Team Round

Trial Competition

Format:

- There will be 10 questions to be solved in 60 minutes.
- Each correct answer is worth 2 points. Each incorrect/blank answer is worth 0 points.
- The difficulty will range from mid AMC 12 to beyond late AIME.
- The team score will be the sum of all individual scores on a team (a total of $4 \times 20 = 80$ points) and the team score (a total of 20 points), for a total of 100 points.

Rules/Submitting:

- Your only tools are a writing utensil and blank scrap paper. You may not use graph paper, protractors, rulers, calculators, or electronic devices (except for one to access the rounds).
- Every answer is an integer between 1 and $2^{21} = 2097152$, inclusive. While taking the round, please ensure that you do not make any typos in any of your answers.
- Each round will be posted on the website at the time indicated in the schedule. Please actively check and refresh the website should any errata/changes be posted during a round.
- If you find any ambiguities or flaws in a problem in a given round, please email us at *after the round is finished* at admccontest@gmail.com with the round and problem number in question. If we decide that the problem is flawed, then we will give everyone full credit for that problem. Otherwise, please use your best judgment to determine what the problem is asking while taking the round.
- You must collaborate with only your team members. Any reported collaboration between members of different teams during this round will be investigated and will likely lead to disqualification.

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Team Round
60 Minutes

T-1. Anthony, Daniel, and Richard have 17, 20, and 26 trading cards, respectively. Every minute, one of the three boys gives away two of his trading cards such that the other two boys get one trading card each. Find the shortest amount of time, in minutes, that it could take for the three boys to each have an equal number of trading cards.

T-2. Find the number of orderings of the six numbers 1, 1, 2, 2, 3, and 6 such that the sum of the first three numbers is twice the sum of the last three numbers.

T-3. Find the largest positive integer n such that when $57^n + 64^n$ is divided by $n!$, the remainder is 1.

T-4. In equilateral $\triangle ABC$, let points D and E be on lines AB and AC, respectively, both on the opposite side of line BC as A. If $CE = DE$, and the circumcircle of $\triangle CDE$ is tangent to line AB at D, find the degree measure of $\angle CDE$.

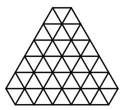
T-5. Find the smallest positive integer n such that

$$\sqrt{5n-1} - \sqrt{5n-2} + \sqrt{5n-3} - \sqrt{5n-4}$$

is less than 0.05.

T-6. In right $\triangle ABC$ with $BC = 7$ and a right angle at A, let the midpoint of side $\overline{AB}$ be D. Suppose that there exists a point E on the circumference of the circumcircle of $\triangle ABC$ such that $\triangle CDE$ is equilateral. Find the square of the side length of $\triangle CDE$.

T-7. In the diagram below, a group of equilateral triangles are joined together by their sides. A parallelogram in the diagram is defined as a parallelogram whose vertices are all at the intersection of two grid lines and whose sides all travel along the grid lines. Find the number of distinct parallelograms in the diagram below.



T-8. There exist complex numbers $z_1, z_2, \ldots, z_{10}$ which satisfy

$$|z_k i^k + z_{k+1} i^{k+1}| = |z_{k+1} i^k + z_k i^{k+1}|$$

for all integers $1 \le k \le 9$, where $i = \sqrt{-1}$. If $|z_1| = 9$, $|z_2| = 29$, and for all integers $3 \le n \le 10$, $|z_n| = |z_{n-1} + z_{n-2}|$, find the minimum value of $|z_1| + |z_2| + \cdots + |z_{10}|$.

T-9. Let $\triangle ABC$ have side lengths $AB = 7$, $BC = 8$, and $CA = 9$. Let D be the projection from A to $\overline{BC}$ and D' be the reflection of D over the perpendicular bisector of $\overline{BC}$. Let P and Q be distinct points on the line through D' parallel to $\overline{AC}$ such that $\angle APB = \angle AQB = 90^\circ$. The value of $AP + AQ$ can be written as $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are positive integers such that b and d are relatively prime, and c is not divisible by the square of any prime. Find $a + b + c + d$.

T-10. For a positive integer n not divisible by 211, let $f(n)$ denote the smallest positive integer k such that $n^k - 1$ is divisible by 211. Find the remainder when

$$\sum_{n=1}^{210} n f(n)$$

is divided by 211.