ANOTHER MOCK CONTEST :O

2022

■ Mock Competition ■

AMC :O

Problems 1-25

HONOR PLEDGE

I pledge to uphold the highest principles of honesty and integrity as a Mathlete. I will neither give nor accept unauthorized assistance of any kind. I will not copy another's work and submit it as my own. I understand that any competitor found to be in violation of this honor pledge is subject to disqualification.

Signature ______________________________

AoPS Username ______________________________

Grade ______________________________

Your Mom's AoPS Username (if existent) ______________________________

Other Random Info ______________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This competition consists of 25 problems. You will have 75 minutes to complete all the problems. You are not allowed to use calculators, books or other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in a private message to DeToasty3. If you complete the problems before time is called, use the remaining time to check your answers.

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1. ____________ Find the value of $2^0 + 2^1 + 2^0 + 2^2$.

2. ____________ How many ways can 2022 be written as the sum of two non-negative integers whose digits are only 0s and 1s?

3. ____________ Penny has 2022 indistinguishable pennies. She colors some (but not all) of the pennies red, and she colors the rest of the pennies blue. In how many ways can she do this such that she has more red pennies than blue pennies?

4. ____________ Find the sum of all two-digit multiples of 5 whose digits sum to 8.

5. ____________ Karate and Judo are fighting in the parking lot. The moment a person gets hit 50 times, they lose the fight. If Karate has already hit Judo 37 times, and Judo has already hit Karate 40 times, what is the largest number of total additional hits possible without one person losing the fight after the last of these hits?

6. ____________ Find the remainder when

$$(2022^2 - 2022) - (2021^2 - 2021)$$

is divided by 1000.

7. ____________ Two rectangles with whole number side lengths have equal areas and perimeters of 18 and 22. What is the area of one of the rectangles?

8. ____________ An arithmetic sequence of positive even integers with 100 terms has exactly four multiples of 66. Find the least possible value of the median of the terms in this sequence.

9. ____________ Rectangle $ABCD$ has side lengths $AB = 52$ and $BC = 10$. Let M be the midpoint of AB, and let N be the midpoint of DM. Find the length of $\overline{AN}$.

10. ____________ Rectangle $ABCD$ has perimeter 1820. Rectangle $EFGH$ is constructed such that $EF = AB - 1$ and $FG = BC - 1$. Find the difference between the areas of $ABCD$ and $EFGH$.

11. ____________ How many ordered triples of (not necessarily distinct) primes (p, q, r) each less than 25 are there such that $p + q = r$?

12. ____________ A 2-liter mixture of water and acid contains 60% acid. When some of the water is removed, the mixture now contains 80% acid. How many milliliters of water were removed? (Note: There are 1000 milliliters in one liter.)

13. ____________ Let R, A, C, K, E, and T be distinct single-digit positive integers. If

$$A \cdot T = 20,$$
$$R \cdot E \cdot K \cdot T = 30,$$
$$R \cdot A \cdot C \cdot K \cdot E \cdot T = 720,$$

what is $A + C + K$?

14. ____________ How many real solutions (x, y) are there to the following system?

$$(x^2 - 4y^2)(4x^2 - y^2) = 0$$
$$2x + 4y = 7$$

15. ____________ Let f be a function such that for any real number x,

$$f(x) + f(2x) = f(x) \cdot f(2x).$$

If $f(256) = 24$, find $f(1)$.

16. ____________ Let $\mathcal{P}(S)$ denote the product of the elements of a set S. Let

$$A_1, A_2, A_3, \ldots$$

denote the distinct non-empty subsets of set $A = \{a, b, c\}$, where a, b, and c are positive integers and $a < b < c$. Given that

$$\mathcal{P}(A_1) + \mathcal{P}(A_2) + \mathcal{P}(A_3) + \cdots = 99,$$

what is the value of c?

17. ____________ Find the largest positive integer n such that $\dfrac{101^n + 1}{n!}$ is an integer.

18. ____________ What is the sum of all integer values of n that satisfy $(n-3)^{36-n^2} = 1$?

19. ____________ Let (a, b) be an ordered pair of positive real numbers such that

$$ab^3 + a^3b = 78.$$

How many possible integer values of $a \cdot b$ are there?

20. ____________ Six distinct points lie in a straight line. First, Cindy draws a red line segment, choosing two of the six points to be the endpoints. Next, Sophia draws a blue line segment, choosing two of the six points to be the endpoints (these can include Cindy's points). Finally, Michael shades any points not on either line segment yellow. If any point on only the red segment is red, any point on only the blue segment is blue, and any point on both the red and blue line segments is purple, find the number of possible colorings of the points. (One coloring is red, purple, blue, blue, yellow, yellow.)

21. ____________ Bill has one penny. He plays a game where he wagers a penny on a positive integer from 1 to 6 (inclusive). Then, he rolls three fair standard six-sided dice. If Bill's number appears on 1, 2, or 3 dice, he will win 1, 2, or 3 pennies, respectively (along with his original penny). Otherwise, if Bill's number appears on none of the three dice, he will not get his original penny back. The expected number of pennies Bill will have after the game is $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m + n$?

22. ____________ A ray from a point P goes through a circle with center O at points A and B, with $PB > PA$. Line segment $\overline{OP}$ intersects the circle at point C. If $PA = 93$, $PC = 51$, and $\angle PBO = 60^\circ$, find the radius of the circle.

23. ____________ Lines with slopes n, $\frac{1}{n}$, and $-\frac{1}{n}$ have y-intercepts $(0, 75)$, $(0, 602)$, and $(0, 700)$, respectively, and all meet at a single point. The value of n can be written as $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

24. ____________ Circles ω_1 and ω_2 have centers O_1 and O_2, respectively, where the radius of ω_2 is 8, and $O_1O_2 = 20$. The circles intersect at two distinct points, one of which is P. The line PO_2 intersects ω_2 at A, distinct from P. Given that there exists a point B on ω_1 such that $BPAO_1$ is a parallelogram, the area of $BPAO_1$ is $m\sqrt{n}$, where m and n are positive integers, and n is not divisible by the square of any prime. Find $m + n$.

25. ____________ Let acute $\triangle ABC$ have $AB = 9$, $AC = 16$, and $\angle ACB = 30^\circ$. Let O be the center of the circumcircle of $\triangle ABC$, and let A' be the reflection of A over point O. Construct line ℓ which passes through O and is parallel to side $\overline{AB}$. Let P be the intersection of ℓ and the bisector of $\angle A'AB$. The perimeter of $ABPA'C$ can be written as $m\sqrt{n}+p$, where m, n, and p are positive integers, and n is not divisible by the square of any prime. Find $m + n + p$.