Individual Round 2

Trial Competition

Format:

- There will be 10 questions to be solved in 60 minutes.
- Each correct answer is worth 1 point. Each incorrect/blank answer is worth 0 points.
- The difficulty will range from early AMC 12 to late AIME.

Rules/Submitting:

- Your only tools are a writing utensil and blank scrap paper. You may not use graph paper, protractors, rulers, calculators, or electronic devices (except for one to access the rounds).
- Every answer is an integer between 1 and $2^{21} = 2097152$, inclusive. While taking the round, please ensure that you do not make any typos in any of your answers.
- Each round will be posted on the website at the time indicated in the schedule. Please actively check and refresh the website should any errata/changes be posted during a round.
- If you find any ambiguities or flaws in a problem in a given round, please email us at *after the round is finished* at admccontest@gmail.com with the round and problem number in question. If we decide that the problem is flawed, then we will give everyone full credit for that problem. Otherwise, please use your best judgment to determine what the problem is asking while taking the round.

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Individual Round 2
60 Minutes

R2-1. Find the smallest positive integer n such that $n! + 1$ is not divisible by any integer between 2 and 9, inclusive.

R2-2. In a plane, eight rays emanate from a point P such that every two adjacent rays form an acute angle with measure $45°$. Next, a line segment with a finite length is drawn in the plane. If the line segment intersects exactly n of the rays, find the sum of all possible values of n. (If the line segment passes through P, then $n = 8$.)

R2-3. In the coordinate plane, let $P = (10, 0)$ and $Q = (a, a)$, for some real number a. It is given that when point P is reflected over point Q, the resulting point P' is 2 times as far from the origin as point P is. Find the sum of all possible values of a.

R2-4. There exists a sequence $a_1, a_2, \ldots, a_6$ of positive integers such that for every term in the sequence, there exists another term in the sequence which is equal to that term. Find the number of possible values of the product $a_1 a_2 \cdots a_6$ less than 1000.

R2-5. Bill and Ben each have 2 fair coins. Each minute, both Bill and Ben flip all their coins at the same time, if they have any. If a coin lands heads, then the other person gets that coin. If a coin lands tails, then that coin stays with the same person. The probability that after exactly 3 minutes, they each end up with 2 coins is $\frac{m}{n}$ for relatively prime positive integers m and n. Find $100m + n$.

R2-6. In $\triangle ABC$ with $AB = 3$ and $AC = 6$, let D be the intersection of the angle bisector of $\angle BAC$ and $\overline{BC}$, and let M be the midpoint of $\overline{AC}$. Let the circumcircle of $\triangle DMC$ intersect line AD again at P, distinct from D. If $DM = 2$, then PC^2 can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

R2-7. There exist positive real numbers a and b such that $\lfloor ab^2 \rfloor = 1$, $\lfloor a^2b^2 \rfloor = 2$, and $\lfloor a^3b^2 \rfloor = 6$, where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to a real number r. It is given that as a approaches its greatest lower bound A, the set of possible values of b approaches a single value B. The value of $A^4 + B^4$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

R2-8. A right $\triangle ABC$ has $AC > AB$ and a right angle at A. A circle is tangent to sides $\overline{AB}$ and $\overline{AC}$ at D and E, respectively, and internally tangent to the circumcircle of $\triangle ABC$ at F. Let line EF intersect the circumcircle of $\triangle ABC$ again at P, distinct from F. If $BP = 20$ and E is the midpoint of $\overline{AC}$, find the area of $\triangle DEF$.

R2-9. Let x and y be distinct real numbers chosen at random from the interval $[-1, 1]$, excluding 0. The probability that

$$\left\lfloor \frac{|x|}{|y|} \right\rfloor \geq \left\lfloor \frac{|x+y|}{|x-y|} \right\rfloor$$

can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers, and $\lfloor r \rfloor$ denotes the greatest integer less than or equal to a real number r. Find $100m + n$.

R2-10. Let $\omega_1, \omega_2, \omega_3, \ldots, \omega_{2020!}$ be the distinct roots of $x^{2020!} - 1$. Suppose that n is the largest integer such that 2^n divides the value

$$\sum_{k=1}^{2020!} \frac{2^{2019!} - 1}{\omega_k^{2020} + 2}.$$

Then n can be written as $a! + b$, where a and b are positive integers, and a is as large as possible. Find $a + b$.