ADMC

Individual Round 1

Trial Competition

Format:

- There will be 10 questions to be solved in 45 minutes.
- Each correct answer is worth 1 point. Each incorrect/blank answer is worth 0 points.
- The difficulty will range from early AMC 12 to mid AIME.

Rules/Submitting:

- Your only tools are a writing utensil and blank scrap paper. You may not use graph paper, protractors, rulers, calculators, or electronic devices (except for one to access the rounds).
- Every answer is an integer between 1 and $2^{21} = 2097152$, inclusive. While taking the round, please ensure that you do not make any typos in any of your answers.
- Each round will be posted on the website at the time indicated in the schedule. Please actively check and refresh the website should any errata/changes be posted during a round.
- If you find any ambiguities or flaws in a problem in a given round, please email us at *after the round is finished* at admccontest@gmail.com with the round and problem number in question. If we decide that the problem is flawed, then we will give everyone full credit for that problem. Otherwise, please use your best judgment to determine what the problem is asking while taking the round.

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Individual Round 1
45 Minutes

R1-1. If n is a positive integer such that $n \times 3^5 = 3^7 - 3^5$, find n.

R1-2. Given a right triangle with legs of lengths 5 and 6, a square is drawn with one side as its hypotenuse such that the triangle is completely inside the square. Find the area of the region inside the square but outside the triangle.

R1-3. Let $f(x) = x^2 - x - 3$ and $g(x) = 2x + 3$ for all real numbers x. Find the absolute value of the sum of all real values of x such that $f(g(x)) = g(f(x))$.

R1-4. In regular hexagon $ABCDEF$, diagonals $\overline{AC}$ and $\overline{BF}$ intersect at a point G. If the area of $\triangle ABG$ is 2, find the area of pentagon $CDEFG$.

R1-5. Let $b > 6$ be an integer. There exist base-b and base-$(b+1)$ numbers such that

$$8 \gcd(600_b, 660_b) = 45 \gcd(100_{b+1}, 110_{b+1}).$$

Find b.

R1-6. 8 people randomly split into 2 groups of four to dance. After that, the 8 people randomly split into 4 pairs of two to talk. The probability that exactly 2 of the 4 pairs contain two people who have danced in the same group of four is $\frac{m}{n}$ for relatively prime positive integers m and n. Find $100m + n$.

R1-7. In trapezoid $ABCD$ with $\overline{AB} \parallel \overline{CD}$, $AB = 4$, and $AD = BC = 5$, let the angle bisector of $\angle ADC$ intersect the diagonal $\overline{AC}$ at a point P. If line BP intersects the side $\overline{CD}$ at a point Q such that $CQ = 8$, find the area of trapezoid $ABCD$.

R1-8. Richard thinks of a positive integer n and writes the base ten representations of $n!$ and $(n+1)!$ on a board. He then erases the zeroes to the right of the last nonzero digit of each number (if any exist), resulting in two numbers a and b. If one of a and b is 4 times the other, find the sum of all possible values of n less than 1000.

R1-9. Let ABC be a right triangle with right angle at A and side lengths $AC = 8$ and $BC = 16$. The lines tangent to the circumcircle of $\triangle ABC$ at points A and B intersect at D. Let E be the point on side $\overline{AB}$ such that $\overline{AD} \parallel \overline{CE}$. Then DE^2 can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

R1-10. Let $P(x)$ be a polynomial with degree 3 and real coefficients such that the coefficient of the x^3 term is 1, and $P(x)$ has roots a, b, and c that satisfy

$$-\frac{(a+b)(b+c)(c+a)}{2022} = abc = 2021.$$

Find the minimum possible value of $|P(1)|$.