KMMC

Karate Masters Mathematics Competitions

Karate Masters Mathematics Competitions

1st Annual

KMMC 10

Karate Masters Mathematics Competitions 10
Tuesday, February 2, 2021

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.
2. This is a twenty-five question multiple choice test. Each question is followed by answers marked A, B, C, D, and E. Only one of these is correct.
3. Mark your answer to each problem on the KMMC 10 Answer Form with a keyboard. Check the keys for accuracy and erase errors and stray marks completely.
4. You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the test, your non-existent proctor will not ask you to record certain information on the answer form.
8. When you give the signal, begin working on the problems. You will have 75 minutes to complete the test. You can discuss only with people that have already taken the test in the private discussion forum until the end of the contest window.
9. When you finish the exam, don't sign your name in the space not provided on the Answer Form.

The Committee on the Karate Masters Mathematics Competitions reserves the right to re-examine students before deciding whether to grant official status to their scores. The Committee also reserves the right to disqualify all scores from a school if it is determined that the required security procedures were not followed.

Students who score well on this KMMC 10 will not be invited to anything because the KMIME (Karate Masters Invitational Mathematics Examination) does not exist. More details about the KMIME and other information are not on the back page of this test booklet.

The publication, reproduction or communication of the problems or solutions of the KMMC 10 during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web or media of any type during this period is a violation of the competition rules.

1. What is the value of
$$\frac{2021^2 - 2021}{2020} + \frac{2021^2 - 2020^2}{2021 - 2020}?$$

   (A) 1010 **(B)** 2020 **(C)** 2021 **(D)** 4041 **(E)** 6062

2. There are 2020 balls in a basket, where 1010 balls are red and the other 1010 balls are blue. Every second, 1 red ball and 3 blue balls are removed from the basket. How many seconds will it take for the number of blue balls left in the basket to be exactly half of the number of red balls left in the basket?

   (A) 10 **(B)** 20 **(C)** 101 **(D)** 202 **(E)** 505

3. A positive two-digit integer n is called *karate-y* if n is prime and the remainder when n is divided by 9 is 8. How many karate-y integers are there?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

4. At a buffet, Karate eats sushi, shrimp tacos, lo mein, and chicken tacos in some order. If he eats sushi sometime before he eats shrimp tacos, but eats lo mein sometime after he eats chicken tacos, in how many possible orders could he have eaten them?

   (A) 3 **(B)** 6 **(C)** 12 **(D)** 15 **(E)** 24

5. Real numbers a, b, and c satisfy $ab^2 = 27$ and $a^5bc^9 = 729$. What is abc?

   (A) 1 **(B)** 3 **(C)** 9 **(D)** 27 **(E)** 81

6. A right triangle has a hypotenuse of length 20 and a perimeter of length 46. What is the area of the triangle?

   (A) 14 **(B)** 28 **(C)** 43 **(D)** 69 **(E)** 138

7. In triangle ABC, $AB = 9$ and $BC = 40$, with a right angle at B. Point P lies on side $\overline{BC}$ such that the line perpendicular to $\overline{BC}$ passing through point P splits the area of $\triangle ABC$ exactly in half. What is the length PC?

   (A) $\frac{9\sqrt{2}}{2}$ **(B)** 20 **(C)** $\frac{41}{2}$ **(D)** $20\sqrt{2}$ **(E)** $\frac{41\sqrt{2}}{2}$

8. Every day in an eight-day interval, the Cents Lord puts a number of cents into her piggy bank. If the number of cents she puts into her piggy bank from one day to the next forms an increasing arithmetic progression and she puts 2008 cents into her piggy bank in total over the eight days, what is the least possible number of cents she could have put into her piggy bank on any one of the days?

   (A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

9. There exist positive integers n which satisfy at least half of the following conditions:

   - n is not a prime number.
   - $n+1$ is equal to a perfect square.
   - $n+2$ is a prime number.
   - $n+3$ is equal to one more than a perfect square.

   What is the sum of all n from 1 to 10, inclusive?

   (A) 4 **(B)** 11 **(C)** 17 **(D)** 20 **(E)** 21

10. Let n be a positive integer between 1458 and 2021, inclusive. What is the largest possible value of the sum of the digits of n when n is expressed in base-9? (Express your answer in base-10.)

    (A) 21 **(B)** 22 **(C)** 23 **(D)** 24 **(E)** 25

11. For a real number x, the median of the list of numbers

    $$10, 14, 18, 22, x+3, x+6, x+9, x+12$$

    is equal to 13.5. What is the sum of the unique mode and range of the list?

    (A) 21 **(B)** 23 **(C)** 25 **(D)** 27 **(E)** 29

12. Three children named Karate, Kung-fu, and Judo meet up. At the meeting, any two children either become friends or do not become friends. For any two children, if their names start with the same letter, there is a two-thirds chance they become friends. If not, there is a one-third chance they become friends. What is the probability that everyone makes at least one friend during the meeting?

    (A) $\frac{2}{9}$ **(B)** $\frac{11}{27}$ **(C)** $\frac{17}{27}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{23}{27}$

13. For a positive integer n, the value of $n^3 + 14n^2 + 35n - 50$ is not divisible by 8. What is the sum of the possible values of the remainder when n is divided by 8?

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

14. Let $f(x) = x^2 - 2x + 2$. What is the minimum value of

$$\underbrace{f(f(\cdots f(x)\cdots))}_{2021 \text{ times}},$$

where x is a real number?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

15. Triangle ABC has $AB = 42$, $BC = 40$, $AC = 26$. Point D is on side $\overline{AC}$ and point E is on side $\overline{BC}$ such that $CD = 10$ and $BE = 14$. What is the area of $ABDE$?

(A) 324 **(B)** 342 **(C)** 360 **(D)** 378 **(E)** 396

16. What is the smallest possible value of

$$\left|\frac{x}{3} - 20\right| + \left|\frac{x}{2} - 10\right| + \left|\frac{x}{3} + 10\right|$$

over all real numbers x?

(A) 20 **(B)** 25 **(C)** 30 **(D)** 35 **(E)** 40

17. If there exist three distinct positive primes p that satisfy the equation

$$p^4 + ap^3 + bp^2 + cp + 2020 = 0$$

for integers a, b, and c, what is the absolute value of $a + b + c$?

(A) 871 **(B)** 1621 **(C)** 2019 **(D)** 2020 **(E)** 2419

18. An infinite number of jars are lined up in a row, where the nth jar from the left has n red marbles and 2 blue marbles. For a positive integer k, Karate randomly selects a marble from each of the first k jars from the left. If the probability that he draws exactly k red marbles is strictly less than $\frac{1}{2020}$, what is the least possible value of k?

(A) 44 **(B)** 45 **(C)** 51 **(D)** 57 **(E)** 63

19. For a real number r, let $\lfloor r \rfloor$ denote the greatest integer less than or equal to r. How many real numbers x satisfy the equation $10\lfloor x \rfloor^2 + 4x^2 = 13\lfloor x \rfloor x$?

(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

20. Karate distributes 23 identical almonds to three distinct children such that each child receives a positive amount of almonds. Next, he takes the three pairwise differences between the three amounts and notes that none of them are divisible by six. In how many ways could Karate have distributed his almonds?

(A) 64 **(B)** 72 **(C)** 96 **(D)** 108 **(E)** 144

21. Triangle ABC has $AB = 8$, $BC = 6$, $AC = 11$. Let points D and E trisect side $\overline{BC}$ such that D is closer to B than C. Let F be the intersection of $\overline{AC}$ and the bisector of $\angle ABC$. Let X be the intersection of $\overline{AD}$ and $\overline{BF}$, and let Y be the intersection of $\overline{AE}$ and $\overline{BF}$. What is the ratio of the area of $\triangle AYF$ to the area of $\triangle AXB$?

(A) $\frac{25}{63}$ **(B)** $\frac{14}{33}$ **(C)** $\frac{16}{35}$ **(D)** $\frac{26}{55}$ **(E)** $\frac{10}{21}$

22. Four people are sitting evenly spaced at a circular table. At once, each person chooses to sit at the seat to their left, the seat to their right, or their current seat, with each seat having a one-third chance of being chosen. If two or more people sit at the same seat, the people who chose that seat leave the table. The people who did not leave sit at their chosen seats. What is the probability that exactly two people are left sitting?

(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{13}{27}$ **(D)** $\frac{16}{27}$ **(E)** $\frac{56}{81}$

23. How many non-congruent, non-degenerate triangles with positive integer side lengths a, b, and c exist such that $ab + ac + bc \leq 50$?

(A) 16 **(B)** 17 **(C)** 18 **(D)** 19 **(E)** 20

24. In triangle ABC with $AB = 13$, $BC = 14$, $AC = 15$, let O be the center of its circumcircle. Let P and Q be the feet of the perpendiculars from O to sides $\overline{AB}$ and $\overline{AC}$, respectively, and let E and F be the feet of the perpendiculars from B and C to line AO, respectively. What is $PE^2 + QF^2$?

(A) 98.5 **(B)** 101.5 **(C)** 104.5 **(D)** 107.5 **(E)** 110.5

25. Let r and s be the distinct roots of the polynomial $P(x) = x^2 - 100x + 76$. Let $Q(x) = x^3 + ax^2 + bx + c$ be the unique polynomial such that

$$Q(\sqrt[3]{r^2+1} + \sqrt[3]{s^2+1}) = 0$$

for integers a, b, and c. What is the absolute value of $a + b + c$?

(A) 5706 **(B)** 6681 **(C)** 7828 **(D)** 8912 **(E)** 9925

2021 SPRING KMMC 10

DO NOT OPEN UNTIL TUESDAY, February 2, 2021

Questions and complaints about problems and solutions for this exam should be sent by private message to:

DeToasty3 and **karate7800**.

The 2021 KMIME will never be held. It would be a 15-question, 3-hour, integer-answer exam if it was held. You will not be invited to participate because this contest does not exist. *A complete listing of our previous publications may be found at our web site:*

Wait, we don't have a website!

****Try Administering This Exam On An Earlier Date. Oh Wait, You Can't.****

1. All the information needed to administer this exam is not contained in the non-existent KMMC 10 Teacher's Manual.
2. YOU must not verify on the non-existent KMMC 10 COMPETITION CERTIFICATION FORM that you followed all rules associated with the administration of the exam.
3. Send **DeToasty3**, **karate7800**, and **nikenissan** a private message submitting your answers to the KMMC 10. AoPS is the only way to submit your answers.
4. The publication, reproduction or communication of the problems or solutions of this exam during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via copier, telephone, e-mail, World Wide Web, sonic waves sent through karate chops, or media of any type during this period is a violation of the competition rules.

The **2020-2021 Karate Masters Mathematics Competitions**
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