

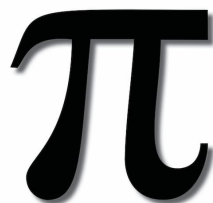
π Pi

Pi Series

1st and Only

Pi

Thursday, March 17, 2022



INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.
2. This is a $\lceil \pi^2 \rceil$ -question test. The answer to each question will be a positive integer.
3. Mark your answers to each problem on the Pi Answer Form with a keyboard. Check the keys for accuracy and erase errors and stray marks completely.
4. SCORING: Each question will have different point values which we will announce when we feel like it.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will not ask you to record your name and other information on the answer sheet.
8. You will have $\lceil 2\pi^2 \rceil$ minutes to complete the competition once you start the test.
9. When you finish the competition, don't sign your name in the space provided on the answer sheet.

The Pi Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

1. What is the smallest positive integer n such that the nearest integer to $n\pi$ is greater than $n\pi$?
2. Foo has n slices of pie and 20 plastic bags. Foo sees that he can arrange his slices of pie and his plastic bags such that a different number of slices of pie are contained within each plastic bag, and no slice of pie is not contained within a plastic bag. What is the smallest possible value of n ?
3. If $\lceil \pi^2 \rceil - \lfloor \pi \rfloor = \frac{m}{n} \cdot \lceil \pi \rceil$, where m and n are relatively prime positive integers, what is $m + n$? (Note that $\lfloor r \rfloor$ denotes the greatest integer not exceeding a real number r , and $\lceil r \rceil$ denotes the least integer greater than or equal to r .)
4. Archimedes has π pounds of pie. He wants to cut the pie into pieces of equal weights to evenly share with himself and his n friends. How many integer values of n are there such that everyone gets at least $\pi - 3$ pounds of pie?
5. In Ryan's really riveting report, he defines the *bicimal point* as the point separating a real number's integer and fractional parts in its binary (base-2) representation. For example, the binary number 10.1_2 's bicimal point is between the 0 and the second 1. How many digits are there in the binary representation of the number π^2 before the bicimal point?
6. Euler has π circular pies, where one whole pie has radius 3. He then combines as much content of his π pies as possible to form a single larger circular pie with an integer radius r . What is the maximum value of r ?
7. Ithaca is selling pies. Unfortunately, he forgot exactly how many pies he sold, so he relies on his memory. He remembers that he sold at least 31 pies between 1:01 and 2:00, exactly 41 pies between 2:01 and 3:00, and exactly 59 pies between 3:01 and 4:00. Furthermore, Ithaca sold as many pies between 1:01 and 2:30 as he did between 2:31 and 4:00. If Ithaca sold n pies between 2:01 and 2:30, how many possible values of n are there?
8. Here on Earth, we have a technique known as PIE, the Principle of Inclusion and Exclusion. However, on the planet of Pi, Pians, the inhabitants of Pi, do not believe in exclusion, so they have PI, the Principle of Inclusion. Thus, if there are 31 Pians who eat bacon for breakfast and 41 Pians who eat eggs for breakfast, then PI would state that $31 + 41 = 72$ of these Pians eat at least one of bacon and eggs, regardless of how many Pians eat both bacon and eggs. Now, if 59 more Pians were to arrive, all of whom eat both bacon and eggs, then the value which PI would give when finding how many Pians eat at least one of bacon and eggs would be exactly twice the value which PIE would give (i.e. the actual value). How many Pians were there who eat both bacon and eggs before the 59 Pians arrived?

9. Syracuse bakes a circular pie. He then cuts the pie into 3 congruent sectors. He cuts out the largest possible circle in each of these sectors to sell as three different pies. He then combines the leftover portions of his pie to make another pie for himself. The ratio of the area of one of the pies that he sold to the area of the pie that he made for himself can be written as $\frac{a\sqrt{b}-c}{d}$, where a , b , c , and d are positive integers, b is square-free, and $\gcd(a, c, d) = 1$. What is $a + b + c + d$? (Assume that no pie is lost while cutting or baking.)
10. Gerolamo has distinct real number values π_1 and π_2 such that

$$\cos(\pi_1) + \cos(\pi_2) = \frac{6}{13} \quad \text{and} \quad \sin(\pi_1) + \sin(\pi_2) = \frac{8}{13}.$$

Let $(\cos(\pi_1) - \cos(\pi_2))^2 + (\sin(\pi_1) - \sin(\pi_2))^2 = \frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$? (Note that π_1 and π_2 are not to be confused with radians!)

π *Pi*

Pi

DO NOT OPEN UNTIL THURSDAY, March 17, 2022

****Administration on an earlier date will disqualify your results.****

- All the information needed to administer this exam is not contained in the non-existent Pi Teacher's Manual. PLEASE READ THE MANUAL BEFORE THURSDAY, MARCH 17, 2022.
- Send **DeToasty3** and **PhunsukhWangdu** a private message submitting your answers to the Pi. AoPS is the only way to submit your answers.
- The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

For more information about the Pi and our other competitions, please visit
Nowhere.

Questions and comments about this competition should be sent to:

DeToasty3 and PhunsukhWangdu.

The problems and solutions for this Pi were prepared by the Pi Editorial Board
under the direction of:

DeToasty3, Offset, & PhunsukhWangdu.