clouds, are responsible for most of these eddy fluxes. A small part is contributed by precipitation. From this diagnostic moisture closure, we compute the trade cumulus ensemble eddy flux by cumulus updrafts and precipitation . We first model the updraft water, clouds, and their conversion to precipitation.

[aside:] A vertical hydrostatic pressure coordinate conserves mass without needing to write the density explicitly. This coordinate and its corresponding velocity retain the familiar vertical geometric interpretation. The average density over the trade cumulus layer is taken to be kg/m3.

## Cloud updraft water

A budget for steady total water in the updraft depends on its entrainment and precipitation sinks. The equations for the suspended liquid and water vapor in the cloud are

andwhere and (km–1) are the lateral entrainment coefficient and the autoconversion of cloud liquid to precipitation, and (<0) is the moist adiabatic condensation rate of water vapor (in a parcel) per unit height,  
This condensation rate is a function of temperature and pressure, taken to equal those of the environment. Summing the liquid and vapor, and taking the Eulerian total specific humidity to be steady,  
This equation for total water is insensitive to the condensation rate . The updraft advects moisture upwards, but the solution is quantitatively insensitive to the mass flux.

Above the cloud*,* , so there is no autoconversion. Only entrainment relaxes humidity to that of the environment:

In the cloud, and the equation for ,   
relaxes vertically toward the intermediate humidity between and ,   
weighted by the precipitation efficiency . Autoconversion removes cloud liquid, relaxing total water toward , and entrainment relaxes total water toward .

Solutions for total cloud specific humidity and are shown in Fig. MOISTURECLOUDS for a range of total moisture sink coefficients from to . Initial specific humidity is 15.3 g kg–1, matching saturation at cloud base = 600 m. Parcels entrain immediately and cloud liquid water autoconverts to precipitation. The parcel may not exceed saturation until it is slightly higher than because of its moisture sinks. Total parcel specific humidity is compared to saturation humidity of the environment to determine cloud liquid water.

The profiles of predicted by this entraining-precipitating updraft model agree with the most humid air sampled on occasion by radiosondes. Clouds height is limited by entrainment of relatively dry air between 1.5-3 km, representing the mean distribution of the trade inversion. The clouds occupy a range of depths representative of trade cumulus clouds, with deeper clouds for lower rates of entrainment and autoconversion. The intermediate total sink rate of gives maximum cloud liquid specific humidity exceeding 0.8 g kg–1 over 0.7-1.5 km and cloud top height of 1.9 km. This total sink will be used in control simulations a single monolithic cloud.

A diagram of weather forecasting

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Figure MOISTURECLOUDS. Observed specific humidity from the ATOMIC *Ron Brown* radiosondes (gray), and their mean (black) and saturation specific humidity . Cloud updraft total specific humidity (blue) and cloud liquid (green) for a range of total moisture sink rates (entrainment and autoconversion coefficients, km–1). Clouds transport moisture up, continuously diluting it toward a mixture between and .

[ /home/deszoeks/Projects/ATOMIC/ATOMIC\_GOES/julia/RHB/sonde/moisture\_clouds.pdf ]

A range of cloud liquid water and cloud top height is produced by varying the total entrainment and precipitation sink, in Fig. CLOUDTOPHT. The lowest cloud from the model, with top at 690 m, is for a sink of = 4 km–1. Clouds vanish for any stronger a sink. The highest cloud top representative of trade cumulus clouds, at 3.54 km, is produced for = 0.523 [WAS 0.44] km–1. Clouds with sink rate of less than 0.523 km–1 transport water up and generate clouds to the full depth of the tropopause like deep convection, with a minimum cloud fraction at the 3.6-km trade inversion. The parcel model generates deep convective clouds above the cloud minimum at the trade inversion. This is an intuitive but unintended result of our model. We caution against quantitative interpretation of clouds above the trade inversion, because the model neglects processes relevant to deep convection such as ice, buoyancy, latent heating, and radiation.

A graph of a line

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Figure CLOUDTOPHT. Clouds of different specific humidity and heights simulated for different total (entrainment + autoconversion) sink rates.

[/home/deszoeks/Projects/ATOMIC/ATOMIC\_GOES/julia/RHB/sonde/cloud\_top\_heights.pdf]

## Precipitation

Precipitation flux (downward > 0) is parameterized by autoconversion from cloud liquid, For the updraft with no storage of cloud liquid water, the precipitation source is   
where ( is the total moisture source in the updraft and is the precipitation efficiency, i.e., the ratio of the autoconversion to the total (entrainment + autoconversion) moisture sink.

## Eddy flux

We write the horizontal mean moisture budget with no total moisture tendency or advection as,  
The first term is the prescribed large-scale advection and subsidence of mean environmental vapor. The second term is the convergence of the total eddy moisture flux by cloud up- and downdrafts, and precipitation. Specific humidity tendency has diurnal cycles that average out over a day. Integrating this balance vertically from cloud base, the eddy moisture flux is  
For cloud base vapor flux of 145 W m–2, the total flux goes to zero at about 8 km (Fig. QMASSFLX).

We parameterize the eddy flux due to clouds by a mass flux scheme consisting of updrafts and downdrafts,  
The cloud updraft mass flux is the product of the cloudy area fraction and the mean velocity in the clouds. The compensating descending mass flux of clear air has much weaker over the larger clear area . The mass flux due to updrafts and downdrafts sums to zero, . As is common, we assume that the humidity of the descending air between the clouds is nearly equal to the mean humidity so   
, and

These fluxes are shown in Fig. QMASSFLXa. The flux at cloud base is set to 145 W m–2 kg m–2 s–1. The cloud-base precipitation is set to the surface precipitation. The mean surface evaporation is 180 W m–2 kg m–2 s–1 and surface precipitation is 25 W m–2 kg m–2 s–1, observed on the *Ron Brown* during EUREC4A/ATOMIC.

To calculate the profiles in Fig. QMASSFLX, the total flux is first estimated which balances the large-scale moisture sink. Precipitation removes water from the total flux, requiring the cloud mass flux to be stronger than the total. Using the precipitation parameterization, the moisture budget allows us to simultaneously solve for the cloud flux and precipitation (Fig. QMASSFLX). Evaluating the analytical integral of still involves a numerical vertical integral, so we directly integrate the differential equation from cloud base with vertical step ,  
with a marching scheme for ,  
where and is the difference between the total water in the updraft and the environment . We use found for (Fig. MOISTURECLOUDS) to solve for the cloud flux and precipitation flux . Precipitation efficiency is set to obtain physical solutions with zero fluxes at cloud top andat cloud base. For , the model returns the observed precipitation W m–2 at cloud base for (autoconversion km–1 and entrainment 0.915 km–1;

WAS (autoconversion km–1 and entrainment 1.125 km–1; Fig. QMASSFLXa).

## Integrating precipitation instead

Integrating the precipitation flux directly is done by substituting in  
Precipitation is numerically integrated from initial value at cloud top, downward through the cloud () with trapezoidal steps,  
where . The cloud-base precipitation is calculated (Fig. PRECIPHEIGHT). Precipitation for trade cumulus clouds depends on the autoconversion . An ensemble of clouds with different precipitation efficiency and total sink rate parameters results in a range of cloud top height. The cloud top height distribution is matched to that of the satellite observations, and the total precipitation is match to

Cloud top height generated by the varying parameters has a bifurcation around 3.6 km. Trade cumulus clouds are vertically bounded below 3.6 km by the trade inversion in the sounding, but clouds with weak sink rate retain enough total moisture to continue past the trade inversion and do not stop until they reach the tropopause. The model predicts this bifurcation based on water closure alone, without considering the effect of buoyancy. While this prediction qualitatively matches expectations for tropical clouds, we caution against interpreting it too strongly, as the model lacks features that would quantitatively constrain deep convection, such as ice and buoyancy. We limit our interpretation to the trade cumulus clouds.

A diagram of a weather diagram

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Figure PRECIPHEIGHT. [mean\_mass\_flux.ipynb 🡪 precip\_height.\*]

The mass flux is (Fig. QMASSFLXb). The mean updraft velocity at each height is then where is the cloud area. The humidity gradient and cloud fraction decay with height.

A graph of height and height

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Figure QMASSFLX. Total moisture flux to balance the large-scale moisture sink (blue), cloud moisture flux (orange), and (downward) precipitation flux (green), and mass flux (red) from the model.

The vertical moisture fluxes balance large scale advection and subsidence of moisture. Most of this moisture flux is below 3 km and most of the mass flux is below 2 km. The cloud and total eddy moisture flux balance subsidence drying at the strong moisture derivative at the top of the trade cumulus layer at 2-3 km. The strong difference at 2-3 km between the dry environment and the cloud updraft, whose specific humidity is proportional to the saturation specific humidity, achieves the moisture flux there. Precipitation is zero at cloud top (1.7 km) and integrates downward proportional to cloud liquid water, confining it in the lower cloud layer.

We model the mean of the cumulus ensemble with a single monolithic cloud of 1.7 km depth (Fig. MOISTURECLOUDS). Clouds still exist above 1.7 km, but the average humidity transported by this monolithic updraft is between saturation and the mean surroundings. We expect the emergent result is accurate that the eddy flux is mostly by vapor transport above 1.7 km. In or out of clouds, eddies transport mostly water vapor.

## Cloud ensemble

In addition to the single monolithic updraft, we also modeled an ensemble of cumulus clouds. Clouds with weaker total moisture sink reach greater maximum height . In the case of this cloud ensemble, we now assume each cloud of height has uniform mass flux between cloud base and cloud top. The mass flux detrains immediately above as the cloud liquid water and updraft mass flux go to zero. The cloud moisture flux of clouds reaching height is a function of the vertical coordinate and the cloud top height . The vertically uniform mass flux is only a function of cloud top height. Entrainment rate and autoconversion also vary with . At cloud top the liquid water goes exactly to zero and so

We bin clouds and their fluxes by the cloud top height . First, we find at cloud top from the total eddy moisture flux Let represent the part of the cloud ensemble corresponding to clouds of height to . The cloud flux is  
at the top of each cloud. From this, the mass flux of the cloud reaching height is then  
The moisture flux profile of the cloud reaching height is   
The moisture quotient becomes somewhat greater than 1 in the moistest part of the cloud. We integrate the flux from the cloud ensemble downward from each cloud top height, starting with the highest trade cumulus top height ( 3.6 km), because the flux found at cloud top requires flux in the cloud below it. Each cloud also generates precipitation at altitude :  
   
The moisture flux is summed from the highest clouds in the ensemble downward.  
To the moisture flux already summed from the higher clouds, add the moisture flux to balance at each additional cloud top height.

THERE IS A RISK the high clouds will completely explain the moisture flux, and the lower clouds will have no to contribute a negative mass flux.

Then using the specific humidity in the cloud, solve for

The precipitation from clouds of height is . The total precipitation is summed over all cloud top heights .

Further approximations can be made to model an equilibrium cloud liquid water in the updraft. The cloud flux is similar if we use the mean environmental saturation to approximate , because the vapor makes up most of the total specific humidity. An equilibrium cloud liquid water is satisfied for a parcel with no storage of cloud liquid water,  
The equilibrium liquid wateris insensitive to mass flux, and then the precipitation flux is given by

## Data

Radiosonde thermodynamic profiles

The humidity profile is the average of the ATOMIC/EUREC4A radiosondes released in Jan-Feb 2020 from the research vessel *Ron Brown*. Among the superset of radiosonde profiles from all platforms in EUREC4A, the *Ron Brown* radiosondes provide a sufficient ensemble mean located northeast of Barbados, away from deeper convection farther south. Figure HUMIDIFF shows the humidity observations from these sondes, and the saturation specific humidity of their mean temperature profile. The mean humidity difference in moisture flux calculation is approximately 2 g kg–1 in the trade cumulus layer (0.5-2 km). The stronger humidity difference of 8 g kg–1 at 3 km, around the maximum height of trade cumulus clouds increases the moisture flux by the fewer deep trade cumulus clouds that reach this level.

GOES satellite cloud height

We use the cloud area as at each height from the GOES-R satellite retrievals. The GOES 2-km resolution makes visible the energy-generating cloud scales responsible for the eddy flux. The dynamics at this scale couple to smaller-scale updrafts and downdrafts, and to turbulence. And yet, the 2 km scale itself is too small to be representative of an ensemble of cloud updrafts and downdrafts. The cloud area fraction is computed as a function of height from the cumulative distribution of GOES cloud top brightness temperature, assuming maximum overlap (i.e., clouds extend from cloud base to the level corresponding to the pixel brightness temperature). Cloud top temperature is mapped to height using the mean temperature profile of radiosondes released from the Ron Brown research vessel during ATOMIC/EUREC4A.

The cloud area fraction reaching each cloud top height indicates how the mass and moisture fluxes by clouds and precipitation are distributed in the domain. We solve for mass flux and updraft velocity profiles,for the cloud top height in the satellite retrieval.

We shall now model the spectrum of cloud top heights by varying , then retune so that the spectrum of clouds gives the surface precipitation Varying generates clouds of different height , defined by the top where liquid water (Fig. MOISTURECLOUDS). Suppose that the total mass flux is a sum of mass fluxes from these clouds of different heights from cloud base to the highest cloud top , with the mass flux density . Then the cloud flux is  
where and depend on cloud height The flux is only due to the highest cloud, and the change of the flux moving downward is  
The last equation uses at cloud top, where exactly. Below cloud top, for each cloud with top   
[[(the equation at bottom of p. 4)]]

A graph of blue bars

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Figure CLOUDBELOWHEIGHT. Schematic of cloud fraction below each height. Tubes represent an ensemble “rank” of cumulus clouds that transport mass and moisture from cloud base to cloud top, for the cumulative cloud fraction below the cloud top height. Mass and moisture fluxes are distributed among the clouds, based on their height.

[ ]

Figure GOESCLOUDAREA. A GOES 2-km IR brightness temperature image.

[ ]

Figure GOESCDF. Vertical cumulative distribution of cloud area fraction.

!!!!!! How does GOES change over the diurnal cycle? !!!!!!

## Sensitivity to the sounding

What effect does warming the sounding have on the mass flux? First, assume that the large-scale forcing is the same, and change only the local humidity profiles. In so doing, we assume radiative-convective equilibrium results in the same large-scale circulation. That is, the large scale deep convective latent heating balances nearly the same radiative cooling, resulting in the same subsidence over the trade cumulus boundary layer.

1. Keep RH constant, increase q\_s(T) and q. With compensations, cloud is almost unchanged.
2. Keep q constant, increase q\_s(T) by adding constant . This results in a sounding with very dry RH.
3. Reduce the lapse rate, so add , and adjust qs, q accordinly

We computed the cloud for our model with relative humidity held constant and a warming of °C. This experiment increases saturation specific humidity to , by +6% by Clausius Clapeyron (at 25 °C). Yet because of compensations, this increases insignificantly, by only 0.12%. [Humidity does not depend on changes in lapse rate of the environment in this experiment.]

If increases 6% at constant RH, 6% less mass flux is requred to balance the same large-scale moisture forcing. Clouds being moister relative to their environment would reduce the equilibrium mass flux that balances the large-scale drying, and probably reduce the cloud fraction. [[This seems basic. Too simple? Has anyone proposed it already? We need to look.]]

If the large-scale circulation also spins down in a warmer climate, then the subsiding environment of the trade cumulus clouds will be relatively drier than the surface, increasing and reducing equilibrium mass flux needed to balance the large scale drying. [[Is this simulated by models with wet-get-wetter behavior???]]

Increasing the updraft-environment humidity difference also increases entrainment. Increasing (6%), corresponding to an increase of 1 °C, but keeping the environment constant reduces cloud liquid water to 41 % of that of the ATOMIC/EUREC4A control case.

### OLD:

## Holes in the trade cumulus

The cloud radiative effect is strongly correlated with the size of holes in the trade cumulus layer (Janssen et al 2022???). We use the distance transform, which gives hole size, and the connectedness of each hole to its neighbors. The holes are not strongly correlated to cloud morphologies described as Sugar, Gravel, Flowers, and Fish. These cloud morphologies are more strongly related to the wind regime and momentum transports by the cumulus ensemble (Nuijens et al. 2022 QJRMS, Savazzi et al. 2022).

In light of our mean water transports computed above, we suppose that the clear regions between clouds act as watersheds, collecting subcloud boundary layer moisture that converges and feeds cloud updraft mass and moisture flux. The area and (physical and optical) depth of clouds are related to their updraft mass flux and their detrainment of radiatively visible and emissive clouds and water vapor.