De Mathematics Competitions

1st Annual

DMC 8 B

Thursday, December 30, 2021

INSTRUCTIONS

1. DO NOT OPEN THIS BOOKLET UNTIL YOU DECIDE TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the DMC 8 Answer Form. Check your answers for accuracy. Only answers properly marked on the answer form will be graded; however, this mock will be graded by people.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will not ask you to record your name and other information on the answer sheet.
8. You will have 40 minutes to complete the competition once you start the test.
9. When you finish the competition, don't sign your name in the space provided on the answer sheet.

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1. Maxim spent \$1.75 on candy, \$2.15 on soda, and \$4.25 on popcorn. If Maxim pays with only 1-dollar bills, what is the least possible amount of change he could get back?

   (A) \$0.15 **(B)** \$0.40 **(C)** \$0.55 **(D)** \$0.60 **(E)** \$0.85

2. What is the value of the expression $\dfrac{4\cdot5\cdot6}{2(0+2+1)}$?

   (A) 20 **(B)** 28 **(C)** 30 **(D)** 36 **(E)** 40

3. Samuel is running forwards. At some point, he turns around and runs backwards. If Samuel ran a total of 60 equal steps and ended up 24 steps behind where he started running, for how many steps was Samuel running backwards?

   (A) 18 **(B)** 36 **(C)** 38 **(D)** 40 **(E)** 42

4. Bill and Ben are playing a game with 100 rounds. Each round, either Bill wins, Ben wins, or they tie. If Bill wins 3 rounds, and the number of rounds Ben wins is a perfect square, what is the least possible number of rounds where they tie?

   (A) 16 **(B)** 19 **(C)** 22 **(D)** 25 **(E)** 28

5. Julia's Ice Cream Shop sells ice cream cones where each wafer cone and each scoop of ice cream costs the same amount for all of the orders. If an ice cream cone with four scoops costs the same as three ice cream cones with one scoop each, what is the ratio between the cost of a wafer cone and the cost of a scoop of ice cream?

   (A) 1 : 3 **(B)** 1 : 2 **(C)** 2 : 3 **(D)** 1 : 1 **(E)** 2 : 1

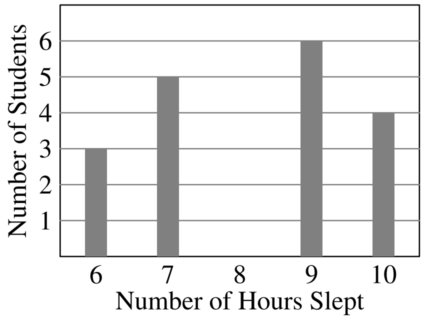
6. A line intersects a rectangle and divides it into two shapes. If one of the shapes has 4 sides, what are all of the numbers of sides the other shape could possibly have?

   (A) 3 **(B)** 4 **(C)** 3 and 4 **(D)** 4 and 5 **(E)** $3, 4,$ and 5

7. How many 5-digit numbers evenly divide 100,000?

   (A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

8. In a survey, Richard asked the students of his third grade class how many hours of sleep each student got last night. The bar graph below shows the results of Richard's survey. However, the bar representing 8 hours has been mysteriously erased. If the median number of hours slept is 8.5, how many students slept for 8 hours?



(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

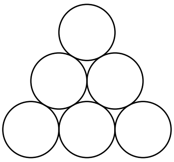
9. Will has 100 bronze coins. As many times as he wants, he can exchange 4 bronze coins for a silver coin or 6 silver coins for a gold coin. What is the smallest number of coins that Will can have after making some number of exchanges?

(A) 5 **(B)** 7 **(C)** 8 **(D)** 10 **(E)** 12

10. What is the probability that a randomly chosen arrangement of the letters of the word $KARATE$ will have an A as the first letter or the last letter (or both)?

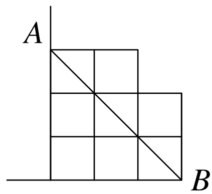
(A) $\frac{7}{15}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

11. What is the minimum number of distinct colors needed to color each of the six circles below such that no two circles with the same color are touching?



(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

12. Peter pushes n unit squares to a wall. Then, he puts three unit squares on top and pushes them to the wall. Finally, he puts two unit squares on top and pushes them to the wall. The figure below shows the resulting shape for $n = 3$. If points A and B represent the top-left and bottom-right vertices of the shape, respectively, what is the largest value of n such that segment $\overline{AB}$ does not go outside of the shape?



(A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 7

13. Anthony has 55 toys that he is packing into boxes. For every box, he wants the number of toys in that box to be equal to the number of boxes (including itself) that have the same number of toys as it does. How many boxes does Anthony need?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 13 **(E)** 15

14. A karate outfit consists of either a beginner robe and a beginner belt or an advanced robe and an advanced belt. Alex has 5 robes and 13 belts in his collection, each of which are beginner or advanced. Alex can currently make 31 possible karate outfits. If he adds a beginner belt and an advanced belt, how many outfits can he then make?

(A) 33 **(B)** 35 **(C)** 36 **(D)** 38 **(E)** 39

15. Each of the vertices of a 10-sided regular polygon is labeled with a positive digit from 1 to 9, inclusive. If the sum of the labels of any four consecutive vertices is constant, what is the greatest possible sum of all of the distinct digits used?

(A) 9 **(B)** 17 **(C)** 24 **(D)** 30 **(E)** 45

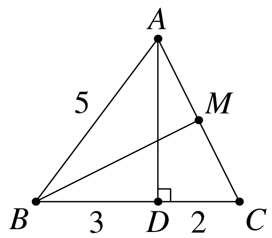
16. A rectangle with a perimeter of 40 is split into four smaller rectangles by two lines that are parallel to the sides of the rectangle. If three of the rectangles have perimeters of 19, 21, and 25, what is the perimeter of the fourth rectangle?

(A) 9 **(B)** 15 **(C)** 17 **(D)** 23 **(E)** 27

17. Starting from the same corner of a square city block, Jack and Jill both start walking around its 800-meter long perimeter in opposite directions. Jack walks at 1.4 meters per second, while Jill walks at 1.8 meters per second. What is the direct distance from their starting point to the point where they first meet again, in meters?

(A) 200 **(B)** $175\sqrt{2}$ **(C)** 250 **(D)** $200\sqrt{2}$ **(E)** 300

18. In $\triangle ABC$, let D be the foot of the altitude from A to side $\overline{BC}$, and let M be the midpoint of side $\overline{AC}$. If $BD = 3$, $CD = 2$, and $AB = 5$, what is the length of $\overline{BM}$?



(A) 4 **(B)** $2\sqrt{5}$ **(C)** $2\sqrt{6}$ **(D)** $3\sqrt{3}$ **(E)** $4\sqrt{2}$

19. If m and n are positive integers and $m^2 - n^2$ is equal to a prime number p, which of the following statements must always be true?

(A) $m+n$ is divisible by 3 **(B)** $p = m - n$ **(C)** $p - m - n$ is odd
(D) $p^2 + m^2 + n^2$ is not prime **(E)** $p - n^2$ is even

20. How many different real numbers x satisfy the equation

$$\left|5|x| - x^2\right| = 10?$$

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

21. Ryan, Emily, and Anna play a game, where each pair of players plays one match. In each match, there is a one-third chance of a tie, and each player has a one-third chance of winning. If players earn two points for winning and one point for a tie, what is the probability that each player has the same number of points after the game?

(A) $\frac{1}{27}$ **(B)** $\frac{2}{27}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{7}{27}$ **(E)** $\frac{1}{3}$

22. Derek thinks of a two-digit number. Dean then asks the following questions in order:

- "Is the number a multiple of 2?"
- "Is the number a multiple of 3?"
- "Is the number a multiple of 4?"

Derek answers "yes," "yes," and "no" to each of the questions, but Dean forgot which order Derek said his responses in. How many possible values of the number are there?

(A) 21 **(B)** 24 **(C)** 29 **(D)** 33 **(E)** 37

23. Define the operation $a \ominus b = a + b - \sqrt{4ab}$. If N is a two-digit number such that

$$(4 \ominus N) \ominus (81 \ominus 196) = 4,$$

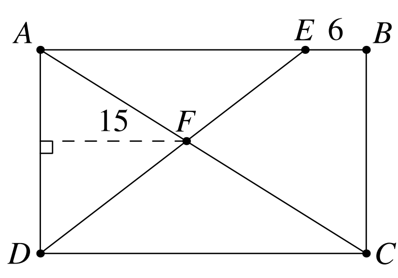
what is the sum of the possible values of N?

(A) 73 **(B)** 106 **(C)** 113 **(D)** 130 **(E)** 145

24. Daniel's favorite positive integer A has B positive integer factors. If the product of A and B is equal to 13,500, what is the sum of the digits of the sum of A and B?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

25. Let $ABCD$ be a rectangle. Let E be on side $\overline{AB}$ such that $BE = 6$, and let F be the intersection of $\overline{AC}$ and $\overline{DE}$. If the distance from F to side $\overline{AD}$ is 15, and the absolute difference between the areas of $\triangle AEF$ and $\triangle CDF$ is 40, what is the area of $BCFE$?



(A) 56 **(B)** 88 **(C)** 100 **(D)** 124 **(E)** 140

2021 DMC 8 B

For more information about the DMC and our other competitions, please visit `https://detoasty3.github.io/dmc.html`.

Questions and comments about this competition should be sent to:

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The problems and solutions for this DMC 8 were prepared by the DMC Editorial Board under the direction of:

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