KMMC

KARATE MASTERS MATHEMATICS COMPETITIONS

Karate Masters Mathematics Competitions

2nd Annual

KMMC 8 A

Thursday, September 9, 2021

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.

2. This is a twenty-five question multiple choice test. For each question, only one answer choice is correct.

3. Mark your answer to each problem on the KMMC 8 Answer Form with a keyboard. Check the keys for accuracy and erase errors and stray marks completely.

4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.

5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.

6. Figures are not necessarily drawn to scale.

7. Before beginning the competition, your competition manager will not ask you to record your name and other information on the answer sheet.

8. You will have 40 minutes to complete the competition once you start the test.

9. When you finish the competition, don't sign your name in the space provided on the answer sheet.

1. Karate's daily training starts at 6:10 PM and ends at 7:45 PM. If Karate takes a 20-minute break in the middle of his training, for how many minutes does he train?

   (A) 65   (B) 75   (C) 85   (D) 95   (E) 105

2. What is the value of the expression $\dfrac{1^3 + 2^2 + 3^1}{1^2 + 2^1}$?

   (A) 1   (B) $\dfrac{6}{5}$ (C) 2   (D) $\dfrac{5}{2}$ (E) $\dfrac{8}{3}$

3. Karate and Judo are shopping for school supplies at a store where pencils cost \$0.10 each, and pens cost \$0.25 each. Given that Karate buys five pencils and two pens, and Judo spends the same amount of money as Karate by buying only pens, how many pencils and pens did Karate and Judo buy all together?

   (A) 10   (B) 11   (C) 12   (D) 13   (E) 14

4. Karate has a recipe for hot chocolate which requires 2 grams of cocoa powder and 5 grams of milk. After adding 5 grams of milk, Karate accidentally adds 3 grams of cocoa powder, so he adds additional milk in the same proportion as the recipe to balance the cocoa powder out. How many grams of additional milk does he add?

   (A) 1   (B) 2.5   (C) 5   (D) 7.5   (E) 10

5. Let $P = 2^2 + 3^2 + 4^2 + \cdots + 10^2$ and $Q = 1^2 + 2^2 + 3^2 + \cdots + 9^2$. What is $P - Q$?

   (A) 0   (B) 1   (C) 2   (D) 99   (E) 100

6. Karate has a bag of sweets consisting of 30% pieces of chocolate, 45% pieces of toffee, and the rest pieces of caramel. After giving half of his caramel to his wife, he has 15 pieces of caramel left. How many pieces of toffee does he have?

   (A) 27   (B) 36   (C) 54   (D) 60   (E) 120

7. A right triangle with a hypotenuse of length 5 inches and a leg of length 3 inches has the same area as a square. What is the side length in inches of the square?

   (A) 2   (B) $\sqrt{6}$ (C) 3   (D) $2\sqrt{3}$ (E) 4

8. One day, Karate hiked through a forest for two hours. The graph below shows his hike, indicating the general time and distance hiked. Which of the following represents a time *m* in minutes after the hike started where Karate is moving the fastest?

Distance / Time
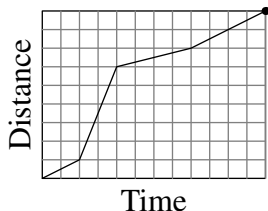
(A) 22   **(B)** 44   **(C)** 66   **(D)** 88   **(E)** 110

9. Given that $a = 0.78$ and $b = 78$, which of the following is equal to 7800?

(A) $\dfrac{a}{b^2}$ **(B)** $\dfrac{a^2}{b}$ **(C)** $\dfrac{b^2}{a}$ **(D)** $\dfrac{b^3}{a}$ **(E)** $\dfrac{b^3}{a^2}$

10. At a station, 12 people board the front car and 40 people board the rear car of a two-car train. Later, some people switch from the rear car to the front car. At the next station, 16 people exit the front car and 20 people exit the rear car, after which there is an equal number of people in both cars. How many people switched cars?

(A) 4   **(B)** 6   **(C)** 8   **(D)** 10   **(E)** 12

11. In the diagram below, square $ABGH$ lies outside of regular hexagon $ABCDEF$. What is the degree measure of $\angle EBH$?
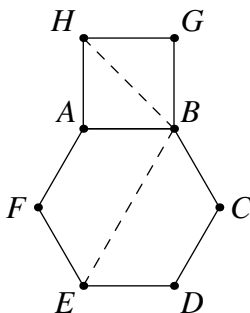
(A) 90   **(B)** 95   **(C)** 100   **(D)** 105   **(E)** 110

12. While driving to work, Karate drove at a constant rate of 60 miles per hour on the highway and a constant rate of 40 miles per hour while driving through the local streets. If Karate drove for 160 miles, and he drove for twice as long on the highway as on the local streets, for how many minutes did he drive on the highway?

(A) 90   (B) 105   (C) 120   (D) 135   (E) 150

13. Karate has 3 coins, each of which is either a penny (a 1-cent coin), a nickel (a 5-cent coin), or a dime (a 10-cent coin). If Karate trades one of his coins for a quarter (a 25-cent coin), the total value of his 3 coins will then be twice the total value of his original 3 coins. What is the least possible total value in cents of his original 3 coins?

(A) 14   (B) 15   (C) 17   (D) 20   (E) 24

14. What is the value of the expression

$$\left(\frac{2^2}{1+3} - \frac{4^2}{3+5}\right) + \left(\frac{6^2}{5+7} - \frac{8^2}{7+9}\right) + \cdots + \left(\frac{98^2}{97+99} - \frac{100^2}{99+101}\right)?$$

(A) −100   (B) −50   (C) −25   (D) −10   (E) −5

15. Karate takes a positive integer and divides it by 12, and the remainder is 4. Without knowing Karate's integer, Judo infers that Karate could have also divided by one of the following numbers and get a remainder of 4 with certainty. Which number is this?

(A) 6   (B) 7   (C) 8   (D) 9   (E) 10

16. A pentagonal prism has two pentagonal faces as bases and five rectangles as faces around the prism. Karate wants to color each of the seven faces of a pentagonal prism so that any two faces that share an edge have different colors. What is the smallest possible number of distinct colors that Karate needs to do this?
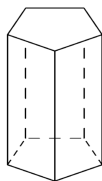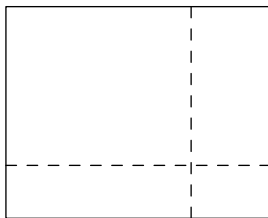

(A) 3   (B) 4   (C) 5   (D) 6   (E) 7

17. Suppose that $a \clubsuit b$ means $a^2 - ab + b^2$. What is the value of x^2 if

$$(x+4) \clubsuit (x-4) = 75?$$

    (A) 18   **(B)** 21   **(C)** 24   **(D)** 25   **(E)** 27

18. A rectangle is cut using only vertical or horizontal cuts, like the cuts shown below. What is the smallest possible number of cuts needed to cut the rectangle into 36 pieces?

    (A) 5   **(B)** 10   **(C)** 12   **(D)** 18   **(E)** 35

19. A certain four-digit number satisfies the following properties:

   - The thousands digit is equal to 4.
   - The sum of the last two digits is 17.
   - The number is divisible by 18.

   What is the sum of the hundreds digit and the ones digit of this number?

    (A) 6   **(B)** 8   **(C)** 10   **(D)** 12   **(E)** 14

20. Three boys named Karate, Judo, and Naruto, as well as two girls named Haruka and Ayaka, sit in a straight line in a randomly chosen order. What is the probability that exactly one boy and one girl are sitting in between Karate and Judo?

   **(A)** $\dfrac{1}{15}$ **(B)** $\dfrac{1}{10}$ **(C)** $\dfrac{2}{15}$ **(D)** $\dfrac{1}{6}$ **(E)** $\dfrac{1}{5}$

21. Karate has five blocks in a row, as shown below, where each letter represents a number, and not all five numbers are equal.

$$\boxed{A}\,\boxed{B}\,\boxed{C}\,\boxed{D}\,\boxed{E}$$

Karate notices that the sums of the numbers on any three consecutive blocks are equal. In which of the following arrangements of the five blocks is it necessarily true that the sums of the numbers on any three consecutive blocks are equal?

(A) $\boxed{A}\,\boxed{C}\,\boxed{D}\,\boxed{B}\,\boxed{E}$ (B) $\boxed{B}\,\boxed{A}\,\boxed{C}\,\boxed{D}\,\boxed{E}$

(C) $\boxed{C}\,\boxed{A}\,\boxed{B}\,\boxed{E}\,\boxed{D}$ (D) $\boxed{D}\,\boxed{E}\,\boxed{C}\,\boxed{A}\,\boxed{B}$

(E) $\boxed{E}\,\boxed{A}\,\boxed{B}\,\boxed{D}\,\boxed{C}$

22. A T-shaped block is inscribed in a circle, as shown below. The T-shaped block is made up of four congruent squares. Given that the area of the circle is 78π, which of the following is closest to the area of the T-shaped block?
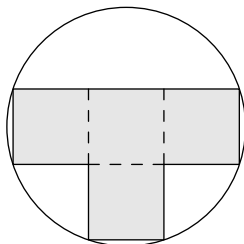
(A) 105   (B) 110   (C) 115   (D) 120   (E) 125

23. At one of Karate's parties, each man shook hands exactly once with everyone other than his own wife, and no two women shook hands with each other. Given that 6 married couples attended, how many handshakes took place among these 12 people?

(A) 15   (B) 36   (C) 45   (D) 60   (E) 66

24. Karate and Judo each draw 5 cards from a deck of 10 cards numbered from 1 to 10. The sum of Karate's cards is 1 less than the sum of Judo's cards, and Judo drew all of the perfect squares. What is the sum of all cards Judo could **not** have drawn?

(A) 17   (B) 20   (C) 27   (D) 28   (E) 32

25. In triangle ABC, let D be the point on the opposite side of line BC as A such that $\overline{AB} \parallel \overline{CD}$. Given that $AB = BD = 8$, $BC = 9$, and $CD = 4$, what is AD^2?
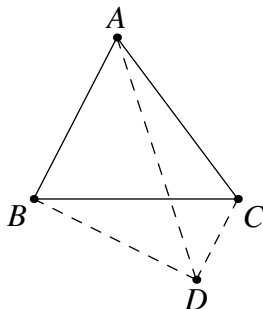
(A) 126   **(B)** 130   **(C)** 134   **(D)** 138   **(E)** 142

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DO NOT OPEN UNTIL THURSDAY, September 9, 2021

Administration on an earlier date will disqualify your results.

- All the information needed to administer this exam is not contained in the non-existent KMMC 8 Teacher's Manual. PLEASE READ THE MANUAL BEFORE THURSDAY, SEPTEMBER 9, 2021.

- Send **DeToasty3**, **karate7800**, and **pandabearcat** a private message submitting your answers to the KMMC 8. AoPS is the only way to submit your answers.

- The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

For more information about the KMMC and our other competitions, please visit
Wait, we don't have a website!

Questions and comments about this competition should be sent to:

DeToasty3, **karate7800**, and **pandabearcat**.

The problems and solutions for this KMMC 8 were prepared by the KMMC Editorial Board under the direction of:

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