2020 DMC 10 Solutions

De Mathematics Competitions

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Answer Key:

1. (E)	2. (D)	3. (E)	4. (D)	5. (C)
6. (B)	7. (B)	8. (A)	9. (B)	10. (C)
11. (A)	12. (A)	13. (D)	14. (A)	15. (A)
16. (D)	17. (D)	18. (E)	19. (E)	20. (B)
21. (C)	22. (A)	23. (D)	24. (D)	25. (C)

Solutions:

1. *Proposed by DeToasty3*

   Answer (E): Evaluating the expression at hand, we get

$$\begin{aligned}\frac{(2^0-2^1)^{2020}}{(2\cdot 0+2^0)^{2021}} &= \frac{(1-2)^{2020}}{(0+1)^{2021}}\\ &= \frac{(-1)^{2020}}{1^{2021}}\\ &= \frac{1}{1}\\ &= 1.\end{aligned}$$

   Our answer is $\boxed{\textbf{(E) } 1}$. ■

2. *Proposed by DeToasty3*

   Answer (D): Since the number of people is always a whole number, we must have that the number of people in the club is a whole number multiple of $9+5=14$. Since 98 is $14\cdot 7$, but 99 is not a whole number multiple of 14, our answer is $\boxed{\textbf{(D) } 98}$. ■

3. *Proposed by jayseemath*

   Answer (E): We see that the figure is composed of 18 unit squares and 6 quarter circles. We find that the area of one unit square is $1 \cdot 1 = 1$ and the area of one quarter circle is $\frac{1}{4} \cdot \pi \cdot 1^2 = \frac{\pi}{4}$, so our answer is

   $$18(1) + 6\left(\frac{\pi}{4}\right) = \boxed{\textbf{(E) } 18 + \frac{3\pi}{2}},$$

   as desired. ■

4. *Proposed by DeToasty3*

   Answer (D): Let x be the number of dogs, and let y be the number of dugs. We get the equation

   $$4x + 3y = 61.$$

   We want $61 - 3y$ to be an integer multiple of 4, where y is as small as possible. Testing from 0 upwards, we find that $y = 3$ gives $x = 13$, which is an integer, so our answer is $\boxed{\textbf{(D) } 3}$. ■

5. *Proposed by DeToasty3*

   Answer (C): Note that we can remove the one extra pizza slice because it does not affect the problem. Then, we have 24 slices of pizza to evenly distribute to n people, which means that we have to find the number of divisors of 24. We see that $24 = 2^3 \cdot 3^1$, so the number of divisors is $(3+1)(1+1) = 8$. However, we have to subtract one case for when $n = 1$, so our answer is $8 - 1 = \boxed{\textbf{(C) } 7}$. ■

6. *Proposed by DeToasty3*

   Answer (B): The key observation is that the line will always pass through the point $(0, 0)$. Relabel the elements as $a \equiv x$ and $b \equiv y$, respectively. The equation of the line can be expressed as

   $$y - b = \frac{b}{a}(x - a),$$

   and plugging in $x = 0$ and $y = 0$ gives us $0 = 0$, a true statement. Then, the slope of the line must be $\frac{2020-0}{1010-0} = 2$. We see that there are two possibilities for (a, b): $(1, 2)$ and $(2, 4)$. There are 12 possibilities, so our answer is $\frac{2}{12} = \boxed{\textbf{(B) } \frac{1}{6}}$. ■

7. *Proposed by DeToasty3*

   Answer (B): Note that since Richard has the most trading cards, it would be optimal for him to give away two trading cards each minute. Also, by the end, each person should have $\frac{17+20+26}{3} = 21$ trading cards. After three minutes of Richard giving 2 trading cards, the number of trading cards that Anthony, Daniel, and Richard have will be 20, 23, and 20, respectively. Then, if Daniel gives away 2 trading cards in the fourth minute, each person will end up with 21 trading cards. We can check to see that 4 minutes is optimal because Anthony starts from 17 trading cards and must go to 21, which must take at least 4 minutes. Therefore, our answer is $\boxed{\textbf{(B) } 4}$. ■

8. *Proposed by DeToasty3*

Answer (A): We will count the complement: the probability that the triangle is obtuse. Since there are infinitely many locations for point C, the probability that the triangle is right is negligible. We will do casework on which angle is obtuse. Since there is at most 1 obtuse angle in any triangle, there is no overlap in these cases.

For $\angle C$ to be obtuse, C must lie on minor arc AB. Otherwise, $\angle C$ is acute. Let D be the point on the circle such that $\angle BAD = 90^\circ$. In addition, let E be the point on the circle such that $\angle ABE = 90^\circ$. If C lies on minor arc AD, then $\angle A$ is obtuse. Otherwise, $\angle A$ is acute. Similarly, $\angle B$ is obtuse when C lies on minor arc BE and acute otherwise.

Based on our cases, we have ruled out all possible locations for C except minor arc DE. This has the same arc length as minor arc AB or 70°. Therefore, the probability that $\triangle ABC$ is acute is $\boxed{\textbf{(A) } \frac{7}{36}}$. ■

9. *Proposed by DeToasty3*

Answer (B): We can set up the following equation:

$$\frac{220}{a} + \frac{180}{\frac{3}{4}a} = \frac{400}{b}$$

Solving, we get

$$\begin{aligned}
\frac{220}{a} + \frac{180}{\frac{3}{4}a} &= \frac{400}{b} \\
\frac{220}{a} + \frac{240}{a} &= \frac{400}{b} \\
\frac{460}{a} &= \frac{400}{b} \\
460b &= 400a \\
\frac{a}{b} &= \frac{460}{400} = \frac{23}{20}.
\end{aligned}$$

Thus, our answer is $\frac{23}{20} = \boxed{\textbf{(B) } 1.15}$. ■

10. *Proposed by DeToasty3*

Answer (C): Since we want to maximize the value of n, we let $p = 2$. Then, q and r must be odd primes. The sum of the squares of two odd numbers always leaves a remainder of 2 when divided by 4, so it adds one to our count of factors of 2. In total, we have 2 factors of 2 from the p^2 and 1 more factor from the $q^2 + r^2$, so our answer is $\boxed{\textbf{(C) } 3}$. ■

11. *Proposed by richy*

Answer (A): Substituting, we get the equation

$$(2x+3)^2 - (2x+3) - 3 = 2(x^2 - x - 3) + 3.$$

Simplifying, we get

$$\begin{aligned}
(2x+3)^2 - (2x+3) - 3 &= 2(x^2 - x - 3) + 3 \\
4x^2 + 12x + 9 - 2x - 3 - 3 &= 2x^2 - 2x - 6 + 3 \\
4x^2 + 10x + 3 &= 2x^2 - 2x - 3 \\
2x^2 + 12x + 6 &= 0 \\
x^2 + 6x + 3 &= 0 \\
(x+3)^2 &= 6 \\
x + 3 &= \pm\sqrt{6} \\
x &= -3 \pm \sqrt{6}
\end{aligned}$$

We see that both $-3+\sqrt{6}$ and $-3-\sqrt{6}$ are real numbers, so the sum of all real values of x is $-3+\sqrt{6}+(-3-\sqrt{6}) = \boxed{\textbf{(A) } -6}$. ■

12. *Proposed by DeToasty3*

Answer (A): Note that for any three non-collinear points A, B, and C, the circumcircle of $\triangle ABC$ is uniquely determined. Therefore, any point not on line AB is on the circumference of some circle passing through both A and B.

Suppose that A, B, and C are collinear. Any line can intersect a circle at most 2 times. Since A and B are fixed, they are the two intersects of line AB and any circle passing through A and B. If C is distinct from A and B, then line AB would have to intersect a circle three times, which cannot happen. Therefore, we need $C = A$ or $C = B$.

In conclusion, the region of points not on the circumference of any circle in $\mathcal{S}$ is $\boxed{\textbf{(A) } \text{Every point on line } AB \text{ excluding } A \text{ and } B}$. ■

13. *Proposed by DeToasty3*

Answer (D): Suppose each of the lazy students pass with probability $p \neq 0$. We are given that 6 students passed, which means 3 of the lazy students passed. This occurs with probability $\binom{7}{3} \cdot p^3 \cdot (1-p)^4$ by binomial probabilities. We must find the probability that Tomo passed given that exactly 3 lazy students passed.

We have $\binom{6}{2}$ ways to choose two lazy students besides Tomo. Therefore, the probability that Tomo is among the 3 lazy students that passed is $\binom{6}{2} \cdot p^3 \cdot (1-p)^4$.

Finally, the desired probability is $\frac{\binom{6}{2} \cdot p^3 \cdot (1-p)^4}{\binom{7}{3} \cdot p^3 \cdot (1-p)^4} = \boxed{\textbf{(D) } \frac{3}{7}}$. ■

14. *Proposed by richy*

Answer (A): Suppose we write the integer as $\overline{a_n a_{n-1} \dots a_1 a_0}$, where the a_k is a digit for $0 \leq k \leq n$. Note that $9 \equiv 1 \pmod 8$, so $9^n \equiv 1 \pmod 8$ for all integers n. Note that for a nonnegative integer N, we have

$$N = a_0 + 9a_1 + 81a_2 + \cdots + 9^n a_n \equiv a_0 + a_1 + a_2 + \cdots + a_n \pmod 8.$$

Consequently, the sum of the digits of N in base-nine is congruent to $N \pmod 8$, so the sum of the digits of N in base-nine is divisible by 8 if and only if N is divisible by 8. Thus, n must be the 2020th positive multiple of 8, so $n = 2020 \cdot 8 = 16160$ and our answer is $1 + 6 + 1 + 6 + 0 = \boxed{\textbf{(A) } 14}$. ■

15. *Proposed by richy*

Answer (A): First, notice that $\triangle BAC$ is a $30-60-90$ triangle. Notice that we can obtain the following through an angle chase: $\angle ABO_1 = 30^\circ$ due to $\triangle ABD$ being an equilateral triangle, and $\angle BO_2C = 2 \cdot (180^\circ - \angle BDC) = 120^\circ$, so the isosceles triangle $\triangle BO_2C$ tells us that $\angle CBO_2 = 30^\circ$. Thus, we deduce that $\angle O_1BO_2 = 90^\circ$. Additionally, we can calculate that BO_1 is the circumradius of a unit equilateral triangle which is $\frac{1}{\sqrt{3}}$, while $BO_2 = O_2D = O_2C = 1$. So, the area of BO_1O_2 is $\boxed{\textbf{(A) } \frac{\sqrt{3}}{6}}$. ■

16. *Proposed by nikenissan*

Answer (D): Observe that $8^3 = 512$, which is greater than 476, so the largest of the three integers has to be at most 7. Suppose that a is divisible by 3. Remember that squares of integers are always 0 or 1 modulo 3. Then, by the second equation, b and c would both have to be divisible by 3. However, this contradicts the first equation. Therefore, none of a, b, and c may be divisible by 3. Assume that 7 is the largest of the three integers. Then, the other two integers must be 2 and 5 in some order to satisfy $a+b+c=14$. Since 2, 5, and 7 also satisfy the second and third conditions, we have our three integers. Finally, we get that

$$\frac{1}{2}+\frac{1}{5}+\frac{1}{7}=\frac{59}{70},$$

so our answer is $m+n=\boxed{\textbf{(D) } 129}$. ■

17. *Proposed by DeToasty3*

Answer (D): Let the four people in one group be A_1, A_2, A_3, A_4, and let the four people in the other group be B_1, B_2, B_3, B_4. Our condition requires two of the four pairs to be some combination of an A and a B, and the other two pairs to be two A's and two B's, respectively. For the pair of two A's, there are $\binom{4}{2}=6$ ways to choose which two subscripts to choose, and the same goes for the pair of two B's. Then, for the other two pairs, there are two ways for the pairs to be matched up. Therefore, the total number of ways to satisfy the requirements is $6\cdot 6\cdot 2=72$. In total, there are $\frac{\binom{8}{2}\cdot\binom{6}{2}\cdot\binom{4}{2}\cdot\binom{2}{2}}{4!}=105$ ways to split the eight people into four pairs, so our answer is $\frac{72}{105}=\boxed{\textbf{(D) } \frac{24}{35}}$. ■

18. *Proposed by DeToasty3*

Answer (E): Let $R=11$, O be the center of the sphere, and P_1 and P_2 be the two planes in question. Then, let P_3 be the plane perpendicular to both P_1 and P_2 and passing through O. Take the cross section of the intersection of P_3 and the sphere.

The cross section becomes a circle with radius 11 and center O with two perpendicular chords representing the planes. Suppose one of the chords intersects the circle at A and B and the other chord intersects the circle at C and D, where D is on the same side of AB as O. Assume that AB corresponds to the image of P_1 and CD corresponds to the image of P_2. Finally, let E be the midpoint of AB, F be the midpoint of CD, and G be the point where the perpendicular chords intersect.

Since the region of P_1 inside the sphere is a circle of area 108π, $AE=\sqrt{108}$. Similarly, $CF=\sqrt{94}$. By Pythagorean Theorem, $EO=\sqrt{AO^2-AE^2}=\sqrt{13}$. Similarly, $FO=\sqrt{CO^2-CF^2}=\sqrt{27}$. Since $EGFO$ is a rectangle, $GO=EF=\sqrt{EO^2+OF^2}=\sqrt{40}$. Finally, let P_4 be the plane perpendicular to P_3 and containing O and G. Take the cross section of the intersection of P_4 and the sphere. The line segment inside the sphere where P_1 and P_2 intersect becomes a chord of the circle in this cross section. The length of this line segment is $2\sqrt{R^2-OG^2}=\boxed{\textbf{(E) } 18}$. ■

19. *Proposed by DeToasty3*

Answer (E): Rewrite the given condition as

$$a+(a+1)+(a+2)+\cdots+(a+n-1)=p \implies na+\frac{n(n-1)}{2}=p.$$

Factoring out n and multiplying both sides of the equation by 2, we get

$$n(2a+n-1) = 2p \implies n(a+27) = 2p.$$

From here, we have three cases for each $n \in \{2, p, 2p\}$:

Case 1. $n = 2$. Then, $a+27$ must be p. Note that the condition $a+n = 28$ forces $a = 26$, and $26+27 = 53$, which is a prime. Therefore, $a = 26$ is a possible value of a.

Case 2. $n = p$. Then, $a+27$ must be 2. This would mean that $a = -25$, and from $a+n = 28$, we get that $n = 53$, which is a prime. Therefore, $a = -25$ is a possible value of a.

Case 3. $n = 2p$. Then, $a+27$ must be 1. This would mean that $a = -26$, and from $a+n = 28$, we get that $n = 54$. However, note that $\frac{54}{2} = 27$, which is not a prime. Therefore, $a = -26$ is not a possible value of a.

We have exhausted all cases, which means that our answer is $26 + (-25) = \boxed{\textbf{(E) } 1}$. ■

20. *Proposed by DeToasty3*

Answer (B): Let E be the intersection of the angle bisector of $\angle BAD$ and BC. In addition, let F be the intersection of the angle bisector of $\angle ADC$ and BC. Let $\angle BAD = \angle ADC = 2\theta$, which implies $\angle BAE = \theta$. Then, $\angle ABE = \angle ABC = 180^\circ - 2\theta$ as $ABCD$ is an isosceles trapezoid and $\angle BAE + \angle ABE + \angle AEB = 180^\circ \implies \angle AEB = \theta$. Therefore, $\triangle ABE$ is isosceles with $AB = BE = 8$. Similarly, $\triangle DCF$ is also isosceles with $CD = CF = 8$.

By parallel lines BF and AD, $\triangle XBF \sim \triangle XAD$. Since $BF = 12$, we have $\frac{XB}{XA} = \frac{BF}{AD} \implies \frac{XB}{XB+8} = \frac{12}{18} \implies XB = 16$. By symmetry, $YC = 16$ and XY is parallel to BC and AD. In particular, $\angle XYA = \angle BEA = \theta$, so $\triangle XYA$ is isosceles with $XA = XY$. Therefore, $XY = \boxed{\textbf{(B) } 24}$. ■

21. *Proposed by DeToasty3*

Answer (C): Taking each of the values k^2+2, k^2+4, and k^2+8 modulo 3, we see that each value reduces to k^2-1, k^2+1, and k^2-1, respectively. If k is 1 or 2, we see that at least one of the values will be 0 modulo 3, which means that it will not be prime. In the case that $k^2+2 = 3 \implies k = 1$, we see that $k^2+8 = 1^2+8 = 9$, which is not a prime. Therefore, both m and n have to be a multiple of 3. Checking each of the answer choices, we see that the only answer which is a multiple of 3 is $\boxed{\textbf{(C) } 72}$. ■

Remark. There indeed exists m and n such that the three values are all prime numbers; an example is 15 and 57, which add to 72.

22. *Proposed by richy*

Answer (A): Checking the first few values of T_n, we see that $T_1 = 1$, $T_2 = \frac{2}{3}$, $T_3 = \frac{4}{4}$, $T_4 = \frac{12}{5}$, $T_5 = \frac{48}{6}$, $T_6 = \frac{240}{7}$, and so on. We conjecture that

$$T_n = \frac{2(n-1)!}{n+1}.$$

We will prove this via induction. Our base case is $n = 1$, which gives $T_1 = 1$. Now, plugging our expression into T_{n+1} yields that

$$T_{n+1} = \frac{2n!}{n+2},$$

which proves that our closed form is correct.

Now, we see that

$$T_{2020} = \frac{2 \cdot 2019!}{2021} = \frac{2020!}{1010 \cdot 2021}.$$

This means that $m = 1010 \cdot 2021 = 2 \cdot 5 \cdot 101 \cdot 43 \cdot 47$, so our answer is $2+5+43+49+101 = \boxed{\textbf{(A) } 198}$. ■

23. *Proposed by DeToasty3*

Answer (D): If Amy knows what n is, then it must have a unique number of divisors. We optimally check for perfect squares first, as those have an odd number of divisors. Listing them out, we see that 1 has 1 divisor, 4 has 3 divisors, 9 has 3 divisors, 16 has 5 divisors, 25 has 3 divisors, and 36 has 9 divisors. Of these, we see that 1, 16, and 36 are unique. We can check to see that no other number in the interval $[1, 40]$ has a unique number of divisors. We also see that the remainders when 1, 16, and 36 are divided by 7 are 1, 2, and 1, respectively. Of these, only 16's remainder is unique, so 16 must have been the wrong value of n that Amy and Sid were thinking of.

Remark. Even though 36 has more than 6 divisors, and Amy's number must have been less than 7 for the remainder, both Amy and Sid think that Amy was told the number of divisors, which can be greater than 6, so it does not affect the logic of the problem.

Now, we see that 16 has 5 divisors and leaves a remainder of 2 when divided by 7, so Amy's number must have been 5 and Sid's number must have been 2. This means that an actual value of n leaves a remainder of 5 when divided by 7 and has 2 divisors.

Testing out values of n, we see that n can be 5 or 19, so our answer is $\boxed{\textbf{(D) } 24}$. ■

24. *Proposed by DeToasty3*

Answer (D): Note that $\triangle BCM$ is equilateral since $\angle B = 60^\circ$. Let Q be the midpoint of MC. By Pythagorean Theorem, $PQ^2 + QB^2 = PB^2$. By the rules of an equilateral triangle, $QB = 4\sqrt{3}$ and we know that $PB = 7$. Therefore, $PQ = 1$. Since Q is fixed, there exists two points P on segment MC such that $PQ = 1$.

Let P_1 be the point on MC such that $P_1Q = 1$ and $MP_1 < P_1C$. In addition, let P_2 be the point on MC such that $P_2Q = 1$ and $MP_2 < P_2C$. Then, let rays BP_1 and BP_2 intersect AC at N_1 and N_2, respectively. Now, we seek $\frac{CN_1}{N_1A} + \frac{CN_2}{N_2A}$. We will find each ratio separately via mass points.

Consider $\triangle ABC$ with cevians BN_1 and CM. Since $CP_1 = 5$ and $P_1M = 3$, place a mass of 6 at C and a mass of 10 at M. Since M is the midpoint of AB, A has a mass of 5. Then, $\frac{CN_1}{N_1A} = \frac{\text{mass}(A)}{\text{mass}(C)} = \frac{5}{6}$.

Now, consider $\triangle ABC$ with cevians BN_2 and CM. Since $CP_2 = 3$ and $P_2M = 5$, place a mass of 10 at C and a mass of C at M. Since M is the midpoint of AB, A has a mass of 3. Then, $\frac{CN_2}{N_2A} = \frac{\text{mass}(A)}{\text{mass}(C)} = \frac{3}{10}$.

Finally, $\dfrac{5}{6} + \dfrac{3}{10} = \boxed{\textbf{(D) } \dfrac{17}{15}}$. ■

25. *Proposed by richy*

Answer (C): Let O be the center vertex and label the vertices on the perimeter of the figure as $A_1, A_2, \ldots, A_5$ in a clockwise order and take all indices modulo 5.

Since no 3 of $OA_1, OA_2, \ldots, OA_5$ may be the same color, the only possibility is that 2 of those 5 edges are one color, some other 2 edges of those 5 are another color, and the final edge is the remaining color. WLOG, let the final color be green and let OA_1 be the green edge. We have 3 ways to choose the final color and 5 ways to rotate the figure to change the location of the edge containing the final color, so we must multiply by 15 at the end.

There are $\binom{4}{2} = 6$ ways to choose the locations of the red edges in which the other 2 edges will be blue. Let $F(i, j)$ with $i < j$ be the number of valid colorings such that OA_1 is green and OA_i and OA_j are both red.

By symmetry, we can see that $F(2, 5) = F(3, 4)$, $F(2, 4) = F(3, 5)$, and $F(2, 3) = F(4, 5)$. Therefore, the number of valid colorings total is given by $15 \cdot 2 \cdot (F(2, 5) + F(2, 4) + F(2, 3))$.

We will compute each value separately through PIE. In addition, call a vertex among $A_1, A_2, \ldots, A_5$ "bad" if the three edges protruding from it are all the same color. Note that if OA_k and OA_{k+1} are different colors, both A_k and A_{k+1} cannot be bad, where indices are taken modulo 5.

Define the function $G_{(i,j)}(S)$, where i and j are positive integers such that $1 \le i < j \le 5$ and S is a subset of $\{1, 2, 3, 4, 5\}$. This function will count the number of colorings such that:

- OA_i and OA_j are red
- OA_1 is green

- The remaining edges protruding from O are blue
- For each element n of S, A_n is a bad vertex

Case 1. Computing $F(2,5)$.

By PIE:

$$F(2,5) = 3^5 - \sum_{1 \le x \le 5} \left(G_{(2,5)}(\{x\})\right) + \sum_{1 \le x < y \le 5} \left(G_{(2,5)}(\{x,y\})\right) - \cdots$$

To evaluate $G_{(2,5)}(\{k\})$, for $1 \le k \le 5$, note that A_kA_{k+1} and $A_{k-1}A_k$ must be the same color as OA_k. Then, the remaining 3 edges can be colored freely. Thus, $G_{(2,5)}(\{k\}) = 3^3$ for $1 \le k \le 5$.

Now, we evaluate the terms where there are 2 bad vertices. If the bad vertices are 3 and 4, then A_2A_3, A_3A_4, and A_4A_5 must be blue. We may color the other 2 edges freely. Thus, $G_{(2,5)}(\{3,4\}) = 3^2$. If the two bad vertices A_p and A_q are not adjacent, then they must be vertices apart. This leaves only 1 edge left that we can color freely. Thus, $G_{(2,5)}(\{p,q\}) = 3$.

The only way for there to be 3 bad vertices is for $1, 3$, and 4 to be bad vertices in which all the edge colors are uniquely determined.

Plugging in the values, we have:

$$F(2,5) = 3^5 - 5 \cdot 3^3 + (3^2 + 5 \cdot 3) - 1 = 131.$$

Case 2. Computing $F(2,4)$.

By PIE:

$$F(2,4) = 3^5 - \sum_{1 \le x \le 5} \left(G_{(2,4)}(\{x\})\right) + \sum_{1 \le x < y \le 5} \left(G_{(2,4)}(\{x,y\})\right) - \cdots$$

Using a similar process as above:

$$F(2,4) = 3^5 - 5 \cdot 3^3 + (2 \cdot 3^2 + 5 \cdot 3) - 2 = 139.$$

Case 3. Computing $F(2,3)$.

By PIE:

$$F(2,3) = 3^5 - \sum_{1 \le x \le 5} \left(G_{(2,3)}(\{x\})\right) + \sum_{1 \le x < y \le 5} \left(G_{(2,3)}(\{x,y\})\right) - \cdots$$

Again, by using a similar process as above:

$$F(2,3) = 3^5 - 5 \cdot 3^3 + (5 \cdot 3) = 123.$$

Therefore, our answer is $30 \cdot (131 + 139 + 123) = \boxed{\textbf{(C) } 11,790}$. ■