



De Mathematics Competitions

1st Annual

DMC 8 A

Thursday, September 9, 2021



INSTRUCTIONS

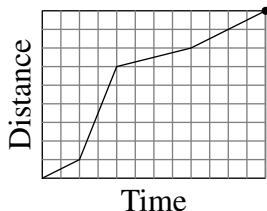
1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.
2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the DMC 8 Answer Form with a keyboard. Check the keys for accuracy and erase errors and stray marks completely.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will not ask you to record your name and other information on the answer sheet.
8. You will have 40 minutes to complete the competition once you start the test.
9. When you finish the competition, don't sign your name in the space provided on the answer sheet.

The DMC Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

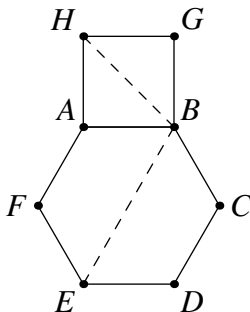
The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

1. Brenda's daily training starts at 6:10 PM and ends at 7:45 PM. If Brenda takes a 20-minute break in the middle of her training, for how many minutes does she train?
- (A) 65 (B) 75 (C) 85 (D) 95 (E) 105
2. What is the value of the expression $\frac{1^3 + 2^2 + 3^1}{1^2 + 2^1}$?
- (A) 1 (B) $\frac{6}{5}$ (C) 2 (D) $\frac{5}{2}$ (E) $\frac{8}{3}$
3. Jackie and Joe are shopping for school supplies at a store where pencils cost \$0.10 each, and pens cost \$0.25 each. Given that Jackie buys five pencils and two pens, and Joe spends the same amount of money as Jackie by buying only pens, how many pencils and pens did Jackie and Joe buy all together?
- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14
4. Johnny has a recipe for hot chocolate which requires 2 grams of cocoa powder and 5 grams of milk. After adding 5 grams of milk, Johnny accidentally adds 3 grams of cocoa powder, so he adds additional milk in the same proportion as the recipe to balance the cocoa powder out. How many grams of additional milk does he add?
- (A) 1 (B) 2.5 (C) 5 (D) 7.5 (E) 10
5. Let $P = 2^2 + 3^2 + 4^2 + \cdots + 10^2$ and $Q = 1^2 + 2^2 + 3^2 + \cdots + 9^2$. What is $P - Q$?
- (A) 0 (B) 1 (C) 2 (D) 99 (E) 100
6. Mike has a bag of sweets consisting of 30% pieces of chocolate, 45% pieces of toffee, and the rest pieces of caramel. After giving half of his caramel to his wife, he has 15 pieces of caramel left. How many pieces of toffee does he have?
- (A) 27 (B) 36 (C) 54 (D) 60 (E) 120
7. A right triangle with a hypotenuse of length 5 inches and a leg of length 3 inches has the same area as a square. What is the side length in inches of the square?
- (A) 2 (B) $\sqrt{6}$ (C) 3 (D) $2\sqrt{3}$ (E) 4

8. One day, Joseph hiked through a forest for two hours. The graph below shows his hike, indicating the general time and distance hiked. Which of the following represents a time m in minutes after the hike started where Joseph is moving the fastest?



- (A) 22 (B) 44 (C) 66 (D) 88 (E) 110
9. Given that $a = 0.78$ and $b = 78$, which of the following is equal to 7800?
- (A) $\frac{a}{b^2}$ (B) $\frac{a^2}{b}$ (C) $\frac{b^2}{a}$ (D) $\frac{b^3}{a}$ (E) $\frac{b^3}{a^2}$
10. At a station, 12 people board the front car and 40 people board the rear car of a two-car train. Later, some people switch from the rear car to the front car. At the next station, 16 people exit the front car and 20 people exit the rear car, after which there is an equal number of people in both cars. How many people switched cars?
- (A) 4 (B) 6 (C) 8 (D) 10 (E) 12
11. In the diagram below, square $ABGH$ lies outside of regular hexagon $ABCDEF$. What is the degree measure of $\angle EBH$?



- (A) 90 (B) 95 (C) 100 (D) 105 (E) 110

12. While driving to work, Zach drove at a constant rate of 60 miles per hour on the highway and a constant rate of 40 miles per hour while driving through the local streets. If Zach drove for 160 miles, and he drove for twice as long on the highway as on the local streets, for how many minutes did he drive on the highway?

(A) 90 (B) 105 (C) 120 (D) 135 (E) 150

13. Daniel has 3 coins, each of which is either a penny (a 1-cent coin), a nickel (a 5-cent coin), or a dime (a 10-cent coin). If Daniel trades one of his coins for a quarter (a 25-cent coin), the total value of his 3 coins will then be twice the total value of his original 3 coins. What is the least possible total value in cents of his original 3 coins?

(A) 14 (B) 15 (C) 17 (D) 20 (E) 24

14. What is the value of the expression

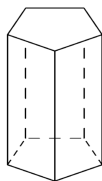
$$\left(\frac{2^2}{1+3} - \frac{4^2}{3+5}\right) + \left(\frac{6^2}{5+7} - \frac{8^2}{7+9}\right) + \cdots + \left(\frac{98^2}{97+99} - \frac{100^2}{99+101}\right)?$$

(A) -100 (B) -50 (C) -25 (D) -10 (E) -5

15. Elaine takes a positive integer and divides it by 12, and the remainder is 4. Without knowing Elaine's integer, Hans infers that Elaine could have also divided by one of the following numbers and get a remainder of 4 with certainty. Which number is this?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

16. A pentagonal prism has two pentagonal faces as bases and five rectangles as faces around the prism. Billy wants to color each of the seven faces of a pentagonal prism so that any two faces that share an edge have different colors. What is the smallest possible number of distinct colors that Billy needs to do this?



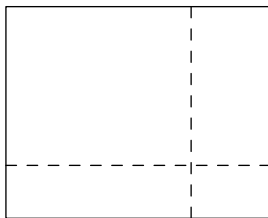
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

17. Suppose that $a \clubsuit b$ means $a^2 - ab + b^2$. What is the value of x^2 if

$$(x+4) \clubsuit (x-4) = 75?$$

- (A) 18 (B) 21 (C) 24 (D) 25 (E) 27

18. A rectangle is cut using only vertical or horizontal cuts, like the cuts shown below. What is the smallest possible number of cuts needed to cut the rectangle into 36 pieces?



- (A) 5 (B) 10 (C) 12 (D) 18 (E) 35

19. A certain four-digit number satisfies the following properties:

- The thousands digit is equal to 4.
- The sum of the last two digits is 17.
- The number is divisible by 18.

What is the sum of the hundreds digit and the ones digit of this number?

- (A) 6 (B) 8 (C) 10 (D) 12 (E) 14

20. Three boys named Ryu, Kai, and Naruto, as well as two girls named Haruka and Ayaka, sit in a straight line in a randomly chosen order. What is the probability that exactly one boy and one girl are sitting in between Ryu and Kai?

- (A) $\frac{1}{15}$ (B) $\frac{1}{10}$ (C) $\frac{2}{15}$ (D) $\frac{1}{6}$ (E) $\frac{1}{5}$

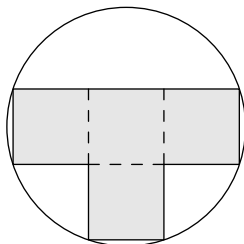
21. Kyle has five blocks in a row, as shown below, where each letter represents a number, and not all five numbers are equal.



Kyle notices that the sums of the numbers on any three consecutive blocks are equal. In which of the following arrangements of the five blocks is it necessarily true that the sums of the numbers on any three consecutive blocks are equal?

- (A) (B)
 (C) (D)
 (E)

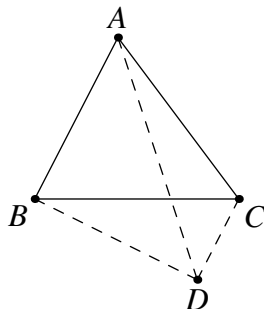
22. A T-shaped block is inscribed in a circle, as shown below. The T-shaped block is made up of four congruent squares. Given that the area of the circle is 78π , which of the following is closest to the area of the T-shaped block?



- (A) 105 (B) 110 (C) 115 (D) 120 (E) 125
23. At one of Sarah's parties, each man shook hands exactly once with everyone other than his own wife, and no two women shook hands with each other. Given that 6 married couples attended, how many handshakes took place among these 12 people?
- (A) 15 (B) 36 (C) 45 (D) 60 (E) 66
24. Don and Doug each draw 5 cards from a deck of 10 cards numbered from 1 to 10. The sum of Don's cards is 1 less than the sum of Doug's cards, and Doug drew all of the perfect squares. What is the sum of all cards Doug could **not** have drawn?

- (A) 17 (B) 20 (C) 27 (D) 28 (E) 32

25. In triangle ABC , let D be the point on the opposite side of line BC as A such that $\overline{AB} \parallel \overline{CD}$. Given that $AB = BD = 8$, $BC = 9$, and $CD = 4$, what is AD^2 ?



- (A) 126 (B) 130 (C) 134 (D) 138 (E) 142



DMC 8 A

DO NOT OPEN UNTIL THURSDAY, September 9, 2021

****Administration on an earlier date will disqualify your results.****

- All the information needed to administer this exam is not contained in the non-existent DMC 8 Teacher's Manual. PLEASE READ THE MANUAL BEFORE THURSDAY, SEPTEMBER 9, 2021.
 - Send **DeToasty3**, **karate7800**, and **pandabearcat** a private message submitting your answers to the DMC 8. AoPS is the only way to submit your answers.
 - The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.
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For more information about the DMC and our other competitions, please visit
<https://detoasty3.github.io/dmc.html>.

Questions and comments about this competition should be sent to:

DeToasty3, karate7800, and pandabearcat.

The problems and solutions for this DMC 8 were prepared by the DMC Editorial Board under the direction of:

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