Pinsky et al (2010) retrieval – Separation Technique

Dopper velocity $V = W + V_g$ downward – negative (1)

$V_g = \int r^6 N(r) V_d(r)\, dr / Z$ $\quad V_f < 0$ (2)

$Z = \int r^6 N(r)\, dr$ (3)

$\langle q \rangle \equiv \frac{1}{N}\sum_{k=1}^{N} q_k$ ensemble mean

$\langle W \rangle = 0 \rightarrow W = W'$ (What about mean divergence?) (4)

$\rho \quad \langle W V_g' \rangle$ correlation; assumed zero, but could relax < 0 (5b)

For time interval T compute $\langle V \rangle (Z)$ for each 0.25 dBZ bin.
(This is where the 6th order polynomial will be applied)

Let $\phi(\langle Z \rangle^{bin}) = \langle V \rangle (Z_k^{min} \leq Z < Z_k^{max})$
$\theta(\langle Z \rangle^{bin}) = [\langle V'^2 \rangle (\quad \leq Z < \quad)]^{1/2}$ } approx. to bins $\quad \phi \leq 0$

find a robust formulation for $\phi, \theta(Z)$

$\zeta = \langle W Z' \rangle$ assumed 0 (5a)

where in (5) $Z' = Z - \langle Z \rangle^{(full)}$; $V_g' = V_g - \langle V_g \rangle^{(full)}$ resolved in h & t

$\langle V \rangle = \cancel{\langle W \rangle}^{0} + \langle V_g \rangle = \phi(\langle Z \rangle)$ fcn of $\langle Z \rangle$ (p 1177)

This is the confusion: The $\langle \rangle$ seems now to refer to a reflectivity bin average. While $\langle W \rangle^{(full)} = 0$, $\langle W \rangle^{bin}$ need not be.

Let there be $\psi(\langle Z \rangle^{bin}) = \langle W \rangle^{bin} (\quad \leq Z < \quad)$

Pinsky assumes $\psi(\langle Z \rangle^{bin}) = 0$ for every bin, but this isn't necessarily so. Problem is we don't have $\langle W \rangle^{bin}$ measurements to constrain what it is.

a) Residual velocity $U(h,t) = V(h,t) - \phi(h, Z(h,t))$ (p 1178)

b) Separate $U = W + V_g'$

Some of $\phi(h, Z(h,t))$ could be attributed to $\overline{W}(Z)$, analogous to $V_g(Z)$

Pinsky assumes residual U is not a fcn. of Z.

... b) [separation] $U = W + V_g'$

$$V_g'(h,t) = V_g(h,t) - \langle V_g(h, Z(h,t))\rangle^{bin} - \phi(h, Z(h,t))$$

but $V_g(h,t)$ is unknown

Let $U(h,t) = \hat{W}(h,t) + \hat{V}_g'(h,t)$;
these are the $\hat{W}$, $\hat{V}_g'$ to be retrieved.

U can be decomposed into $\hat{W}(h,t) = a[Z(h,t)]\, U(h,t) = aU$
$\hat{V}_g'(h,t) = (1-a)U$

weight factors $a(h, Z(h,t))$

$$\rho(h) = \frac{\langle W V_g'\rangle^{(full)}}{\sigma_W \sigma_{V_g'}}$$ correlation is assumed a priori

Coefficients a are found so that $\dfrac{\langle \hat{W}\hat{V}_g'\rangle^{(full)}}{\sigma_W \sigma_{V_g'}} = \rho(h)$

and $(\langle U^2\rangle^{bin})^{1/2} = \Theta(h, Z)$. ???

For $\rho = 0$ $\langle \hat{W}^2\rangle^{(full)} + \langle \hat{V}_g'^2\rangle^{(full)} = \langle U^2\rangle^{(full)}$
the variances simply add.

All retrievals done independently as a function of height h.

Appendix B: $S_1 = \langle \theta^{-1}(h, Z(h,t))\rangle^{(full)}$ $\quad a(Z) = a_0/\Theta(Z)$
$S_2 = \langle \theta^{-2}(h, Z(h,t))\rangle^{(full)}$

¿ Then $\theta^p(h,Z)$ never appears in appendix B except in S_1 & S_2 ?

All sums $\langle\ \rangle = \langle\ \rangle^{(full)}$ are full, over all Z, not for particular Z bins.

$((U^2)'$ uncorr. to $\theta, \theta^{-1}, \theta^{-2})$

$$\langle \hat{W}^2\rangle = \langle a^2 U^2\rangle = a_0^2\langle U^2\theta^{-2}\rangle \doteq a_0^2\langle U^2\rangle S_2 \qquad (B1)$$

$$\langle \hat{V}_g'^2\rangle = \langle (1-a)^2 U^2\rangle = (1 - 2a_0 S_1 - a_0^2 S^2)\langle U^2\rangle \qquad (B2)$$

$$\langle \hat{W}\hat{V}_g'\rangle = \langle a(1-a)U^2\rangle = a_0(S_1 - a_0 S_2)\langle U^2\rangle \qquad (B3)$$

$$\text{correlation } \rho^2 = \frac{\langle \hat{W}\hat{V}_g'\rangle^2}{\langle \hat{W}^2\rangle\langle \hat{V}_g'^2\rangle} = \frac{(S_1 - a_0 S_2)^2}{S_2(1 - 2a_0 S_1 + a_0^2 S_2)} \quad \text{quadratic} \quad (B4)$$

$$\langle x\,y\rangle = \langle (\langle x\rangle + x')(\langle y\rangle + y')\rangle = \langle x\rangle\langle y\rangle + \cancel{\langle x'\langle y\rangle\rangle} + \cancel{\langle y'\langle x\rangle\rangle} + \langle x'y'\rangle$$

So in B1 what happened to $\langle (U^2)'(\theta^{-2})'\rangle$?

Consider this inner product as a function of Z:

$$\langle U^2\,\theta^{-2}\rangle^{(full)} = \frac{1}{N_z}\sum_z \langle (\langle U^2\rangle^{bin} + (U^2)')(\langle\theta^{-2}\rangle^{bin} + (\theta^{-2})')\rangle^{bin}$$

departures from Z bins

$$= \frac{1}{N_z}\sum_z \left\{\langle U^2\rangle^{bin}\langle\theta^{-2}\rangle^{bin} + \langle (U^2)'\cancel{(\theta^{-2})'}\rangle^{bin}\right\}$$

And $\theta' = (\theta^{-2})' = 0$ because $\theta \equiv \langle U^2\rangle^{bin}$

θ is a bin average — it has no fluctuations.

$$\text{so } \langle U^2\,\theta^{-2}\rangle^{(full)} = \frac{1}{N_z}\sum_z \left\{\langle U^2\rangle^{bin}\langle\theta^{-2}\rangle^{bin}\right\}$$

$$= \langle U^2\rangle^{(full)}\langle\theta^{-2}\rangle^{(full)}$$

$$= \langle U^2\rangle^{(full)} S_2$$

$\langle U^2\rangle^{bin}\langle\theta^{-2}\rangle^{bin} = 1$

$$\underbrace{\frac{1}{N_z}\sum_z}_{\text{weighted sum of bins}} \langle\ \rangle^{bin} = \langle\ \rangle^{full}$$

Try to be clearer:

$$\langle U^2\theta^{-2}\rangle^{full} = \frac{1}{N}\sum_z n_z\langle U^2\rangle^{bin}\langle\theta^{-2}\rangle^{bin} + \underbrace{\frac{1}{N}\sum_k (U^2)'\langle\theta^{-2}\rangle^{bin}}_{\text{each sum w/in a Z bin} = 0}$$

$$\langle x\,y\rangle = \mathrm{corr}(x,y)\,\sigma_x\sigma_y$$

$$\frac{1}{N}\sum_k \left(\langle U^2\rangle^{bin} + (U^2)'\right)\theta^{-2}$$

$$\langle x\,y\rangle = \langle x\rangle\langle y\rangle + \langle x'y'\rangle$$

perturbations from full.

$$\frac{1}{N}\sum_k (U^2\theta^{-2}) = \langle U^2\rangle\underbrace{\langle\theta^{-2}\rangle}_{S_2} + \underbrace{\langle (U^2)'(\theta^{-2})'\rangle}_{\text{uncorrelated so } \langle (U^2)'(\theta^{-2})'\rangle = 0}$$

full avgs.

$$\frac{1}{N}\sum\left[(1 - 2a_0\theta^{-1} + a_0^2\theta^{-2})U^2\right] = \langle 1 - 2a_0\theta^{-1} + a_0^2\theta^{-2}\rangle\langle U^2\rangle$$

$$+ \langle (2a_0\theta^{-1} + a_0^2\theta^{-2})'(U^2)'\rangle$$

Assume

Residual variance $(U^2)' = U^2 - \langle U^2\rangle$ uncorrelated to $\theta, \theta^{-1}, \theta^{-2}$ $\rightarrow$ neglect nonlinear products $\langle (U^2)'\,\theta^{-2}\rangle, \langle (U^2)'\,\theta^{-1}\rangle$

quadratic (B4) $\longrightarrow$

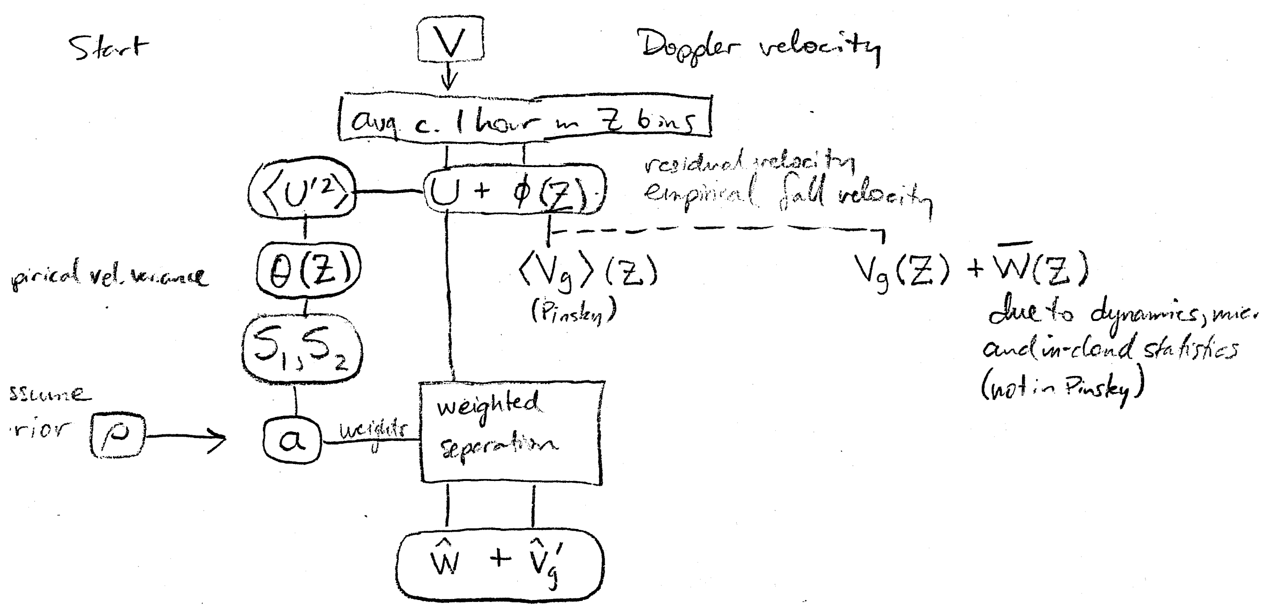
$$S_2^2(1-\rho^2)a_0^2 - 2S_1S_2(1-\rho^2)a_0 + (S_1^2 - S_2\rho^2) = 0 \qquad (B5)$$

solve by quadratic formula

$$a_0 = \frac{S_1}{S_2} - \frac{\rho}{S_2}\sqrt{\frac{S_2 - S_1^2}{1-\rho^2}} \qquad (B6)$$

for $\rho = 0$ $\qquad a_0 = \frac{S_1}{S_2} \qquad (B7)$

Flowchart for separation technique



$U = \hat{W} + V_g'$

Let $\begin{cases} \hat{W} = a(Z,h)\ U(h,t) \\ \hat{V}_g' = (1 - a(Z,h))\ U(h,t) \end{cases}$

$a(Z,h) = \underline{a_0(h)}\ \underline{\theta^{-1}(Z)}$ separation-of-variables product

Thus ① Compensating $var(U)$'s dependence on Z $[var(U) = \theta^2($

so that $var(\hat{W})$ is independent of Z.

The $\theta^{-1}(Z)$ factor in $a(Z,h)$ achieves this: $\hat{W} = a_0(h)\theta^{-1}(Z)U$

$var(\hat{W}(h)) = a_0(h)\ var(\theta^{-1}(Z)U(h,t))$ full variance

$var_Z(\hat{W}(h)) = a_0(h)\theta^{-2}(Z)\ var_Z(U(h,t))$ take var over cond. Z

$= a_0(h)\frac{\cancel{\theta(Z)}}{\cancel{\theta^3(Z)}} = a_0(h^2)$ □ does not depend on Z. ✓

② The full correlation between $\hat{W}(h,t)$ and $\hat{V}_g'(h,t)$ is as desired

Appendix B solves for $a_0(h)$ so that this condition is met.