



berfoer Series

1st and Only

berfoer

Thursday, February 24, 2022



INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.
2. This is a four-question test. For each question, there will be multiple parts. For each part, unless it says to show work or explain, you only have to give an answer.
3. Mark your answers/explanations to each problem on the berfoer Answer Form with a keyboard. Check the keys for accuracy and erase errors and stray marks completely.
4. SCORING: Each question will have point values indicated underneath it.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will not ask you to record your name and other information on the answer sheet.
8. You will have 50 minutes to complete the competition once you start the test.
9. When you finish the competition, don't sign your name in the space provided on the answer sheet.

The berfoer Committee reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

1. Consider the number 2022.

- What is the sum of the digits of the number?
- What is the closest perfect square to the number?
- Let the perfect square found in (b) be N^2 . What is the closest perfect square to N ?
- Is the fractional part of \sqrt{N} or $\sqrt[4]{2022}$ greater? Explain.

Point Values: a. 1 point for correct answer. b. 1 point for correct answer. c. 1 point for correct answer. d. 1 point for correct answer, 1 point for correct explanation.

2. Two sports teams, the Planes and the Helicopters, play against each other in a 24-game series. In each game, either the Planes or the Helicopters win, and there are no ties. At the end of the 24 games, whichever team has won more games is the winner. Otherwise, the series ends in a tie. Suppose that there have been eight games so far, and the Planes have won five of them.

- What percent of the first eight games did the Planes win?
- How many of the remaining 16 games would the Planes need to win in order to beat the Helicopters in the series?
- Is it possible for the Helicopters to win more games than the Planes after the first 10 games (given the results of the first eight games)? Explain.

Now, suppose that the series is split into quarters, where quarter 1 consists of the first six games, quarter 2 consists of the 7th to 12th games, and so on.

- Is it possible for the Helicopters to have lost every game in quarter 1 (given the results of the first eight games)? Explain.
- If the Planes beat the Helicopters in the series, what is the smallest possible number of games the Planes could have won in the quarter where they won the most games? Explain and give a valid construction.

Point Values: a. 1 point for correct answer. b. 1 point for correct answer. c. 1 point for correct answer, 1 point for correct explanation. d. 1 point for correct answer, 1 point for correct explanation. e. 1 point for correct answer, 1 point for correct explanation, 1 point for valid construction.

3. There are some number of people in a room, where everyone is either male or female. A group of three people from the room is *commendable* if the group has at least one person of each gender. Suppose that two males, Ryan and Richard, and two females, Katherine and Katie, are in the room.
- How many possible commendable groups can be formed with three people chosen from only Ryan, Richard, Katherine, and Katie? (The order in which the people are chosen does not matter.)
 - Can a group be formed with three people from these four that is **not** commendable? Explain.

Suppose that there are a total of 5 males in the room, including Ryan and Richard, and the total number of people in the room is $3N$, where N is a positive integer.

- How many possible commendable groups of three people can be formed among the 5 males and Katherine and Katie if neither Ryan nor Richard are in it? (The order in which the people are chosen does not matter.)
- If the people in the room can be split into N groups of 3 people such that every group is commendable, at least how many females must be in the room (including Katherine and Katie)?

Next, some more people join the room, so there are now 13 males and a total of $3M$ people in the room, where M is a positive integer.

- If the people in the room can be split into M groups of 3 people such that every group is commendable, how many possible values of M are there, and what is the smallest possible value of M ? Explain.
- For the smallest value of M found in (e), suppose that the $3M$ people are split into M groups of 3 again. What is the largest possible number of groups that are **not** commendable that can be formed?

Point Values: a. 1 point for correct answer. b. 1 point for correct answer, 1 point for correct explanation. c. 2 points for correct answer. d. 2 points for correct answer. e. 1 point for correct answers (0.5 for each), 2 points for correct explanation. f. 3 points for correct answer.

4. Let $ABCD$ be an isosceles trapezoid with $\overline{AB} \parallel \overline{CD}$, $AB < CD$, and $AB + CD = AD$. Let P be the point on side \overline{AD} such that $AB = AP$ and $DC = DP$. Suppose that $ABCD$ has perimeter 36.

- If $\angle ABC = \theta$, what is the measure of $\angle ADC$ in terms of θ ?
- What is the length of \overline{AD} ?

Let the perpendicular bisectors of \overline{PB} and \overline{PC} intersect at a point Q . Suppose that Q lies on side \overline{BC} .

- What is the measure of $\angle AQD$? Explain.
- What must the measure of $\angle BPC$ be? Explain.
- What is the length of \overline{PQ} ?
- Is \overline{PQ} parallel to \overline{AB} and \overline{CD} ? Explain.
- Is the circle passing through P , B , and C tangent to line AD ? Explain.
- Let M be the midpoint of \overline{PQ} . Let \overline{BM} and \overline{AQ} intersect at a point X , and let \overline{CM} and \overline{DQ} intersect at a point Y . What is the length of \overline{XY} ?

Point Values: a. 1 point for correct answer. b. 1 point for correct answer. c. 1 point for correct answer, 2 points for correct explanation. d. 1 point for correct answer, 2 points for correct explanation. e. 1 point for correct answer, 2 points for correct explanation. f. 1 point for correct answer, 2 points for correct explanation. g. 3 points for correct answer.



berfoer

DO NOT OPEN UNTIL THURSDAY, February 24, 2022

****Administration on an earlier date will disqualify your results.****

- All the information needed to administer this exam is not contained in the non-existent berfoer Teacher's Manual. PLEASE READ THE MANUAL BEFORE THURSDAY, FEBRUARY 24, 2022.
- Send **DeToasty3** a private message submitting your answers to the berfoer. AoPS is the only way to submit your answers.
- The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

For more information about the berfoer and our other competitions, please visit
Nowhere.

Questions and comments about this competition should be sent to:

DeToasty3.

The problems and solutions for this berfoer were prepared by the berfoer Editorial Board under the direction of:

DeToasty3 & Nashoba.